Quark-Gluon Plasma Formation in Holographic Shock Waves Model of Heavy-Ion Collisions

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Holographic Methods for Strongly Coupled Systems

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Outlook

- Physical picture of formation of Quark-Gluon Plasma in heavy-ions collisions
- Why holography?
- Results from holography (fit of experimental data via holography:

  top-down

  bottom-up)

  - Holography description of static QGP
  - Holography description of QGP formation in heavy ions collisions

Experimental data

- Thermalization time
- Multiplicity

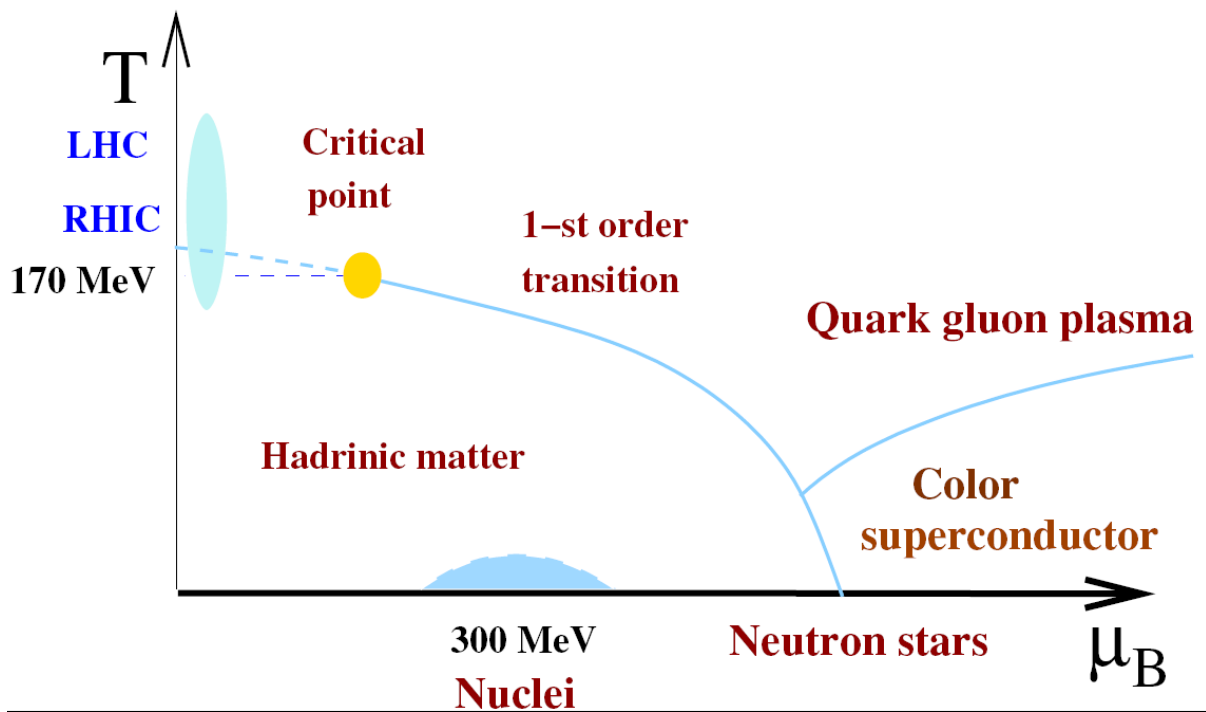
Quark-Gluon Plasma (QGP): a new state of matter

QGP is a state of matter formed from deconfined quarks, antiquarks, and gluons at high temperature

QCD: asymptotic freedom, quark confinement

T increases, or density increases

nuclear matter → Deconfined phase



Experiments: Heavy Ions collisions produced a medium

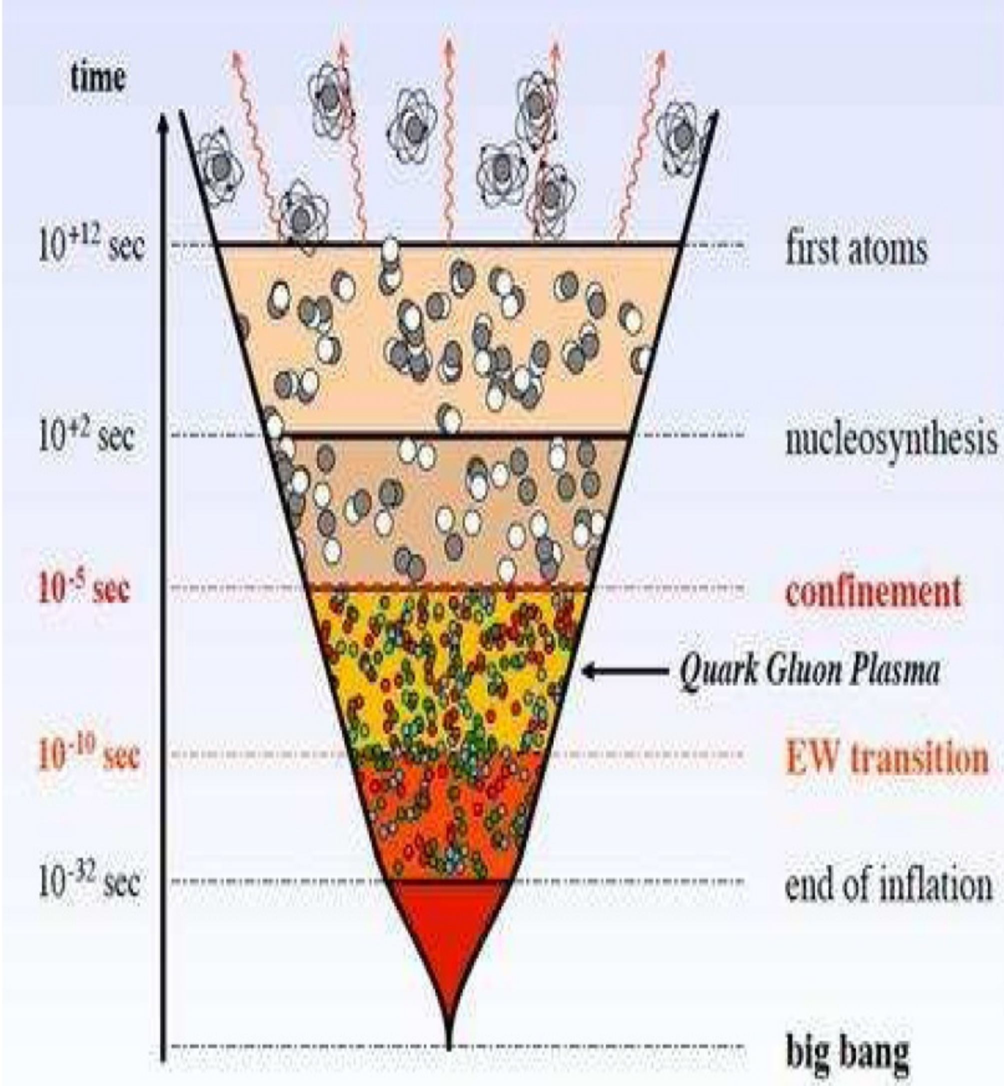
HIC are studied in several **experiments:**

- started in the 1990's at the Brookhaven Alternating Gradient Synchrotron (AGS), $\sqrt{s_{NN}} = 4.75\,GeV$
- the CERN Super Proton Synchrotron (SPS) $\sqrt{s_{NN}} = 17.2\,GeV$
- the Brookhaven Relativistic Heavy-Ion Collider (RHIC) $\sqrt{s_{NN}} = 200\,GeV$
- the LHC collider at CERN. $\sqrt{s_{NN}} = 2.76\,TeV$

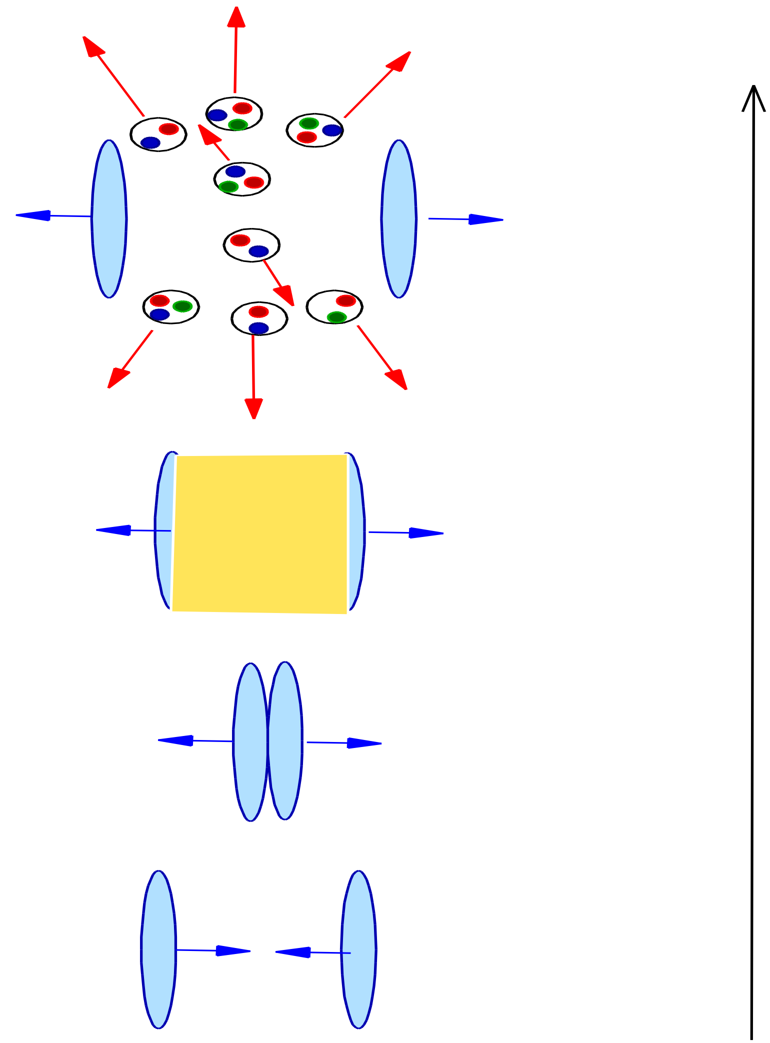
There are strong experimental evidences that RHIC or LHC have created some medium which behaves collectively:

