GGI Firenze

March 23 2015

Workshop on

Holographic methods for strongly coupled systems

STRONG INTERACTIONS WITH MANY FLAVORS

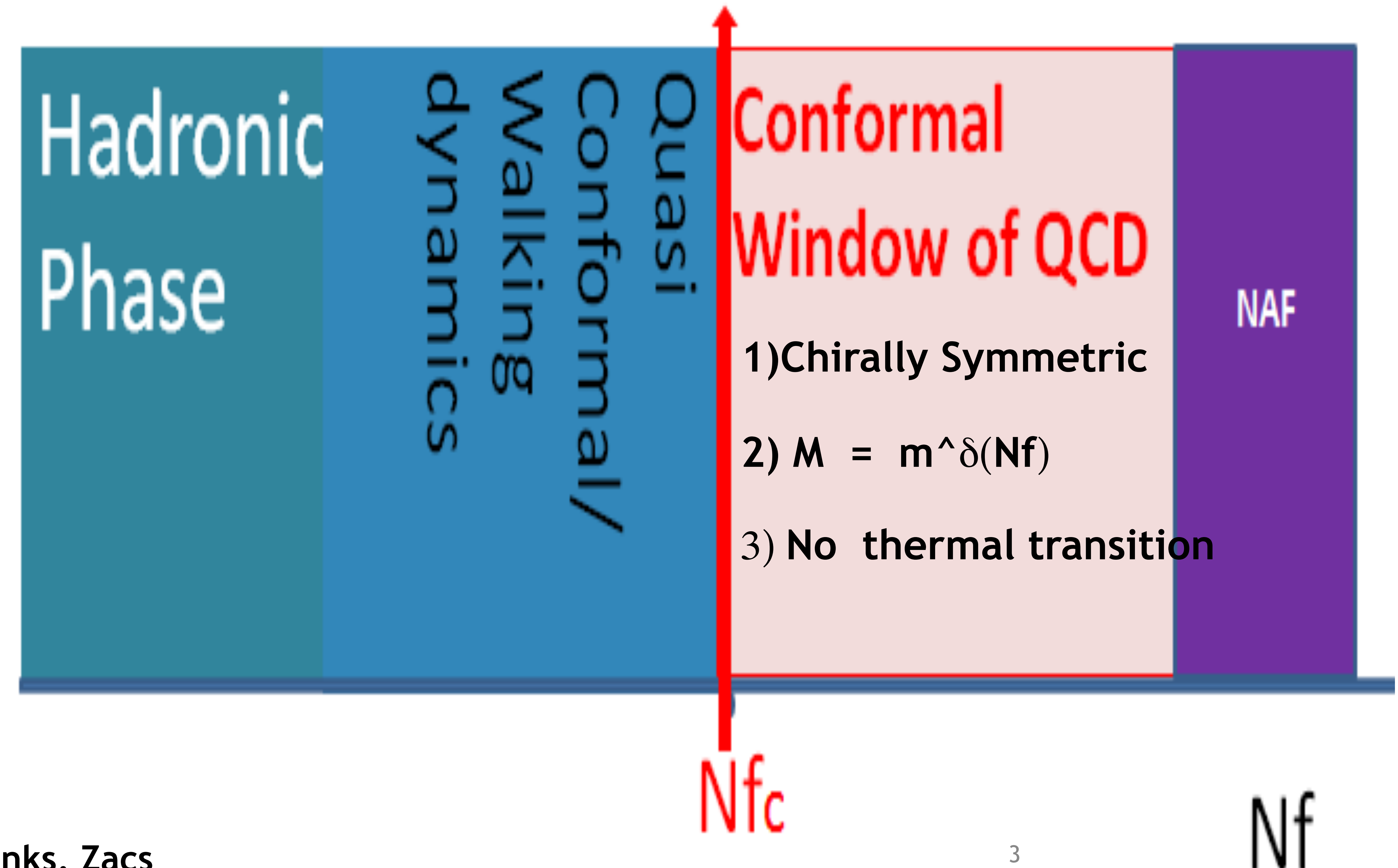
Lattice results

Maria Paola Lombardo

INFN

Based on **M.P.L, K. Miura, T. Nunes da Silva, E.Pallante, Int.J.Mod.Phys. A29 (2014) arXiv:1410.2036**
M.P.L, K. Miura, T. Nunes da Silva, E.Pallante, JHEP 1412 (2014) 183 arXiv:1410.0298

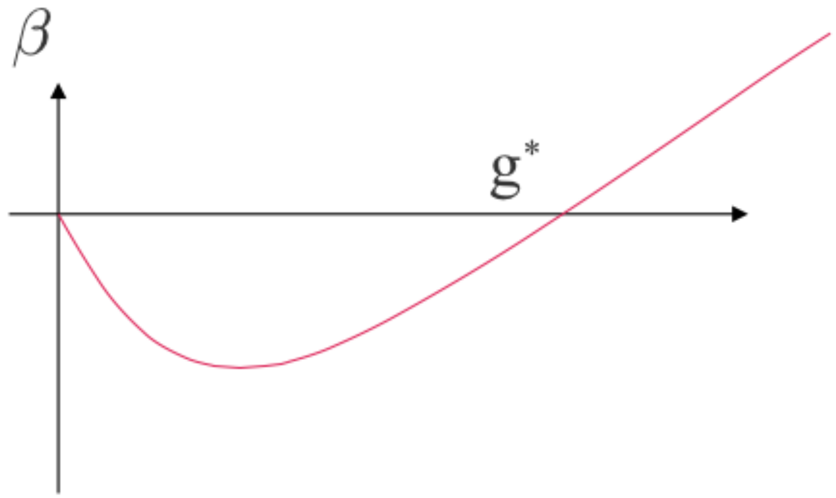
Nf

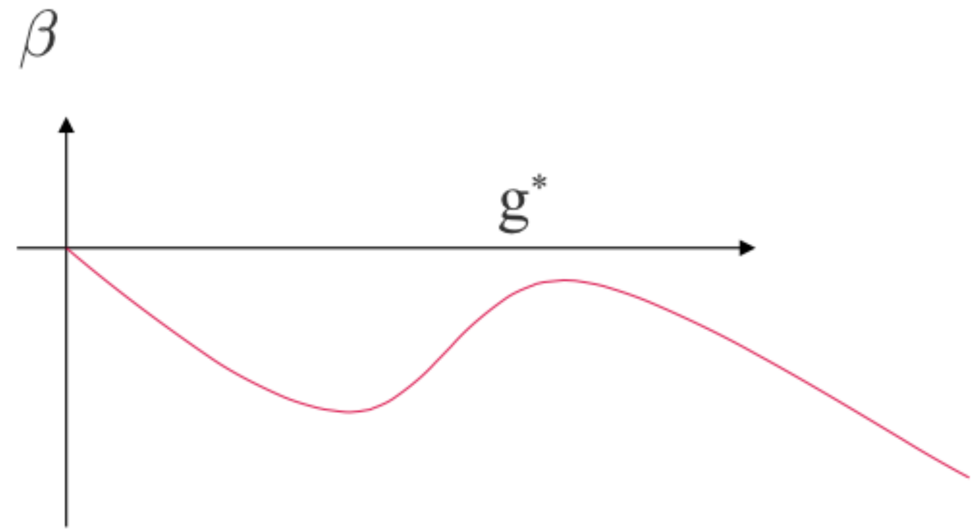


Banks, Zacs
Appelquist et al
Miransy-Yamawaki

Adding flavors to QCD: Conformal Window

- An IR fixed point can emerge already in the two-loop β function as you increase the number N_f of fermions. (Gross and Wilczek, Banks and Zaks, ...)





Conformal: $N_f > N^*_f$

- Long distance (IR) Conformal theory.
- Chiral symmetry $SU(N_f) \times SU(N_f)$
- No Confinement
- But asymptotical free in the UV.

Walking: $N_f < N^*_f$, but close to N^*_f

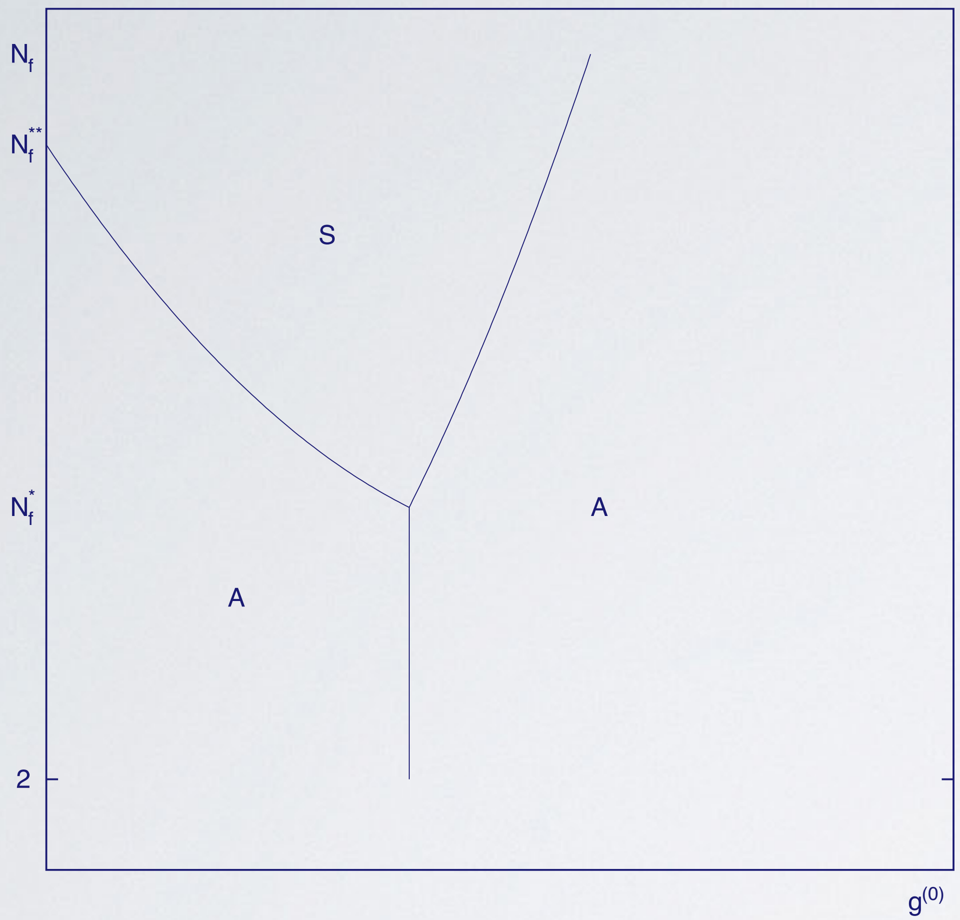
- Spontaneous breaking of chiral symmetry $SU(N_f) \times SU(N_f)$
- Confinement
- Spontaneous breaking of an approximate (IR) conformal symmetry

Slide by R. Brower

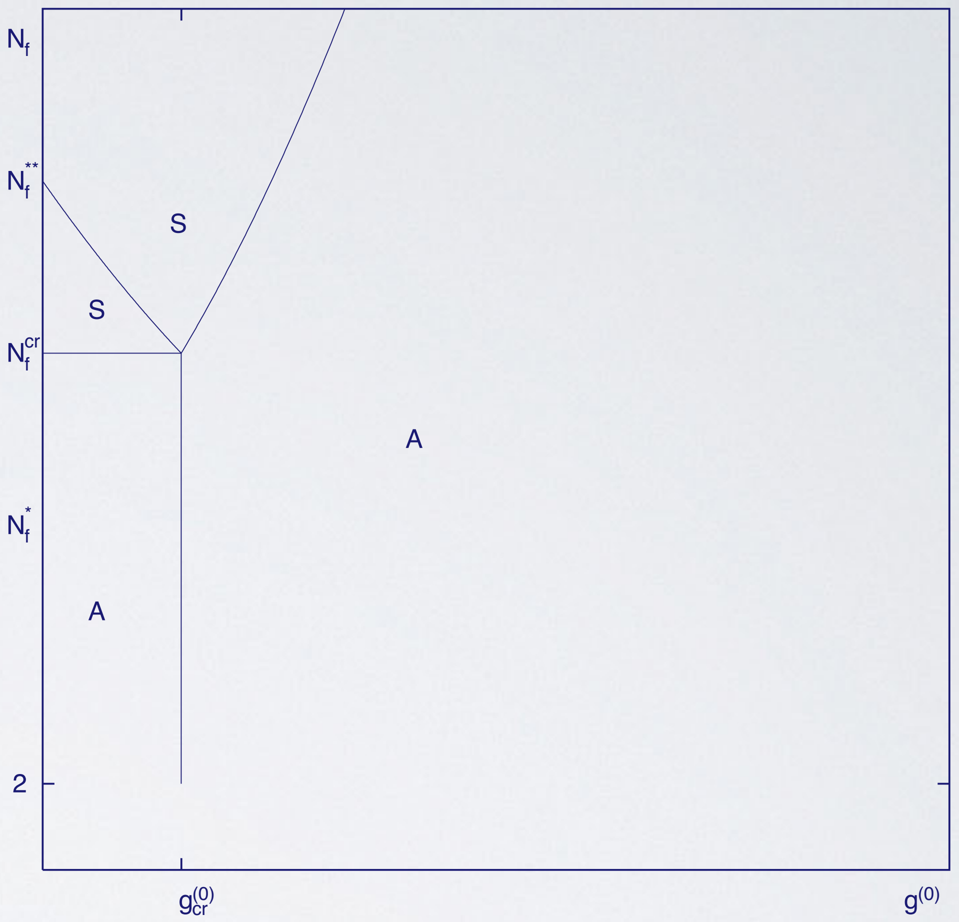
CHIRAL SYMMETRY BREAKING NEEDS LARGE GAUGE COUPLING

- Chiral symmetry breaking is possible when the gauge coupling exceeds a critical value
$$\alpha_c \equiv \frac{\pi}{3\,C_2(R)} = \frac{2\pi\,N}{3\,(N^2-1)}\,,$$
where $C_2(R)$ is the quadratic Casimir of the the representation R.
- Thus we would expect that when N_f is decreased below the value N_f^c at which $\alpha_* = \alpha_c$, the theory undergoes a transition to a phase where chiral symmetry is spontaneously broken. The critical value N_f^c is given by
- $$N_f^c = N\left(\frac{100N^2-66}{25N^2-15}\right)\,.$$
For large N, N_f^c approaches $4N$, while for $N=3$, N_f^c is just below 12.

Hystorical overview

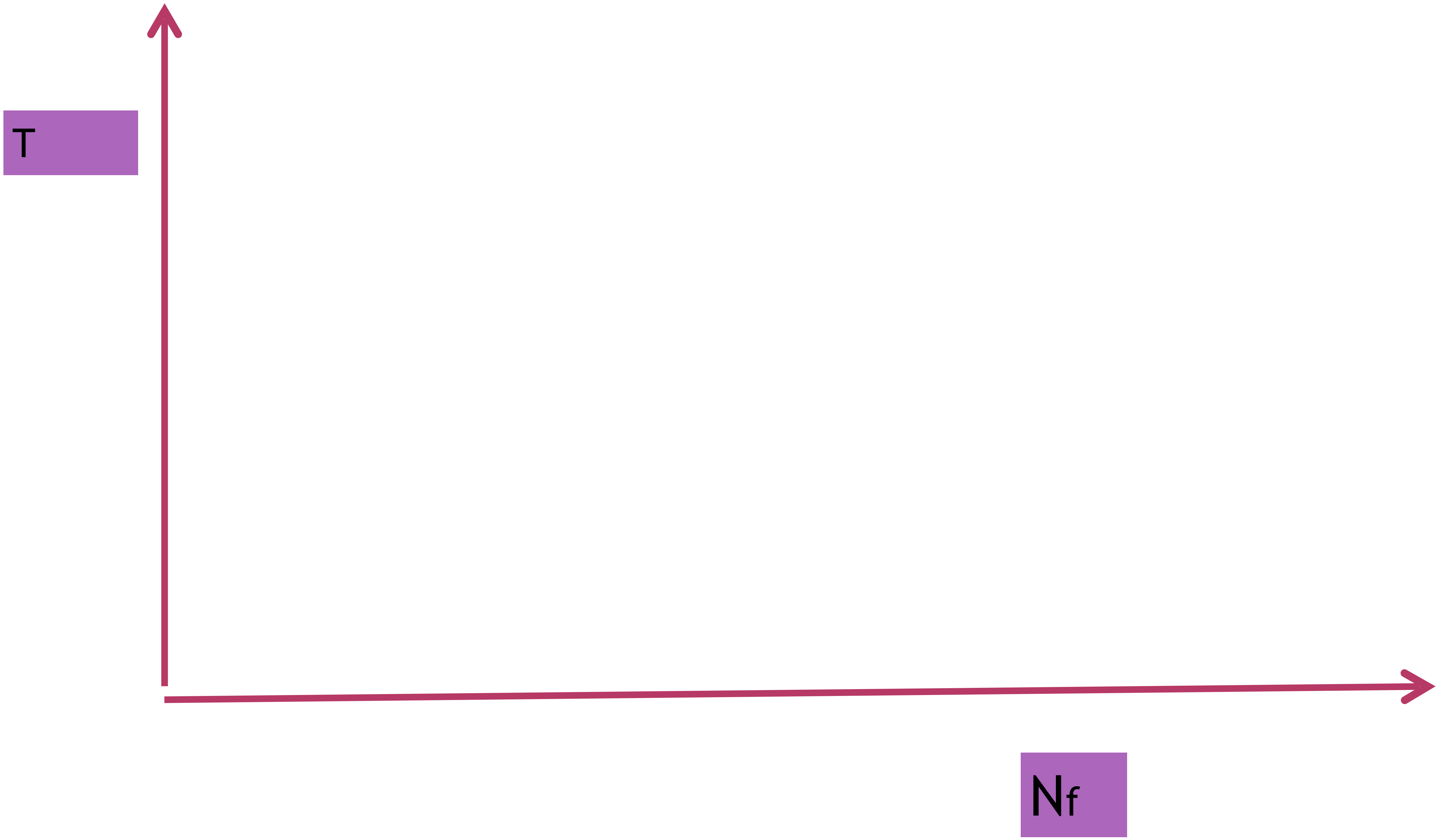


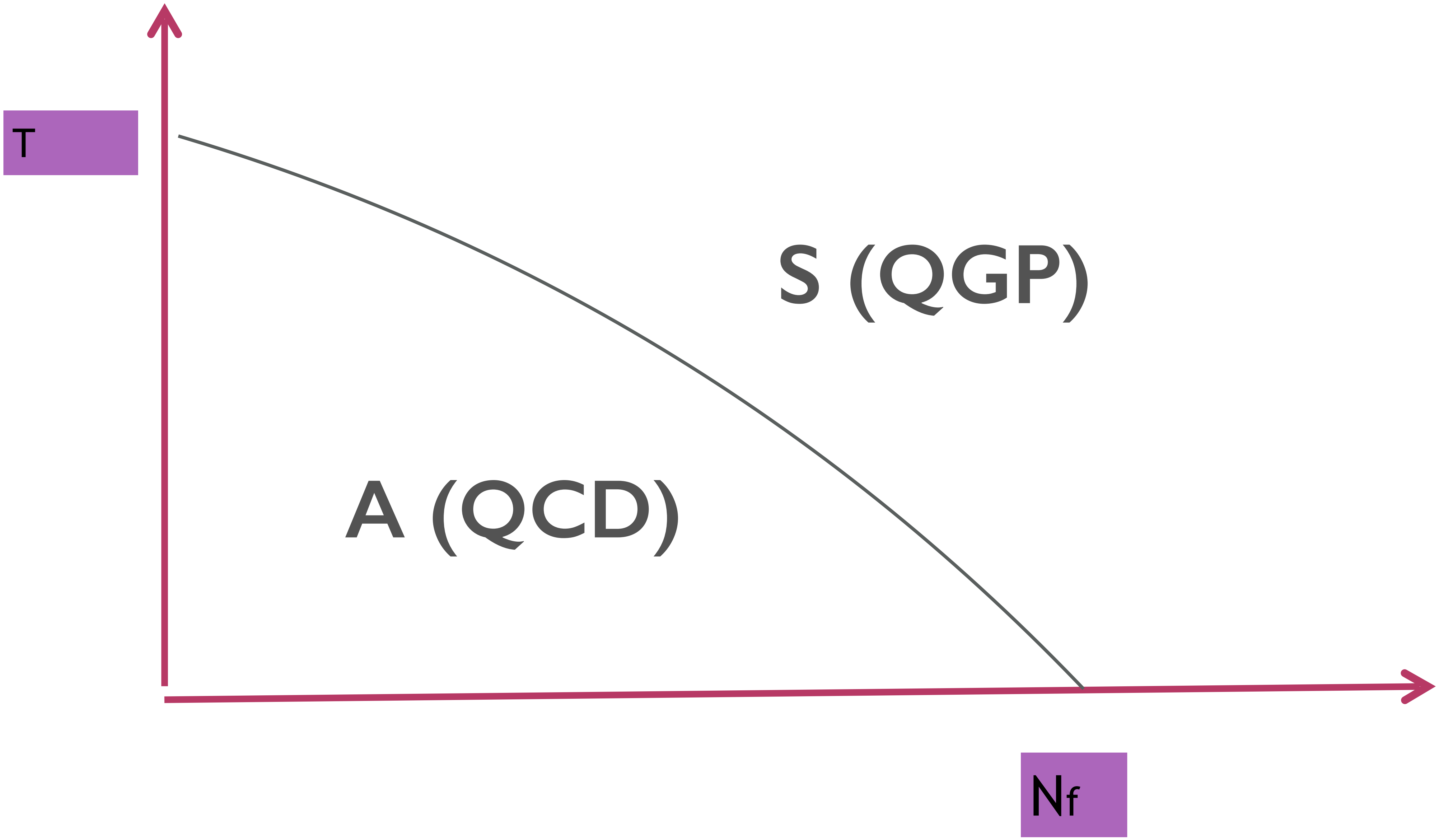
PHASES OF QCD -- BANKS ZACS



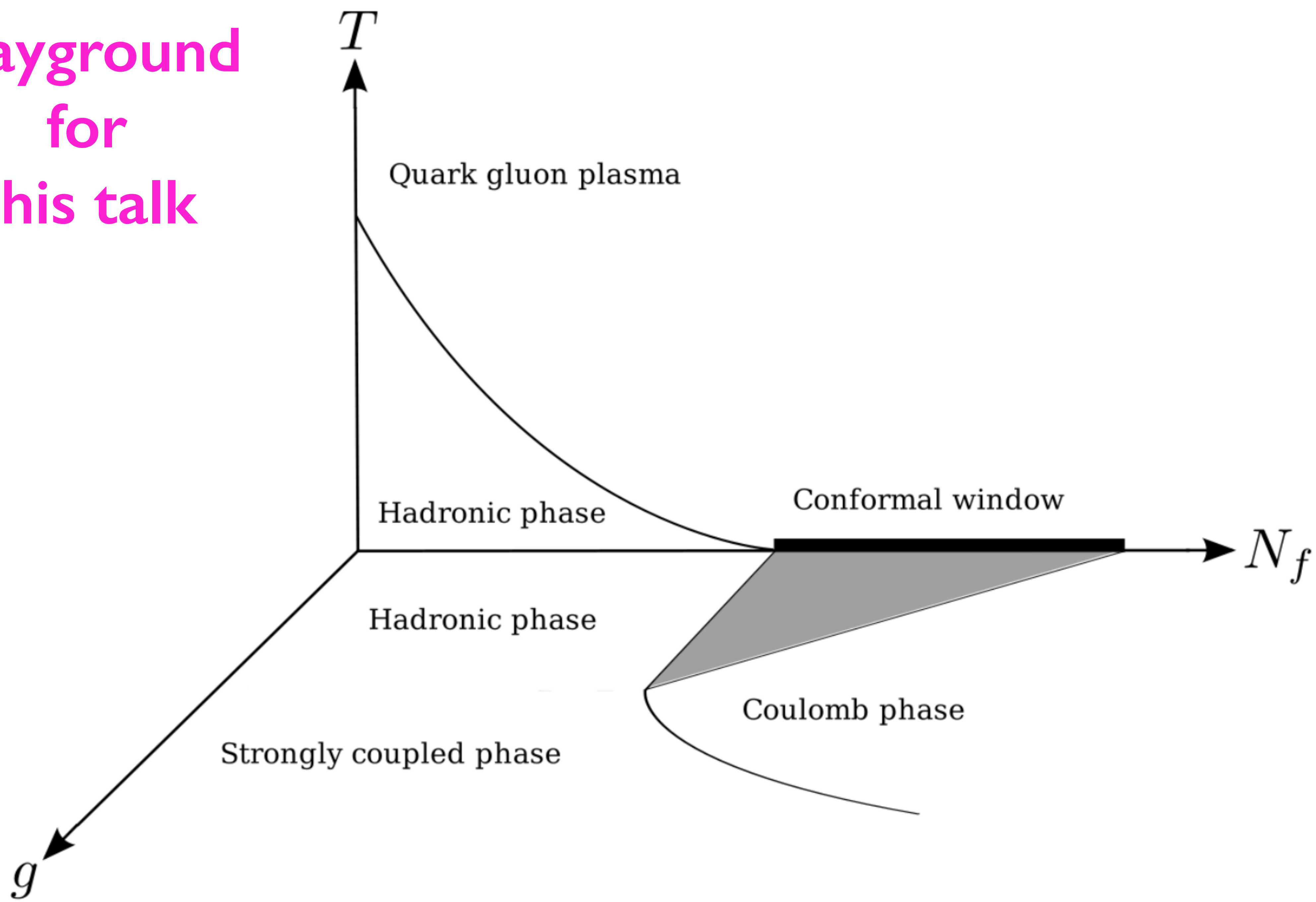
PHASES OF QCD - THE CONFORMAL WINDOW

Yet another axis..




