Black Brane Steady States

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Technion

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Based on collaboration with Amos Yarom, arXiv:1501.01627

Motivation

- Behavior of strongly correlated systems out of equilibrium

- In general far from equilibrium is challenging ...

- Thermalization of 1+1 systems: universal steady state !

  theory and experiment: [Bernard,Doyon '12] [Karrasch et al. '12] [Brantut et al '13] [Schmidutz et al '13]

- Ansatz for 1+D relativistic CFT [Basheen '13] [Chang,Karch,Yarom '13] [Bhaseen,Doyon,Lucas,Schalm '13]

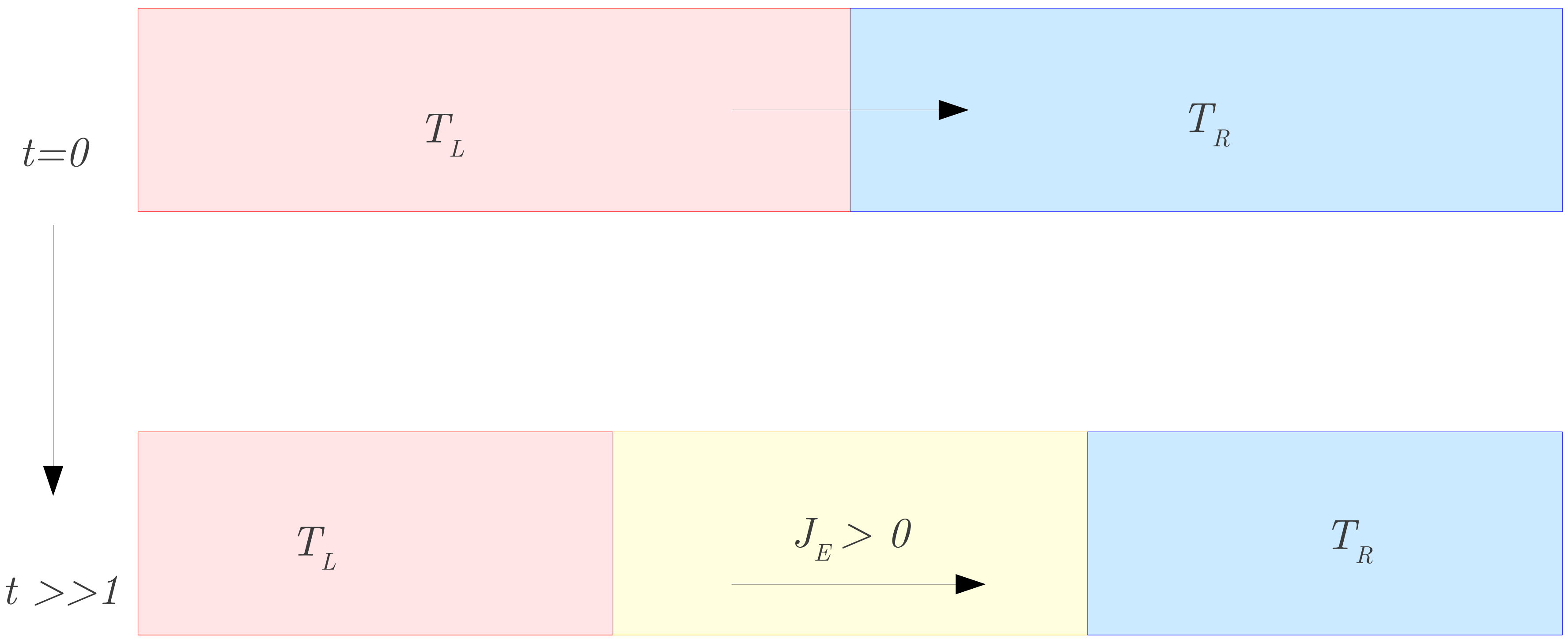
- Gauge/gravity duality: real time, non-equilibrium, finite T interacting systems

  → dynamically construct dual of 1+D conjectured steady state

Outline

- 1+1 steady state
- 1+D steady state
- Black brane steady state

- Two isolated quantum systems at different T in instantaneous thermal contact



- Large systems: late time steady state forms

- In 1+1 CFT the heat flow is universal

[Bernard,Doyon '12; Basheen '13; Chang,Karch,Yarom '13]

Two dimensional steady state

Following [Chang,Karch,Yarom '13]

- 1+1 CFT flat space: $ds^2 = -dt^2 + dx^2 = dz\, d\bar{z}$

- Conformal : $T^{\mu}{}_{\mu} = 0 \Rightarrow T_{z\bar{z}} = 0$

- Conservation : $\nabla_\mu T^{\mu\nu} = 0 \Rightarrow T_{zz} = T_+(z)\,, \quad T_{\bar{z}\bar{z}} = T_-(\bar{z})$

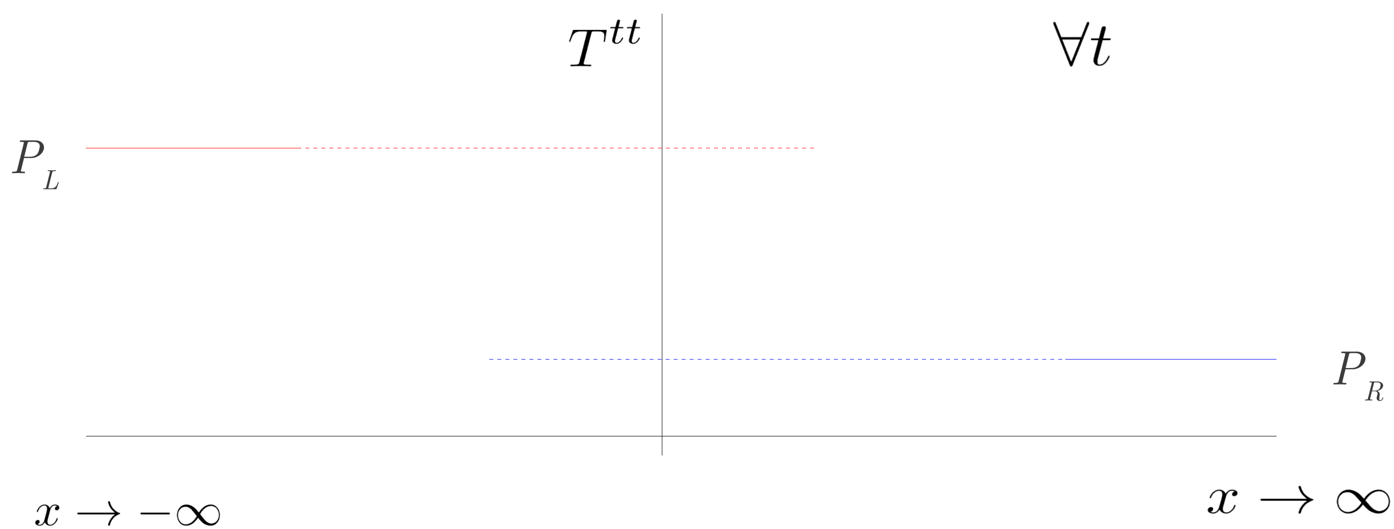
- In Cartesian : $T^{\mu\nu} = \begin{pmatrix} T_+(x+t) + T_-(x-t) & T_-(x-t) - T_+(x+t) \\ T_-(x-t) - T_+(x+t) & T_+(x+t) + T_-(x-t) \end{pmatrix}$