- modification of particle spectra (compared to p+p)
- jet quenching
- high p_T-suppression of hadrons
- elliptic flow
- suppression of quarkonium production

Study of this medium is also related with study of Early Universe



Evolution of the Early Universe



Evolution of a Heavy Ion Collision

Study of QGP is related with one of the fundamental questions in physics: what happens to matter at extreme densities and temperatures as may have existed in the first microseconds $10^{-5} s,\ \ T \sim 10^{12}\, K$ after the Big Bang.

QGP as a strongly coupled fluid

- **Conclusion from the RHIC and LHC experiments: appearance of QGP (not a weakly coupled gas of quarks and gluons, but a strongly coupled fluid).**

- This makes perturbative methods inapplicable

- The lattice formulation of QCD does not work, since we have to study real-time phenomena.

- This has provided a motivation to try to understand the dynamics of QGP through the **gauge/string duality**

Dual description of QGP as a part of Gauge/string duality

- There is not yet exist a gravity dual construction for QCD.
- Differences between N = 4 SYM and QCD are less significant, when quarks and gluons are in the deconfined phase (because of the conformal symmetry at the quantum level, N = 4 SYM theory does not exhibit confinement).

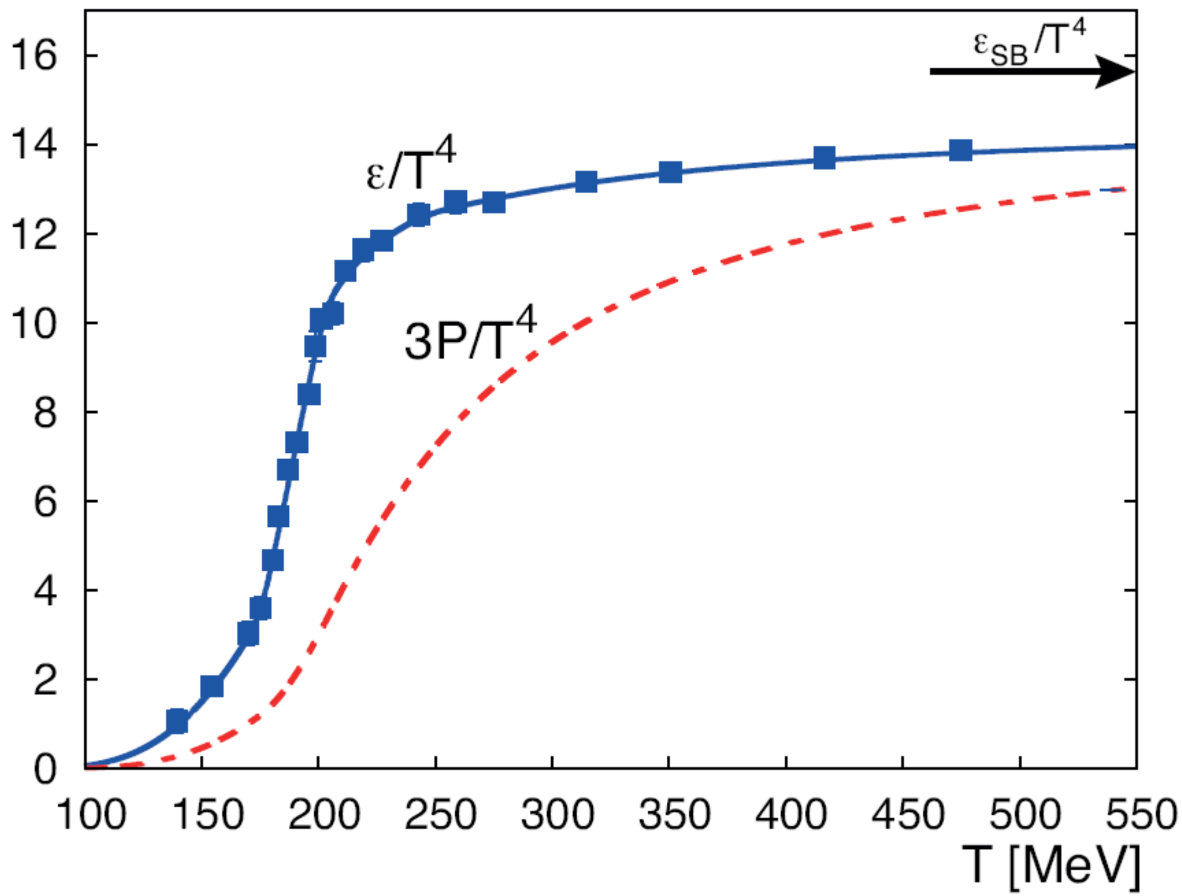
- Lattice calculations show that QCD exhibits a quasi-conformal behavior at temperatures T >300 MeV and the equation of state can be approximated by $E = 3P$ (a traceless conformal energy-momentum tensor).

- This motivates to use the AdS/CFT correspondence as a tool to get non-perturbative dynamics of QGP.

- There is the considerable success in description of the static QGP.

Reviews: Solana, Liu, Mateos, Rajagopal, Wiedemann, 1101.0618 **+ AFTER**

I.A., Holographic approach for QGP in HIC, UFN, 184, 2014;
DeWolfe, Gubser, Rosen,Teaney, HI and string theory, Prog. Part.Nucl.Phys., 75, 2014
P.M.Chesler, W. van der Schee, Early thermalization, ….. 1501.04952 [nucl-th]



lattice calculation of QCD thermodynamics $N_f = 3$

S. Borsanyi et al., "The QCD equation of state with dynamical quarks," arXiv:1007.2580

Holography for QGP formation

Based on two conjectures:

1) TQFT in M_D-spacetime = Black hole in AdS_{D+1}-space-time

TQFT = QFT with temperature

Holography for QGP formation

2) Thermalization of QFT in Minkowski D-dim space-time ⟷ Black Hole formation in Anti de Sitter (D+1)-dim space-time

Models of BH creation in D=5 and their meaning in D=4

To initiate the process of BH formation one has to perturb the initial metric.

$$g_{MN} \Rightarrow g_{MN}^{(0)} + g_{MN}^{(1)}$$

- **AdS/CFT correspondence**

$$Z_{ren}(z_0)\; g_{\mu\nu}^{(1)} \Big|_{\substack{boundary \\ z_0 \to 0}} = T_{\mu\nu}$$

Main idea: make some perturbation of AdS metric that near the boundary mimics the heavy ions collisions and see what happens.

Hologhraphic thermalization

How to “mimic” the heavy ions collision

Models:

shock waves/ collision in AdS

infalling shell

colliding ultrarelativistic particles in AdS_3 (toy model)