T
S (QGP)
A (QCD)
Nf

Playground for this talk



Outline

Scaling and its probes

Inside the conformal window

The pre-conformal behavior

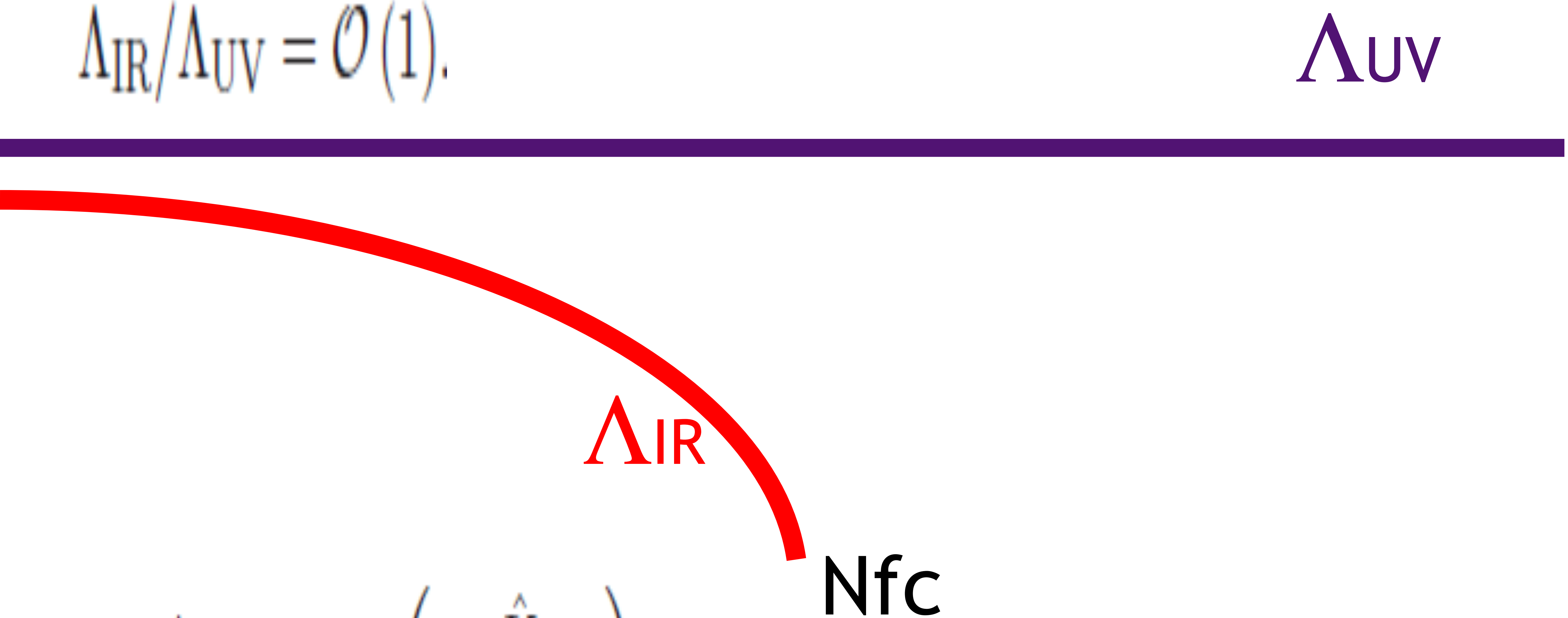
Pre-conformality as a tool for QGP

Scale separation

Discussion and Outlook

From UV to IR

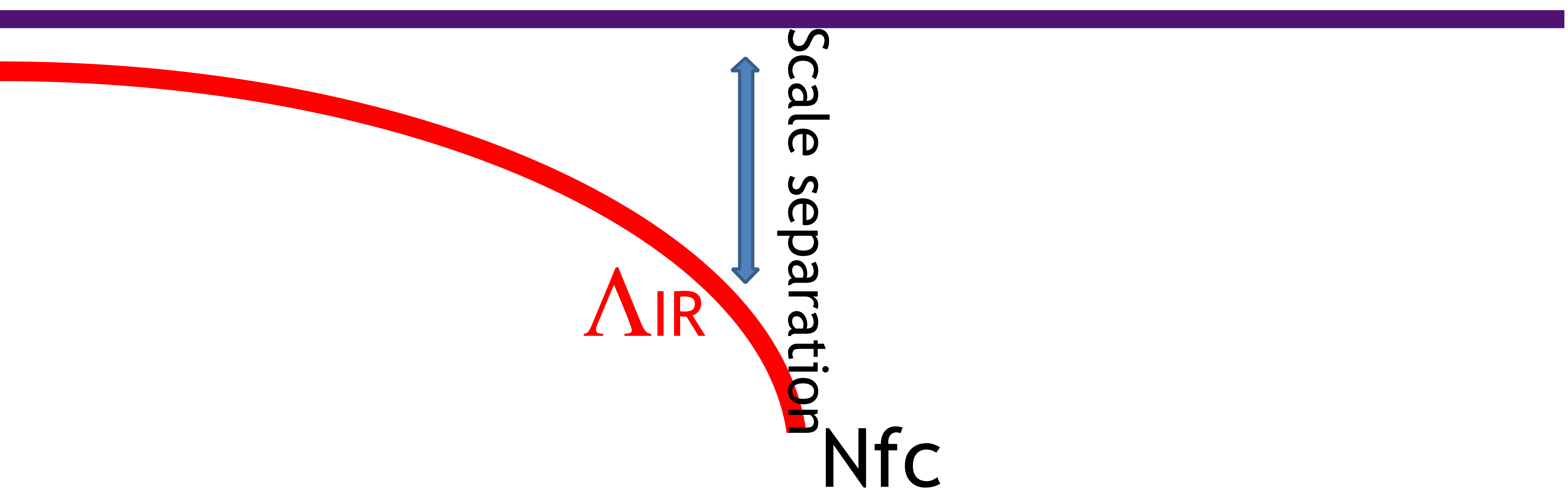
$$\Lambda_{\rm IR}/\Lambda_{\rm UV} = \mathcal{O}(1).$$



$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

From UV to IR

$$\Lambda_{\rm IR}/\Lambda_{\rm UV} = \mathcal{O}\left(1\right).$$



$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\hat{K}}{\sqrt{x_c - x}}\right)$$

Physical scales & Lattice scales

- ✓ Lattice introduces two further technical scales a and L obscuring the UV and IR behaviour respectively

- ✓ Ratios of homogeneous quantities

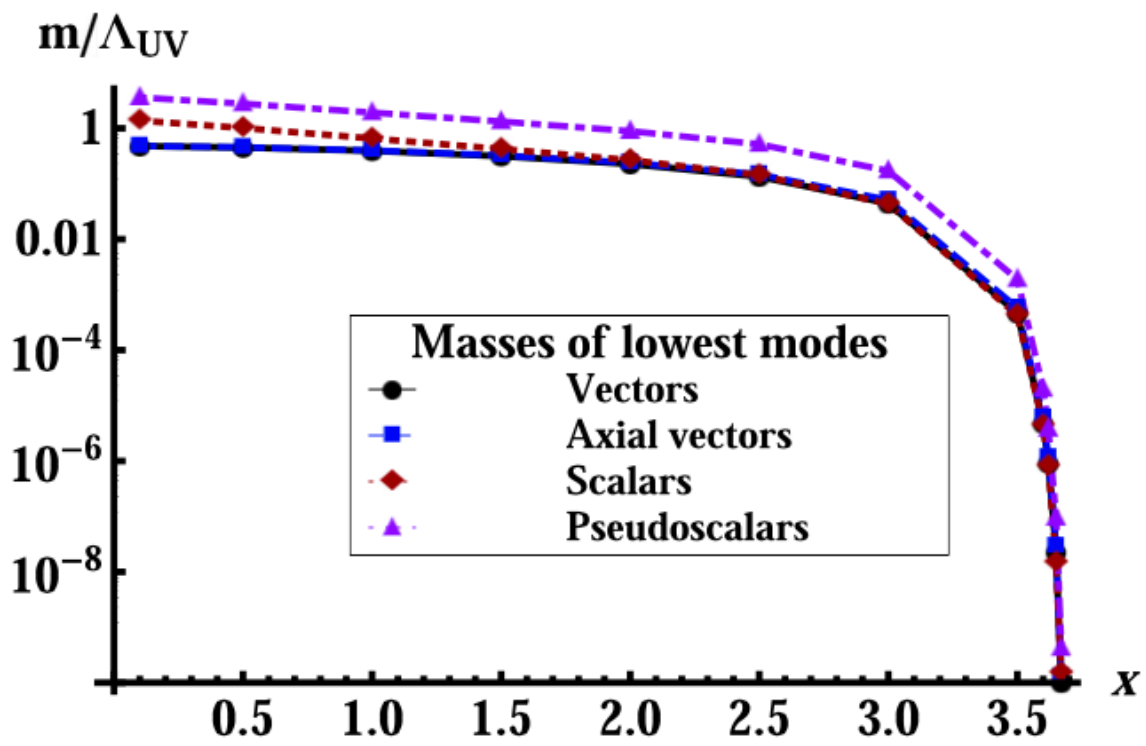
Miranski , Yamawaki
Braun, Gies
Kiritsis et al

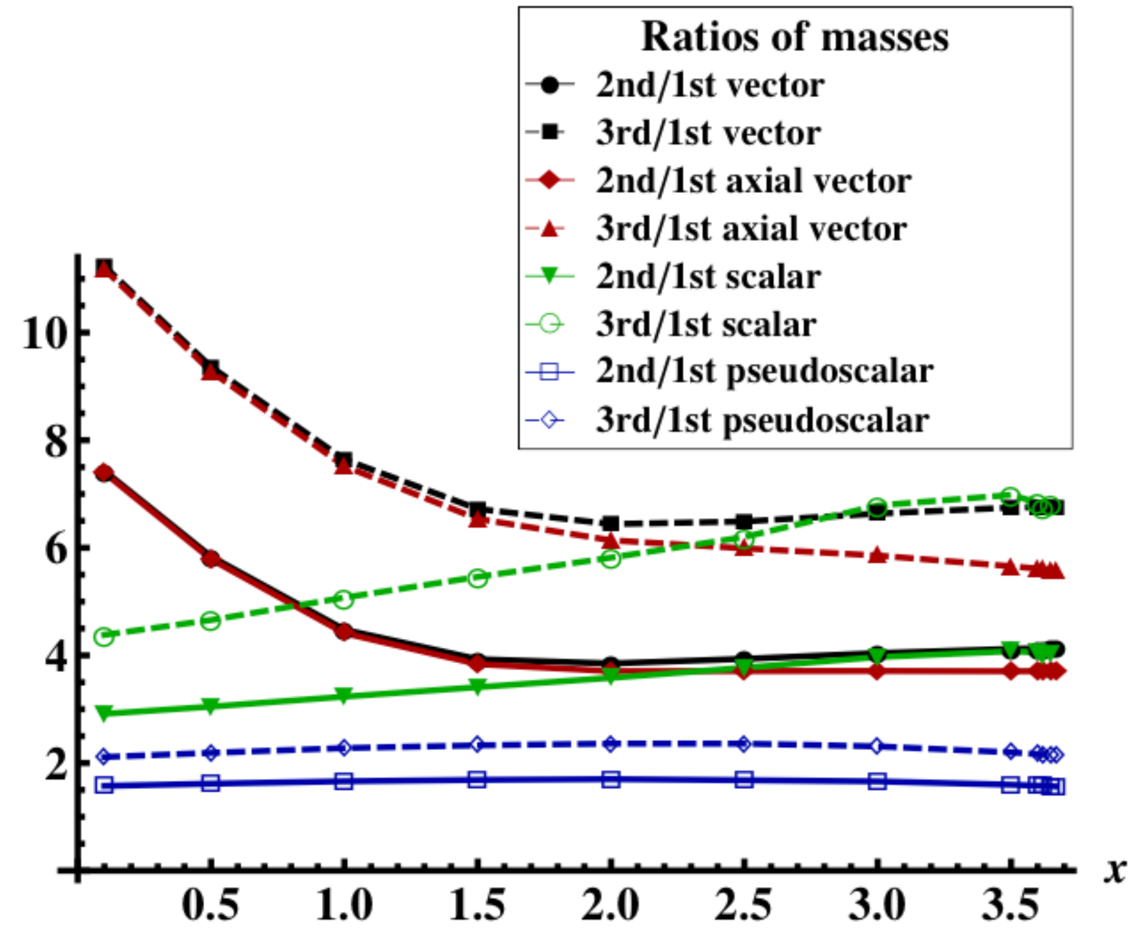
R = O1/O2

useful: Help controlling a and L systematic effects Display scale hierarchy with no need to fix the scale across different theories

- ✓ When **O2** is an UV quantity – non critical at Nfc -- taking the ratio is de facto a scale fixing procedure for **O1**

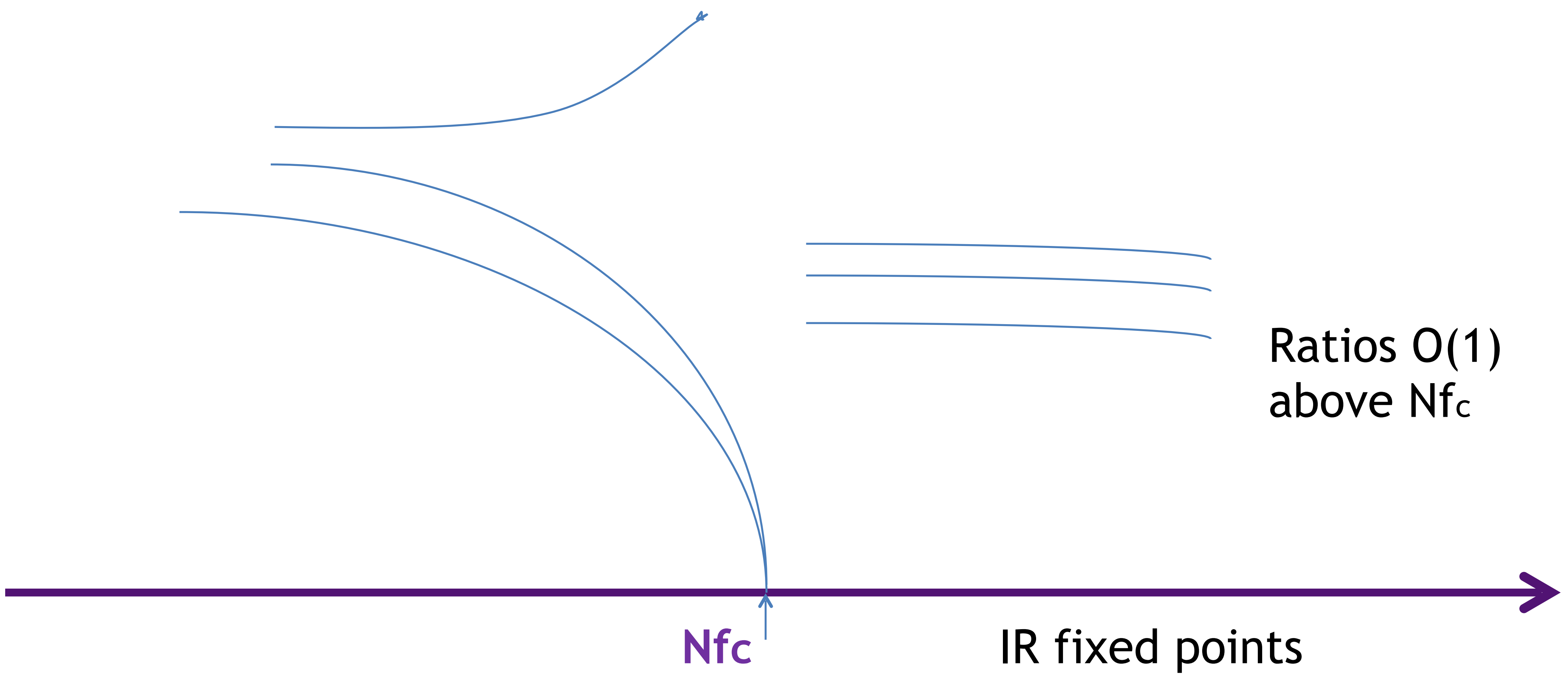
✓





Arean, Iatrakis, Jarvinen, Kiritsis 2013

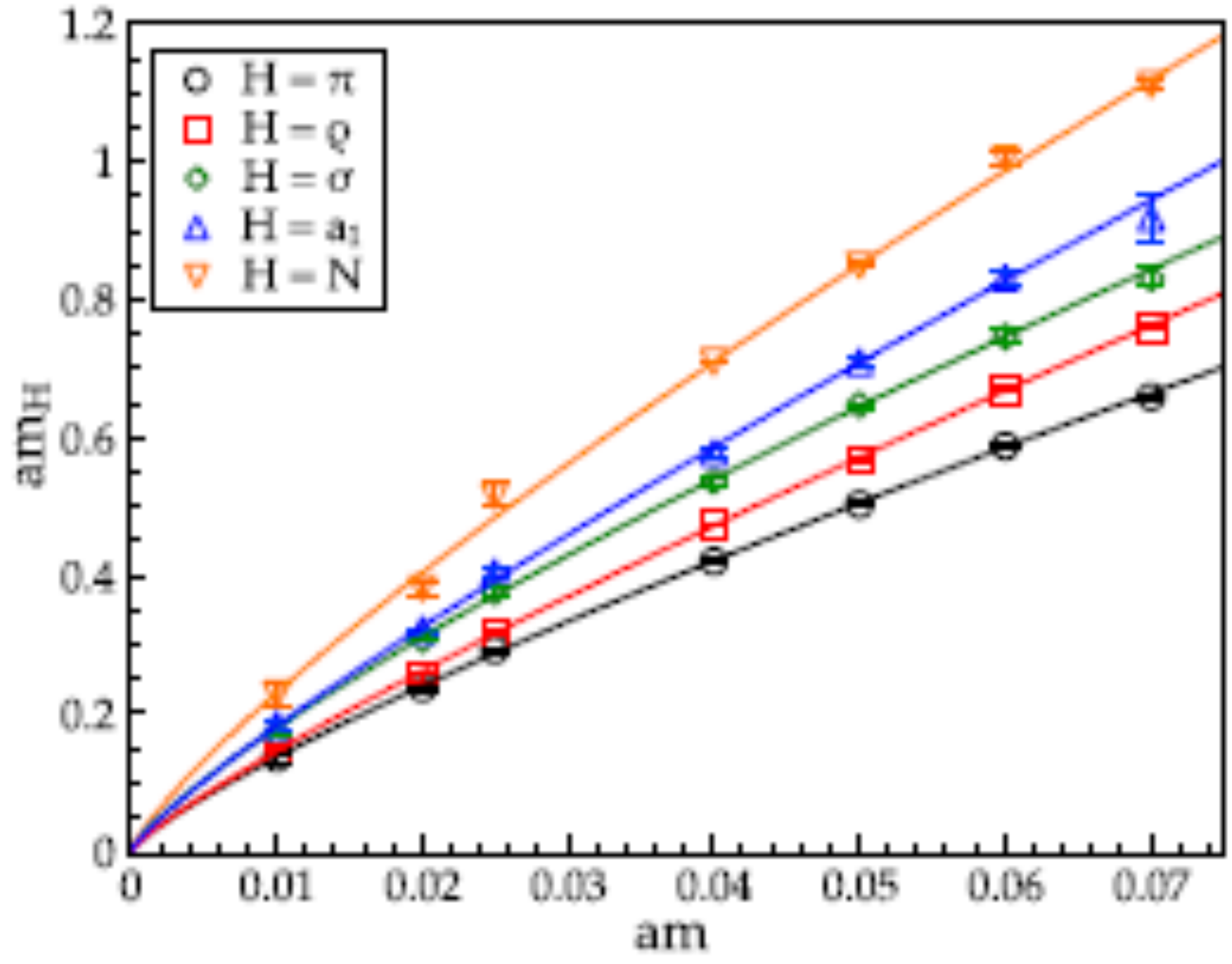
Adimensional ratios below and above Nf_c



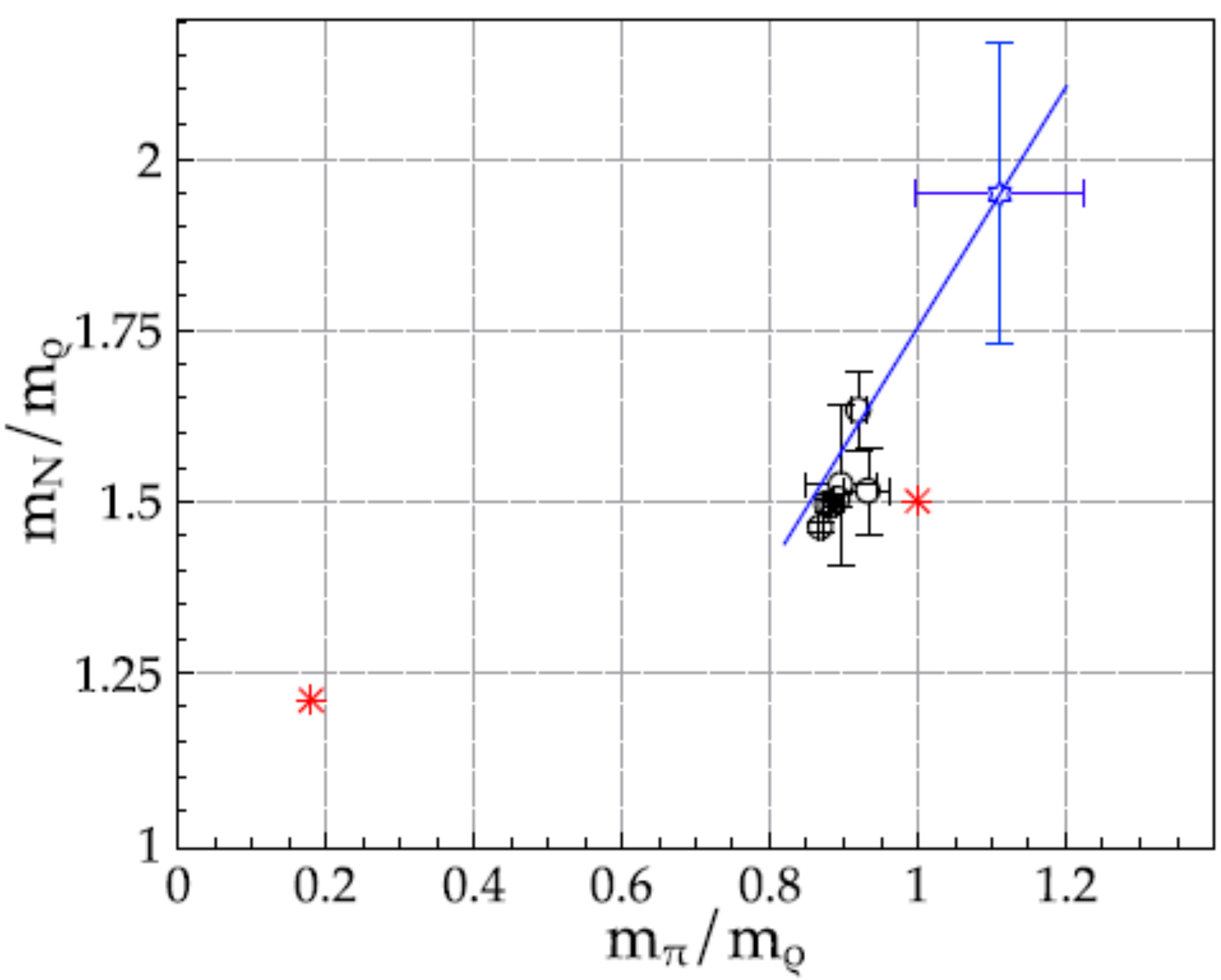
$O(Nf,m) = c_o(Nf)\ m^{\delta(Nf)}$

Inside the conformal window

Hadron spectrum



Ratios in the conformal window at a glance: the Edinburgh plot



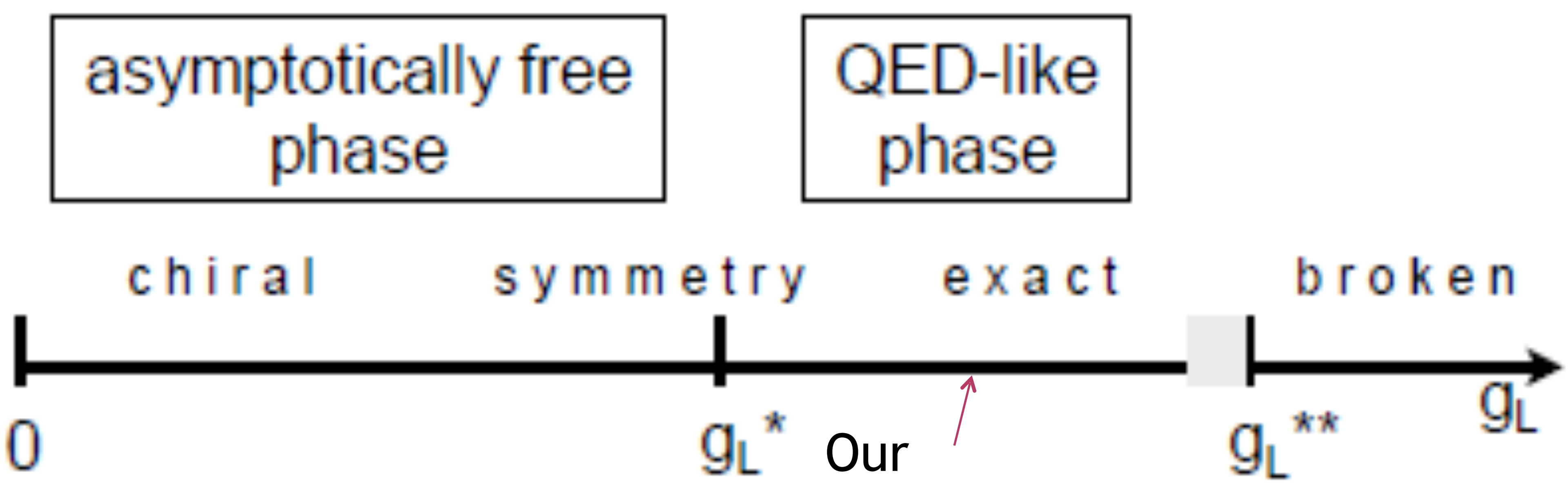
Lattice corrections to conformal scaling

1: Size $M_H = L^{-1} f_H(x)$ $\quad x \equiv L m^{1/y_m}$

2: Coupling $M_H = L^{-1} f_H\left(x, g_0 m^{\omega}\right)$

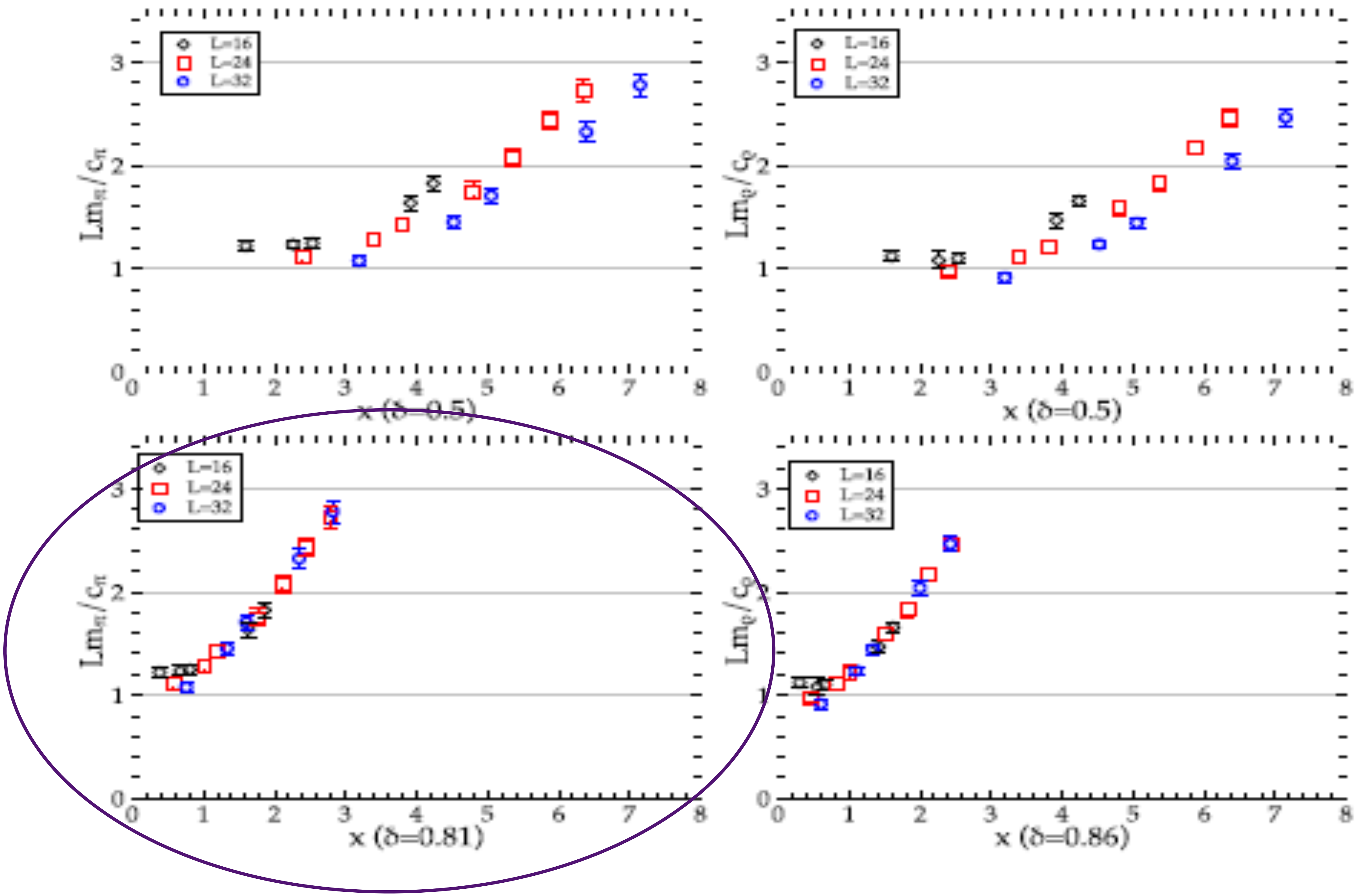
Del Debbio, Zwicky;
Hasenfratz et al;
MpL, da Silva, Miura, Pallante

$$LM_H = F_H(x)\left\{1 + g_0 m^{\omega} G_H(x) + \mathcal{O}\left(g_0^2 m^{2\omega}\right)\right\}$$