- The energy density (pressure) satisfies a wave equation:

$\nabla_t^2 T^{tt} - \nabla_x^2 T^{tt} = 0 \longrightarrow$ Left and Right moving wavefronts at $v=c$

- Late time is fully determined by initial profile and BC.

- Fixed pressure at spatial infinity

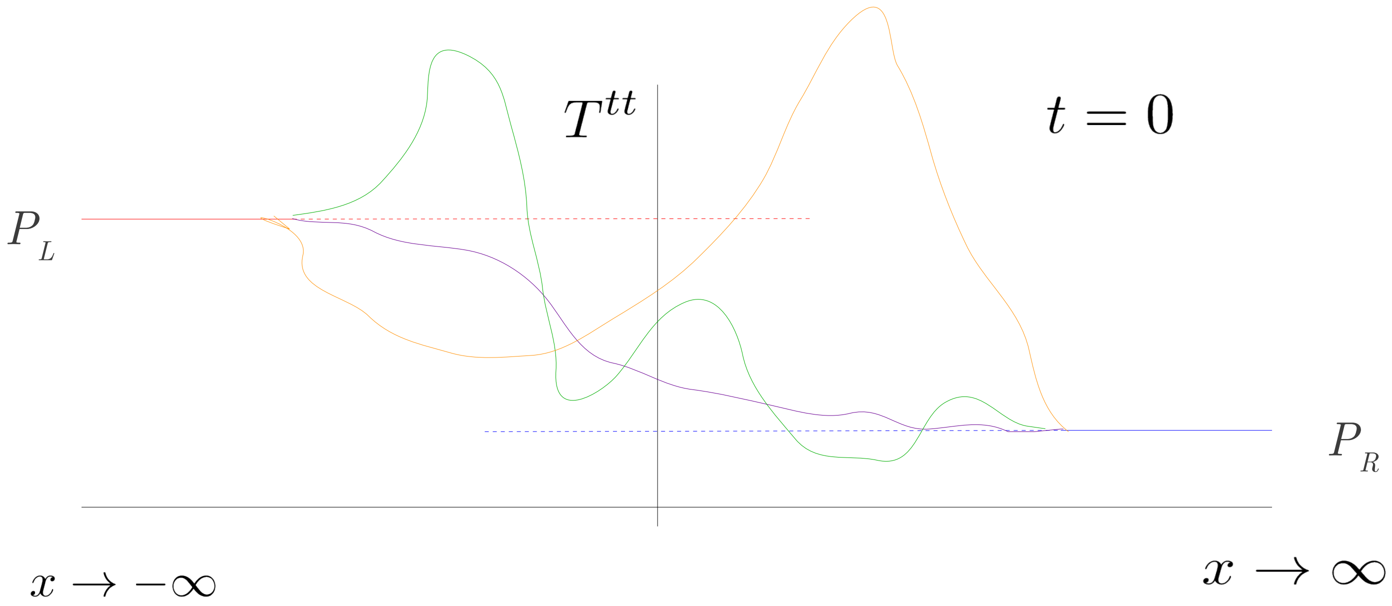


- Boundary conditions:

$$T^{\mu\nu}(t, x \to \pm\infty) = \begin{pmatrix} P_{R/L} & 0 \\ 0 & P_{R/L} \end{pmatrix}$$

- Late time is fully determined by initial profile and BC.

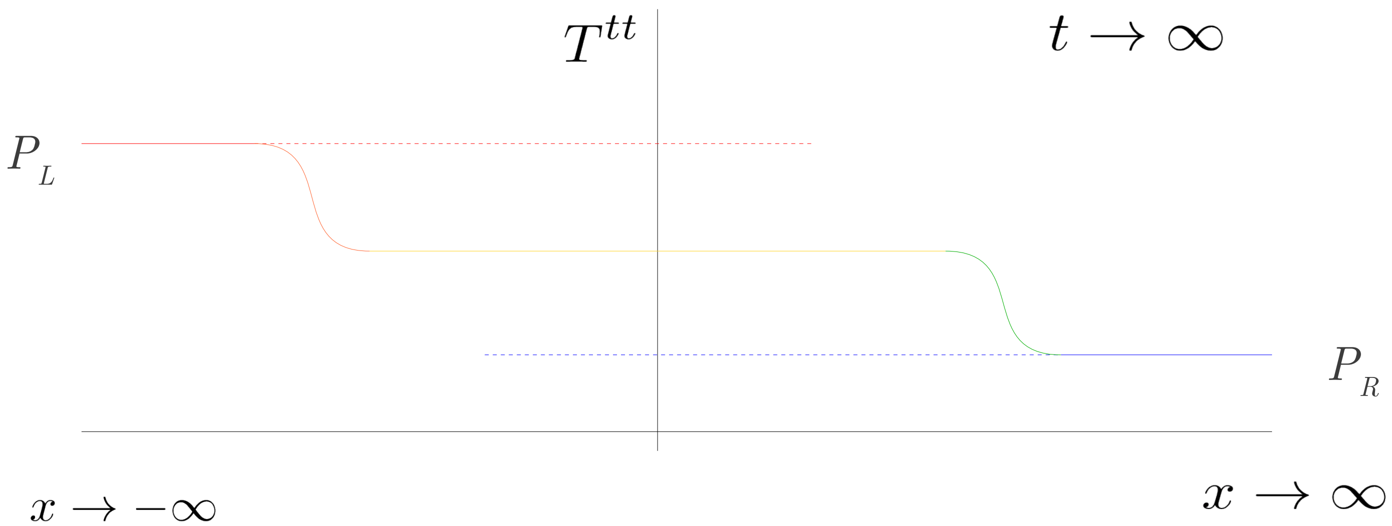
- Fixed pressure at spatial infinity



- No matter the interpolating initial profile

- Late time is fully determined by initial profile and BC.