Nucleus collision in AdS/CFT

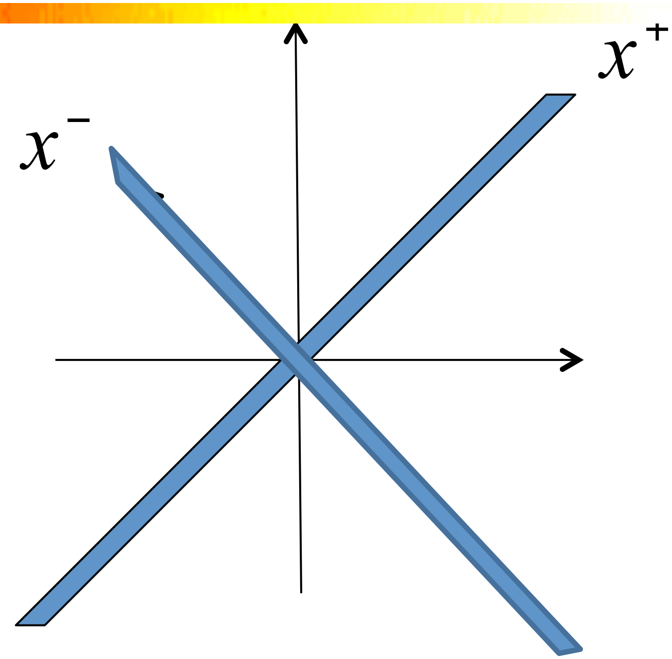
An ultrarelativistic nucleus is a shock wave in 4d with the energy-momentum tensor

$$\langle T_{--}\rangle \sim \mu\,\delta(x^-)$$

$$\langle T_{++}\rangle \sim \mu\,\delta(x^+)$$

$$\langle T_{--}\rangle \sim \frac{1}{(L^2+x_\perp^2)^3}\,\delta(x^-)$$

Woods-Saxon profile

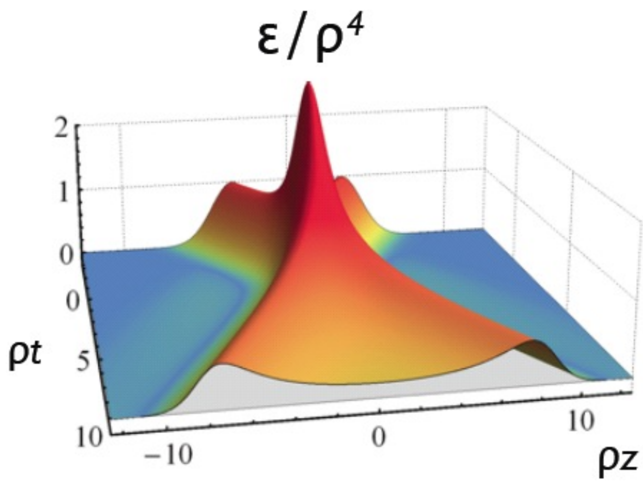


$$ds^2 = \frac{L^2}{z^2}\left[-2\,dx^+dx^- + \frac{2\pi^2}{N_C^2}\langle T_{--}(x^-)\rangle z^4\,dx^{-2} + +\frac{2\pi^2}{N_C^2}\langle T_{++}(x^+)\rangle z^4\,dx^{+2} + dx_\perp^2 + dz^2\right]$$

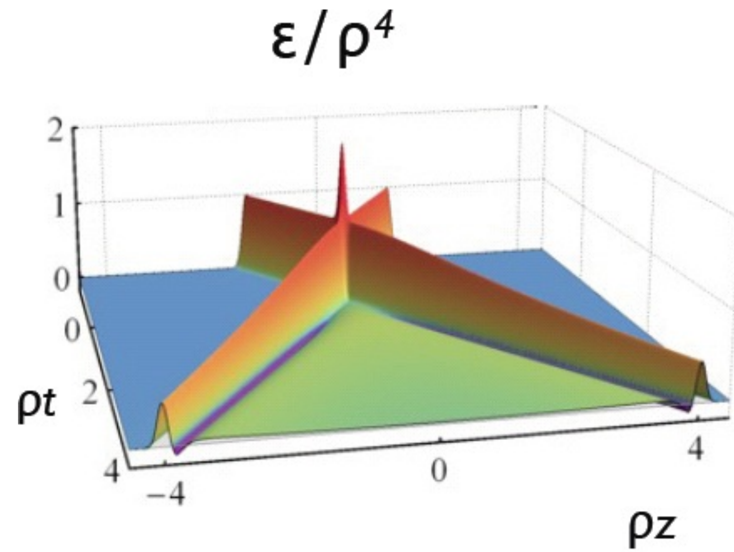
The metric of two shock waves in AdS corresponding to collision of two ultrarelativistic nucleus in 4D

Holographic collision of two gaussian shocks

From Chesler & Yaffe



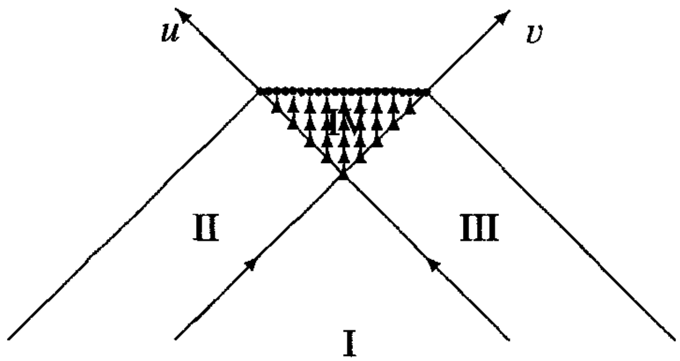
Low Energy Shocks



High energy shocks

Shocks pass through each other

Collision of plane waves in M_4



I.A., K.S. Viswanathan, I. Volovich
Nucl.Phys. B 452 (1995) 346

$$ds^2 = 4m^2[1 + \sin(u\theta(u)) + v\theta(v)]^2 du dv$$
$$- [1 - \sin(u\theta(u)) + v\theta(v)][1 + \sin(u\theta(u)) + v\theta(v)]^{-1} dx^2$$
$$-[1 + \sin(u\theta(u)) + v\theta(v)]^2 \cos^2(u\theta(u)) - v\theta(v)) dy^2,$$

where $u < \pi/2,\ v < \pi/2,\ v + u < \pi/2.$

$$r = m[1 + \sin(u+v)], \quad t = x, \quad \theta = \pi/2 + u - v, \quad \phi = y/m,$$

Interior of BH

Generalization to ADS?

Hologhraphic thermalization

Physical quantities that we expect to estimate:

D=5 AdS		D=4 Minkowski
• Black hole formation time	⟺	• Thermalization time
• Entropy		• Multiplicity

Thermalization time

Experimental data (just estimations)

$$\epsilon(y) = \frac{1}{A\tau_{therm}} \frac{dN}{dy} < m_{tr} >, \qquad m_{tr} = \sqrt{m_\pi^2 + k_{tr}^2}$$

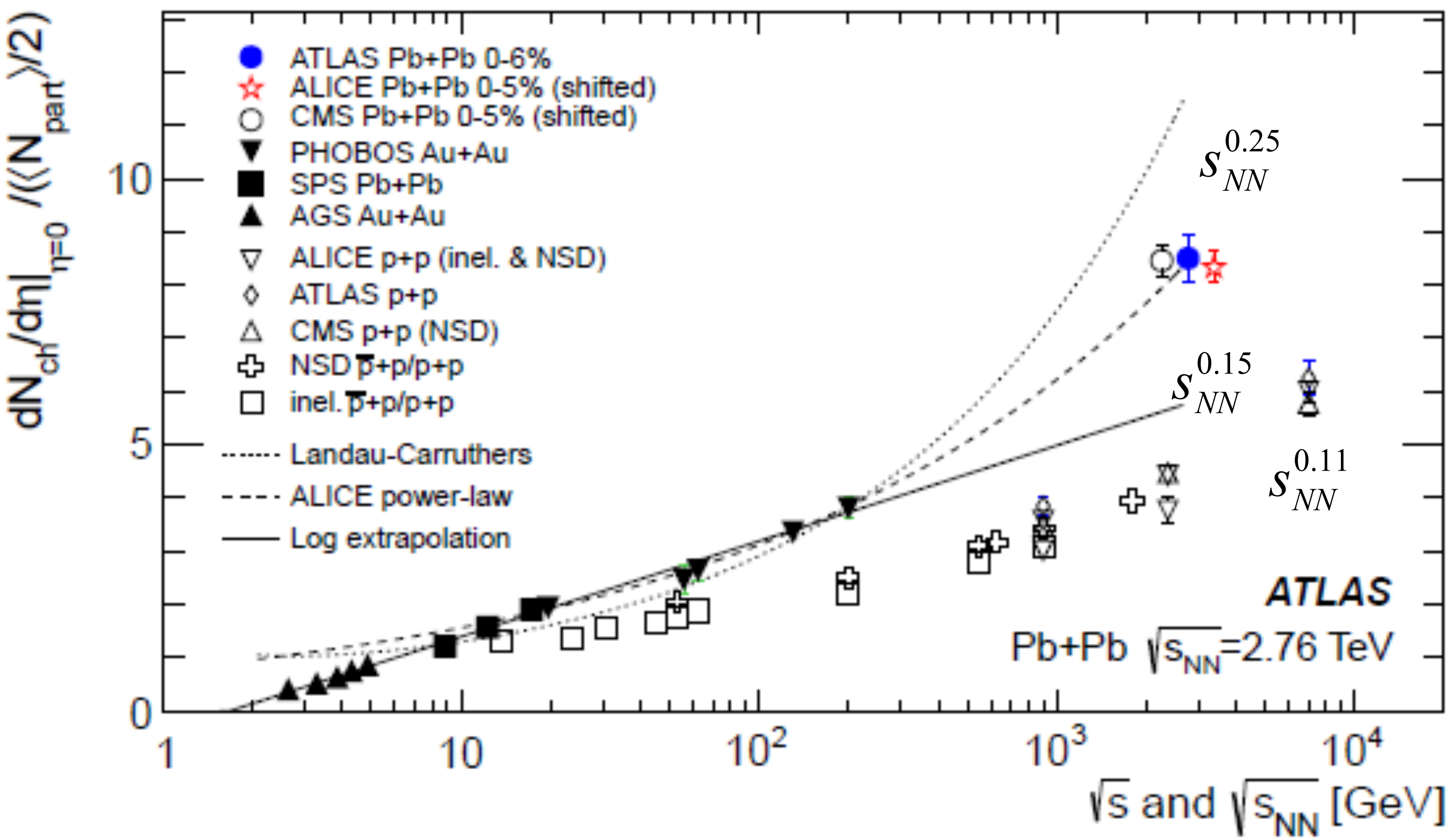
Distribution of energy density ϵ over rapidity y