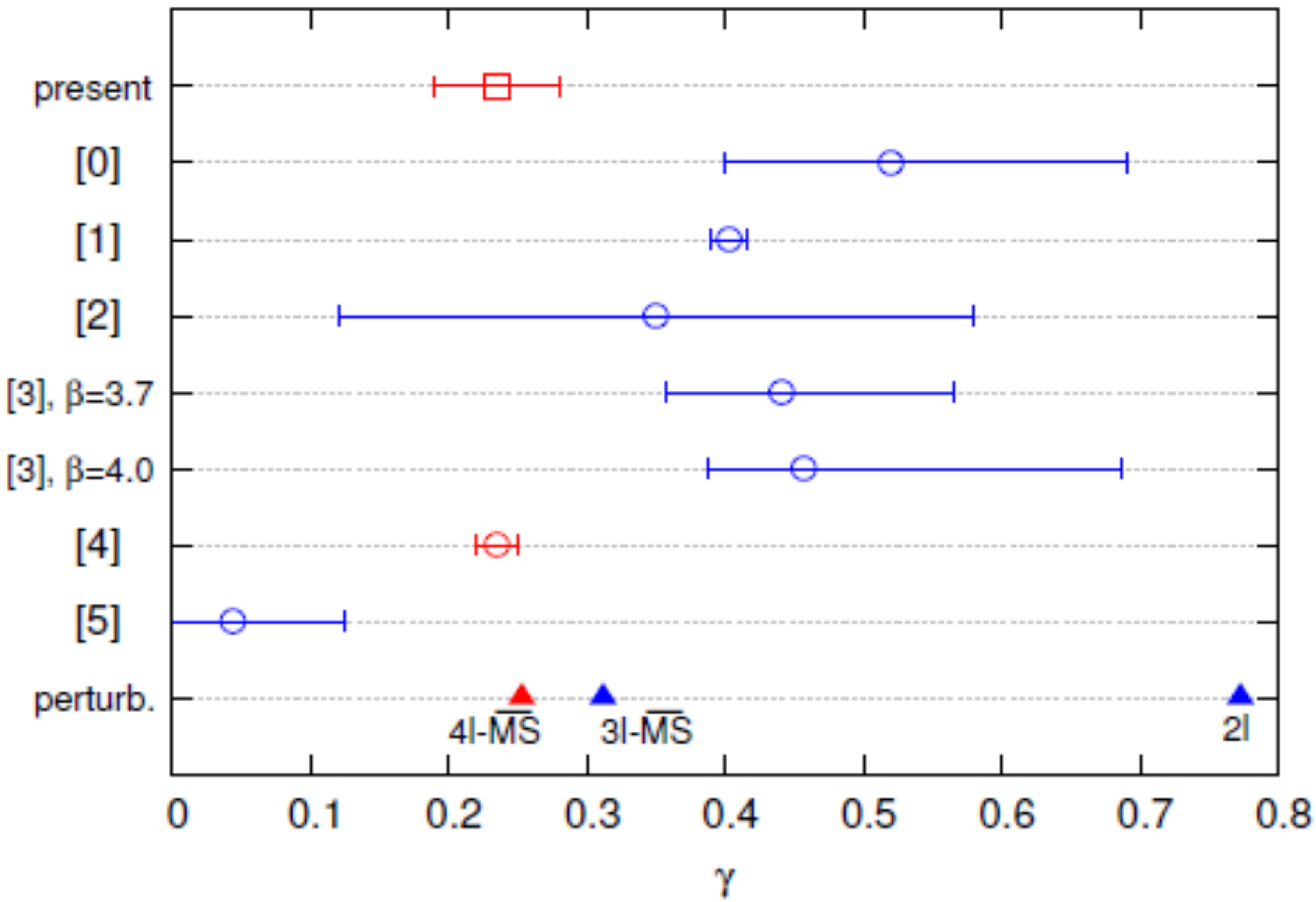


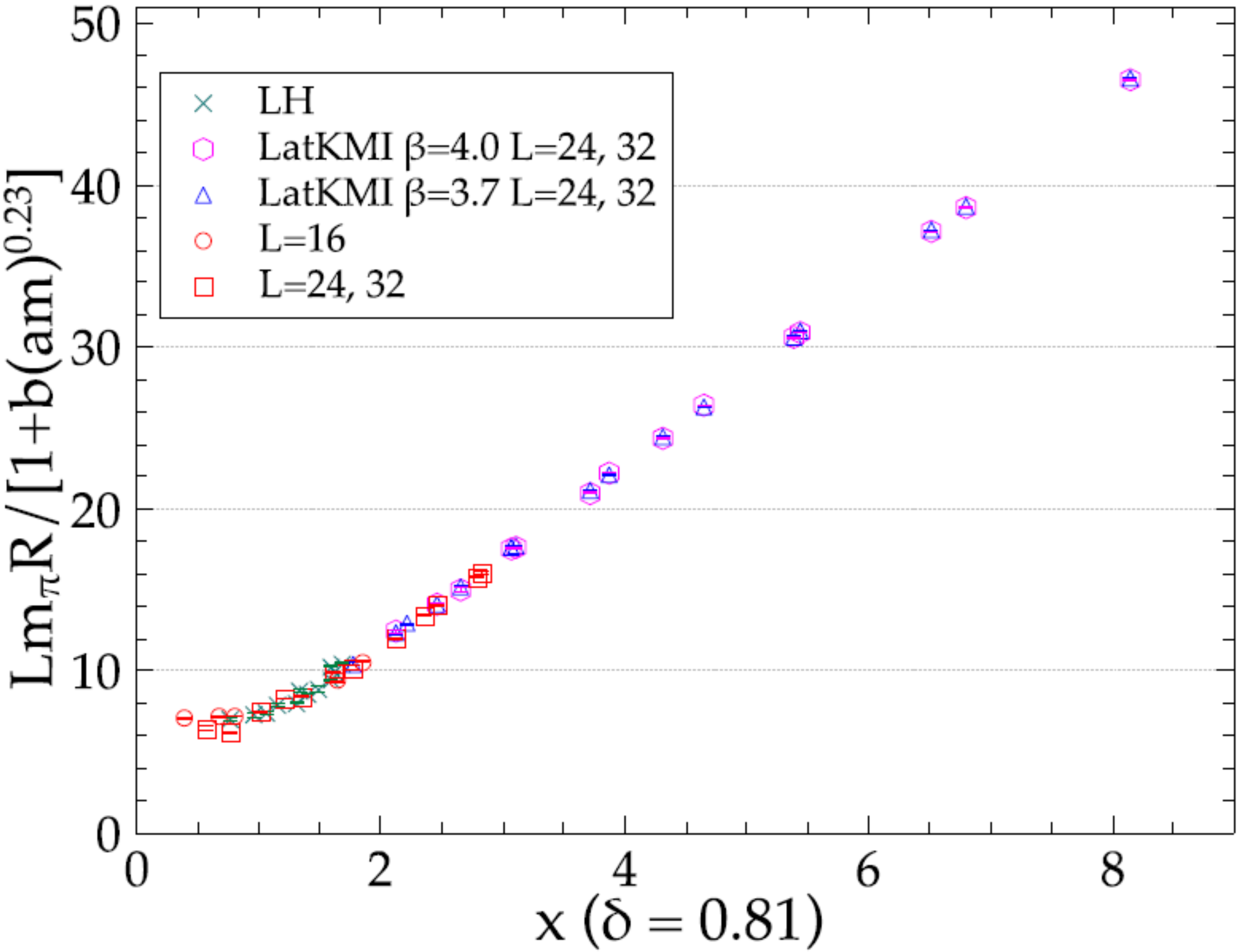
Our simulations are here

Anomalous dimension from the QED phase?



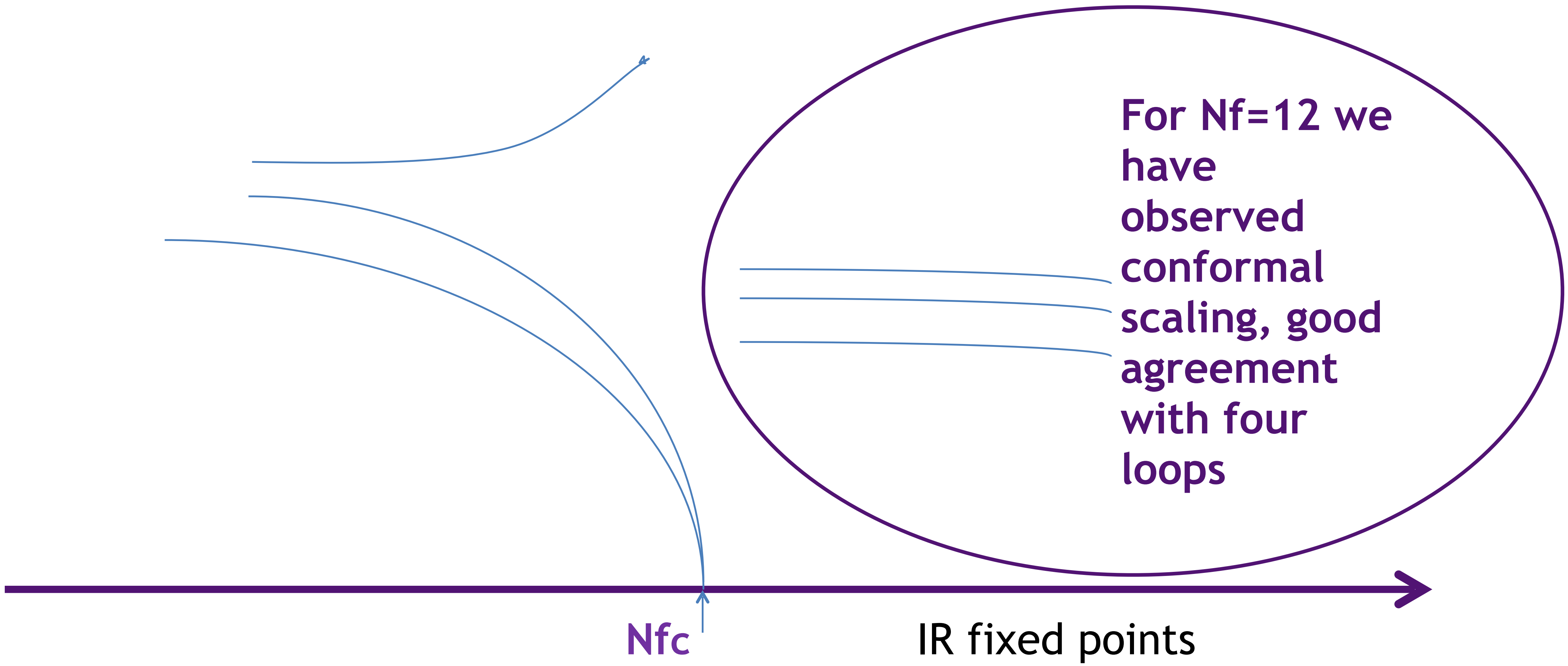
Summary of the results: accidental agreement??





LH
LatKMI β=4.0 L=24, 32
LatKMI β=3.7 L=24, 32
L=16
L=24, 32
0
10
20
30
40
50
0
2
4
6
8
Lm$_\pi$R/[1+b(am)$^{0.23}$]
x (δ = 0.81)

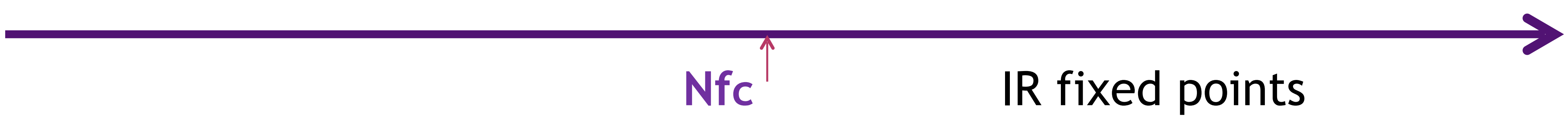
Adimensional ratios below and above Nfc



$O(Nf,m) = c_o(Nf)\ m^\wedge\delta(Nf)$

Conformal scaling observed for Nf=12

For Nf=12, T=0, we have observed conformal scaling, agreement among different groups once corrections to scaling are taken into account



$O(Nf,m) = c_o(Nf)\ m^{\wedge}\delta(Nf)$

Conformal phase at T=0 established

The preconformal behavior

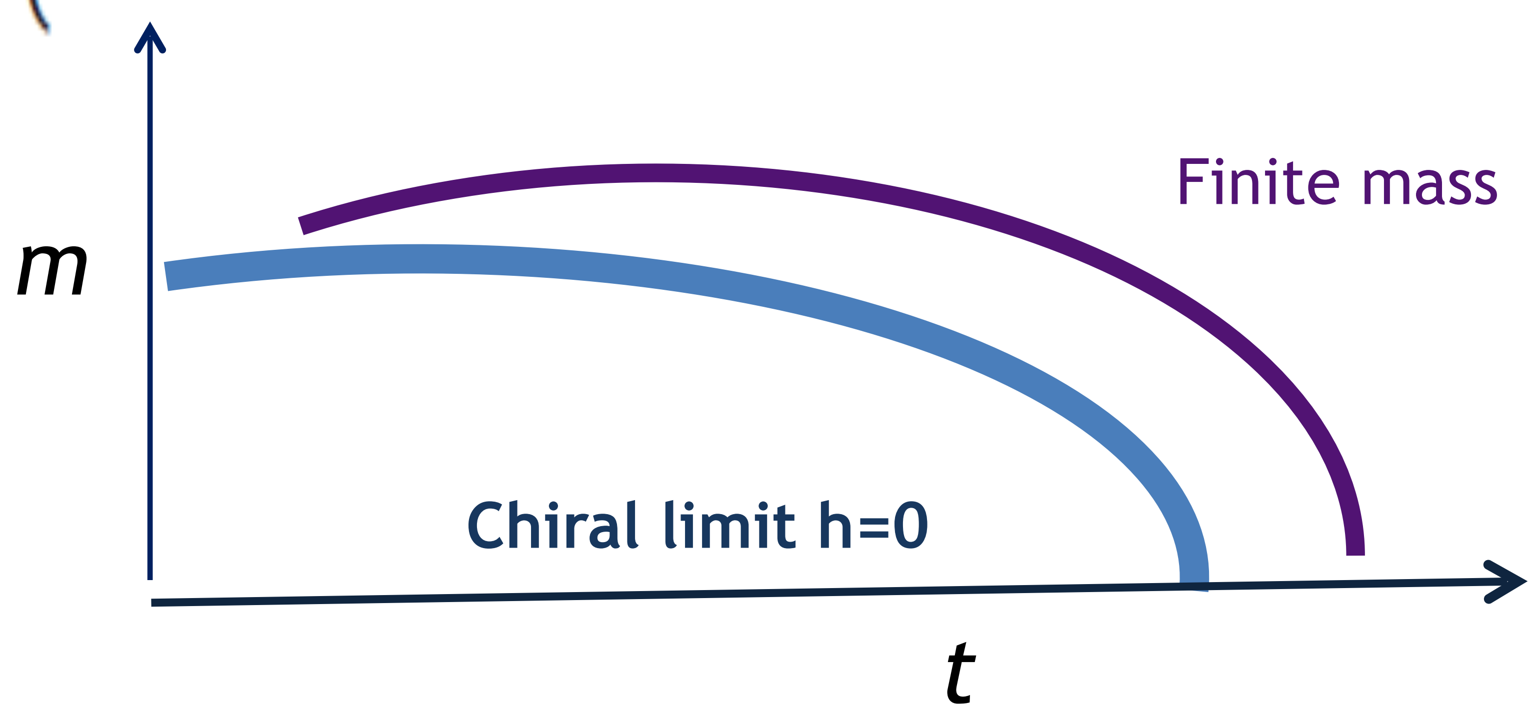
Scaling for essential singularities

Nogada, Hasegawa, Nemoto,PRL 2012

$$g(t,h,N^{-1}) = b^{-1}\hat{g}(e^{-(t/t_0)^{-x_t}}b, hb^{y_h}, N^{-1}b).$$

m <-> Chiral Condensate
h <-> bare mass
t <-> Nfc – Nc

$$m \propto \begin{cases} e^{-(1-y_h)(t/t_0)^{-x_t}} & \text{for } he^{y_h(t/t_0)^{-x_t}} \ll 1 \\ h^{y_h^{-1}-1} & \text{for } he^{y_h(t/t_0)^{-x_t}} \gg 1 \end{cases}$$



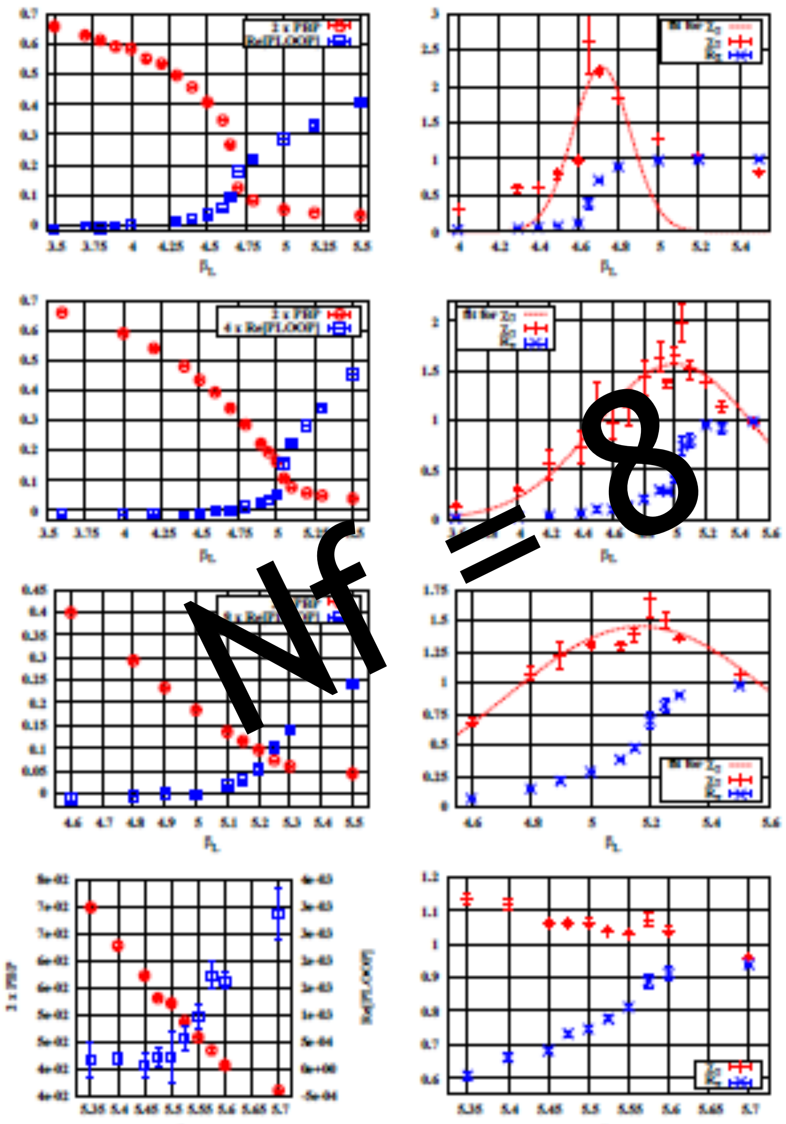
Within the scaling window data at finite mass contain information on the critical behaviour . They can can be approximativelydescribed as zero mass ones, but with a larger apparent critical point.

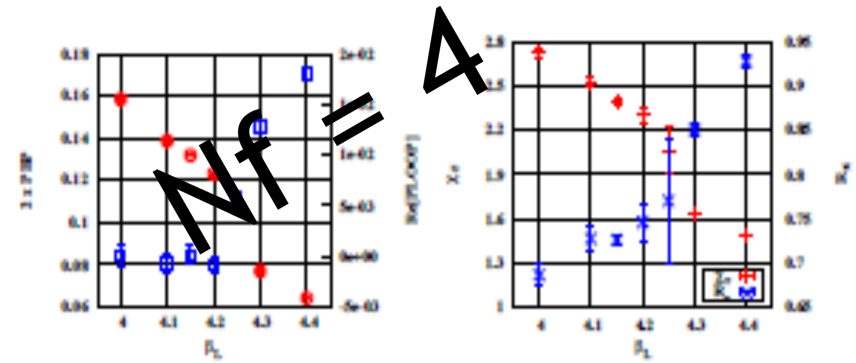
Choose an observable to monitor the approach to the conformal window, and work on the lattice first

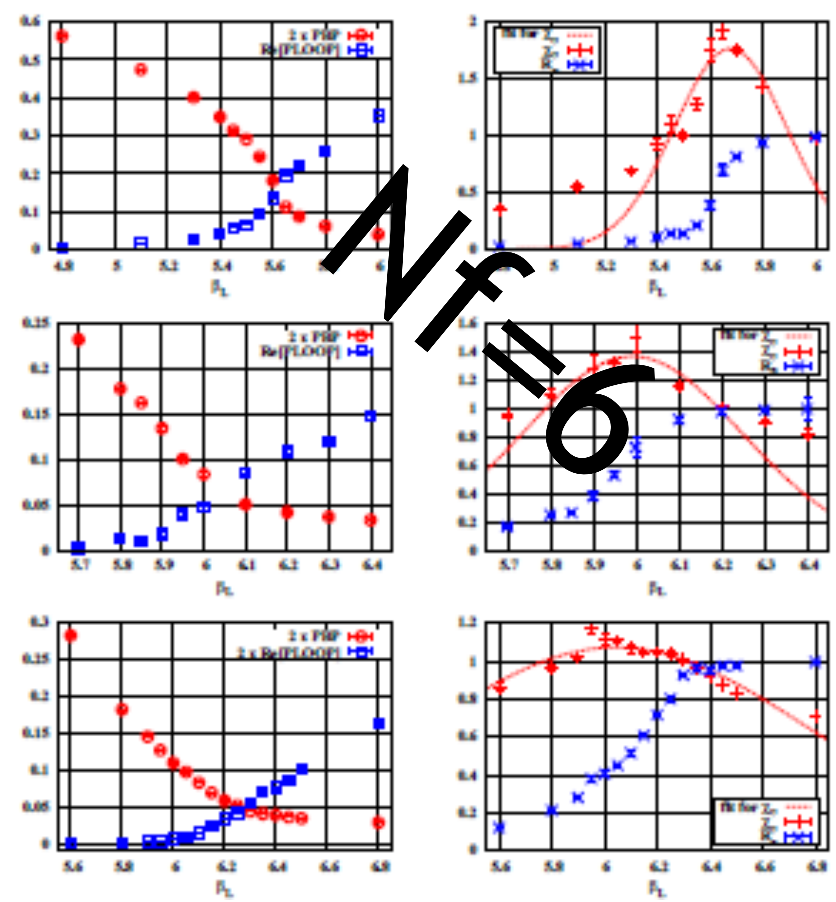
Our choices

Critical temperature
String Tension
Wilson flow and w0

We studied the thermal transition for several Nf and several Nt







All simulations :
Gauge Action one loop Sym.
Tadpole improved AsqTad

The critical number of flavor from lattice results

$$T_c \equiv \frac{1}{a(\beta_{\mathrm{L}}{}^{\mathrm{c}}) \cdot N_t} \; .$$

$$\beta(g) = -(b_0 g^3 + b_1 g^5) \; ,$$

$$b_0 = \frac{1}{(4\pi)^2}\left(\frac{11 C_2[G]}{3} - \frac{4T[F]N_f}{3}\right) \; ,$$