- Fixed pressure at spatial infinity



- Steady state forms:

$$T^{tt}|_{t\to\infty} = \frac{1}{2}(P_L + P_R)$$

$$T^{tx}|_{t\to\infty} = \frac{1}{2}(P_L - P_R)$$

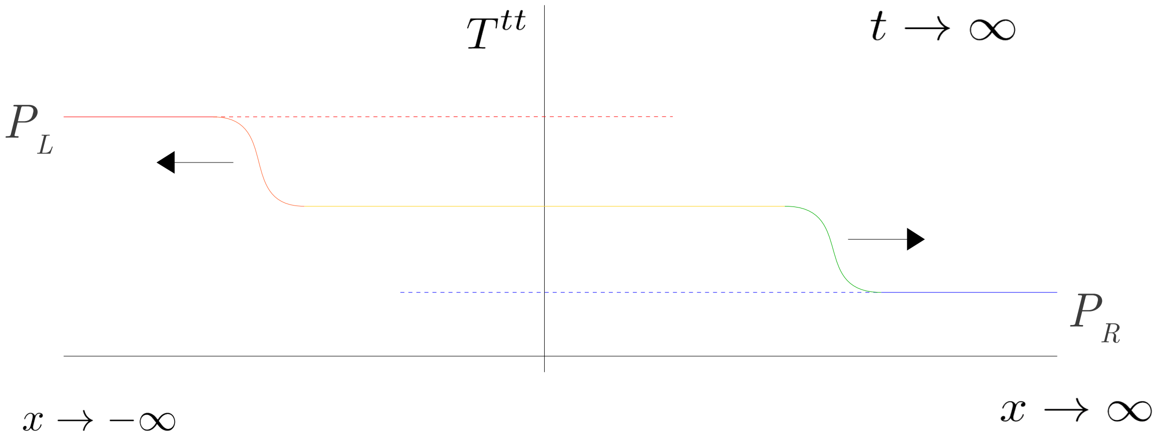
1+1 steady state

Asymptotic heat baths in thermal equilibrium

- Pressure and

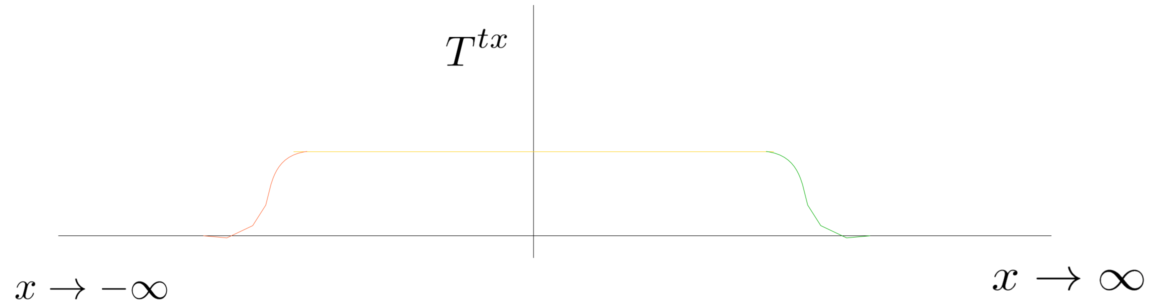
  energy density:

$$T^{tt} = \frac{\pi}{12}\left(c_- T_L^2 + c_+ T_R^2\right)$$



- Heat flow:

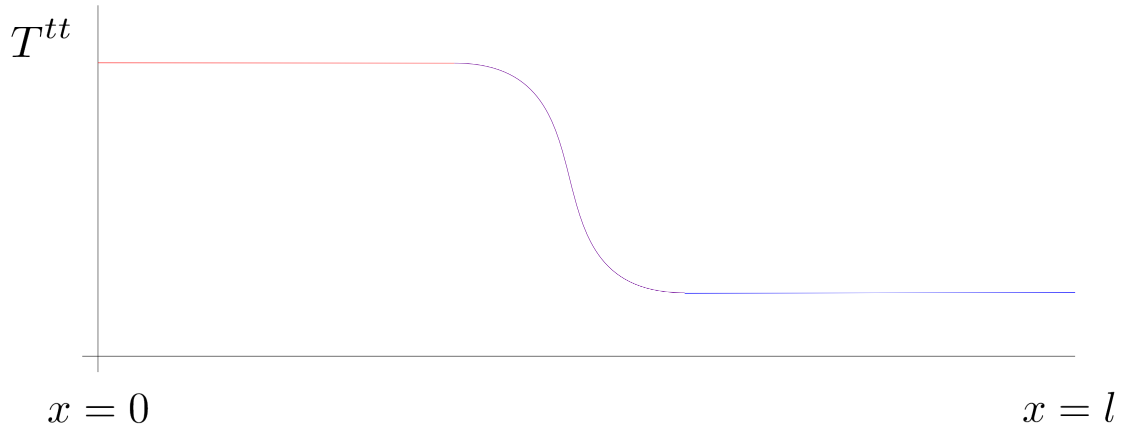
$$T^{tx} = \frac{\pi}{12}\left(c_- T_L^2 - c_+ T_R^2\right)$$



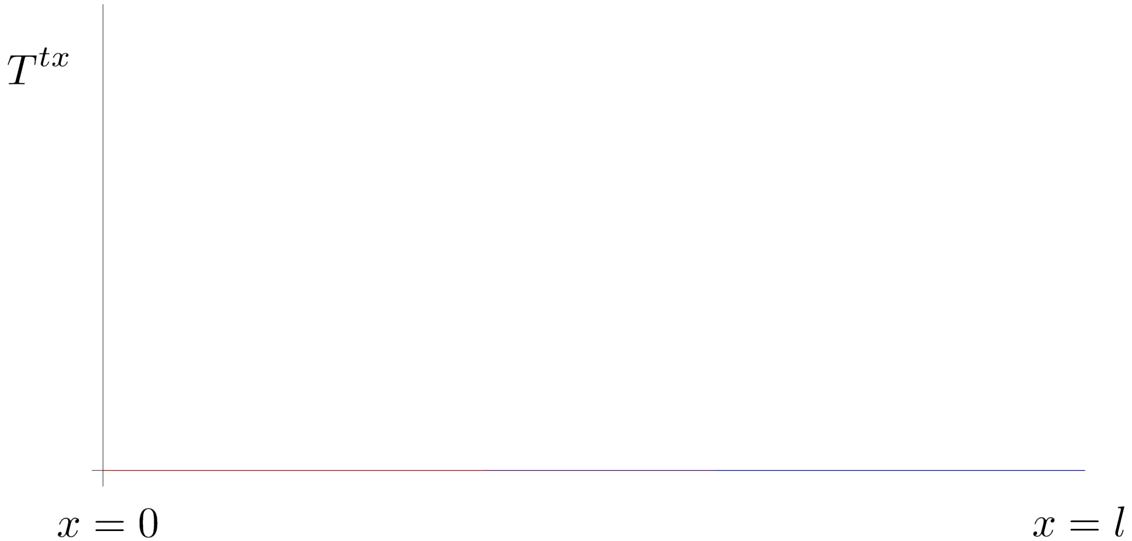
1+1 finite system

- Pressure and energy density:

$t/l = 0$



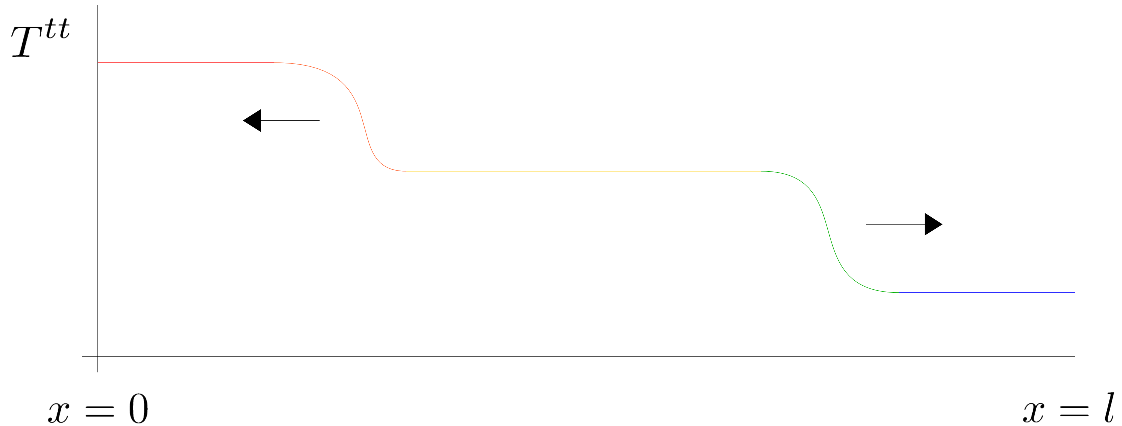
- Heat flow:



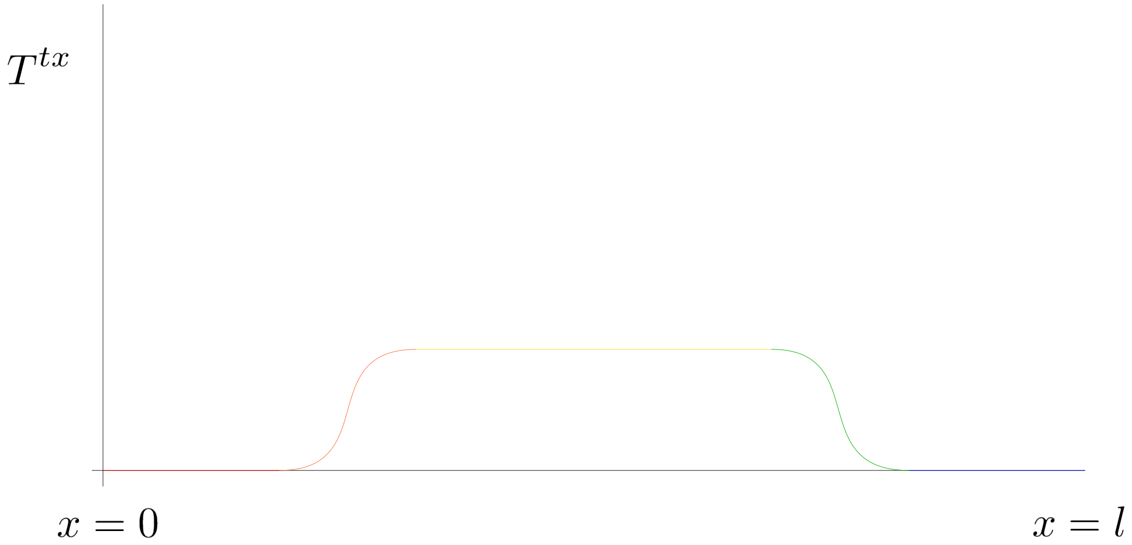
1+1 finite system

- Pressure and energy density:

$t/l = 0.3$



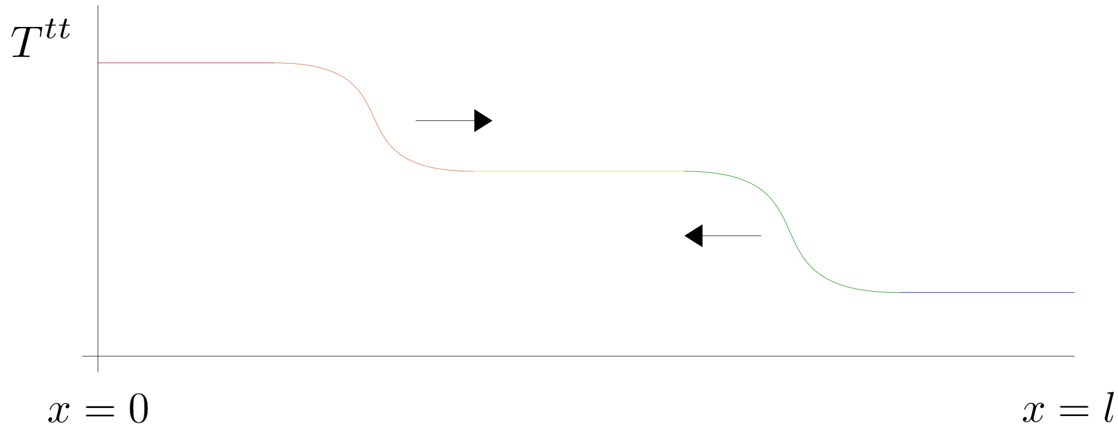
- Heat flow:



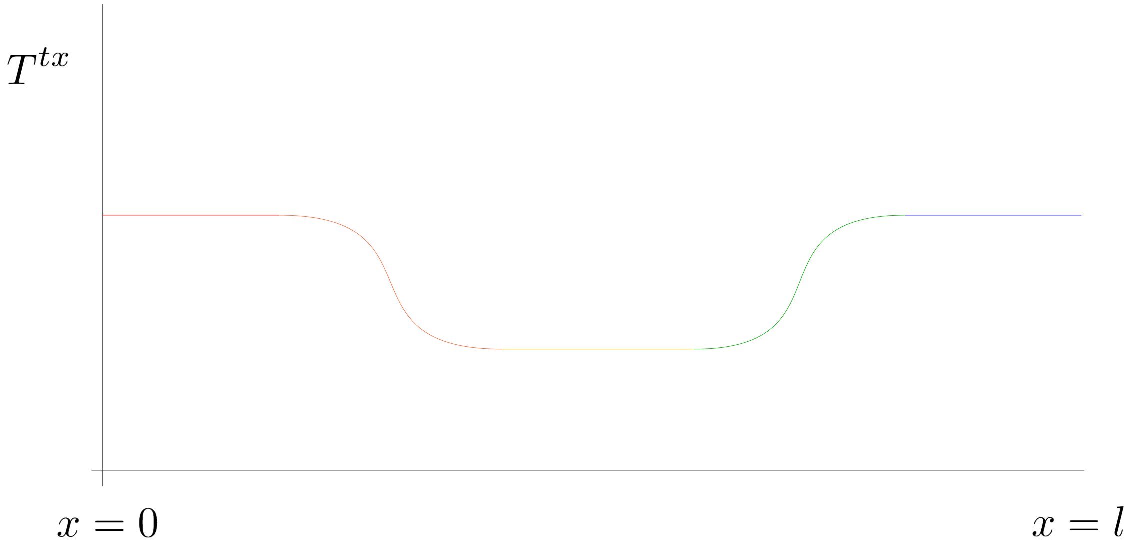
1+1 finite system

- Pressure and energy density:

$t/l = 0.8$



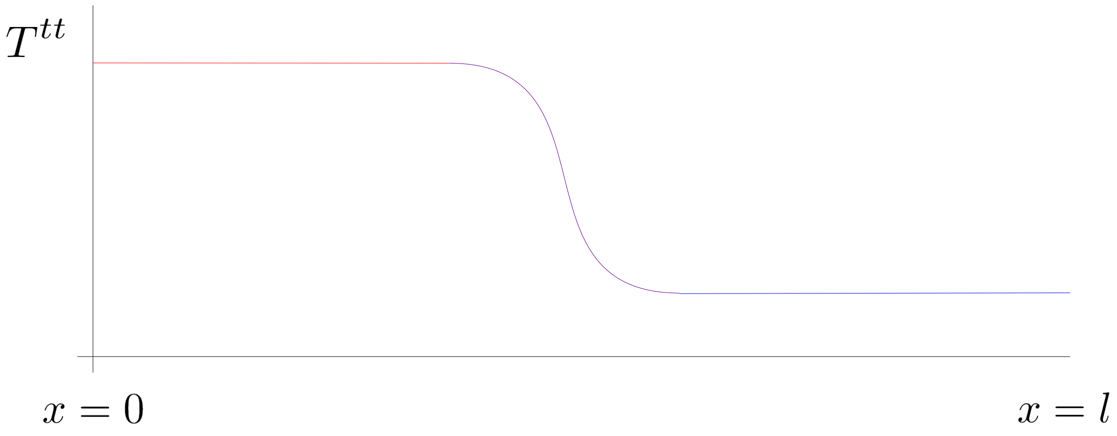
- Heat flow:



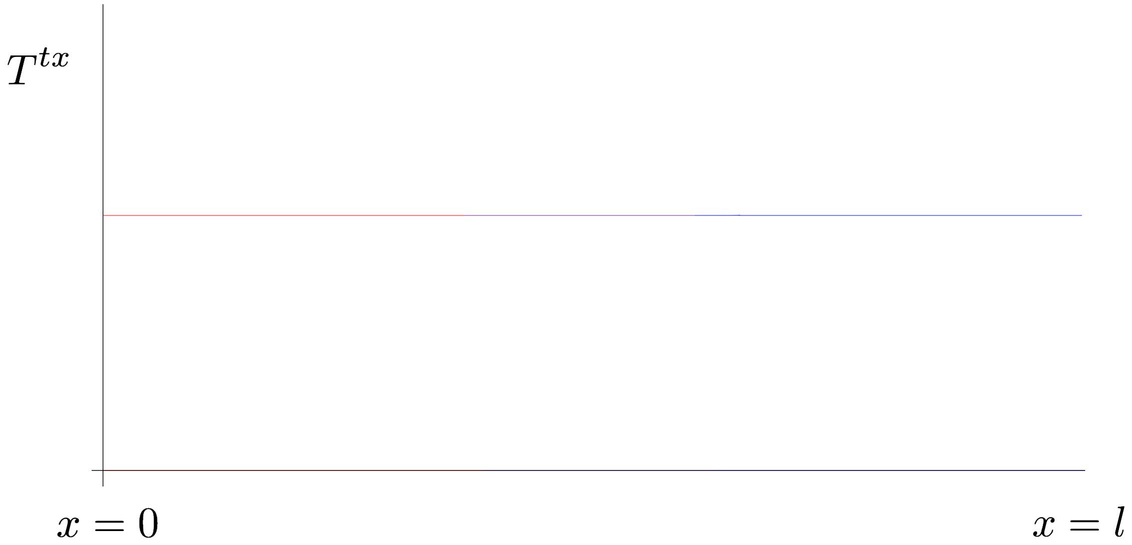
1+1 finite system

$t/l = 1$

- Pressure and energy density:



- Heat flow:

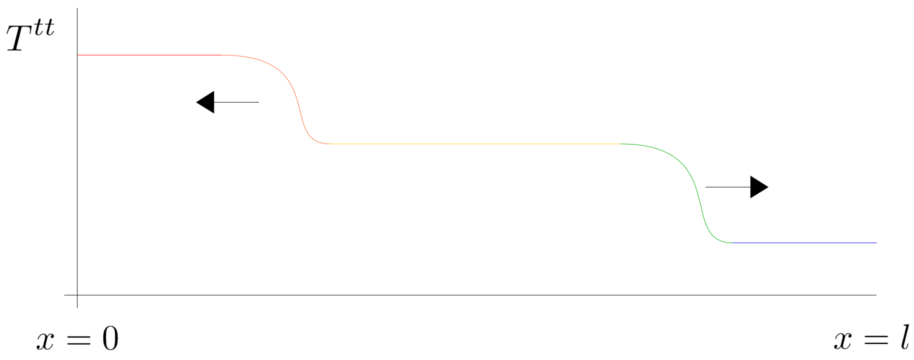


1+1 universal steady state

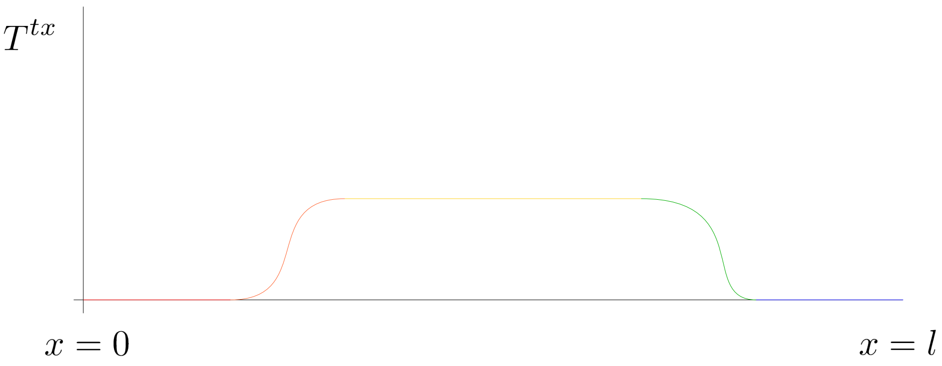
- Conformal and conservation of stress tensor