Bjorken, 1983

Multiplicity

Experimental data

Plot from: ATLAS Collaboration 1108.6027



PbPb

$$\mathcal{M} \sim s_{NN}^{0.15}$$

pp:

$$\mathcal{M} \sim s^{0.11}$$

Multiplicity as entropy

D=4. Macroscopic theory of high-energy collisions

Landau(1953); Fermi(1950)

thermodynamics, hydrodynamics, kinetic theory, …

D=5. Holographic approach

Main conjecture: multiplicity is proportional to entropy of produced D=5 Black Hole

$$\mathcal{M} \sim S$$

Gubser et al: 0805.1551

The mininal black hole entropy can be estimated by trapped surface area

$$S \geq S_{trapped} = A_{trapped} / 4G_N$$

Gubser, Pufu, Yarom, JHEP , 2009

Alvarez-Gaume, C. Gomez, Vera,

Tavanfar, Vazquez-Mozo, PLB, 2009

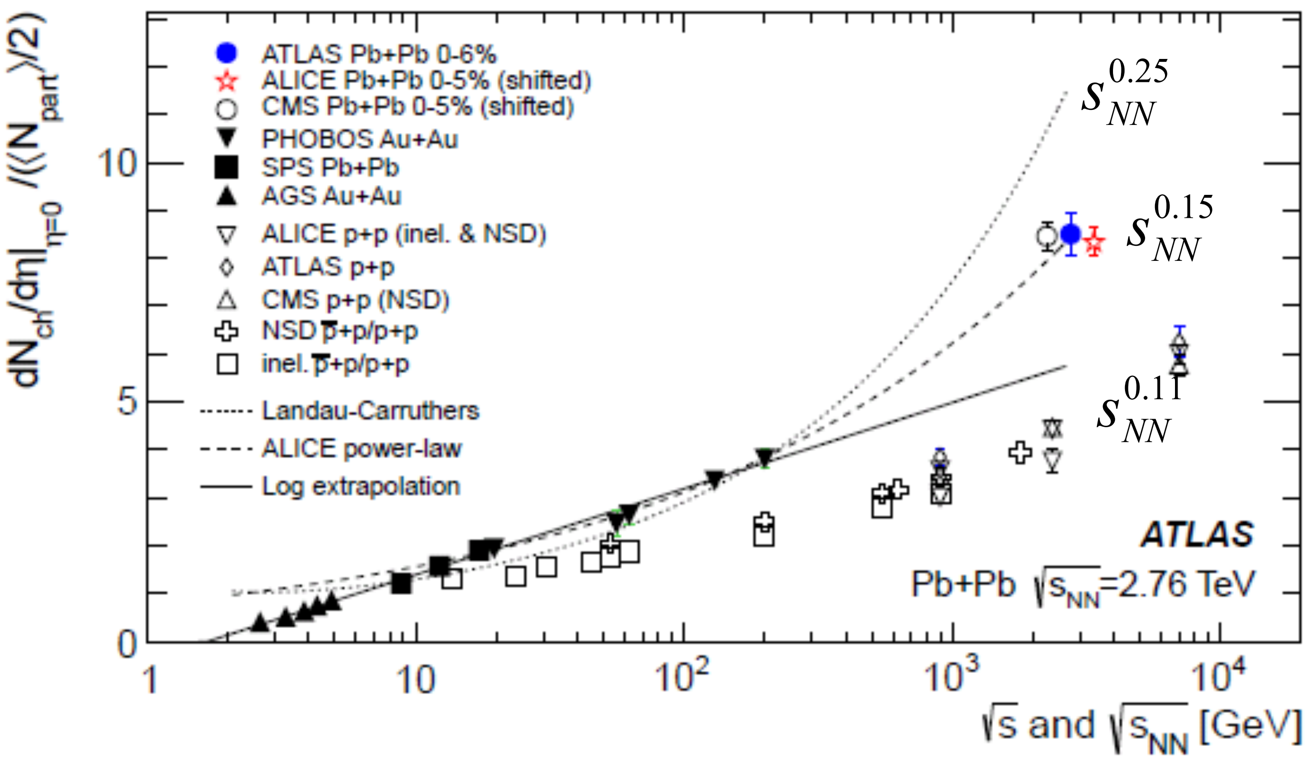
IA, Bagrov, Guseva, JHEP, 2009

Kiritsis, Taliotis, JHEP, 2011

Multiplicity: Hologhrapic formula vs experimental data

The simple holographic model gives

$$\mathcal{M} \sim s_{NN}^{1/3}$$



Search for models with suitable entropy

Metric with modified b-factor **IHQCD** Gursoy, Kiritsis, Nitti

$$S_5 = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[R + \frac{d(d-1)}{L^2} - \frac{4}{3}(\partial\Phi)^2 + V(\Phi_s)\right]dx^5$$

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2)$$

Reproduces 2-loops QCD beta-function

Reproduce an asymptotically-linear glueball spectrum

Search for models with suitable entropy

Kiritsis, Taliotis, **JHEP(2012)**

Shock wave metric with modified b-factor

$$ds^2 = b^2(z)\big(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z,x^1,x^2)\delta(x^+)(dx^+)^2\big)$$

Typical behavour

$$b(z) = \frac{L}{z}e^{-z^2/z_0^2}$$

$$s_{NN}^{\delta_1}\ln^{\delta_2} s_{NN}$$

$$\delta_1 \approx 0.225, \quad \delta_2 \approx 0.718$$

not 0.15

Shock walls collision with modified by b-factor

Description of HIC by the wall-wall shock wave collisions

S. Lin, E. Shuryak, 0902.1508
I. A., Bagrov and E.Pozdeeva, JHEP(2012)

$$ds^2 = b^2(z)\left(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi^w(z, x^1, x^2)\delta(x^+)(dx^+)^2\right)$$

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z)$$

I. A., E.Pozdeeva,T.Pozdeeva (2013, 2014)

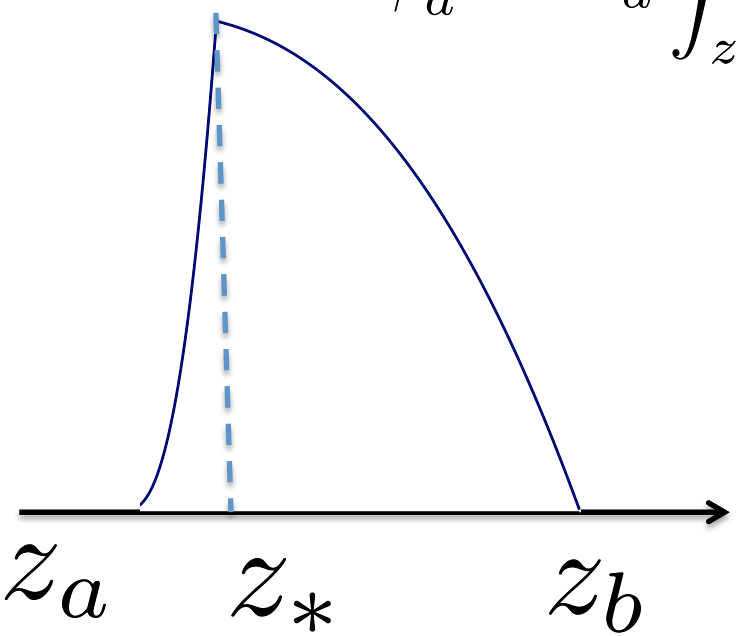
$S_{points} \sim s_{walls}$

Shock walls collision with modified by b-factor

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z \right) \phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z)$$

$$\phi^\omega(z) = \phi_a^\omega \theta(z_* - z) + \phi_b^\omega \theta(z - z_*)$$

$$\phi_a^w = C_a \int_{z_a}^{z} b^{-3} dz, \quad \phi_b^w = C_b \int_{z_b}^{z} b^{-3} dz.$$



$$C_a = C \frac{\int_{z_b}^{z_*} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz}$$