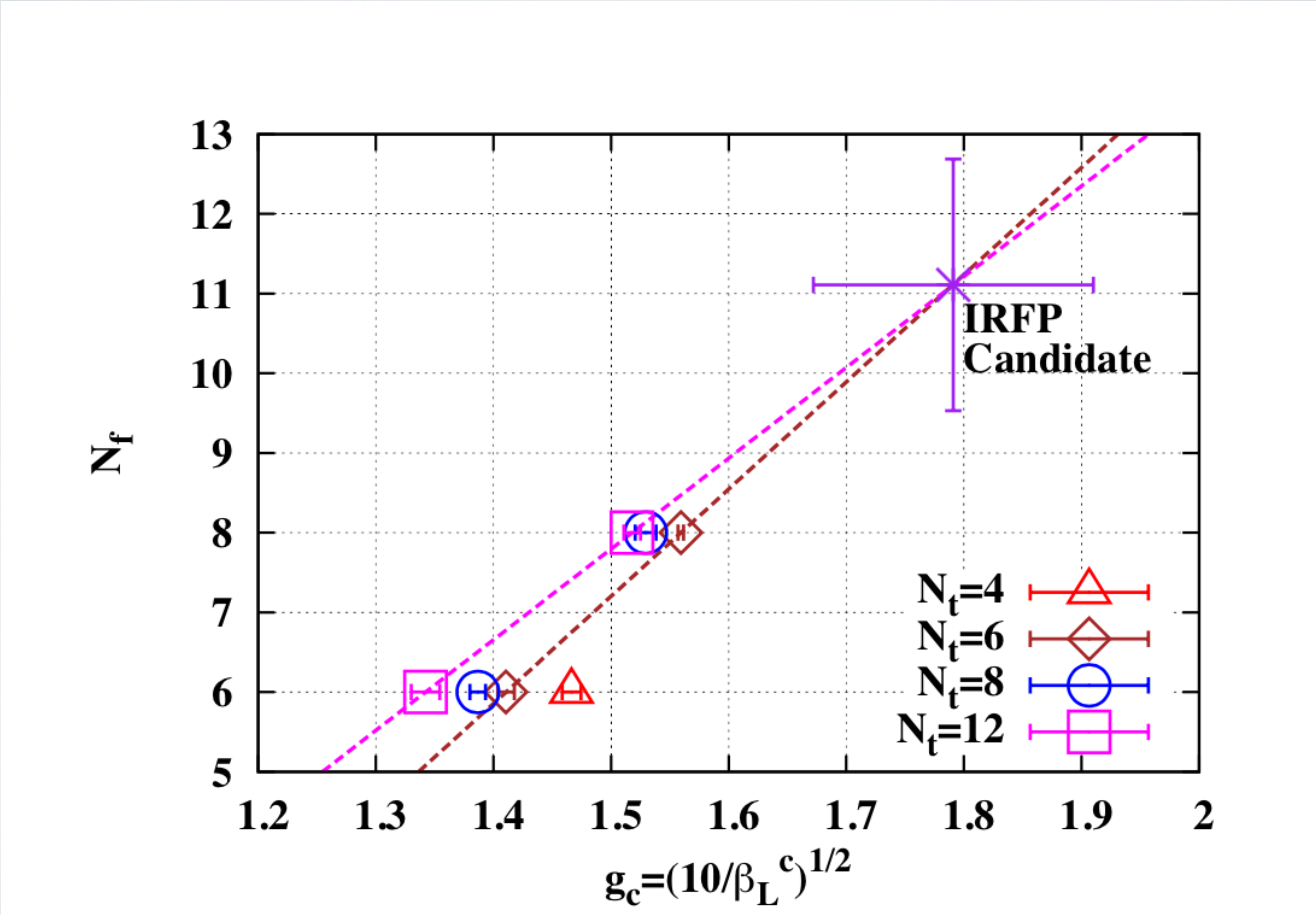
$$b_1 = \frac{1}{(4\pi)^4}\left(\frac{34(C_2[G])^2}{3} - \left(\frac{20C_2[G]}{3} + 4C_2[F]\right)T[F]N$$

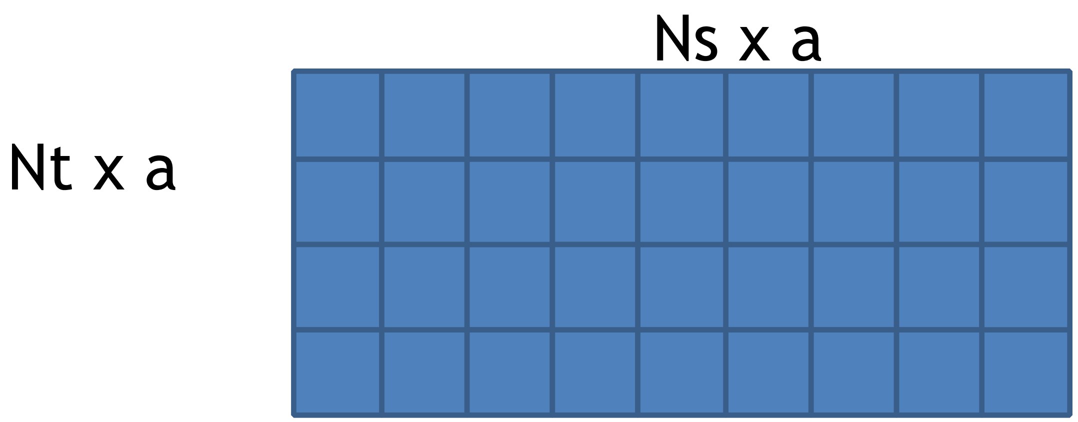
$N_f \backslash N_t$	4	6	8	12	h	β_0
0	18.11 ± 0.65	18.21 ± 0.91	16.56 ± 0.71	—	—	—
	16.29 ± 0.75	17.81 ± 1.02	16.56 ± 0.78	—	0.05	8.26
4	21.99 ± 1.04	19.98 ± 0.95	17.12 ± 2.43	—	—	—
	16.56 ± 1.44	18.67 ± 1.38	17.12 ± 3.41	—	0.30	6.15
6	25.41 ± 1.43	25.33 ± 1.43	22.94 ± 1.29	22.30 ± 2.52	—	—
	21.66 ± 1.64	23.87 ± 1.58	22.21 ± 1.40	22.30 ± 2.66	0.03	5.55
8	—	50.05 ± 0.87	47.06 ± 3.28	34.34 ± 1.91	—	—
	—	34.32 ± 1.40	42.67 ± 6.33	34.34 ± 3.90	1.08	4.34

$$R(g_{\mathrm{L/E}}) \equiv a(g_{\mathrm{L/E}})\Lambda_{\mathrm{L/E}} = \left(b_0 g^2_{\mathrm{L/E}}\right)^{-b_1/(2b_0^2)} \exp\left[\frac{-1}{2b_0 g_{\mathrm{L/E}}}\right]$$

$$R^{\mathrm{imp}}(\beta_{\mathrm{L/E}}) = \Lambda^{\mathrm{imp}}_{\mathrm{L/E}} \; a(\beta_{\mathrm{L/E}}) \equiv \frac{R(\beta_{\mathrm{L/E}})}{1+h} \times \left[1 + h \; \frac{R^2(\beta_{\mathrm{L/E}})}{R^2(\beta_0)}\right]$$

The critical number of flavor from bare results





From the Lattice..

$$T \equiv \frac{1}{a(\beta_{\mathrm{L}}) \cdot N_t} ,$$

..to the continuum

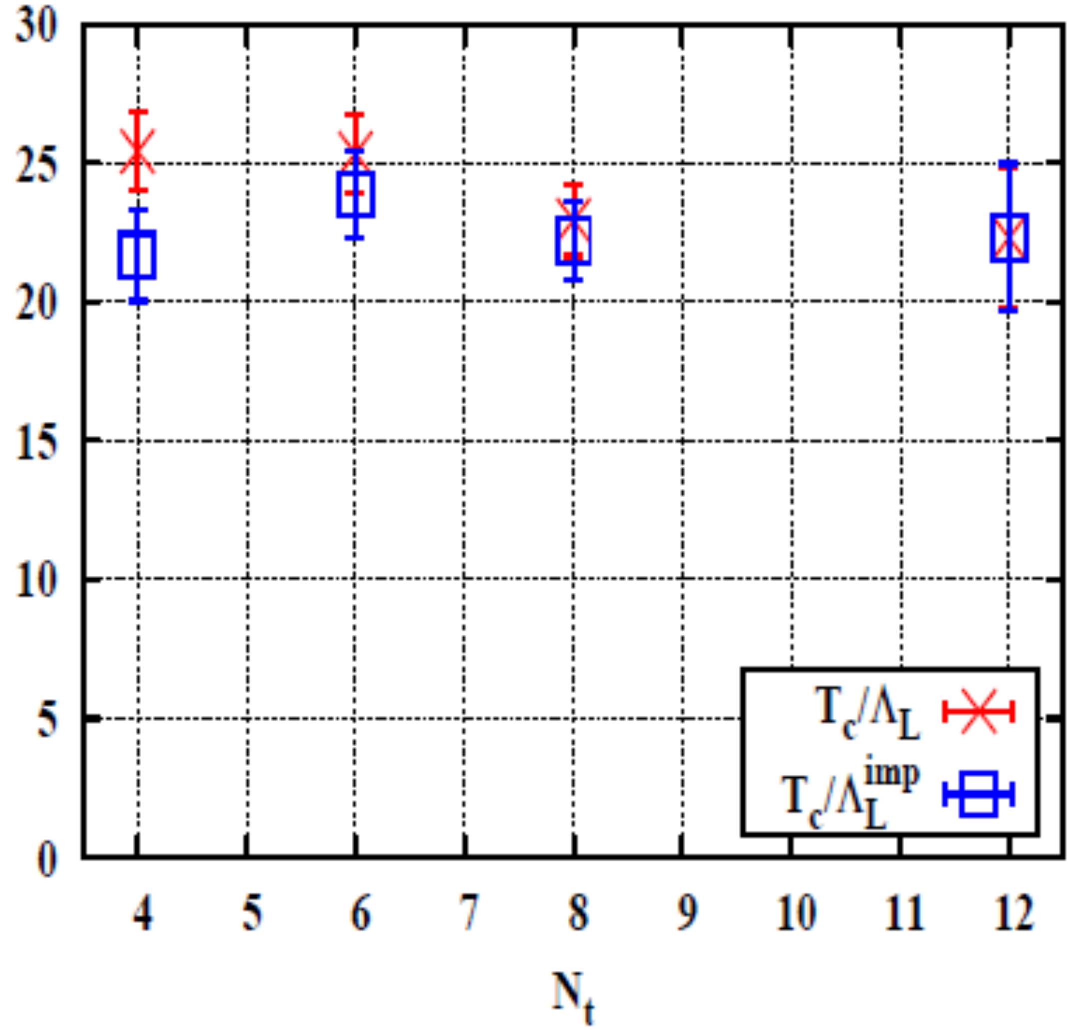
Via old fashioned asymptotic scaling

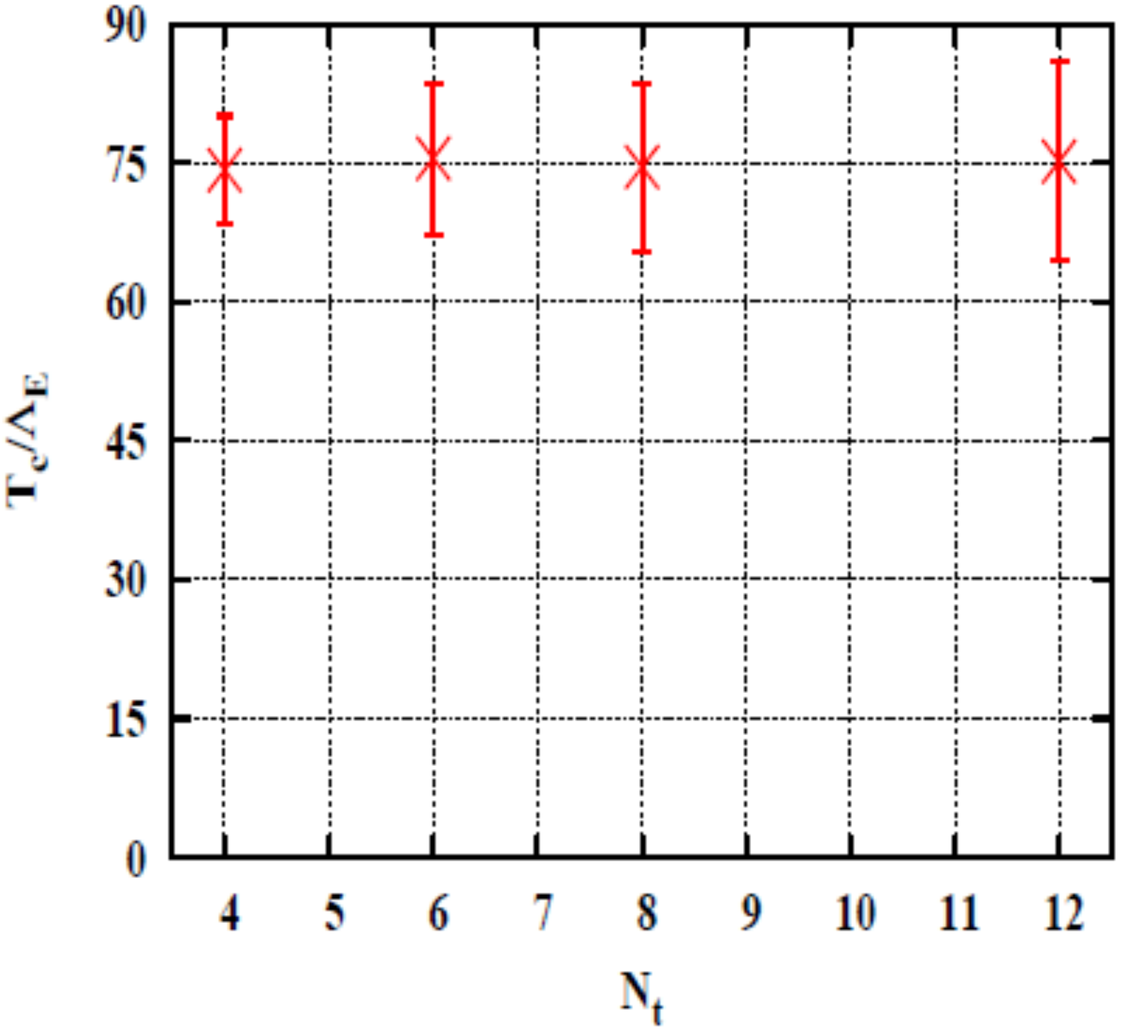
$$\Lambda_{\mathrm{L}}\, a(\beta_{\mathrm{L}}) = \left(\frac{2N_c b_0}{\beta_{\mathrm{L}}}\right)^{-b_1/(2b_0^2)} \exp\left[\frac{-\beta_{\mathrm{L}}}{4N_c b_0}\right] .$$

$$\frac{1}{N_t} = \boxed{\frac{T_c}{\Lambda_{\mathrm{L}}}} \times \left(\Lambda_{\mathrm{L}}\, a(\beta_{\mathrm{L}}^{c})\right) .$$

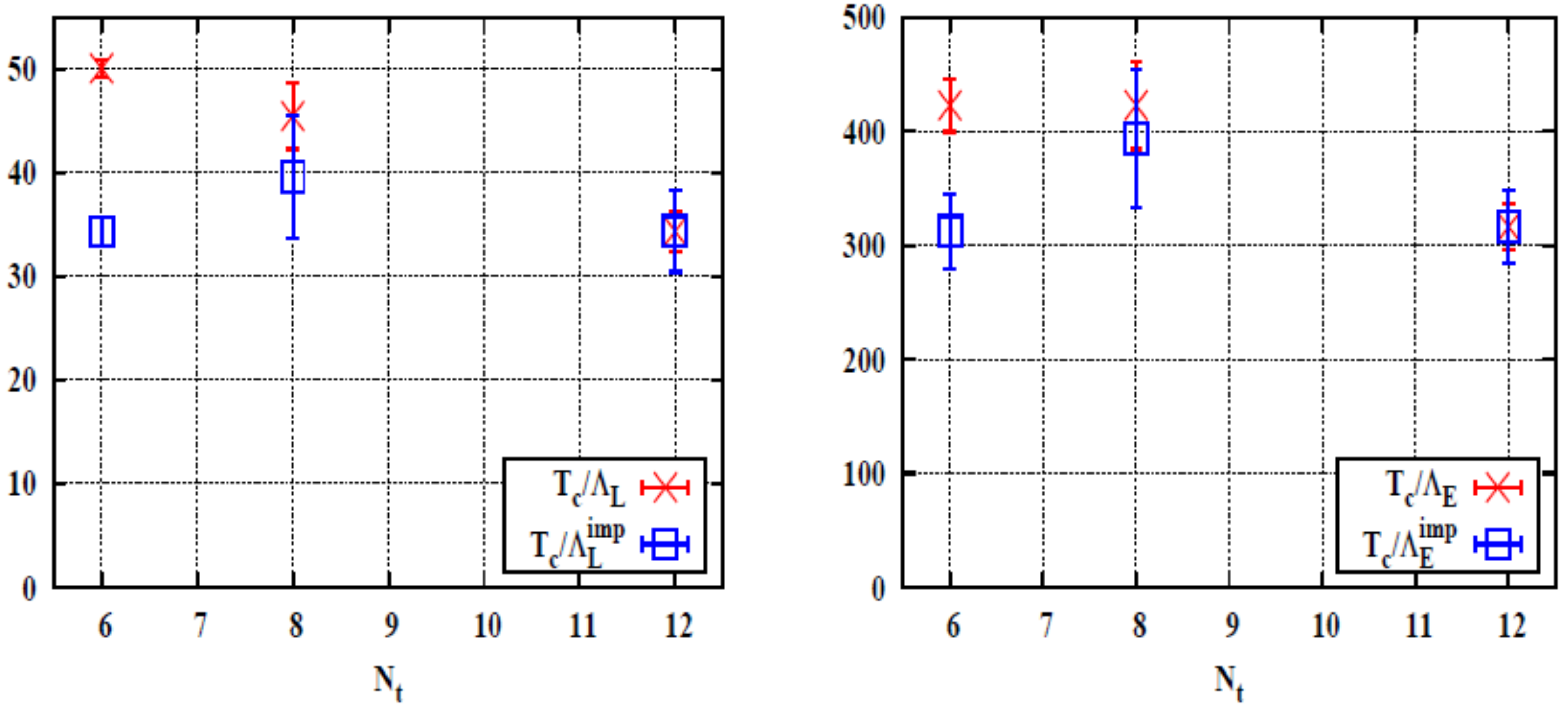
Must be approx. constant for several Nt

Nf = 6, asympt. scaling





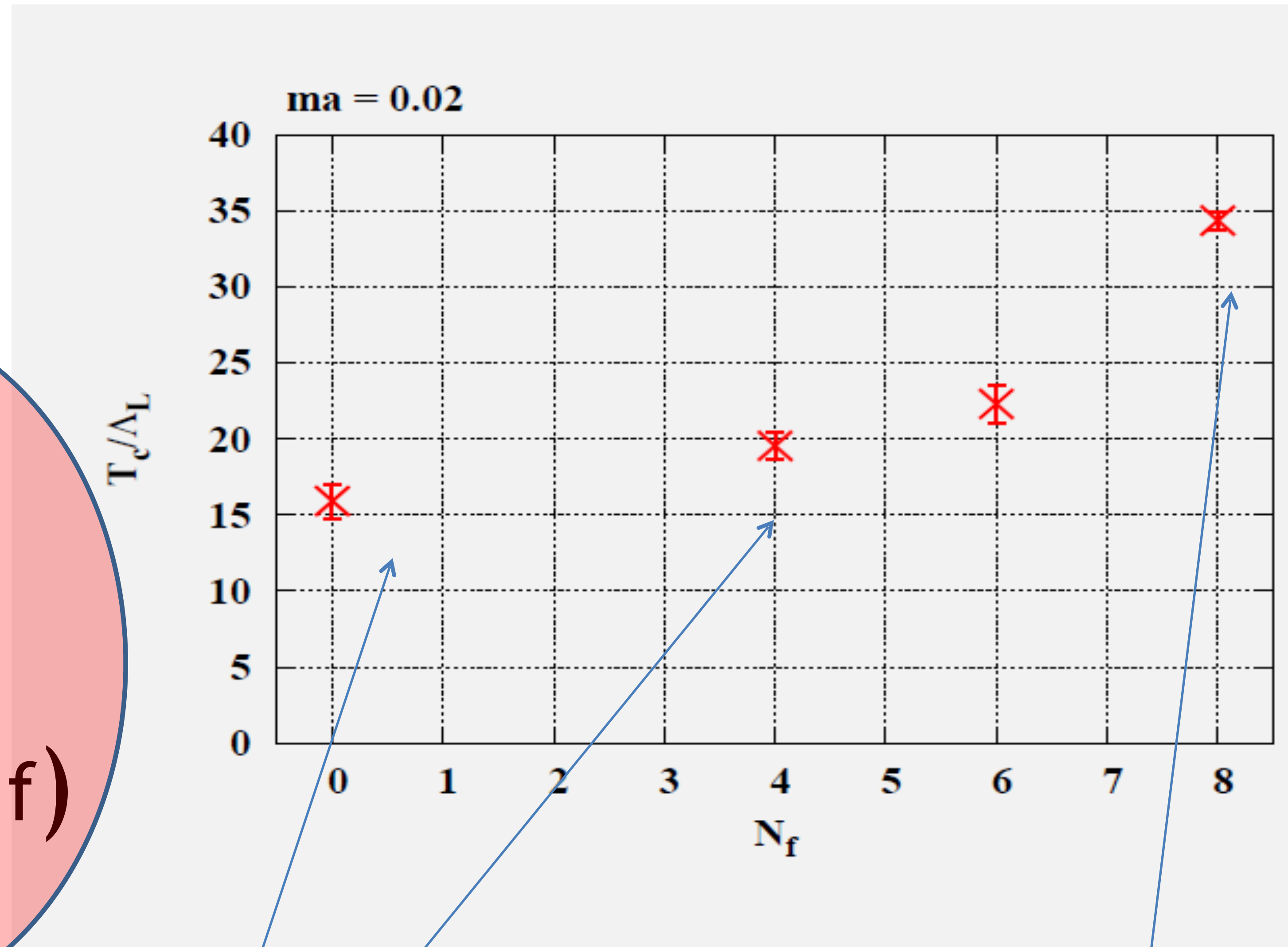
Nf = 8, asympt scaling



Tc/Λ as a function of Nf

Tc/Λ

Λ = Λ(Nf)

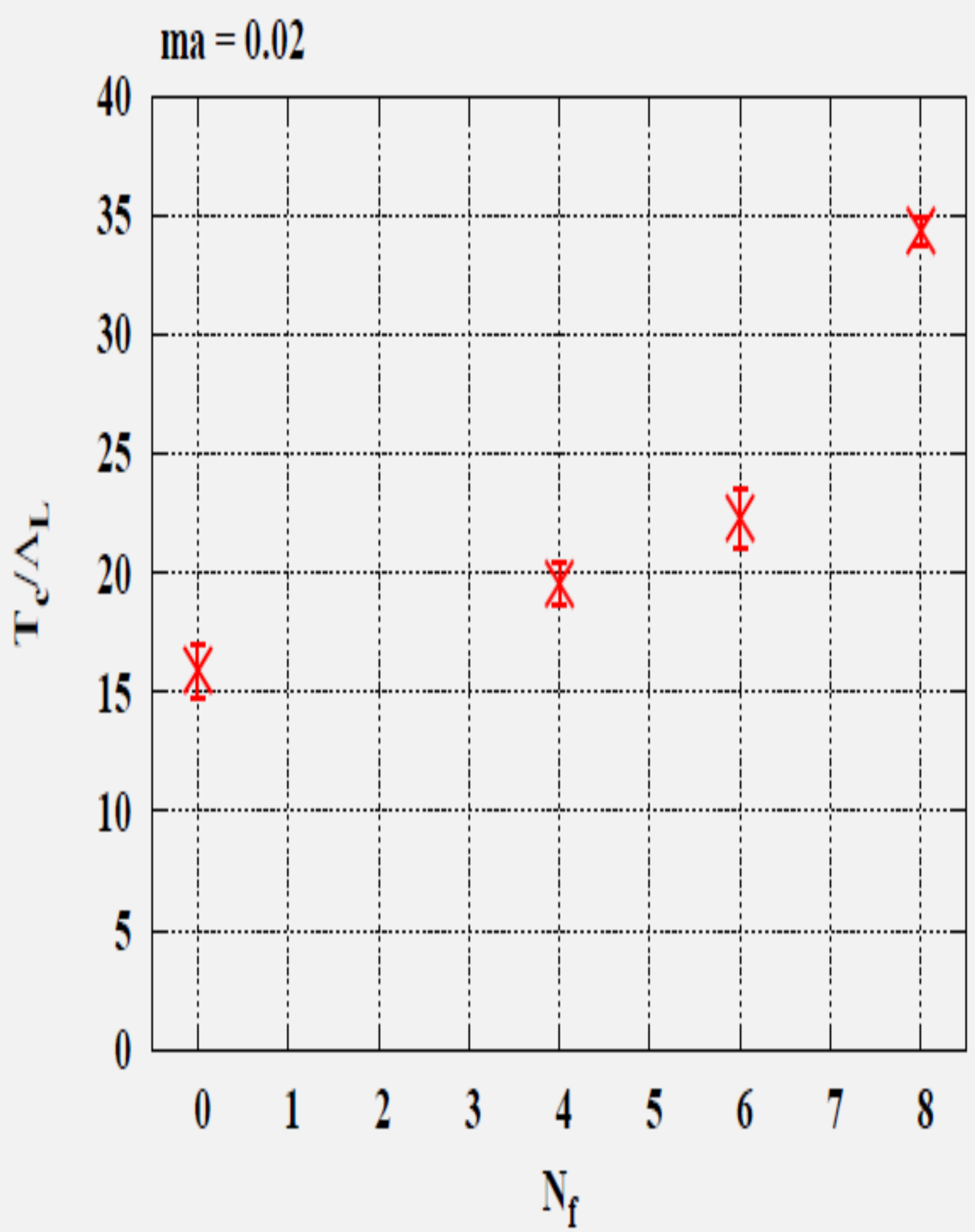


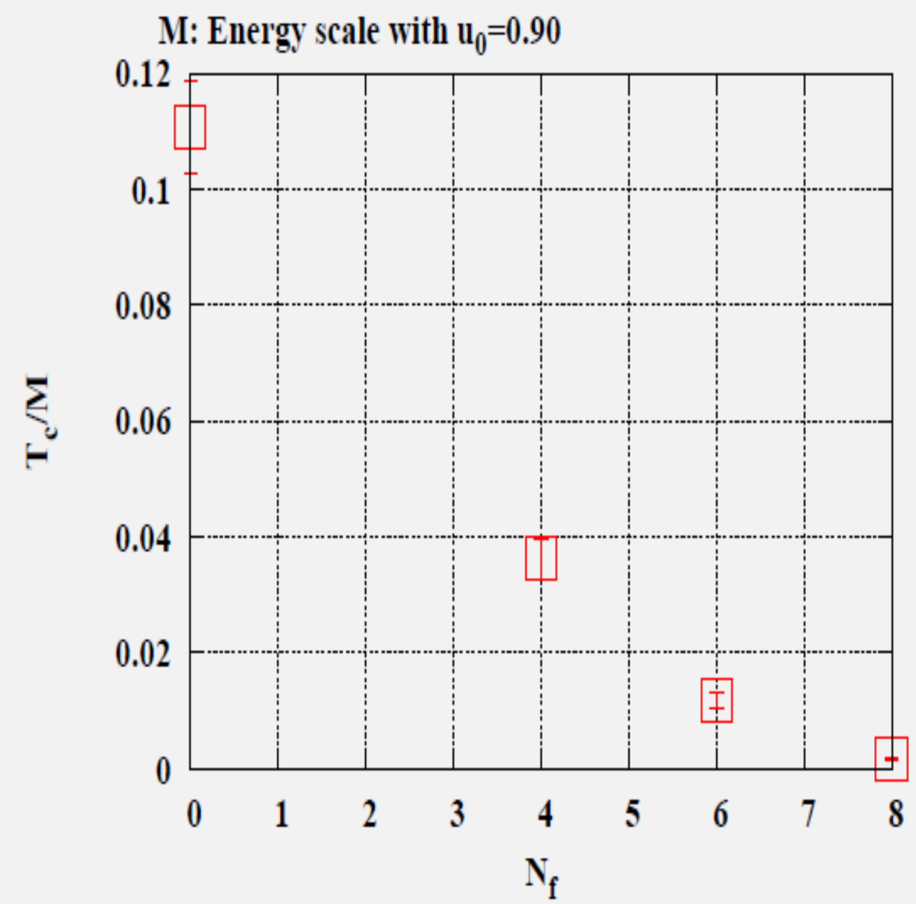
Scale separation ?