- $t/\ l <<1$

Pressure and energy density:



Heat flow:



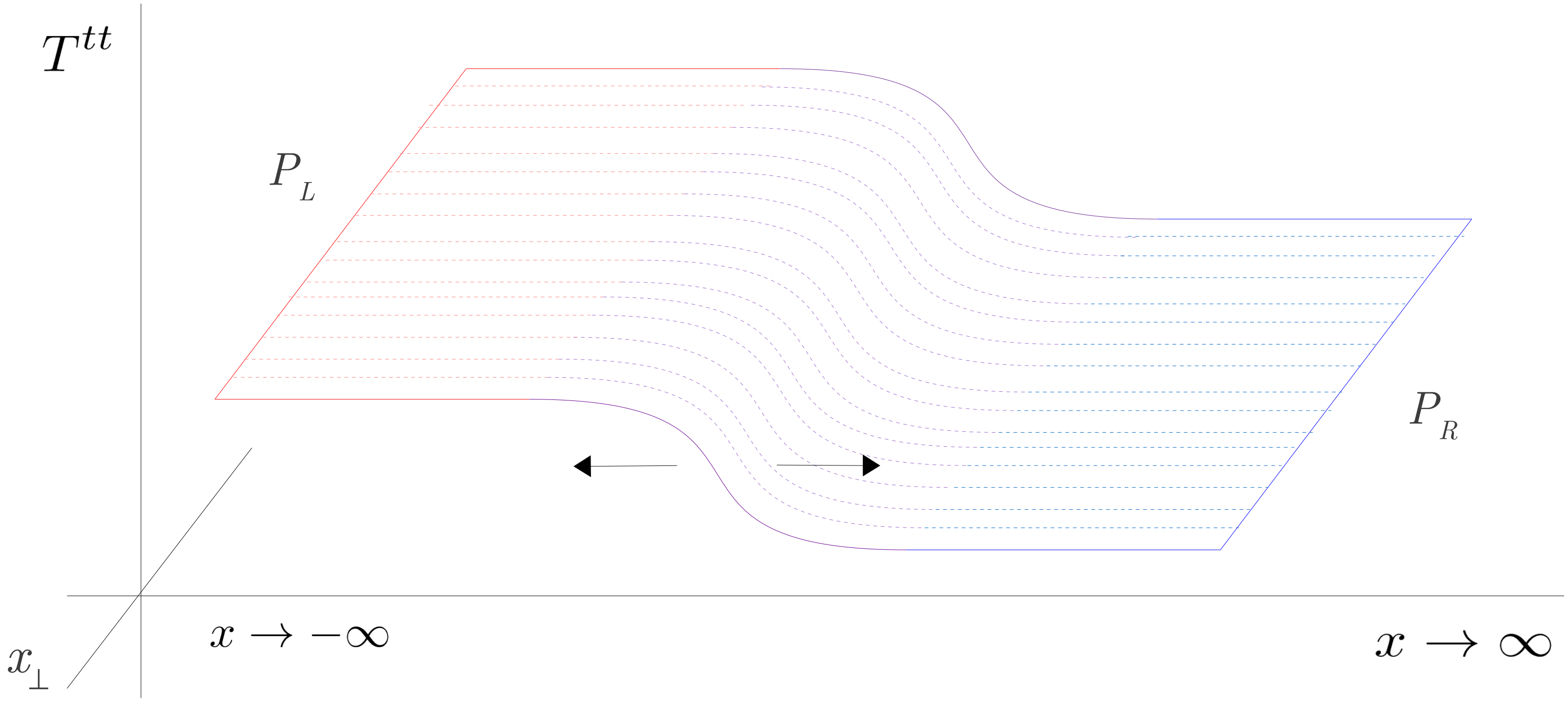
$$T^{tt}|_{t\to\infty} = \frac{1}{2}(P_L + P_R)$$

$$T^{tx}|_{t\to\infty} = \frac{1}{2}(P_L - P_R)$$

Higher dimensional universal flow

Following [Chang,Karch,Yarom '13]

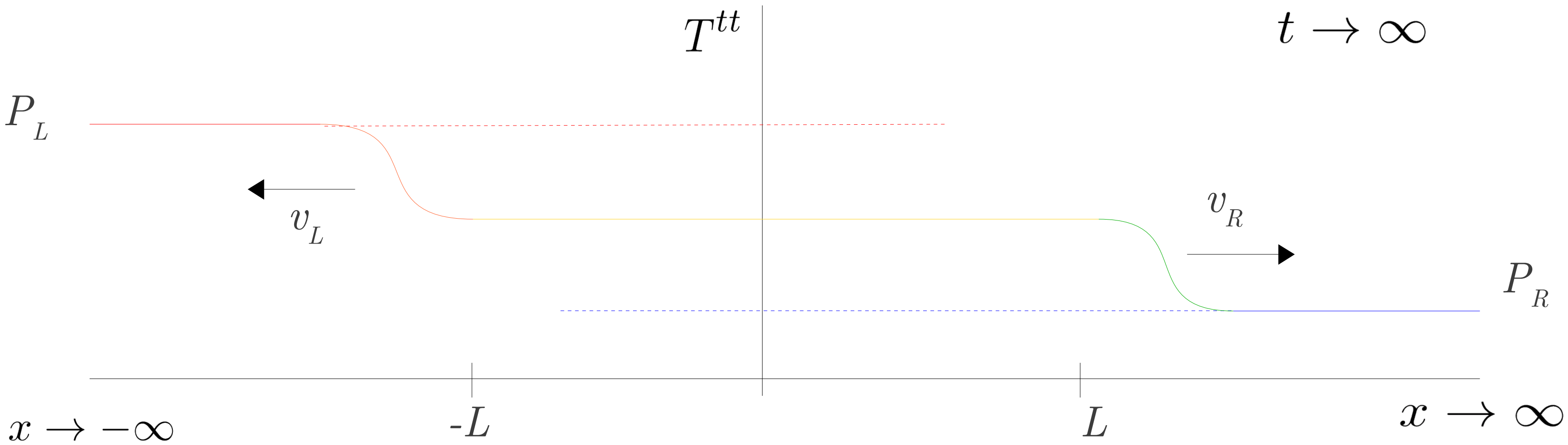
- Assumption: same structure of L and R moving waves describes the system
- 1+D CFT with pressure gradient in x direction but homogeneous in $x_\perp$



- Do the 2 steps and the steady state plateau form?