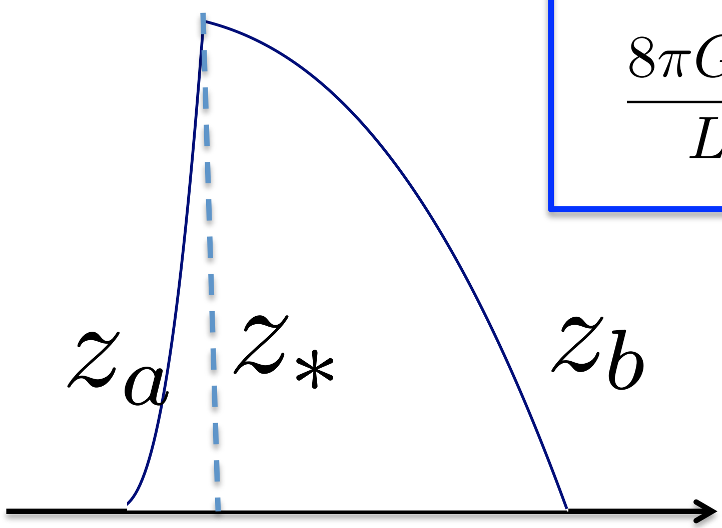
$$C_b = C \frac{\int_{z_a}^{z_*} b^{-3} dz}{\int_{z_b}^{z_a} b^{-3} dz}$$

Shock walls collision with modified by b-factor

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z \right) \phi^w(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z)$$

$$\frac{8\pi G_5 E}{L^2} b^{-3}(z_a) \int_{z_b}^{z_*} b^{-3} dz = \int_{z_b}^{z_a} b^{-3} dz,$$

$$\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) \int_{z_a}^{z_*} b^{-3} dz = -\int_{z_b}^{z_a} b^{-3} dz,$$



$$b^3(z_a) + b^3(z_b) = \frac{8\pi G_5 E}{L^2}$$

$$s = \frac{1}{2G_5} \int_{z_a}^{z_b} b^3 \, dz$$

Power-law b-factor

$$b(z) = \left(\frac{L}{z}\right)^a$$

$$S_{walls} = \frac{L}{2G_5}\left(\frac{8\pi G_5}{L^2}\right)^{\frac{3a-1}{3a}} E^{\frac{3a-1}{3a}}$$

The multiplicity depends as $s^{0.15}_{NN}$ in the range $10\text{-}10^3$ GeV

Power-law b-factor coinsides with experimental data at *a*≈0.47.

Let us take $$b(z) = \left(\frac{L}{z}\right)^{1/2}$$

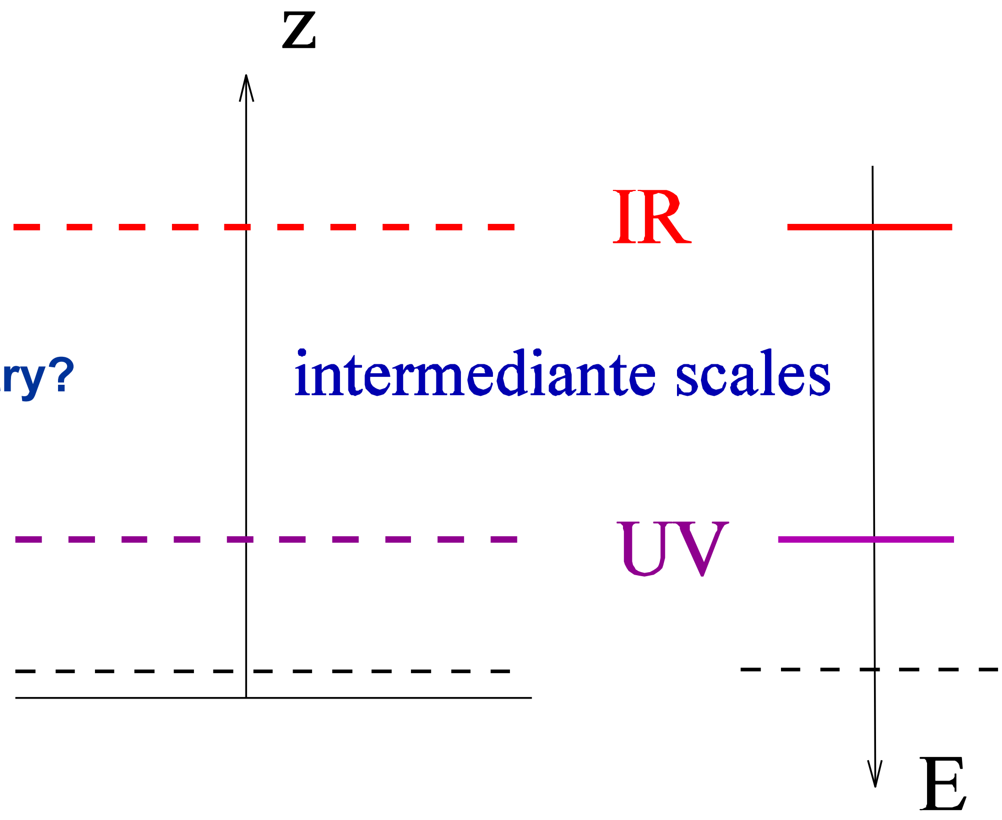
Price: non standard kinetic term!

Multiplicity vs quark potential

Soft/hard wall

Interpolating geometry?

AdS_5



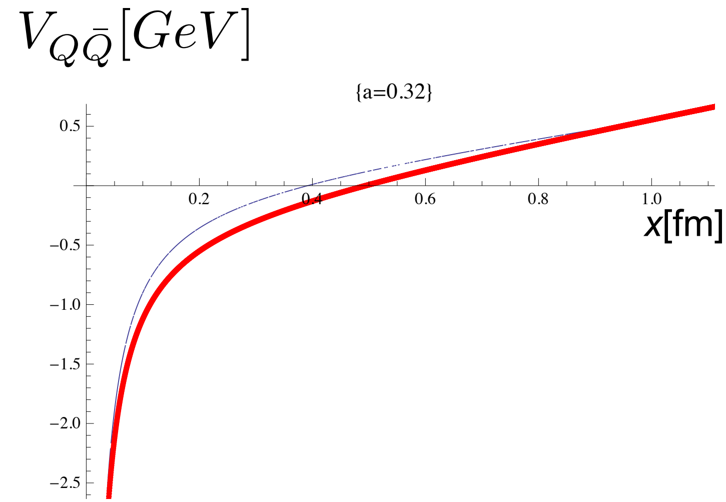
Question: can we fit this background with other data?

Multiplicity vs quark potential

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2)$$

$$b^2(z) = \frac{L^2 h(z)}{z^2}$$

$$h = e^{\frac{a z^2}{2}}$$



AdS with soft-wall

$$V_{Cornell}(x) \equiv V_{Q\bar{Q}}(x) = -\frac{\kappa}{x} + \sigma_{str} x + V_0$$

$$\kappa \approx 0.48, \quad \sigma_{str} = 0.183 GeV^2, \quad C = -0.25 GeV$$

Coulomb term

Confinement linear potential

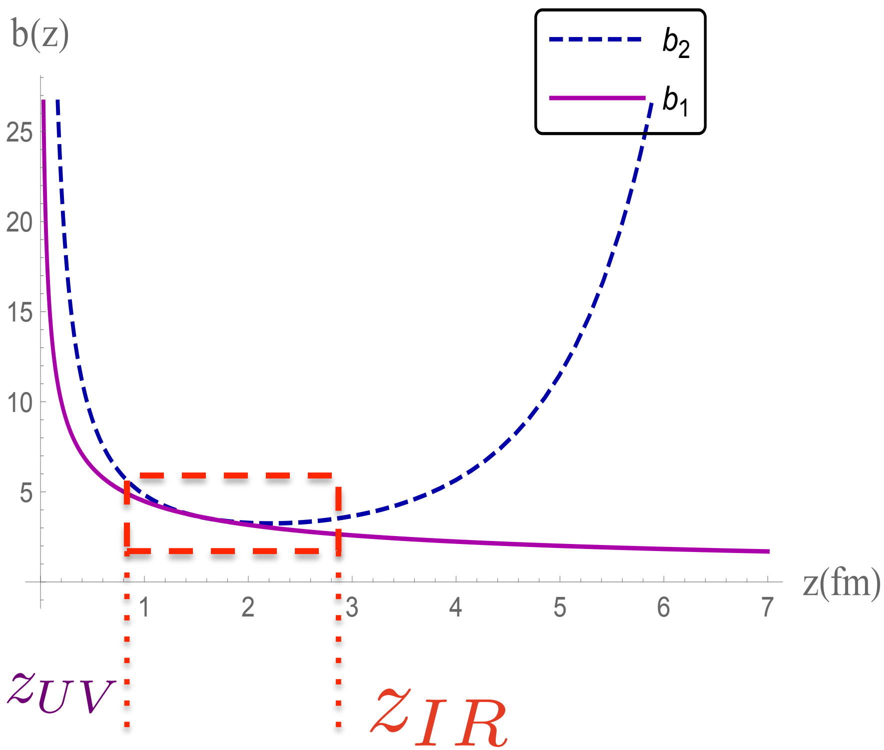
O. Andreev and V. Zakharov
hep-ph/0604204
R.Galow at al, 0911.0627
S.He, M.Huang, Q.Yan
1004.1880