Nf

Conventional running

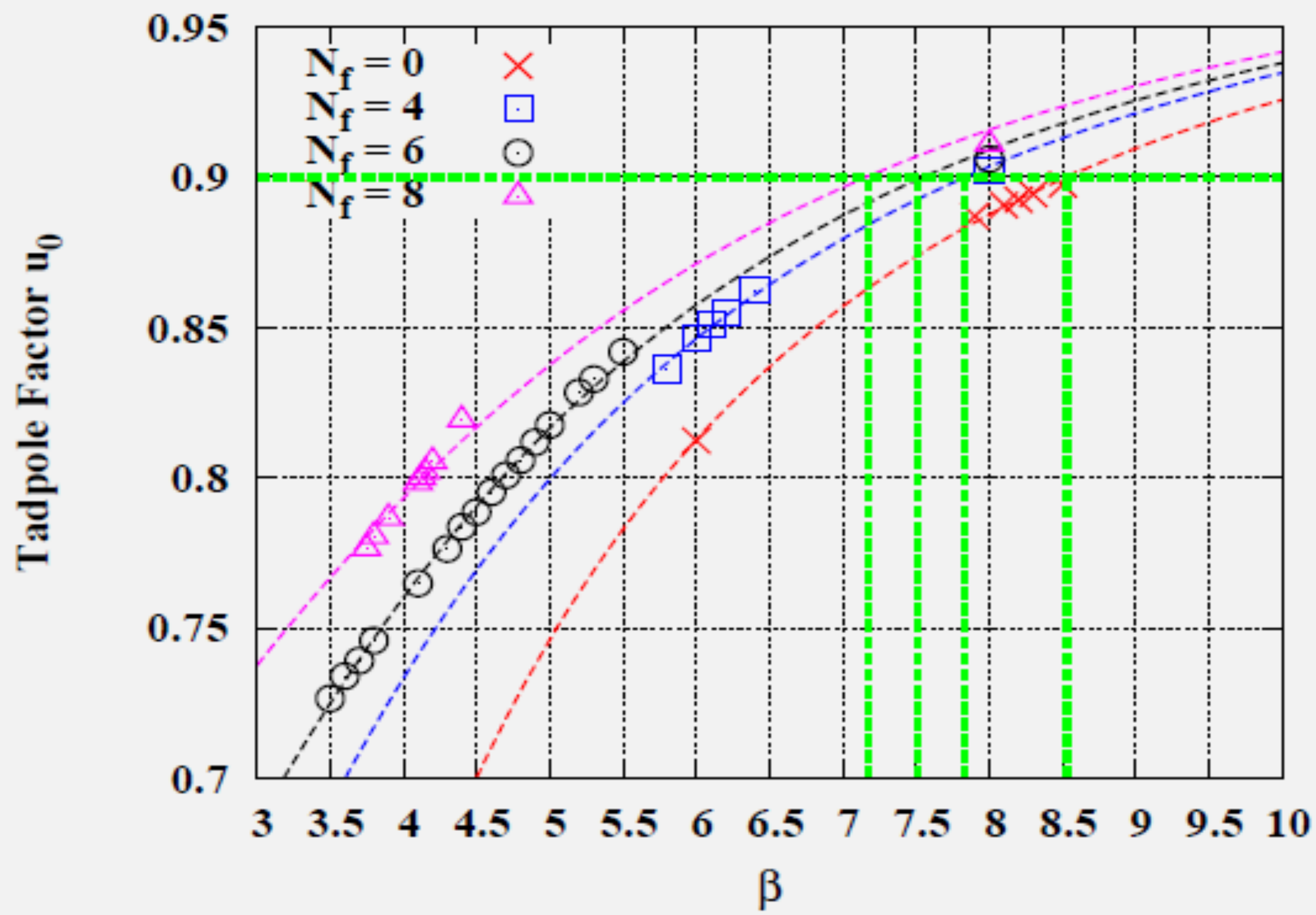
Solution: Λ = Λ (Nf) ; use UV scale





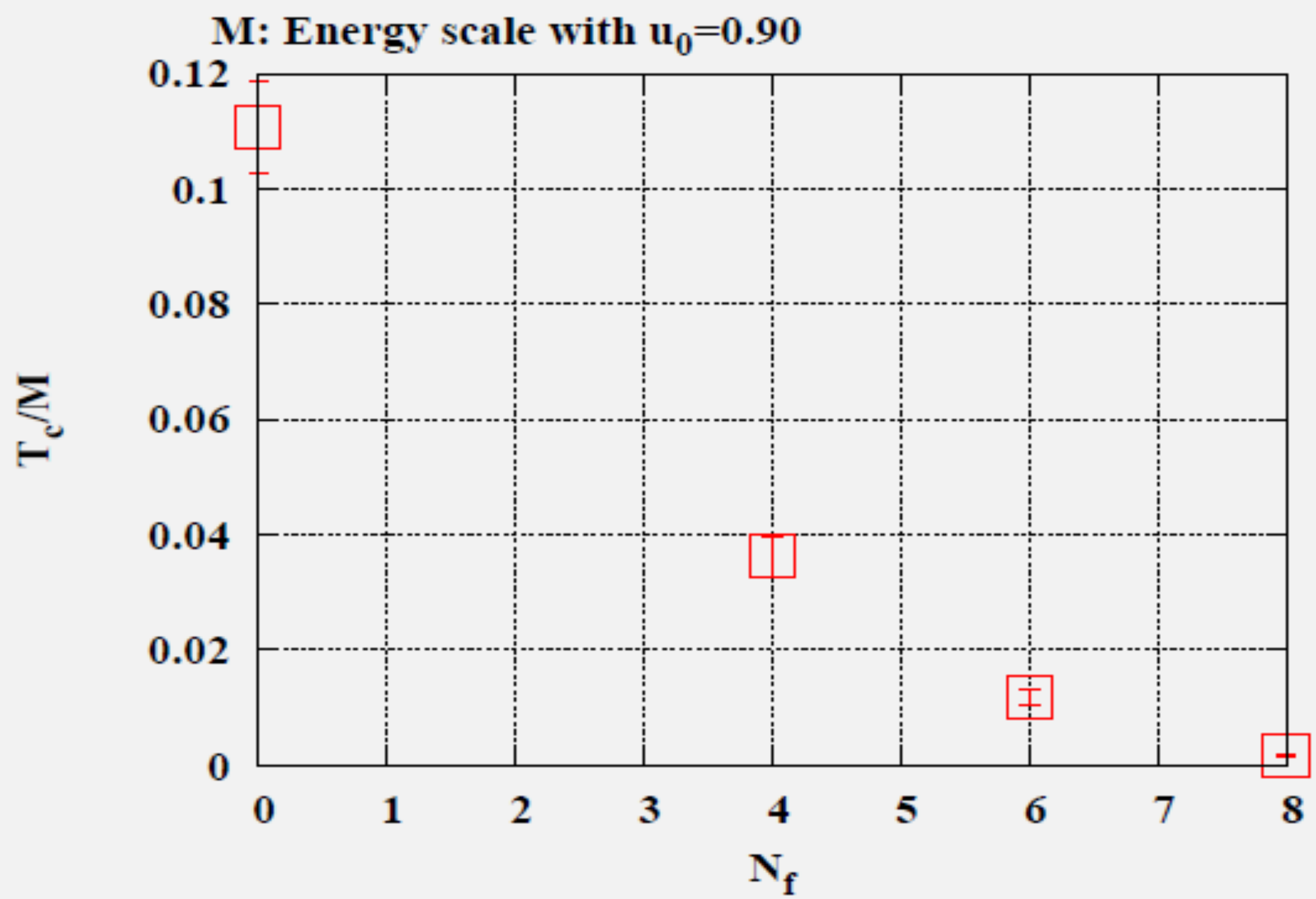
$$\frac{T_c}{M} = \frac{1}{N_t}\exp\left[\int_{g_{\rm ref}}^{g_c}\frac{dg}{B(g)}\right].$$

Fixing an UV scale



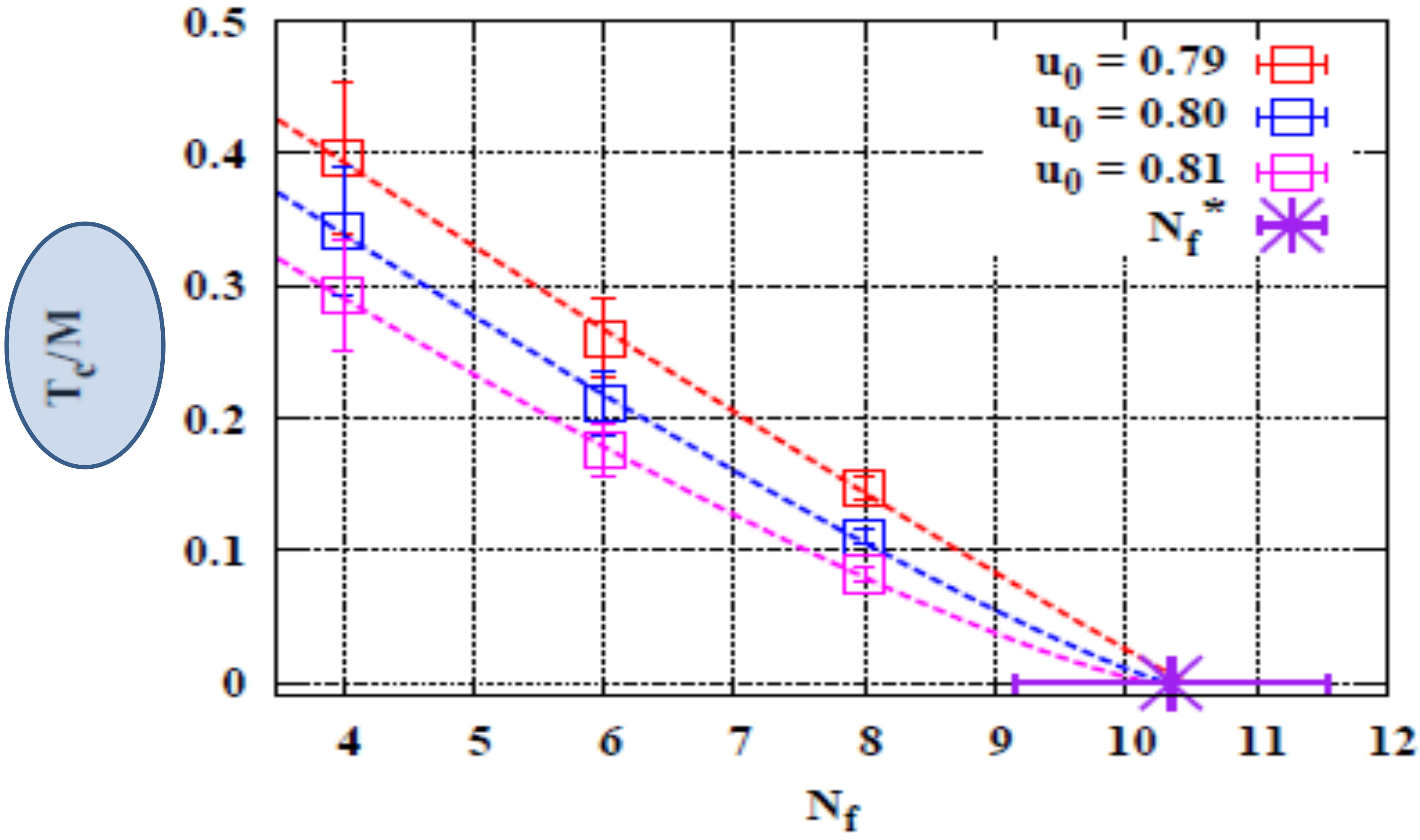
- We have measured the tadpole factosr $u_0 = \langle \Box \rangle^{1/4}$ at $T = 0$.
- We use the couplings obtained by the constant u_0 line to define a UV reference scale M.

Tc / M_{UV}



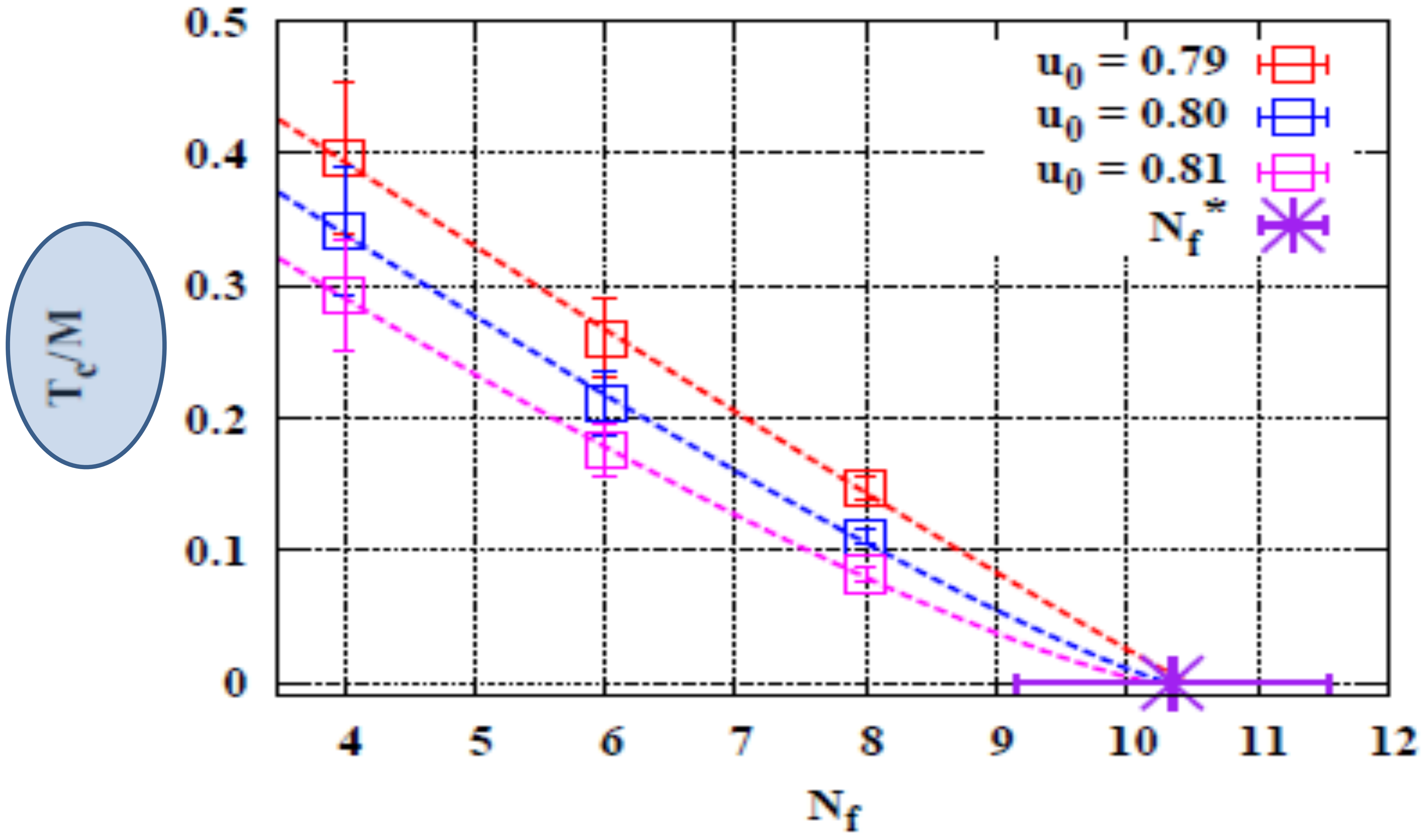
$$\frac{T_c}{M} = \frac{1}{N_t} \exp\left[\int_{g_{\rm ref}}^{g_c} \frac{dg}{B(g)}\right] .$$

Tc/M extrapolates to zero for Nf ~ 10.5*



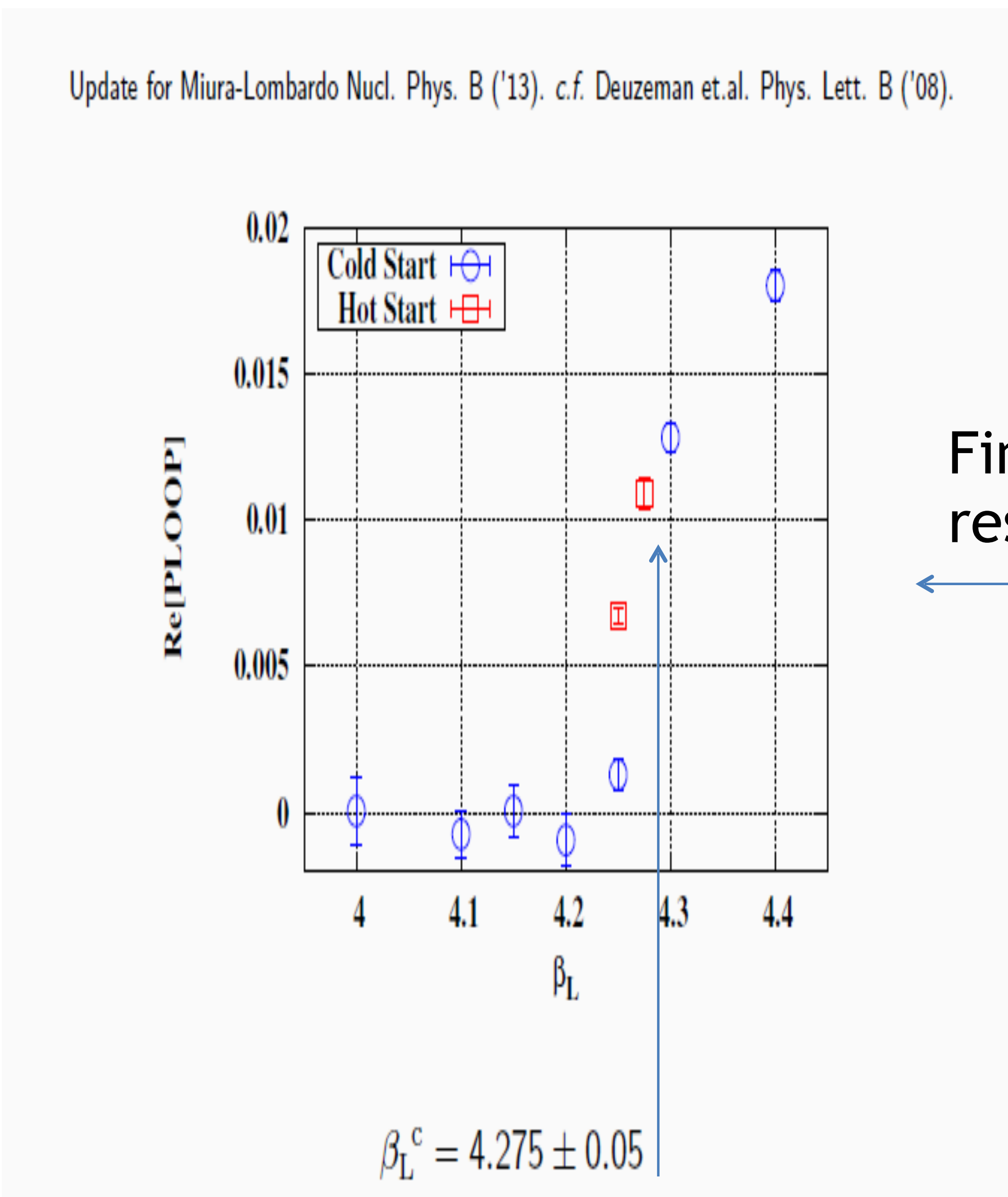
Tc/M extrapolates to zero for Nf ~ 10.5*