Higher dimensional universal flow

Conjecture: late time generic CFT connected to asymptotic heat baths



I	II	III
L moving wave	Steady state	R moving wave

Universal heat flow and energy density determined imposing only $\nabla_\mu T^{\mu\nu} = 0$

Ansatz for steady states

Regions I and III

- BC :
$$T^{\mu\nu}(x \to \mp\infty) = \begin{pmatrix} \epsilon(P_0 \pm \Delta P) & 0 & 0 \\ 0 & P_0 \pm \Delta P & 0 \\ 0 & 0 & P_0 \pm \Delta P \ldots \end{pmatrix}$$

- Ansatz :
$$T_I^{\mu\nu} = \begin{pmatrix} -\frac{1}{v_L} W_L(x+v_L t) + \epsilon(P_0 + \Delta P) & W_L(x+v_L t) \\ W_L(x+v_L t) & -v_L W_L(x+v_L t) + P_0 + \Delta P \end{pmatrix}$$

$$T_{III}^{\mu\nu} = \begin{pmatrix} \frac{1}{v_R} W_R(x-v_R t) + \epsilon(P_0 - \Delta P) & W_R(x-v_R t) \\ W_R(x-v_R t) & v_R W_R(x-v_R t) + P_0 - \Delta P \end{pmatrix}$$

$$W_L(-\infty) = 0\,, \quad W_L(\infty) = J_L\,, \quad W_R(\infty) = 0\,, \quad W_R(-\infty) = J_R$$

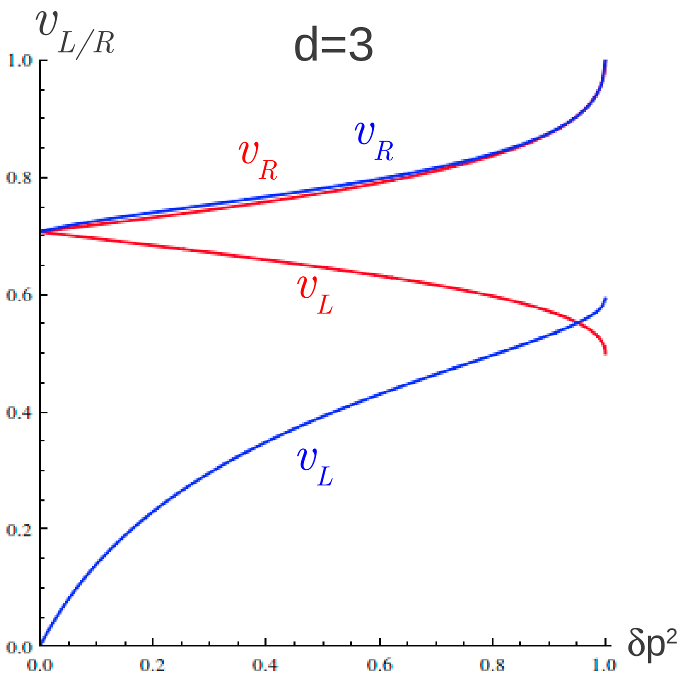
Region II

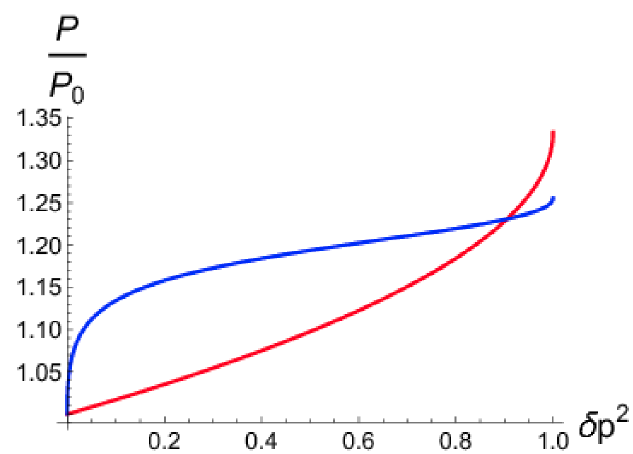
- Ansatz:
$$T_{II}^{\mu\nu} = \begin{pmatrix} T_{II}^{tt}(x) & J \\ J & P_{II} \end{pmatrix}$$

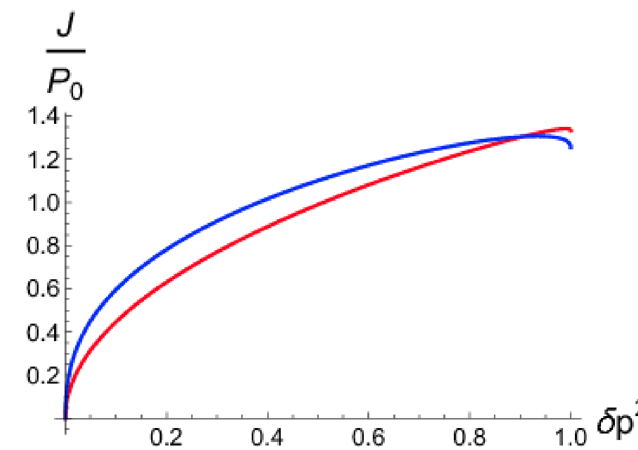
- Matching cond: $T_I^{\mu\nu}(t\to\infty) = T_{II}^{\mu\nu}(x\to-\infty)\,,\quad T_{III}^{\mu\nu}(t\to\infty) = T_{II}^{\mu\nu}(x\to\infty)$

- Solution: $J = \dfrac{2\Delta P}{v_L+v_R}\,,\quad P_{II} = P_0 - \dfrac{v_L - v_R}{v_L+v_R}\Delta P\,,\quad T^{tt}(\pm\infty) = \pm\dfrac{J}{v_{R/L}} + \epsilon(P_0 \mp \Delta P)$