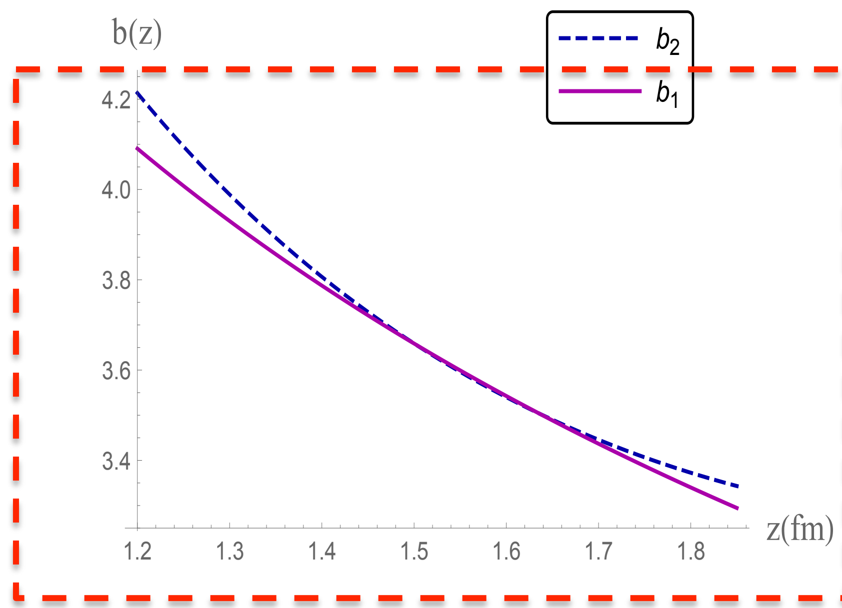
Multiplicity and quark potential

with D.Ageev
arXiv:1409.7558

$$\frac{L^2 e^{\frac{a z^2}{2}}}{z^2} \approx \frac{L^2}{z L_{eff}}$$

$$z_{UV} < z < z_{IR}$$





But: there is a problem with the available energy

$$b^3(z_a) + b^3(z_b) = \frac{8\pi G_5 E}{L^2}$$

Multiplicity and quark potential

$$\frac{L^2 e^{\frac{a z^2}{2}}}{z^2} \approx \frac{L^2}{z L_{eff}}$$

Trapped surface

$$z_{UV} < z < z_{IR}$$

$$z_a < z < z_b$$

Pack the trapped surface in the interval

$$z_{UV} < z_a < z < z_b < z_{IR}$$

$$s \sim (L_{eff} E)^{1/3}$$

But restriction on energy

$$E_{IR} < E < E_{UV}$$

Small energies!

Thermalization time

BH creation in two shock waves collisions is modeled by Vaidya metric with a horizon corresponding to the location of the trapped surface

Thermalization time is estimated within standard prescription with the Vaidya metric

Danielsson, Keski-Vakkuri, Kruczenski

hep-th/9905227,

………………

I.A. arXiv: 1503.02185

Thermalization time via Vaidya metrc

$$ds^2 = b^2(z)(-dt^2 + dz^2 + dx_i^2)$$

Blackening function

$$f(z_h, z) = 1 - K(z_h, z)$$

$$K(z_h, z) = \frac{K(z)}{K(z_h)} \qquad K(z) = \int_0^z \frac{dz}{b(z)^3}$$

$$ds^2 = b^2(z)\left(-f(z_h, z)\, dt^2 + \frac{dz^2}{f(z_h, z)} + d\vec{x}^2\right)$$

$$dv = dt - \frac{dz}{f(z_h, z)}$$

Vaidya metric

$$ds^2 = b^2(z)\left(-f(z_h, z, v)\, dv^2 - 2dvdz + d\vec{x}^2\right)$$

$$f(z_h, z, v) = 1 - \theta(v)\, K(z_a, z)$$

Thermalization time in confining background

Vaidya metric

$$ds^2 = b^2(z)\left(-f(z_h, z, v)\,dv^2 - 2dvdz + d\vec{x}^2\right)$$

$$f(z_h, z, v) = 1 - \theta(v)\,K(z_h, z)$$

$$\ell = 2s\int_0^1 \frac{b(s)}{b(sw)}\frac{dw}{\sqrt{(1-K(z_h, sw))\cdot\left(1-\frac{b^2(s)}{b^2(sw)}\right)}}$$

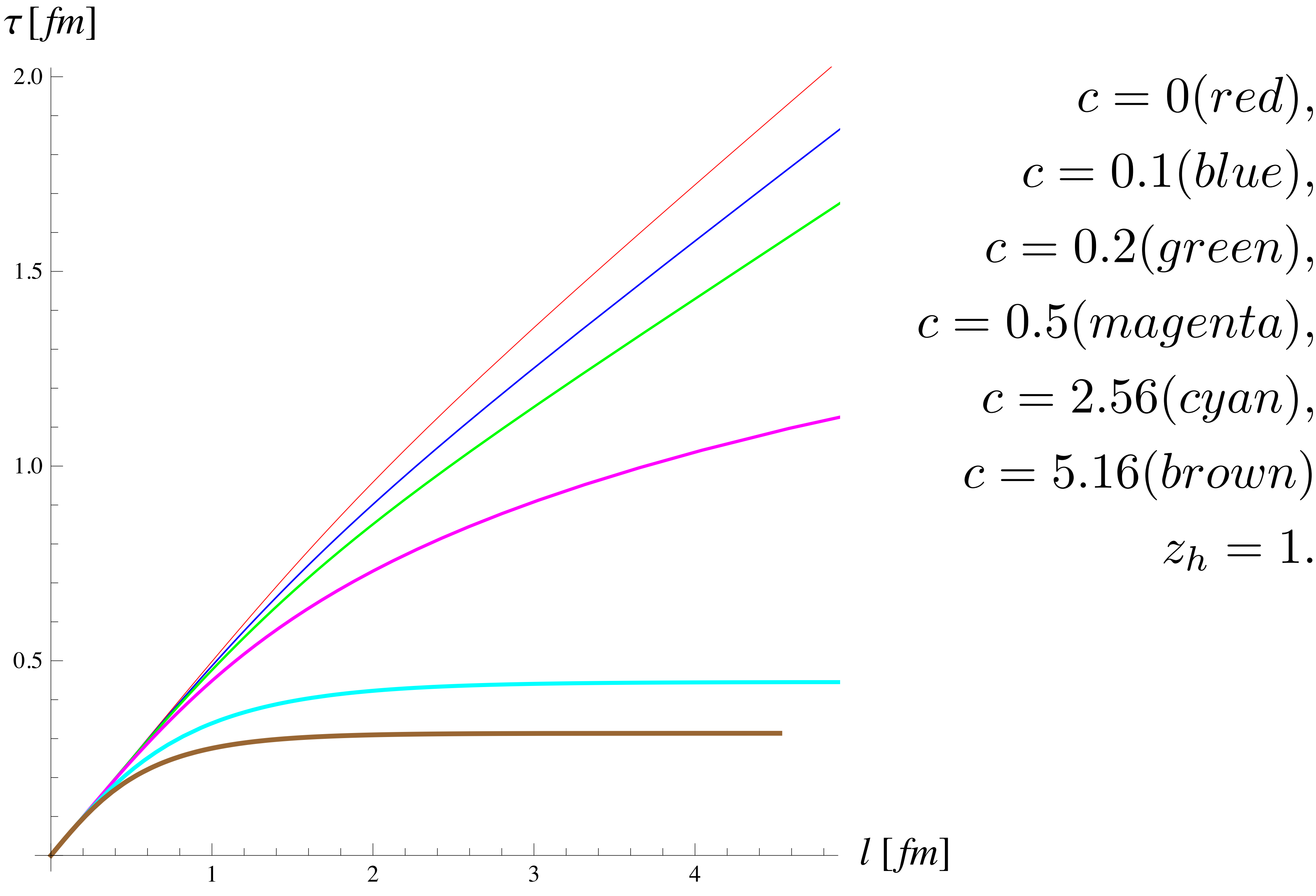
$$\tau = s\int_0^1 \frac{dw}{1-K(z_h, sw)}$$