M fixed with the help of perturbation theory

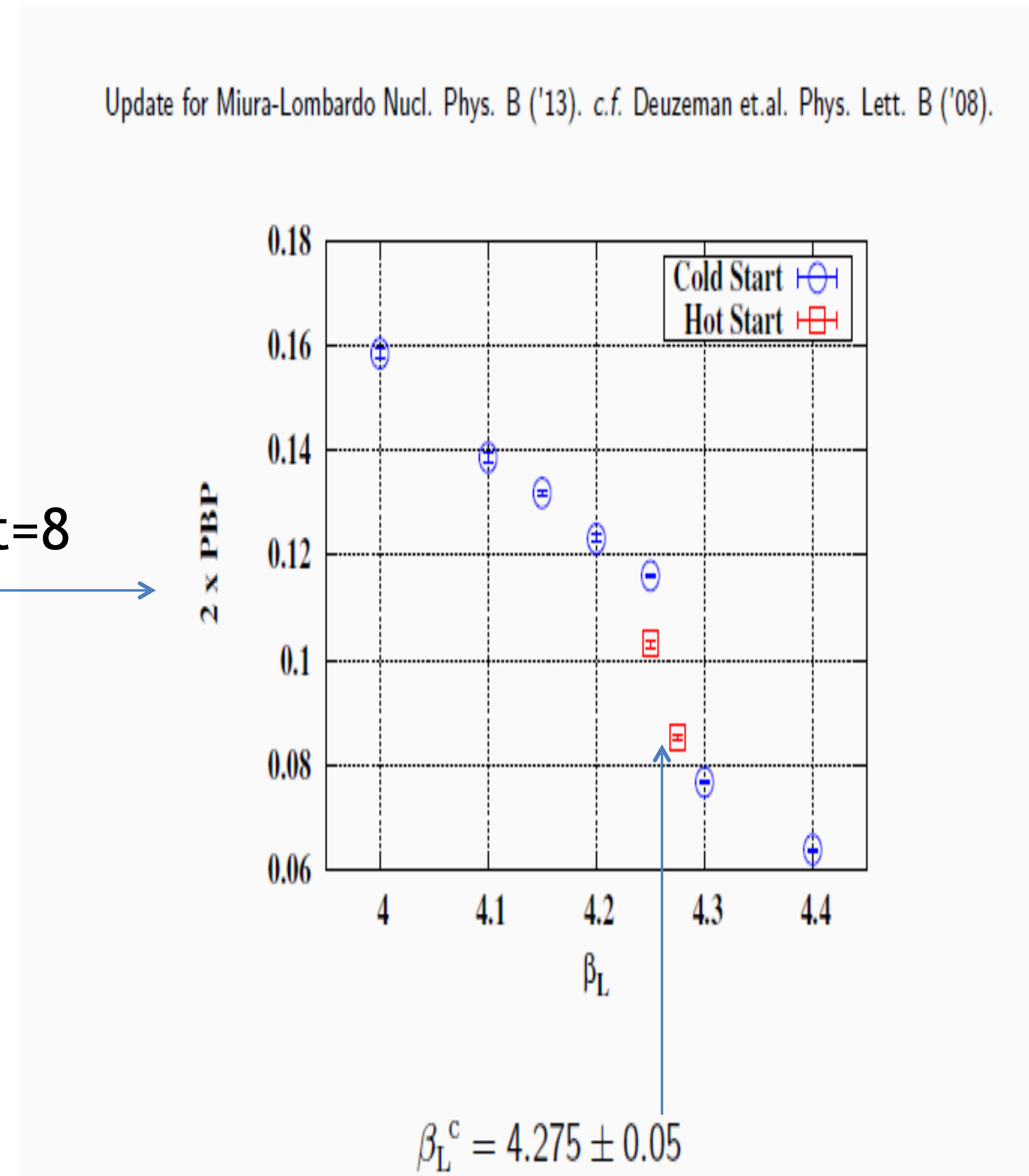


String tension and w0

Lattice setup: β for Nf=8

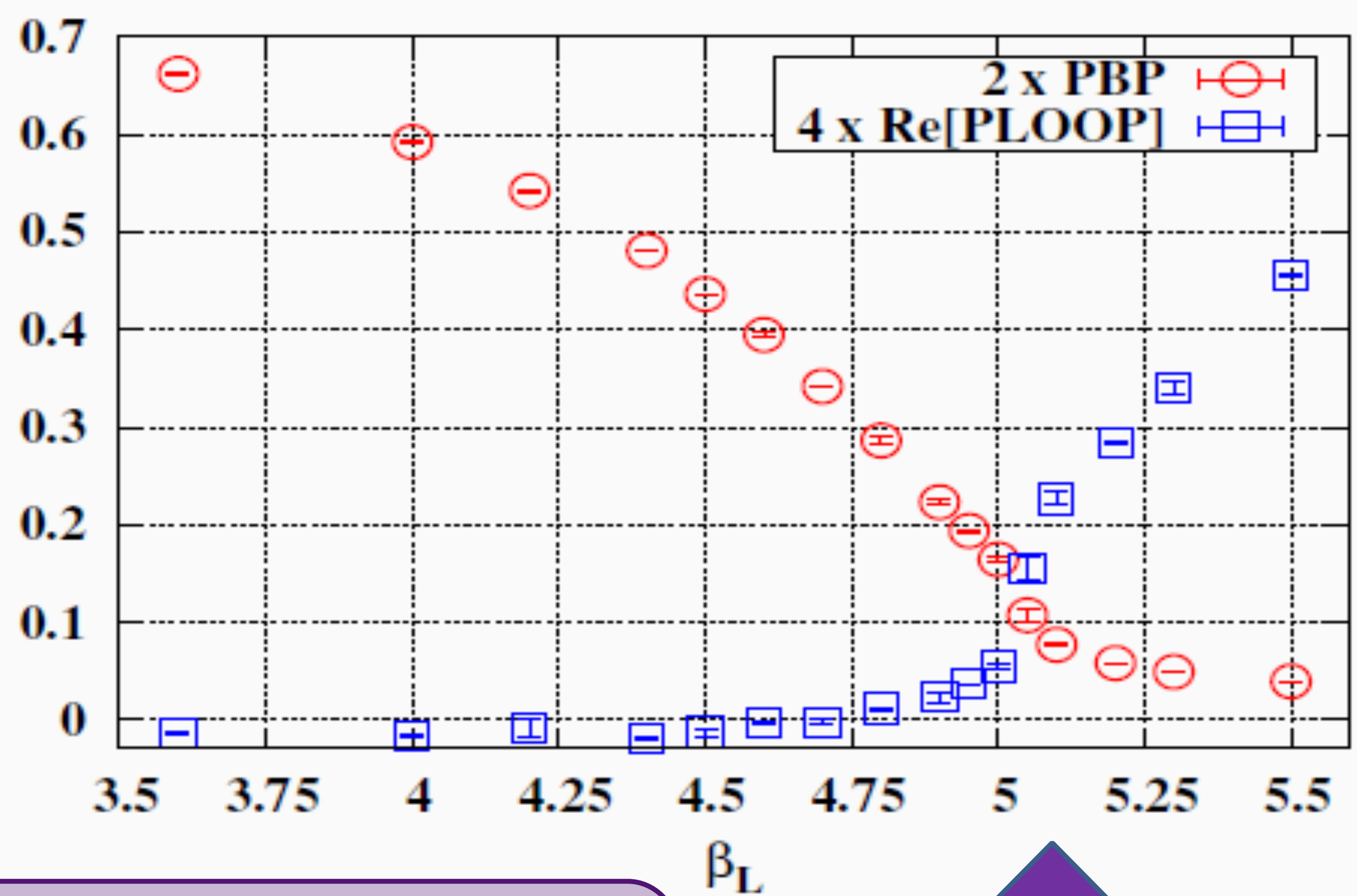


Finite T results Nt=8



Choice for the T=0 simulation

Lattice setup: β for Nf=6



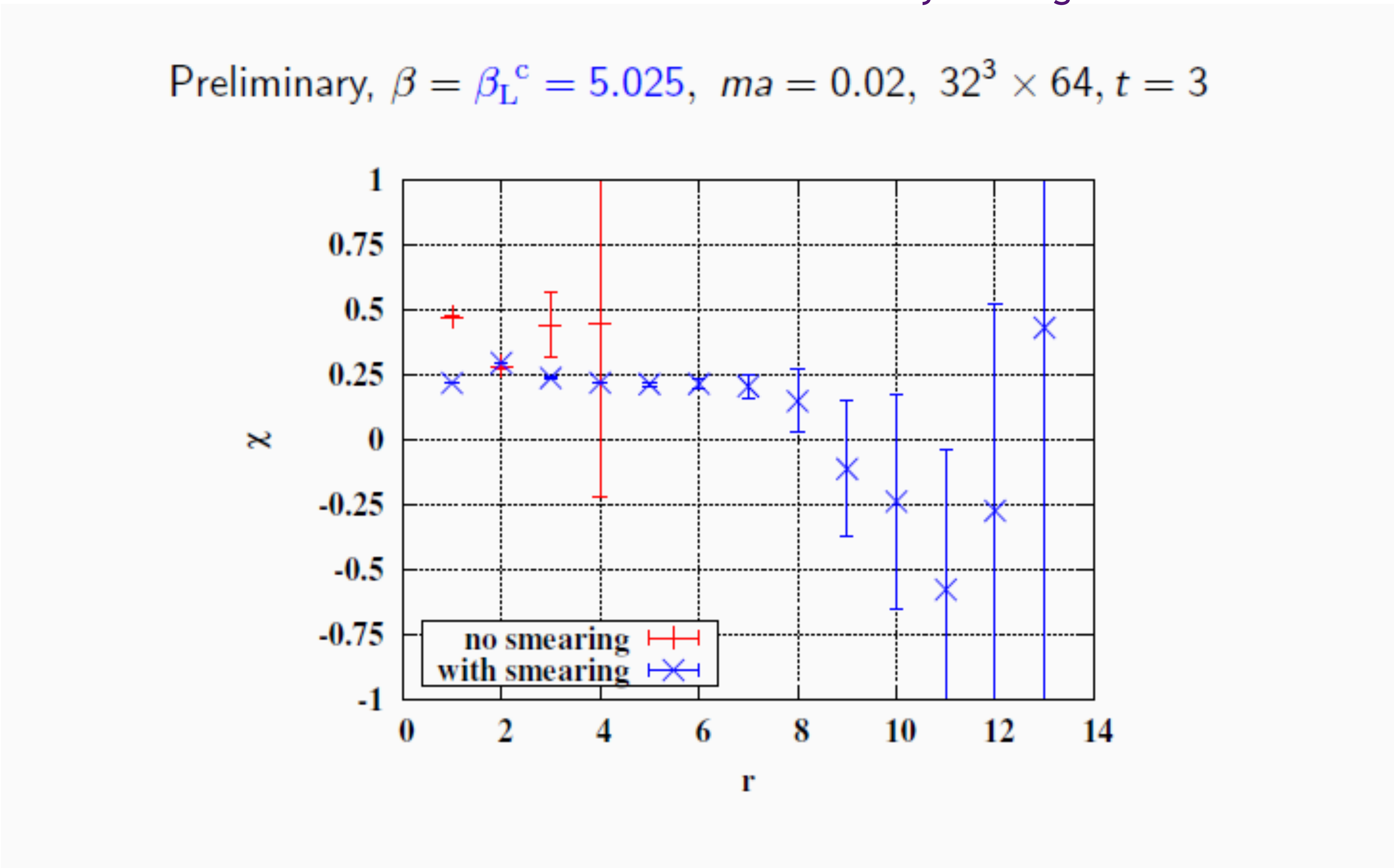
Finite temperature , Nt=6 results

βc = 5.025: Choice for the T=0 simulations

And analogously for Nt=8

Nf=6: Creutz ratios

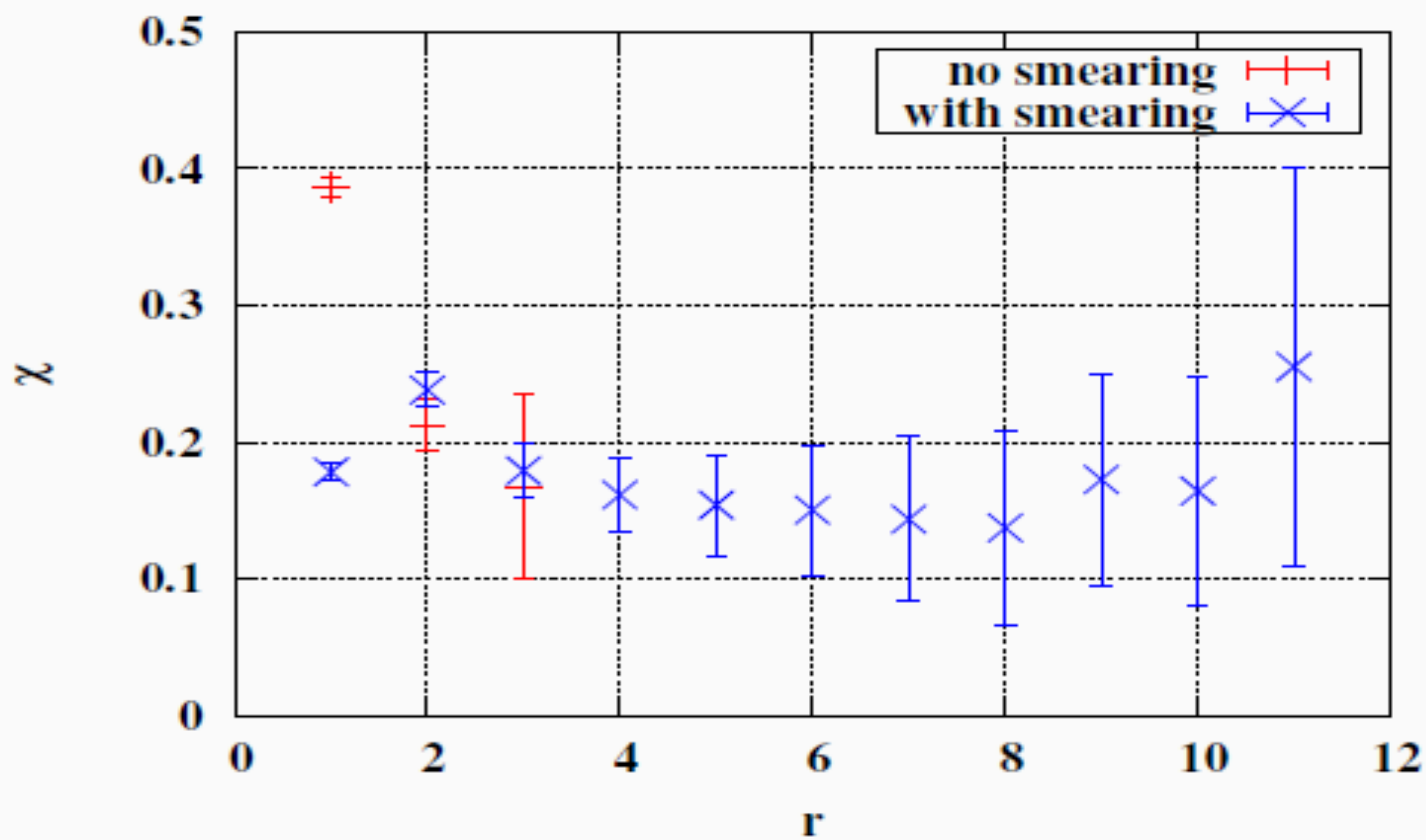
Measurements' code by M. Wagner and collaborators



Nf=8: Creutz ratios

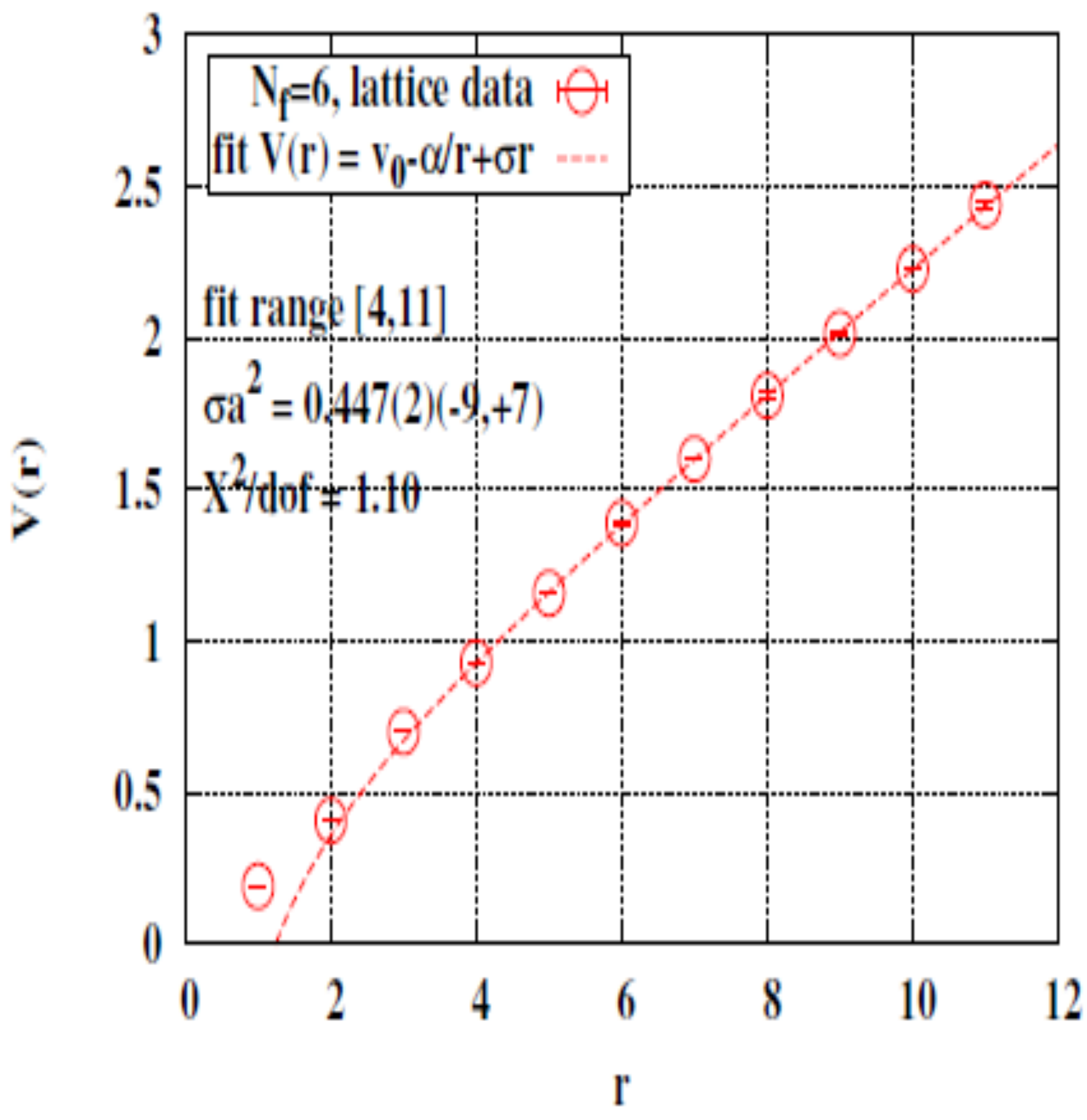
Measurements' code by M. Wagner and collaborators

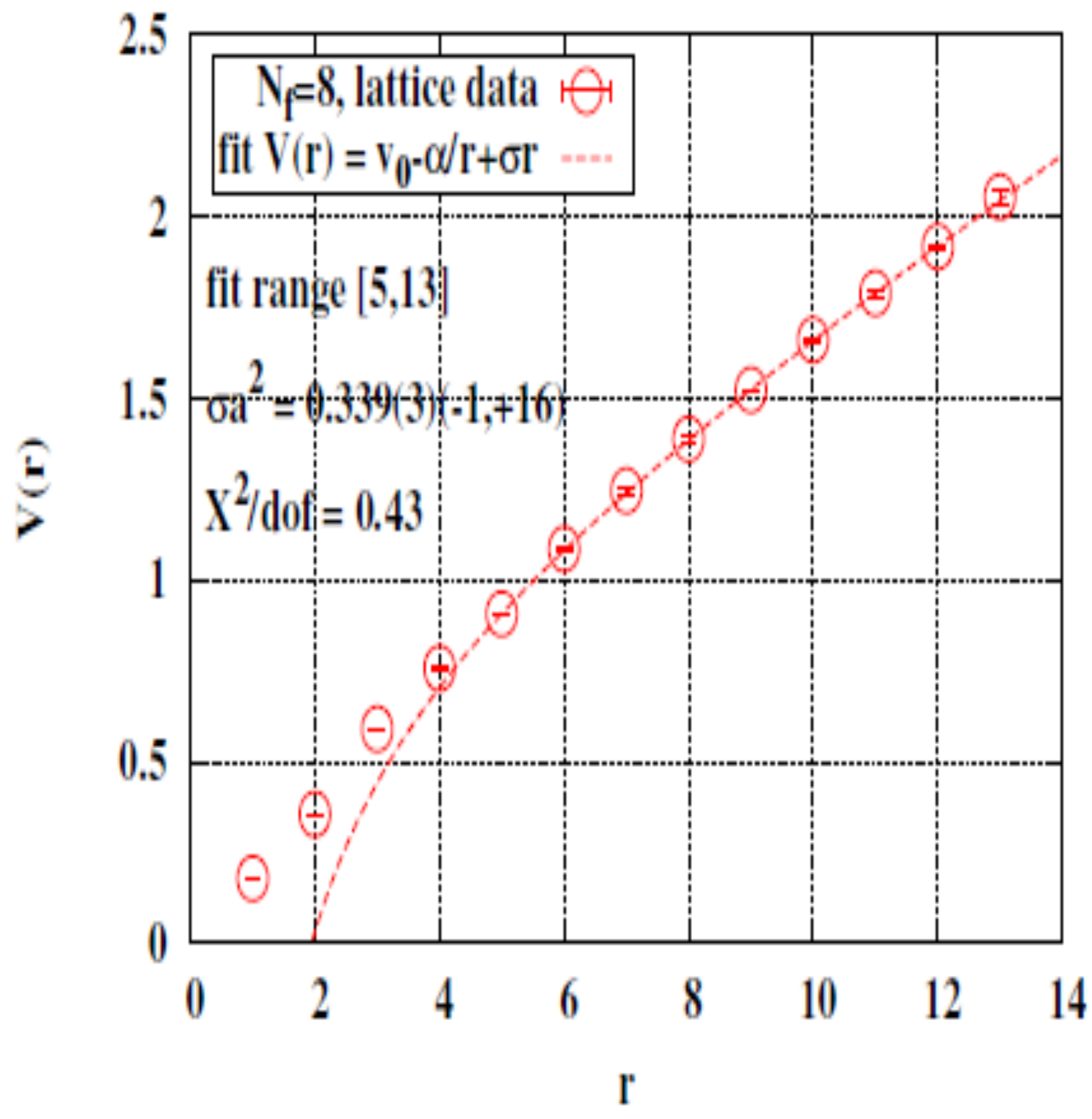
Preliminary, $\beta = \beta_L{}^c = 4.275$, $ma = 0.02$, $32^3 \times 64, t = 3$



$$\chi_{r,t} = -\log\left[\frac{W_{r,t}W_{r+1,t+1}}{W_{r,t+1}W_{r+1,t}}\right] = \frac{\alpha}{\hat{r}(\hat{r}+1)} + \hat{\sigma}\ .$$

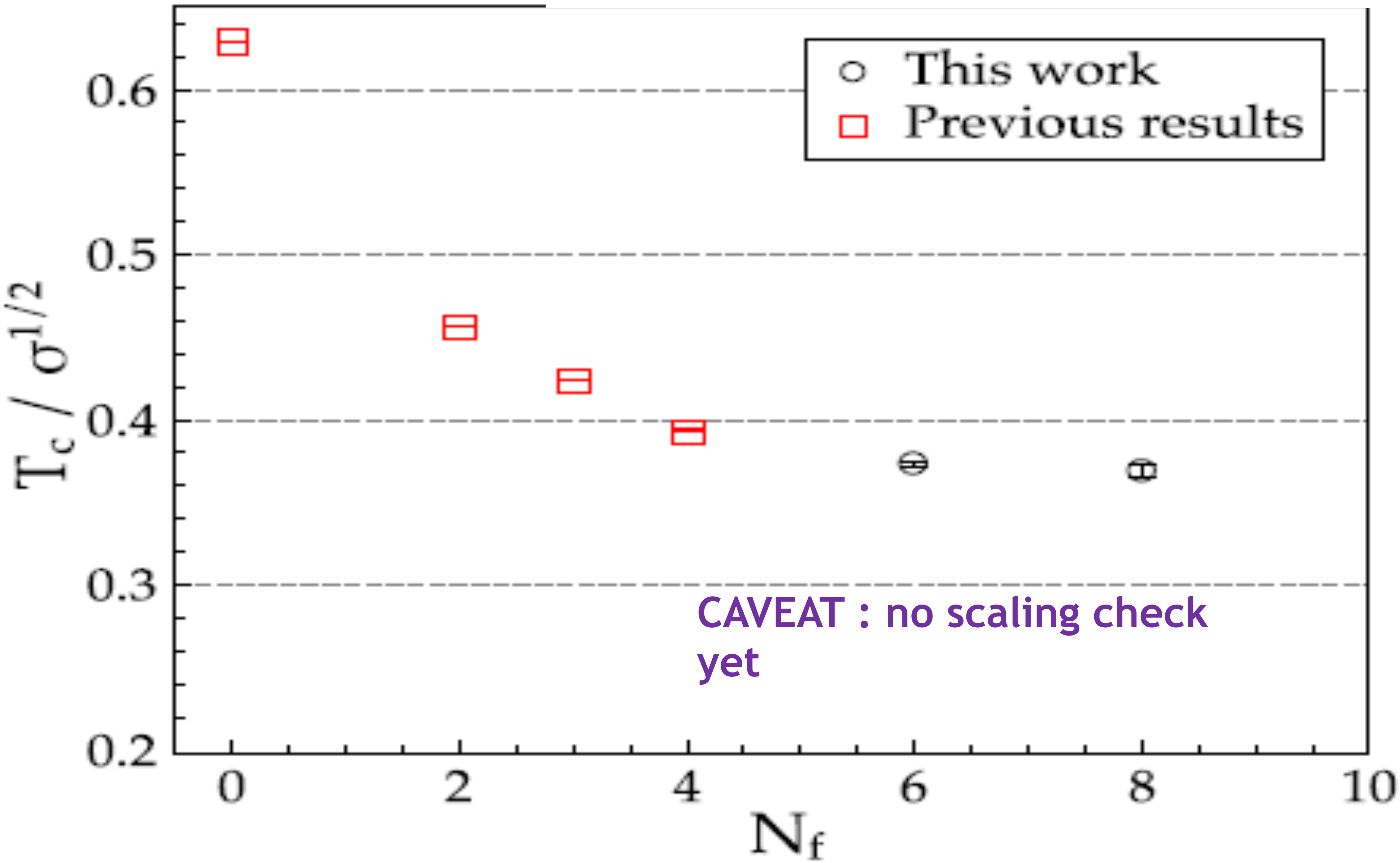
Heavy Quark Potential





$T_c/\sqrt{\sigma}$

$$T_c/\sqrt{\sigma} = \begin{cases} 0.373(2)(+5,-6), & N_f = 6, \beta = 5.025, \\ 0.369(4)(+1,-5), & N_f = 8, \beta = 4.275. \end{cases}$$



CAVEAT : no scaling check yet

Wilson flow

$$\mathcal{E}(t) = t^2 \langle E(x,t) \rangle \,, \quad E(x,t) \equiv -\frac{1}{2} \mathrm{tr}\, G_{\mu\nu}(x,t) G_{\mu\nu}(x,t)$$

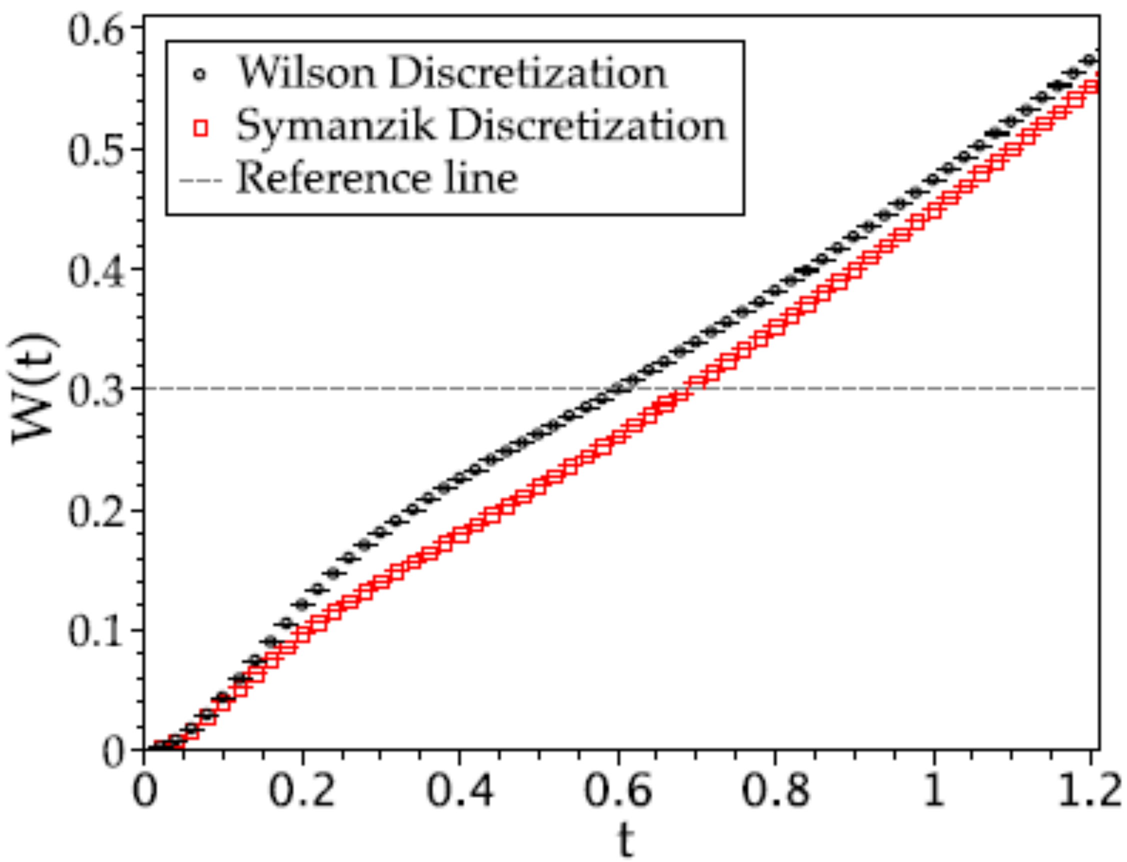
$$w_0 \;:\; w_0^2 \mathcal{E}'(w_0^2) = 0.3$$