$$b(z) = \frac{e^{cz^2}}{z}$$

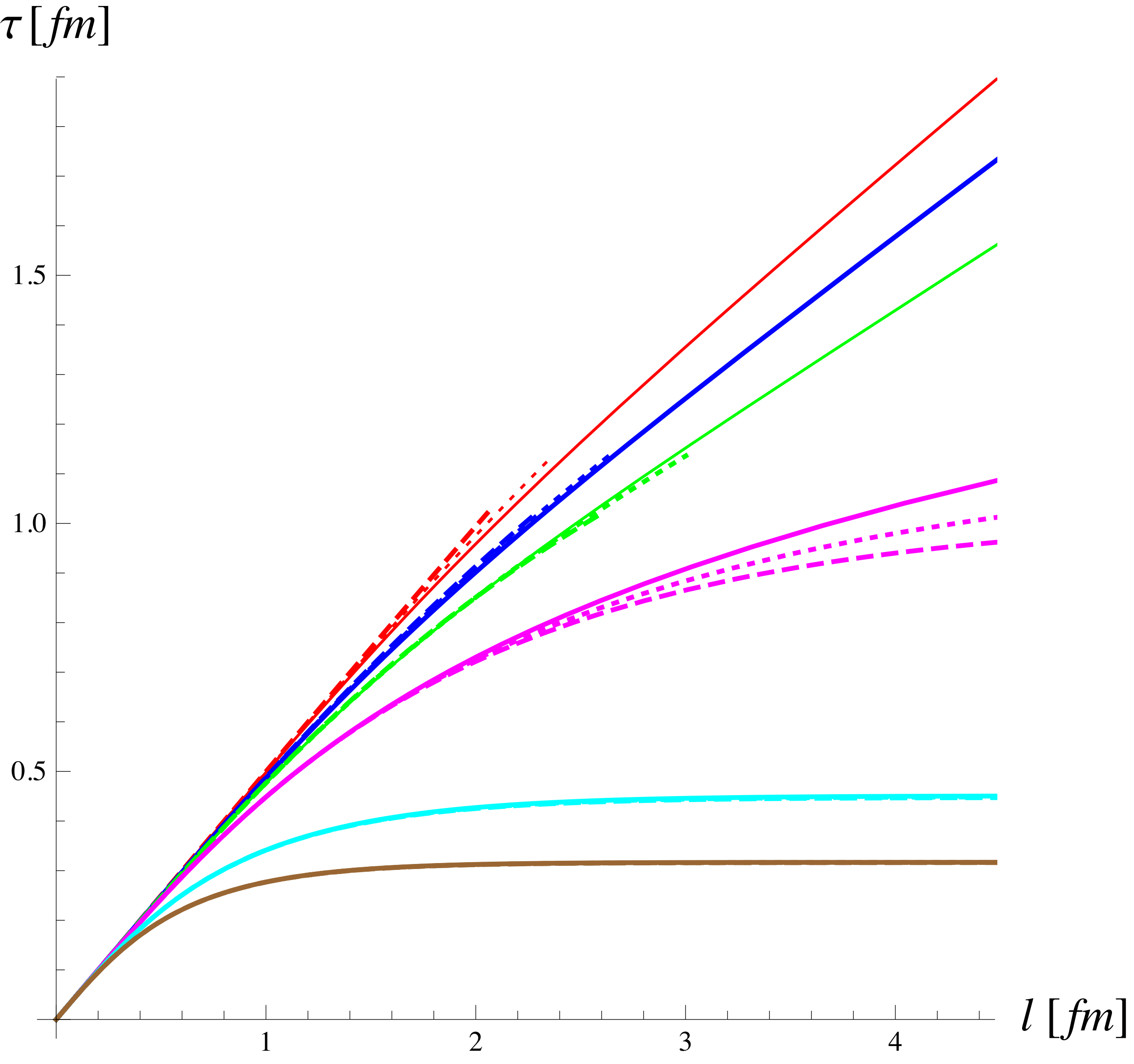
$$K(z_h, z) = \frac{-1+\mathrm{e}^{-3\,cz^2}+3\,\mathrm{e}^{-3\,cz^2}cz^2}{-1+\mathrm{e}^{-3\,cz_h{}^2}+3\,\mathrm{e}^{-3\,cz_h{}^2}cz_h{}^2} \approx k_4 z^4 + k_6 z^6 + \mathcal{O}\left(z^8\right)$$

$$k_4 = 1/z_h^4 + \ldots$$

Thermalization time in confining background

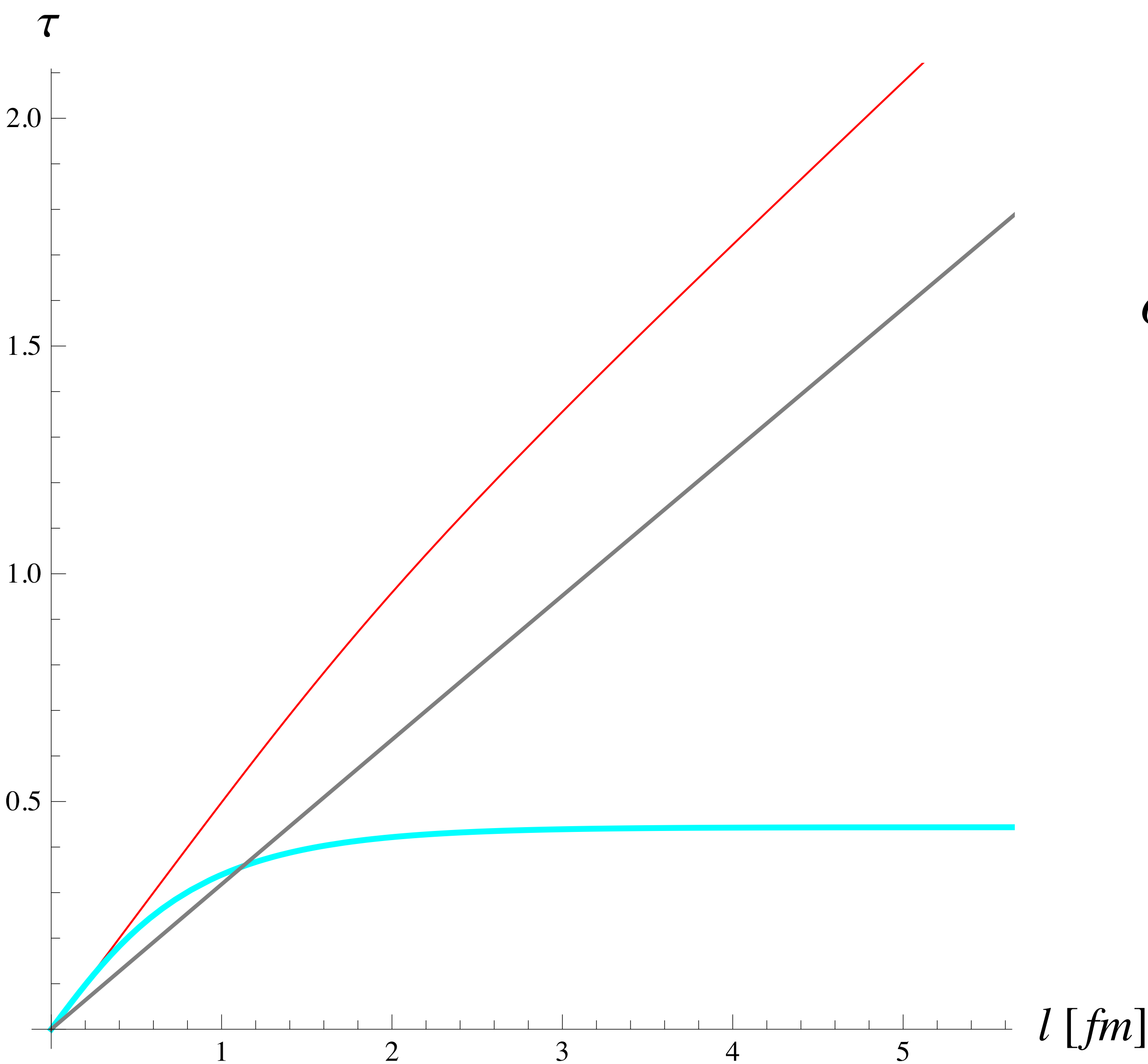


Thermalization time in confining background



$c = 0(red),$
$c = 0.1(blue),$
$c = 0.2(green),$
$c = 0.5(magenta),$
$c = 2.56(cyan),$
$c = 5.16(brown)$
$z_h = 1fm(solid\,lines),$
$z_h = 1.2fm(dotted\,lines),$
$z_h = 1.8\,fm(dashed\,lines)$

Thermalization time in confining background



$$c = 0, \quad a = 1\ (red),$$
$$c = 0, \ a = 0.5\, (gray),$$
$$c = 2.56, \ a = 1\, (cyan)$$

Anisotropic thermalization

In the past: it has been claimed that the pre-equilibrium period can only exist for up to 1 fm/c and

after that, the QGP becomes isotropic.

Now: QGP is created after very short time after the collision $\tau_{therm} \sim 0.1 fm/c$ and it is anisotropic for a short time $0 < \tau_{therm} < \tau < \tau_{iso}$

The time of locally isotropization is about $\tau_{iso} \sim 2 fm/c$

M. Strickland, 1312.2285 [hep-ph]

Anisotropic thermalization

- Experimental evidence for anisotropies:

jet quenching,

changes in R-mod.factor,

photon and dilepton yields

D.Giataganas, 1306.1404,

D.Trancanelli, 1311.5513

Created QGP is anisotropic

This gives a reason to consider BH formation in anisotropic background

Duality with Lifshitz

Kachru, Liu, Mulligan, 0808.1725
…..
Azeyanagi, Li, Takayanagi, 0905.0688
…..