✓Computationally easy

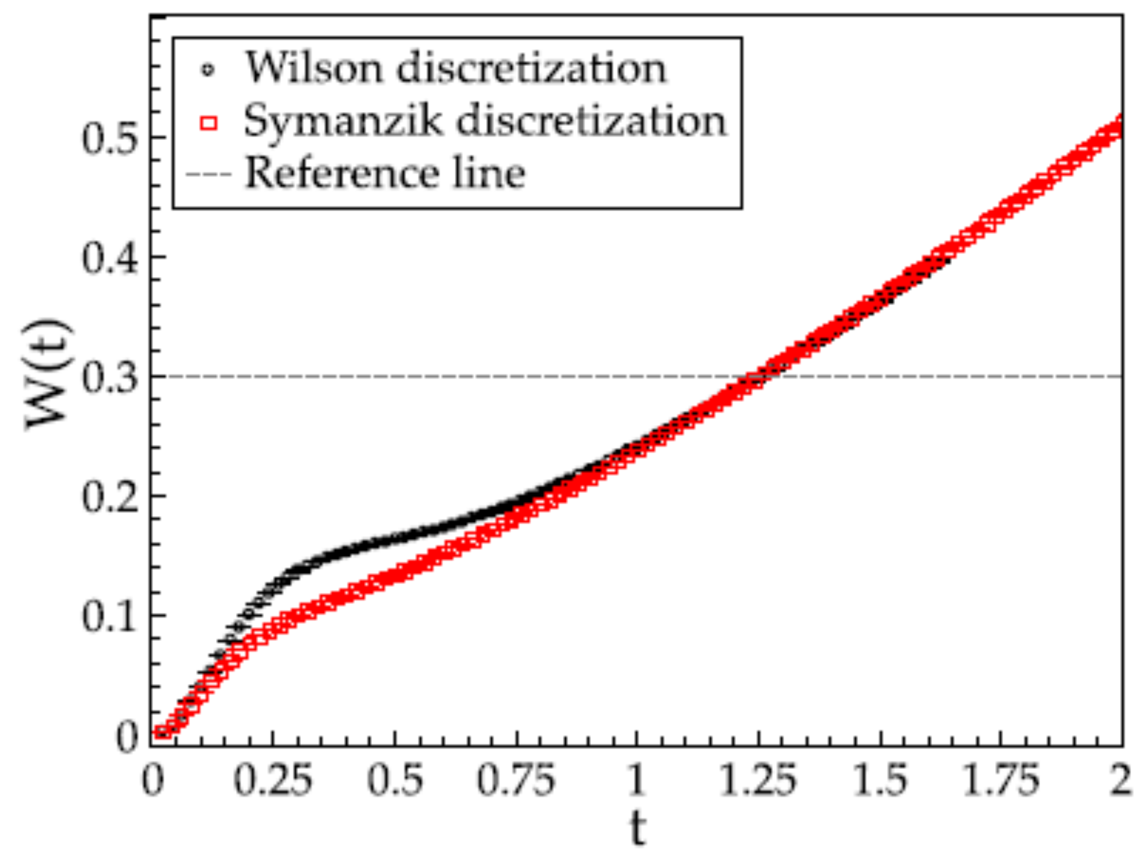
✓Naturally smooth

✓Well behaved at short distance

Scale from the flow, Nf=6

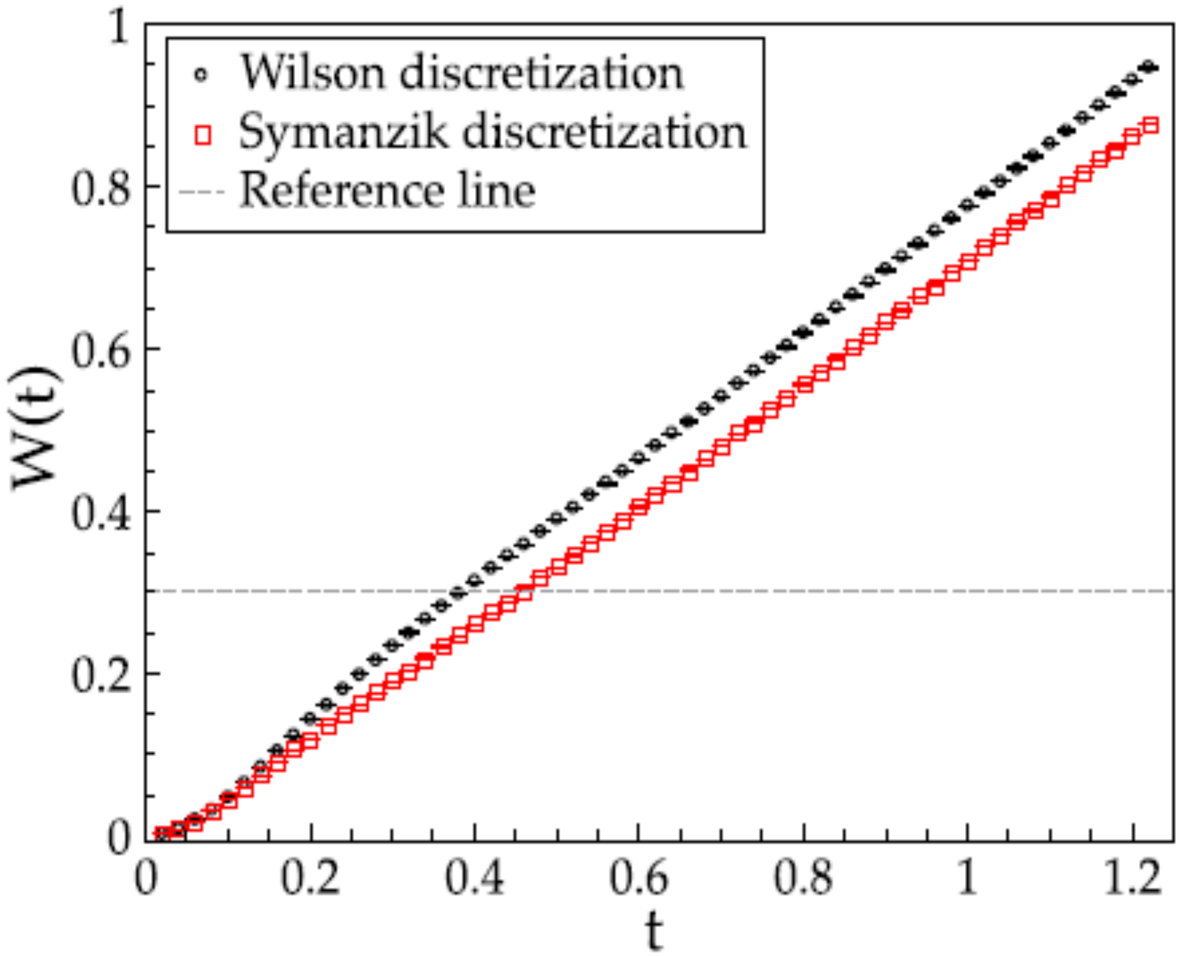


$\beta = 5.025$

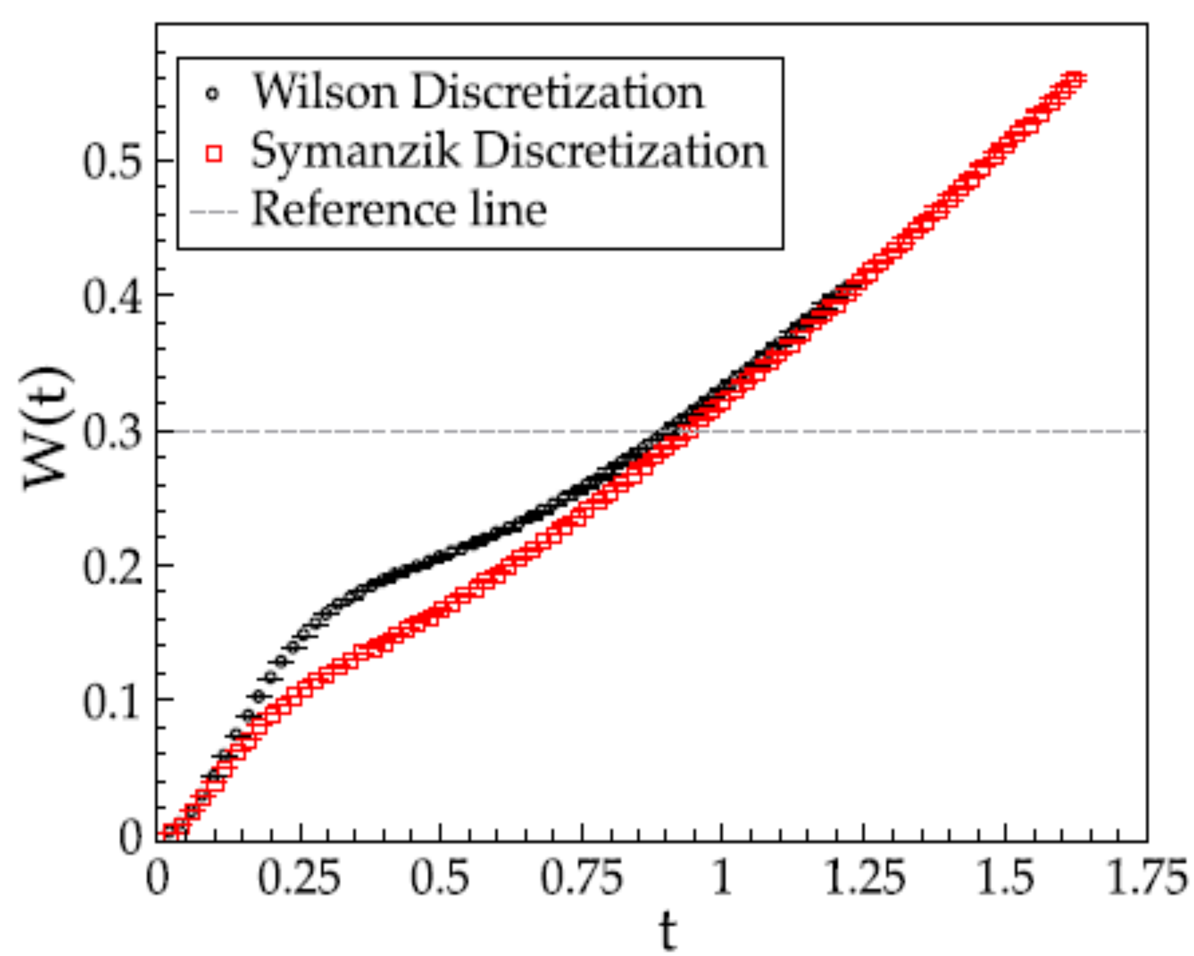


$,\beta = 5.200$

Scale from the flow, Nf=8

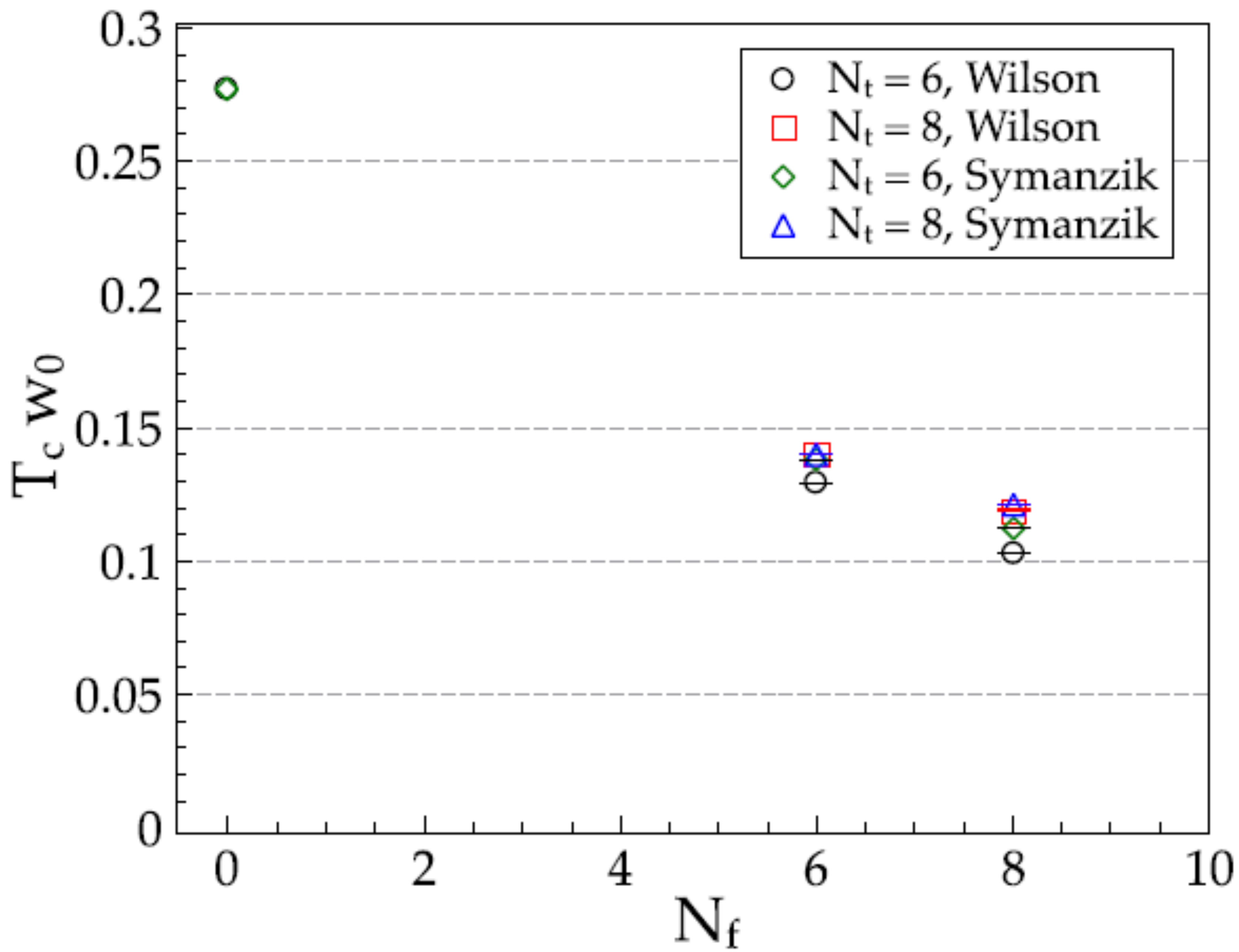


$,\beta = 4.1125$

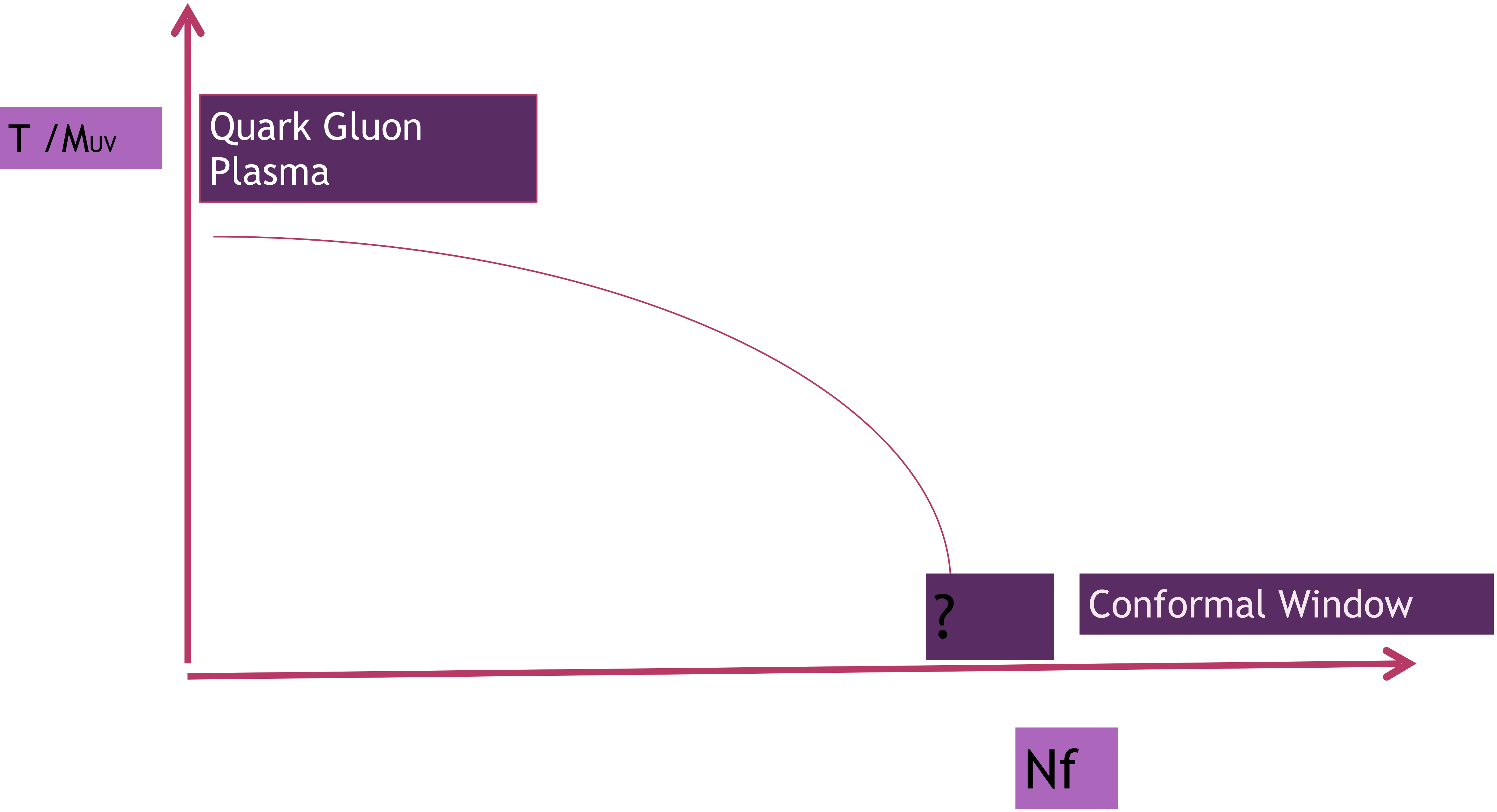


$,\beta = 4.275$

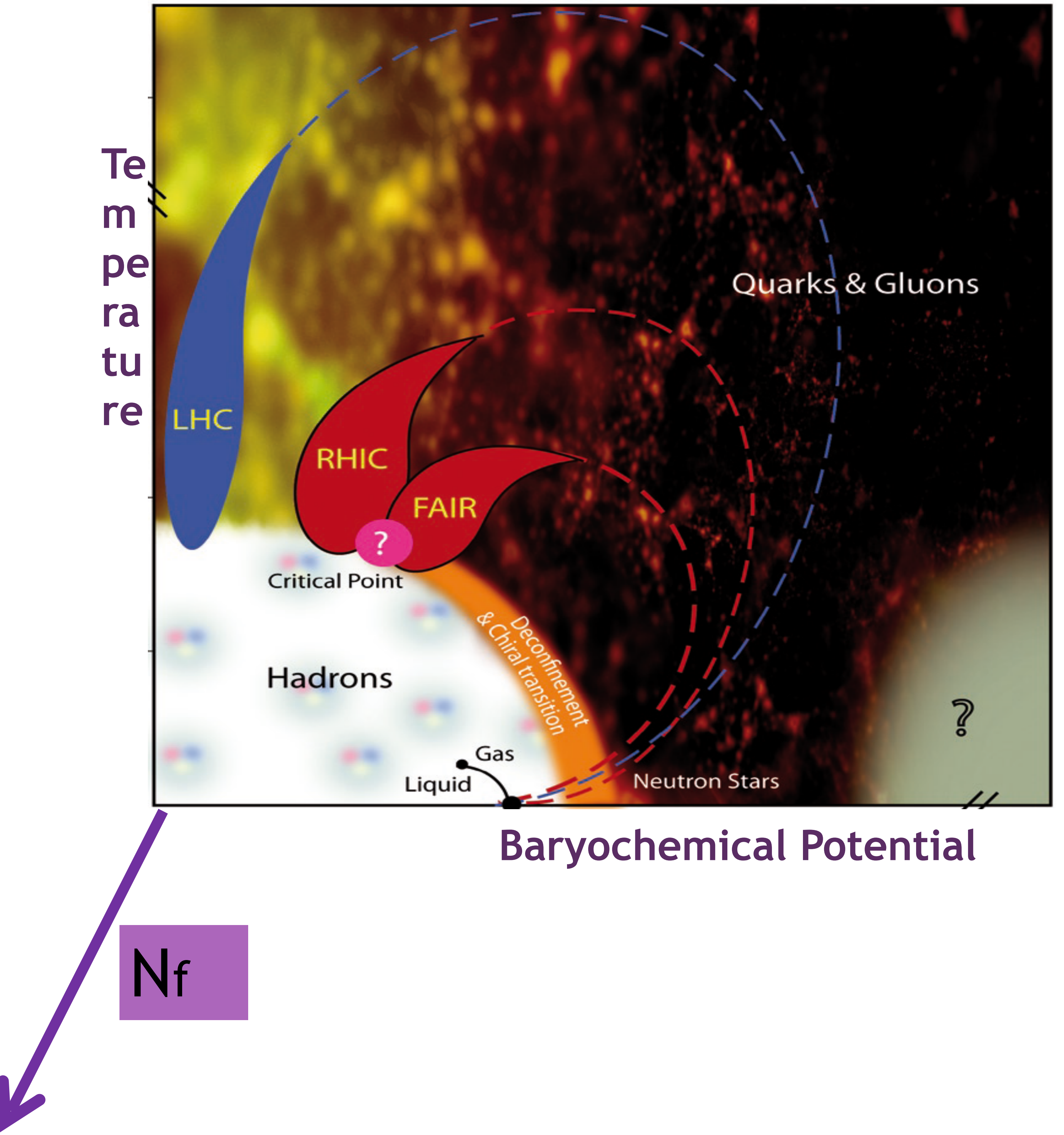
Results for Tc on the 1/w0 scale



Preconformality as a tool for the QGP


T / Muv
Quark Gluon
Plasma
?
Conformal Window
Nf

Phases of Strong Interactions with **many flavors**



Conformal/scale (quasi)invariance ubiquitous

Approach to the free field

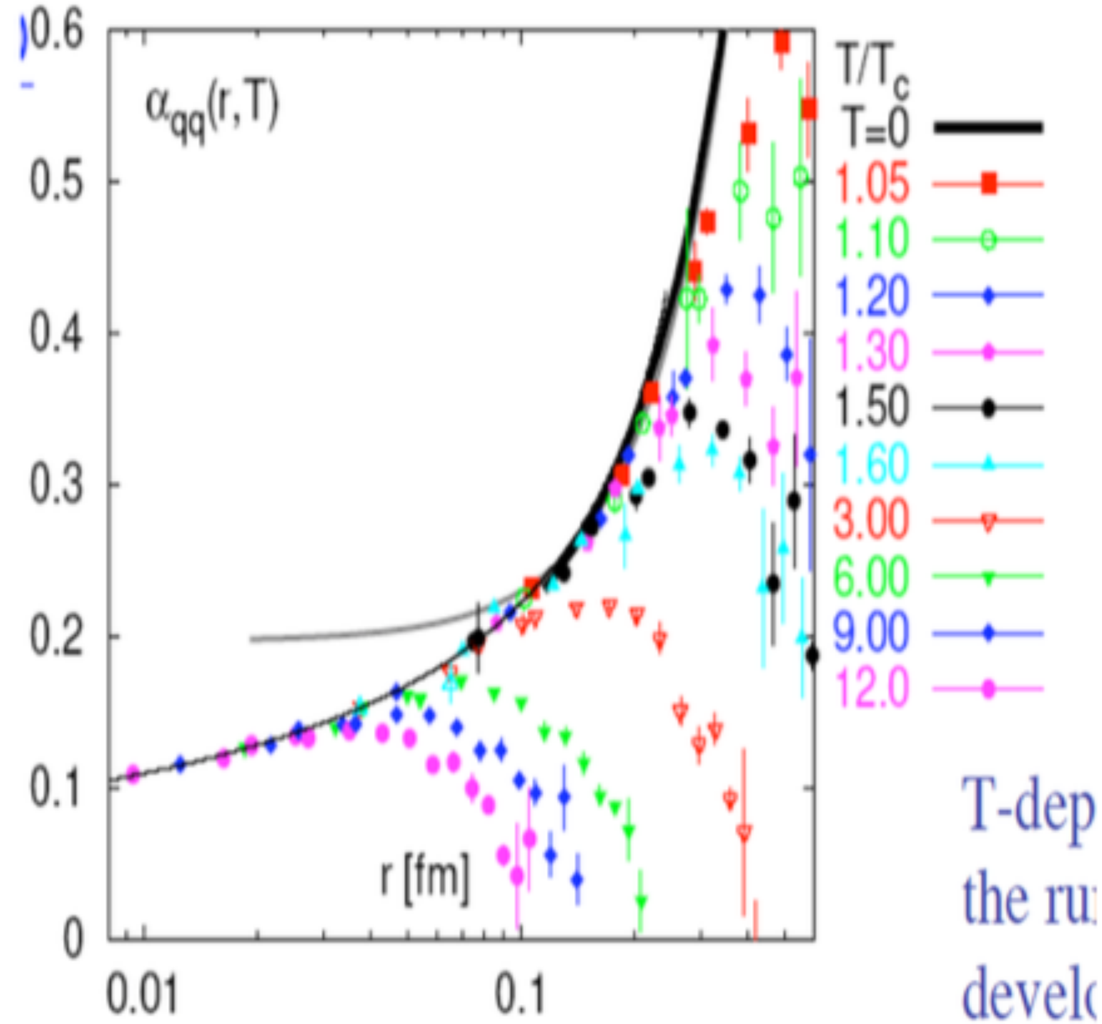
Bulk viscosity set to zero

AdS/CFT methods

Speed of sound close to 0.3

Coupling slowly running :

Hints of conformality also at Strong coupling



Karsch Zantov Kaczmarek 2006

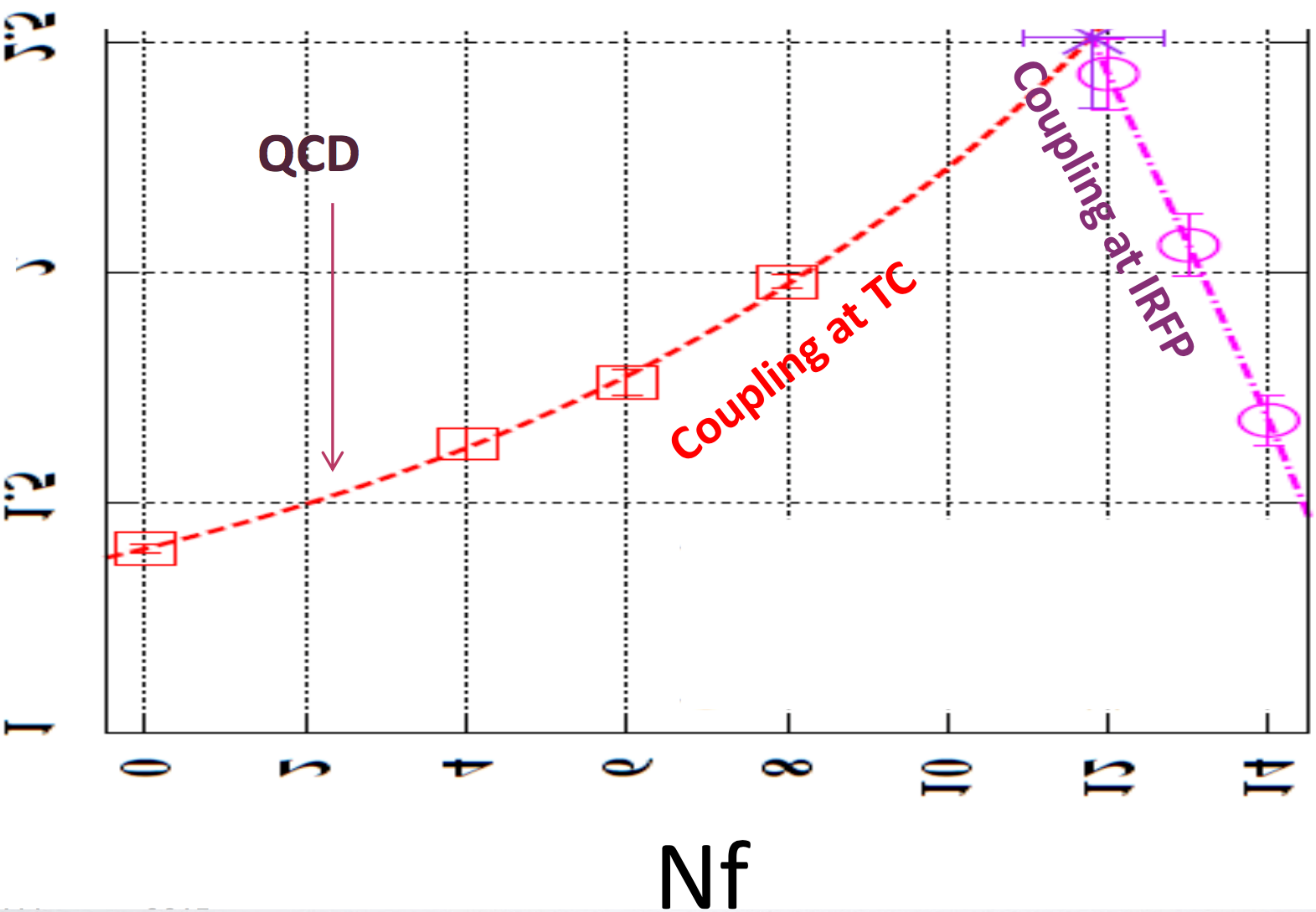
Strength of the QGP at Tc and IRFP

We consider the critical coupling at the temperature scale 1/Nt [**as proposed by Shuryak et al.]**