Gravity background

$$ds^2 = L^2(-r^{2\nu}dt^2 + r^2 d\vec{x}^2_{d-1} + \frac{dr^2}{r^2})$$

$$t \to \lambda^\nu t, \quad \vec{x} \to \lambda \vec{x}, \quad r \to \frac{1}{\lambda} r$$

Lifshitz-like

$$ds^2 = L^2 \left(r^{2\nu} \left(-dt^2 + dx^2 \right) + r^2 \sum_{j=1}^{q} dy_j^2 + \frac{dr^2}{r^2} \right)$$

Multiplicity with anisotropic Lifshitz background

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{|g|}\left[R - 2\Lambda - \frac{1}{12}H_3^2 - \frac{m_0^2}{2}B_2^2\right]$$

M.Taylor, arXiv: 0812.0530

$$H_3 = 2\sqrt{\frac{\nu-1}{\nu}}\rho d\rho\wedge dt\wedge dx, \quad B_2 = \sqrt{\frac{\nu-1}{\nu}}\rho^2 dt\wedge dx$$

$$\Lambda = 5 + \frac{6}{\nu} + \frac{3}{\nu^2}$$

$$ds^2 = \rho^2\left(-dt^2 + dx^2\right) + \rho^{2/\nu}\left(dy_1^2 + dy_2^2\right) + \frac{d\rho^2}{\rho^2}$$

IA, A. Golubtsova arXiv:1410.4595

Shock wave

$$z = 1/\rho$$

$$ds^2 = \frac{\phi(y_1, y_2, z)\delta(u)}{z^2}du^2 - \frac{1}{z^2}dudv + z^{-2/\nu}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2}$$

Solves E.O.M. if

$$\Box_3$$

$$\delta(u)\left[\Box_3 - \left(1 + \frac{2}{\nu}\right)\right]\frac{\phi(y_1, y_2, z)}{z} = -2zT_{uu}$$

$$ds^2 = \rho^{2/\nu}\left(dy_1^2 + dy_2^2\right) + \frac{d\rho^2}{\rho^2}$$

Multiplicity with anisotropic Lifshitz background

Domain walls

$$\left[\Box_{Lif_3} - \frac{1}{L^2}\left(1+\frac{2}{\nu}\right)\right]\frac{\phi(z)}{z} = -16\pi G_5 z J_{uu}$$

$$J_{uu} = E\left(\frac{z}{L}\right)^{1+2/\nu}\delta(z-z_*)$$

$$\frac{\partial^2\phi(z)}{\partial z^2} - \left(1+\frac{2}{\nu}\right)\frac{1}{z}\frac{\partial\phi(z)}{\partial z} = -16\pi G_5 J_{uu}$$

$$\phi = \phi_a\theta(z_* - z) + \phi_b\theta(z - z_*)$$

$$\phi_a(z) = C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1\right)\left(\frac{z^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1\right),$$

$$\phi_b(z) = C_0 z_a z_b\left(\frac{z_*^{2(\nu+1)/\nu}}{z_a^{2(\nu+1)/\nu}} - 1\right)\left(\frac{z^{2(\nu+1)/\nu}}{z_b^{2(\nu+1)/\nu}} - 1\right),$$

$$C_0 = -\frac{8\nu\pi G_5 E z_a^{1+2/\nu} z_b^{1+2/\nu}}{(\nu+1)L^{3+\frac{2}{\nu}}\left(z_b^{2(\nu+1)/\nu} - z_a^{2(\nu+1)/\nu}\right)}.$$

Multiplicity with anisotropic Lifshitz background

Colliding Domain Walls

$$ds^2 = L^2\Big[-\frac{1}{z^2}dudv + \frac{1}{z^2}\phi_1(y_1,y_2,z)\delta(u)du^2 + \frac{1}{z^2}\phi_2(y_1,y_2,z)\delta(v)dv^2 + \frac{1}{z^{2/\nu}}\left(dy_1^2 + dy_2^2\right) + \frac{dz^2}{z^2}\Big]$$

$$S \sim \frac{\nu}{4G_5}(8\pi G_5)^{2/(\nu+2)}E^{2/(\nu+2)}$$

To get $S \sim E^{0.3}$

$$\boxed{\nu = 4}$$

Blackening of anisotropic background

$$ds^2 = b^2(z)(-\frac{dt^2}{z^{2(\nu-1)}} + d\vec{x}^2 + dz^2)$$

Blackening

$$ds^2 = b^2(z)\left(-\frac{f(z_h,z)}{z^{2(\nu-1)}}dv^2 - 2\frac{dv dz}{z^{\nu-1}} + d\vec{x}^2\right)$$

$$dv = dt - \frac{dz}{z^{1-\nu}f(z_h,z)} \qquad f(z_h,z) = 1 - \frac{K(z)}{K(z_a)}$$

$$ds^2 = b^2(z)(\frac{-dt^2 + dx^2}{z^{2(\nu-1)}} + dy_1^2 + dy_2^2 + dz^2)$$

Blackening ?

Thermalization time in anisotropic background

For power-law b-factor

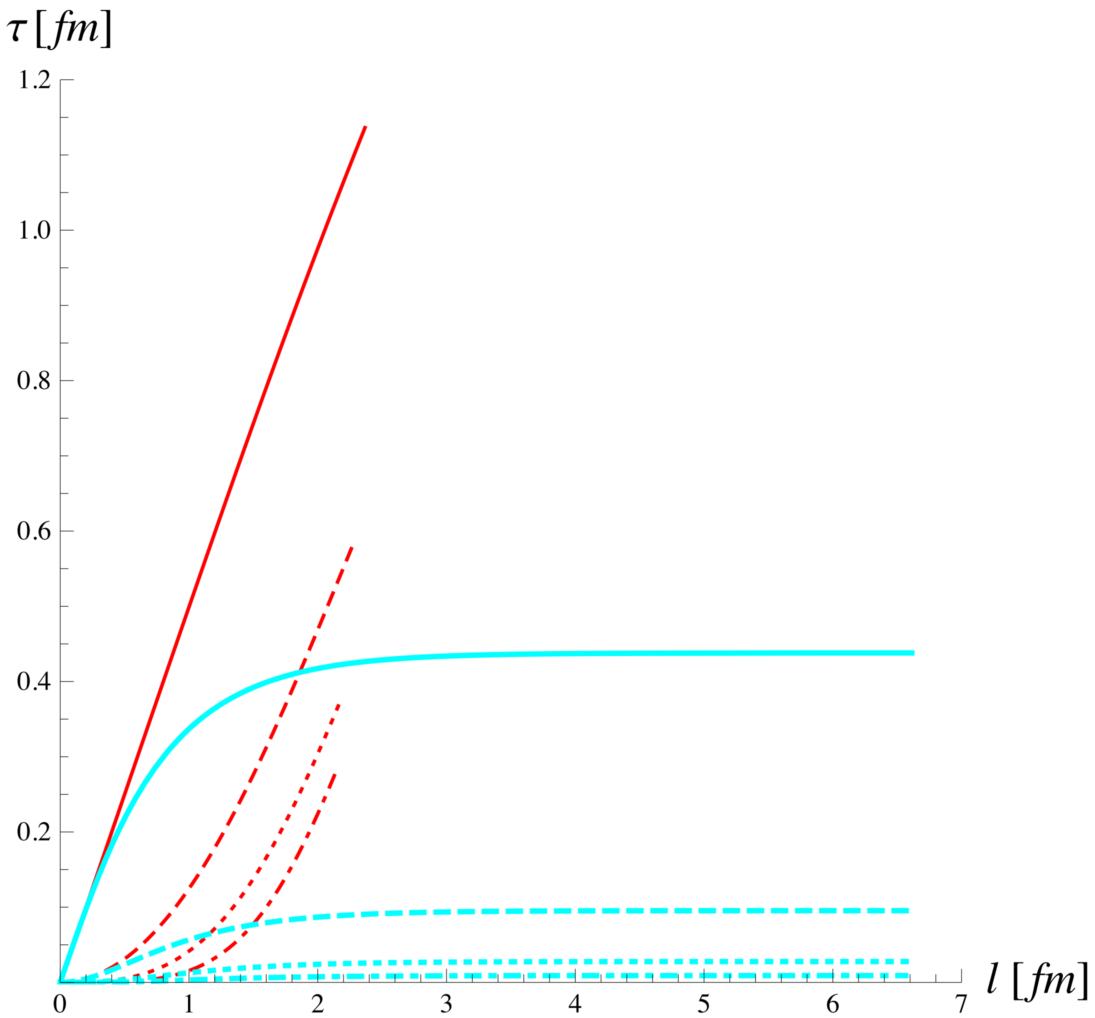
Alishahiha, Astaneh, Mozaffar, 1401.2807; Fonda, Franti, Keranen, Keski-Vakkuri, Thorlacius, Tonni, 1401.6088

Arbitrary b-factor

$$\ell = 2s \int_0^1 \frac{b(s)}{b(sw)} \frac{dw}{\sqrt{(1 - K(z_h, sw)) \cdot \left(1 - \frac{b^2(s)}{b^2(sw)}\right)}}$$

$$\tau = s \int_0^1 \frac{dw}{1 - K(z_h, sw)}$$

Thermalization time in confining background with anisotropy



$c = 0\ (red)$

$c = 2.56\, fm^{-2}\ (cyan)$

$\nu = 1 (solid\ lines)$

$\nu = 2 (dashed\ lines)$

$\nu = 3 (dotted\ lines)$

$\nu = 4 (dotdashed\ lines)$

Nice picture, but not that we want!