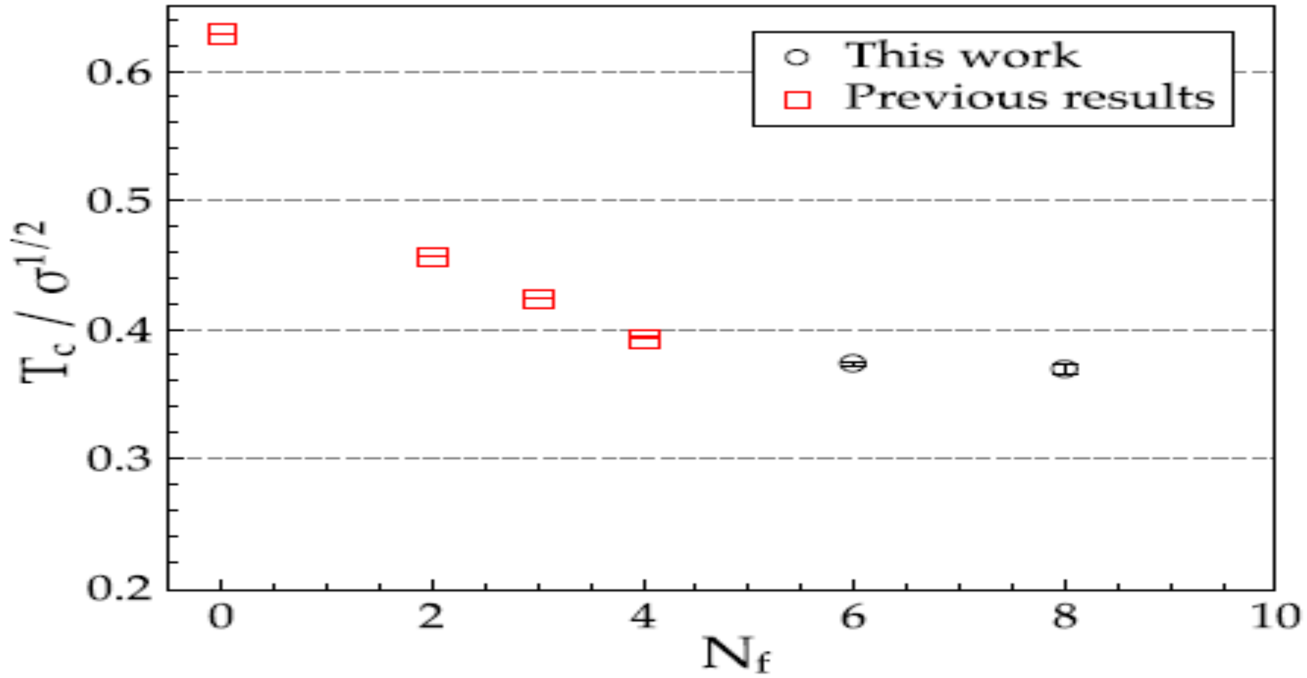
$$R(g_L^c, g_T^c) = 1/N_t\,,$$

$$\bar{R}(g_L{}^c, g_L{}^{\rm ref}) \equiv \frac{M(g_L{}^{\rm ref})}{a^{-1}(g_L{}^c)} = \exp\left[\int_{g_L{}^c}^{g_L{}^{\rm ref}} \frac{dg_L}{\beta(g_L)}\right]$$
$$\simeq \left(\frac{(g_L{}^c)^2}{(g_L{}^c)^2 b_1 + b_0}\frac{(g_L{}^{\rm ref})^2 b_1 + b_0}{(g_L{}^{\rm ref})^2}\right)^{-b_1/(2b_0^2)}$$
$$\times \exp\left[\frac{1}{2b_0}\left(\frac{1}{(g_L{}^{\rm ref})^2} - \frac{1}{(g_L{}^c)^2}\right)\right],$$

$$M(g_L{}^{\rm ref}) = 1/N_t\, a(g_L^c).$$


QCD
Coupling at TC
Coupling at IRFP
2.5
2
1.5
1
0
2
4
6
8
10
12
14
Nf

Scale separation

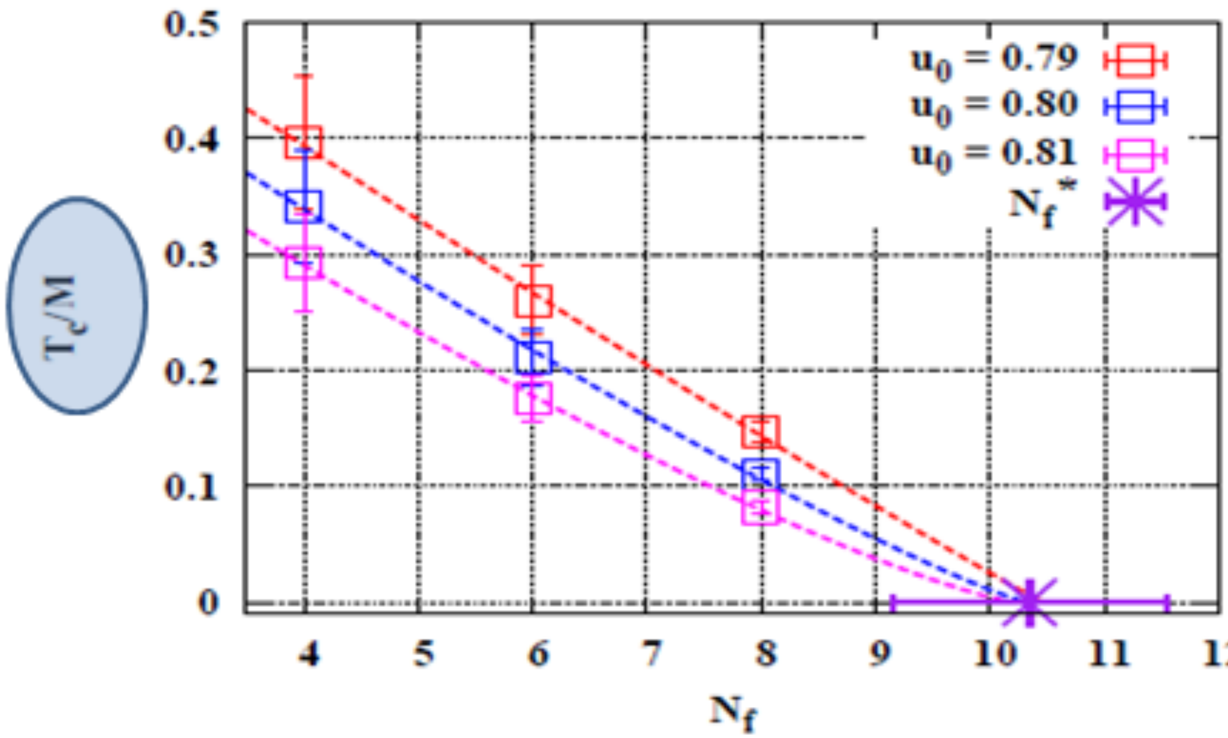


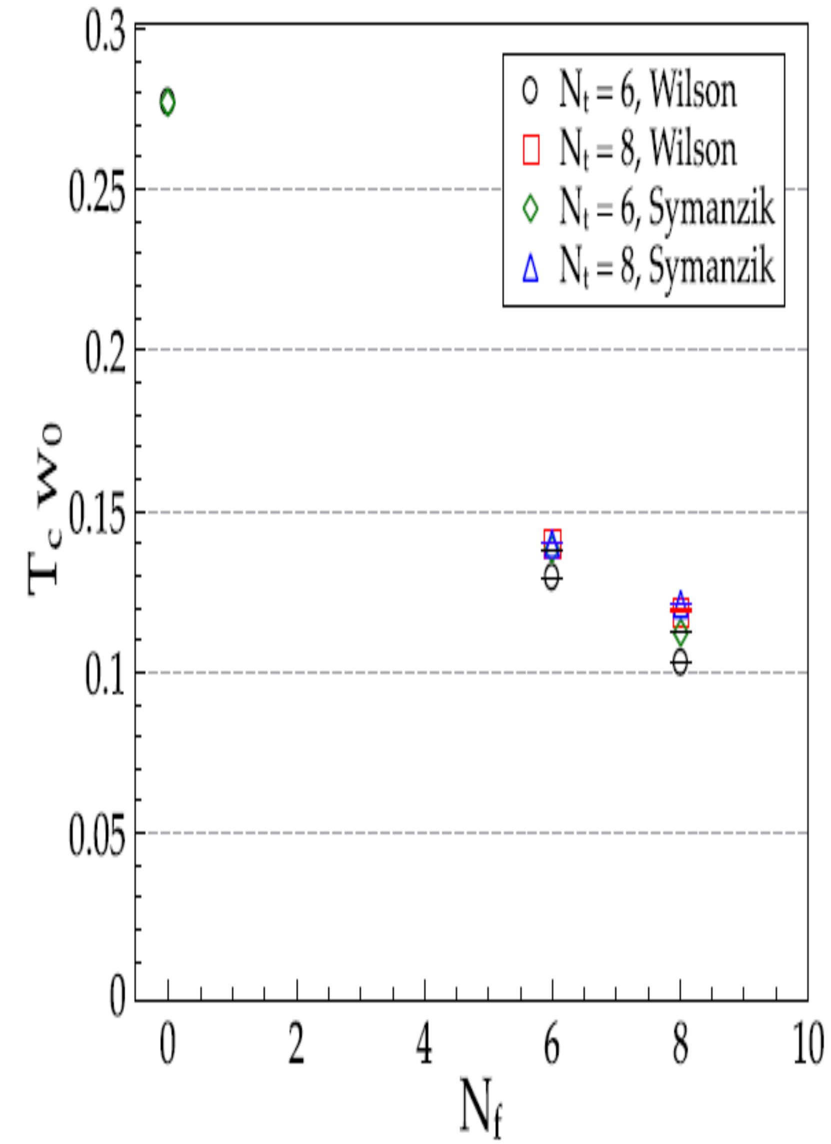
Nf -> Nfc :

Tc/M = 0

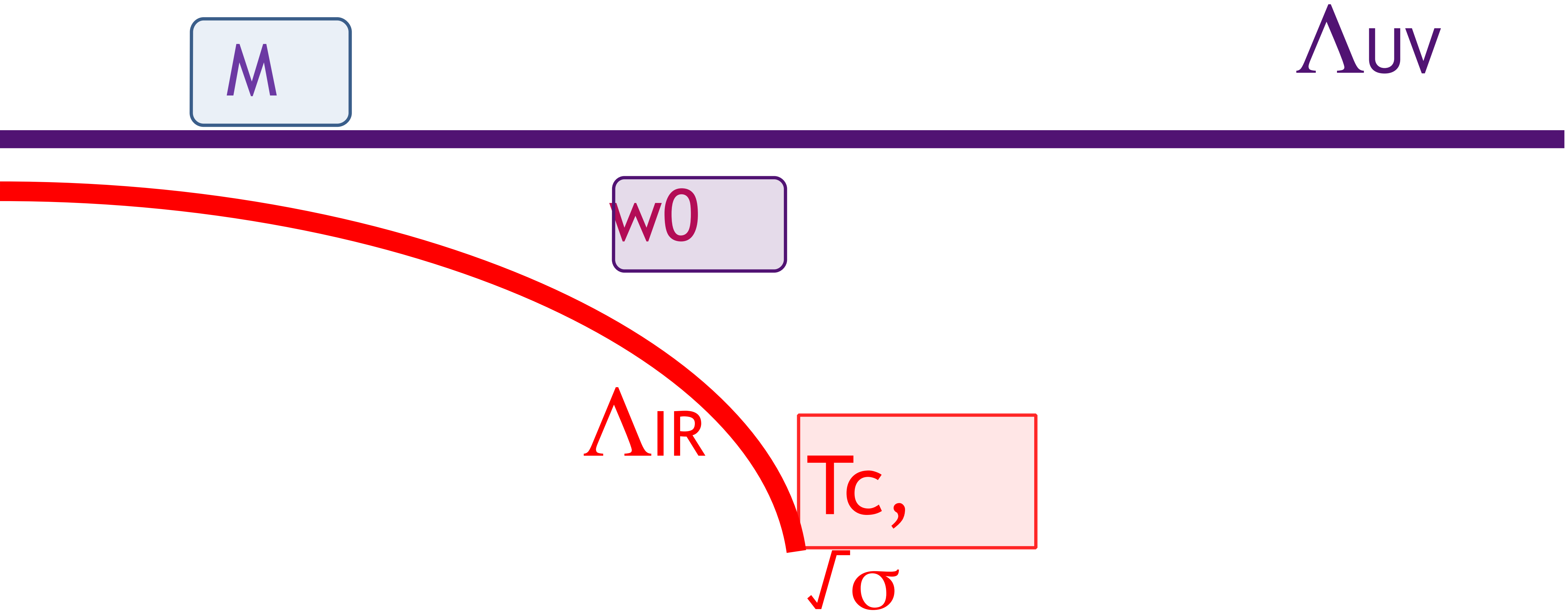
Tc w0 = 0 (?)

Tc / √σ ~ 0.3



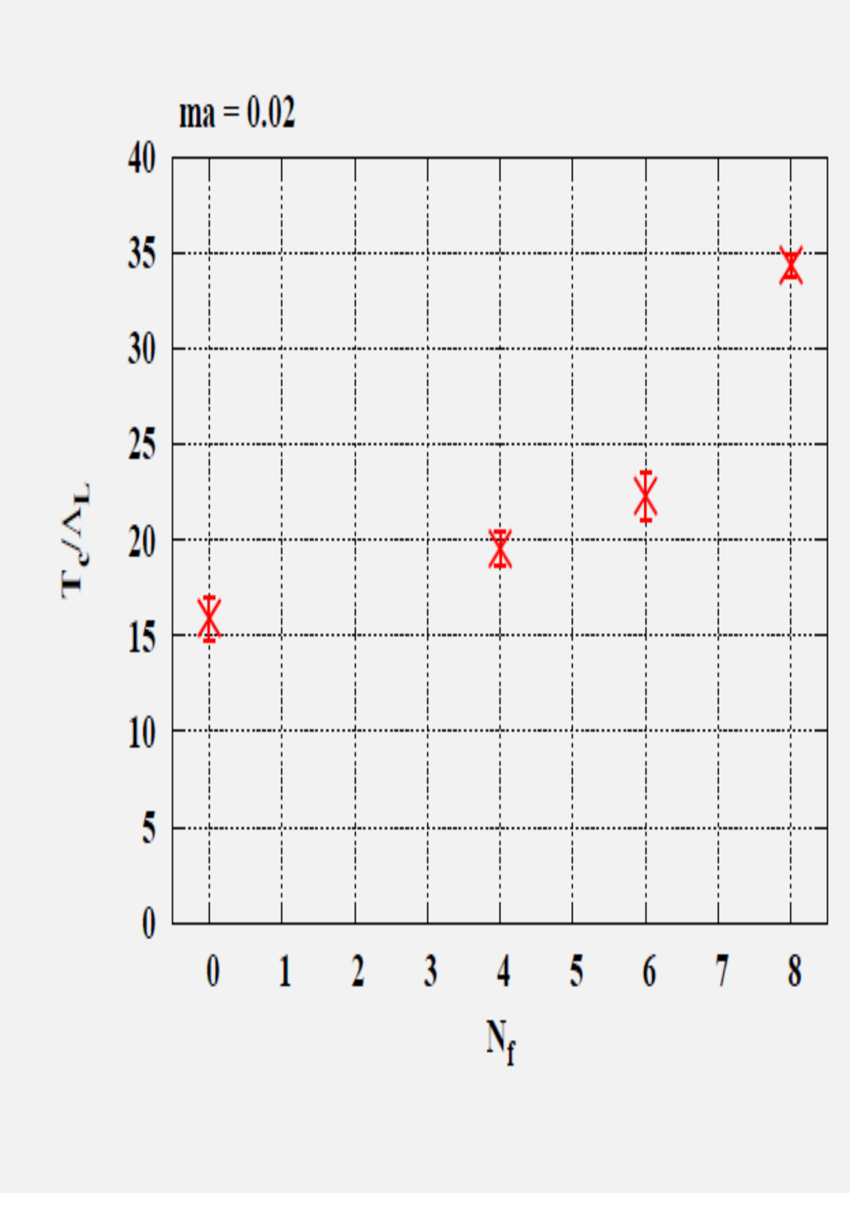


Different scales



Towards a quantitative comparison with holography

$$\frac{2\pi T_c}{M_{KK}} = 1 - \frac{1}{126\pi^3}\lambda_4^2\frac{N_f}{N_c}\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right)$$



Bigazzi and Cotrone, JHEP 2015

$$\left(1 + \frac{12\pi^{3/2}}{\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{6}\right)}\right) \approx -1.987$$

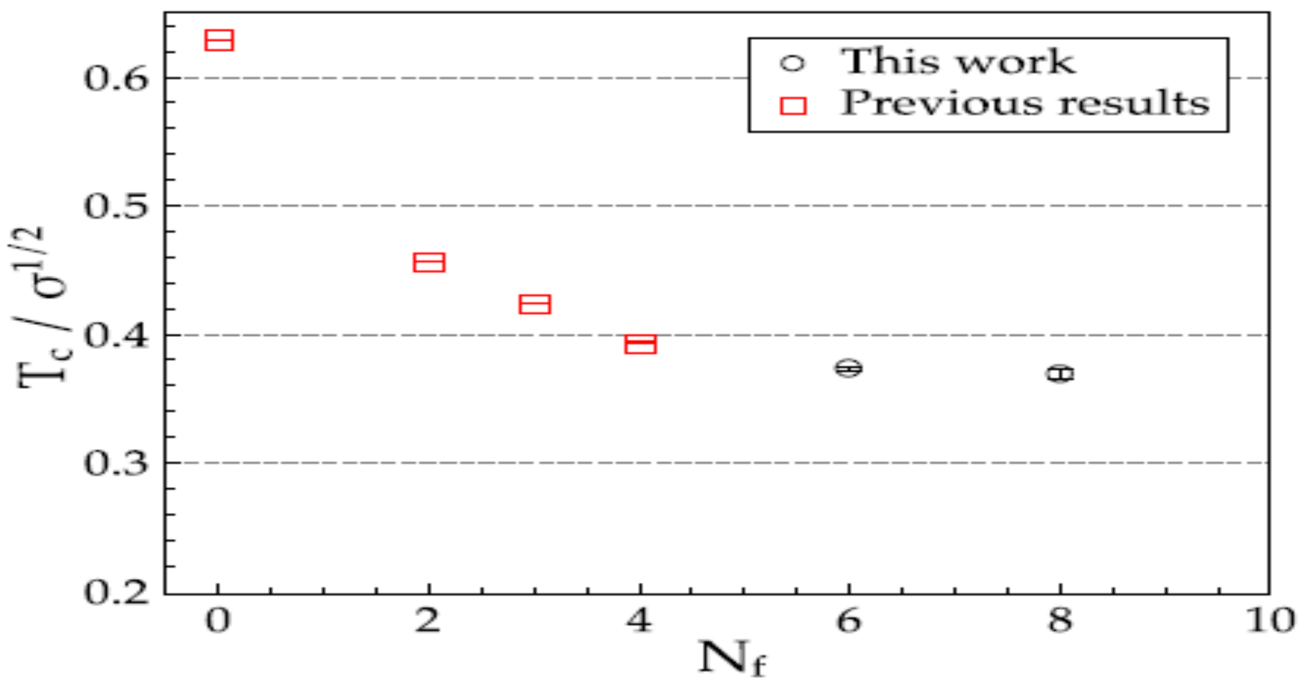
T increases with Nf on the scales used in these two studies

String tension, ratio Tc/sqrt(sigma)

$$T_s = \frac{1}{2\pi\alpha'} e^{2\lambda}|_{x=0} = \frac{2}{27\pi}\lambda_4 M_{KK}^2 \left[1 + \epsilon_f(3A_1 - A_2 - 28k)\right]$$

$$3A1 - A2 - 28k \approx 1.13.$$

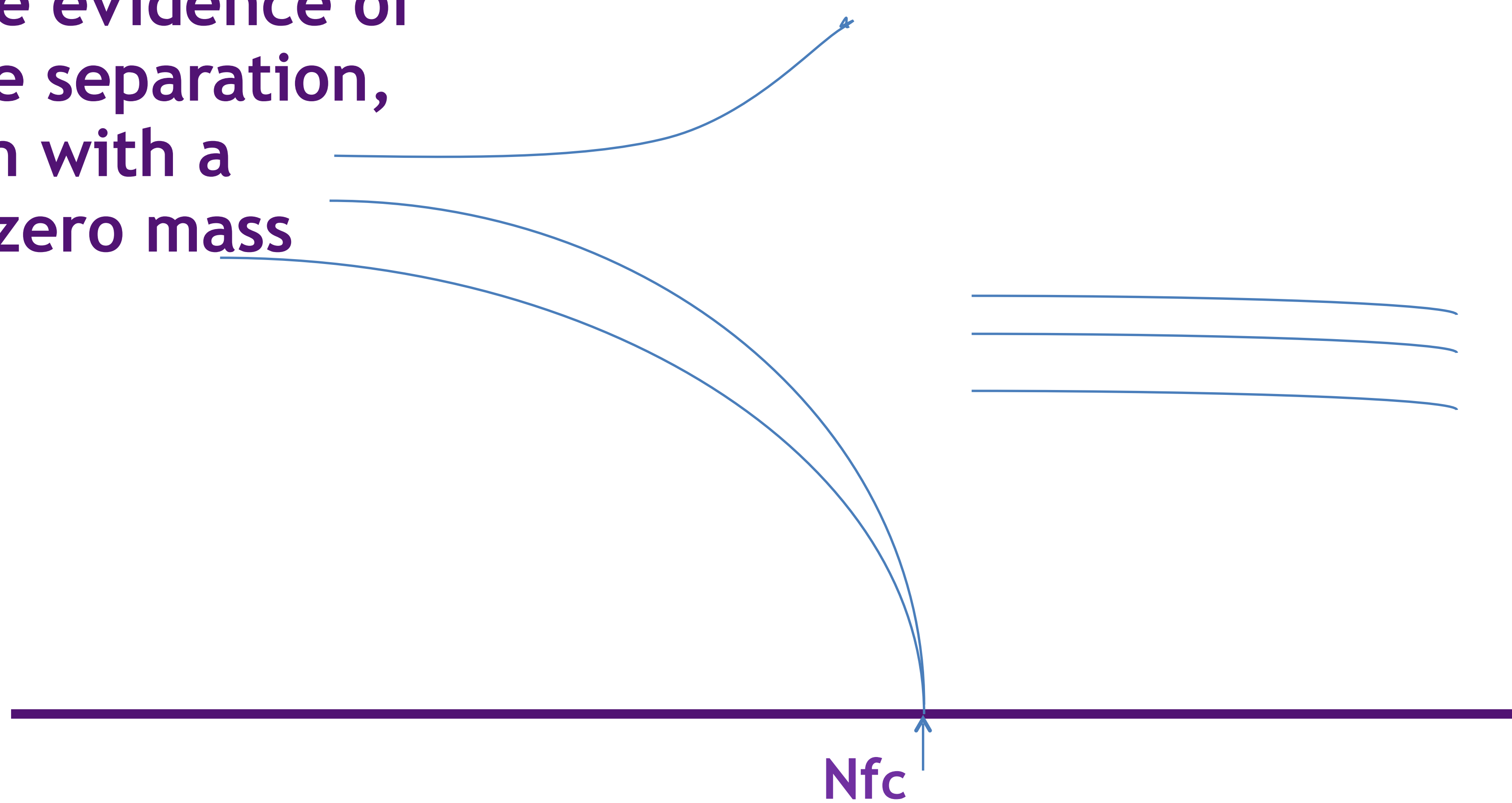
Adimensional ratios are free from the ambiguities of scale setting and might help comparing different approaches

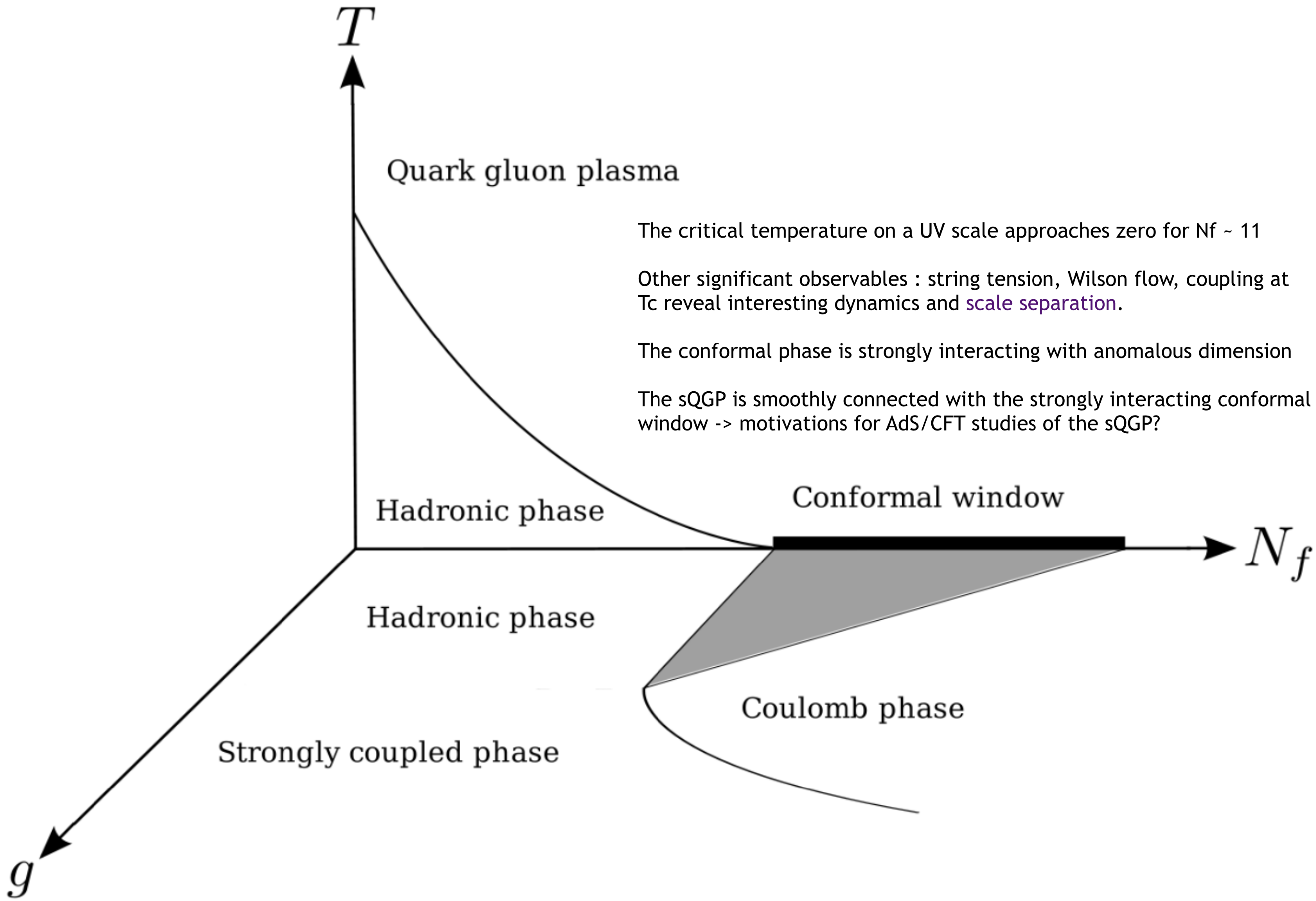


Summary

For Nf=8 we have observed some evidence of scale separation, even with a nonzero mass

For Nf=12 we have measured conformal scaling, in good agreement with four loops and other lattice estimates . Perhaps surprisingly, the scaling appears to persist in the QED-like region





The critical temperature on a UV scale approaches zero for Nf ~ 11

Other significant observables : string tension, Wilson flow, coupling at Tc reveal interesting dynamics and scale separation.

The conformal phase is strongly interacting with anomalous dimension

The sQGP is smoothly connected with the strongly interacting conformal window -> motivations for AdS/CFT studies of the sQGP?