- Conformal $\epsilon(P) = (d-1)P$ + thermal eq. at ends of II $\longrightarrow$ solve for $v_{L/R}$





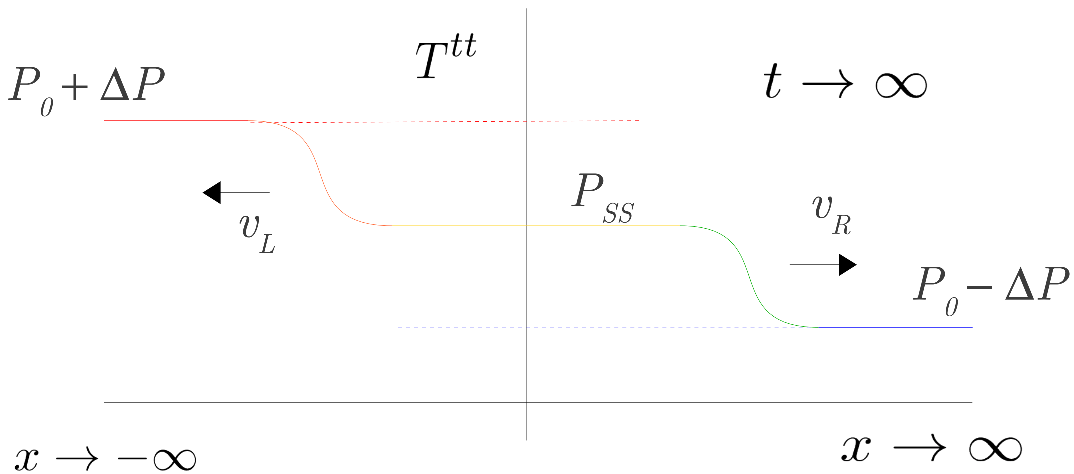


Thermodynamic branch

$$\boxed{\begin{aligned}\frac{P}{P_0} &= \frac{1}{d}\left(2(d-1) - (d-2)\sqrt{1-\delta p^2}\right)\\ \frac{J}{P_0} &= \frac{\sqrt{2(d-1)}}{d}\sqrt{(d^2-2d+2)\delta p^2 - (d-2)^2(1-\sqrt{1-\delta p^2})}\end{aligned}}$$

Higher dimensional universal steady state

Conjecture: late time generic CFT connected to asymptotic heat baths



$$\left\{ \frac{P_{SS}}{P_0}, \frac{J_{SS}}{P_0} \right\} = f\left(\frac{\Delta P}{P_0}, d \right)$$

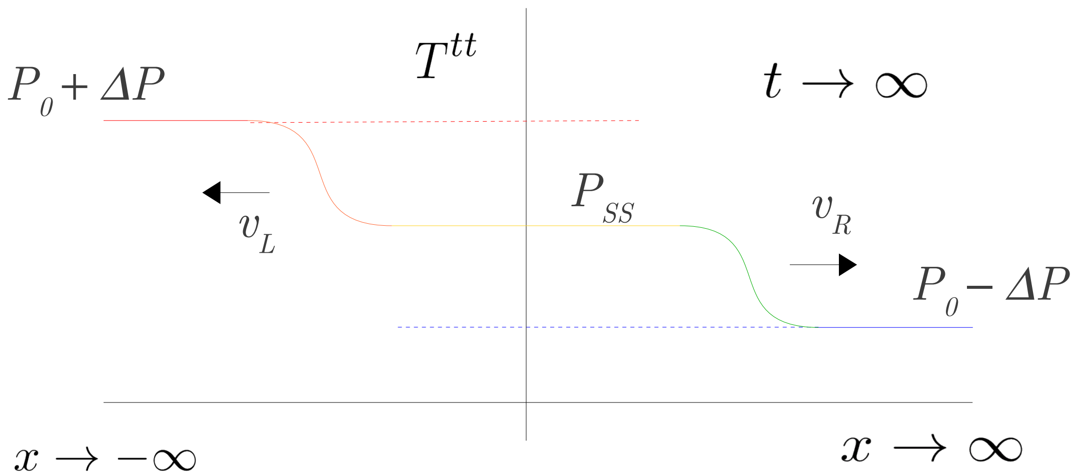
Assumptions : $\nabla_\mu T^{\mu\nu} = 0$ and thermal equilibrium at $x = \pm L$

Flow driven steady state

- **What about diffusion?**
- **Which branch is realized?**

Higher dimensional universal steady state

Conjecture: late time generic CFT connected to asymptotic heat baths



$$\left\{ \frac{P_{SS}}{P_0}, \frac{J_{SS}}{P_0} \right\} = f\left(\frac{\Delta P}{P_0}, d \right)$$

Assumptions : $\nabla_\mu T^{\mu\nu} = 0$ and thermal equilibrium at $x = \pm L$

Flow driven steady state

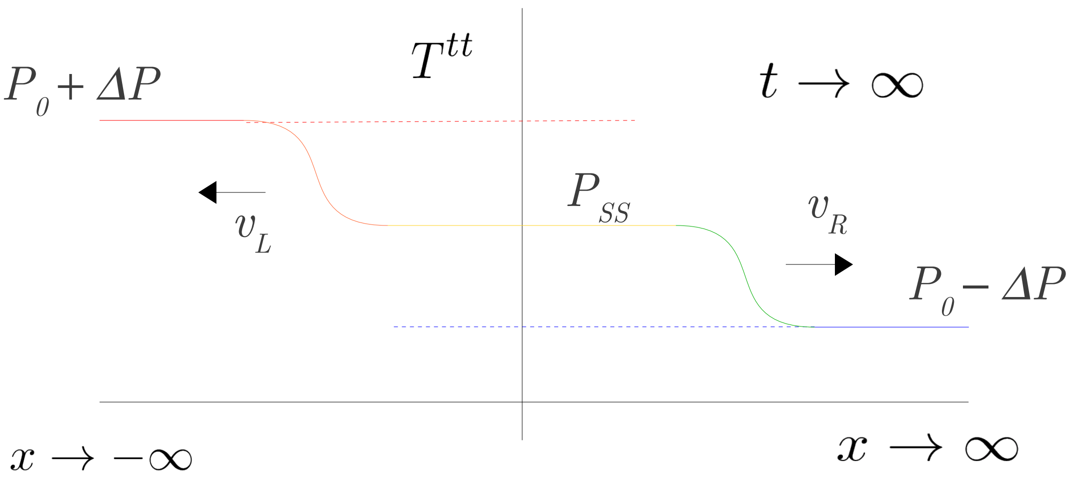
- **What about diffusion?**
- **Which branch is realized?**

2^{nd} Order Hydrodynamics

- **close to equilibrium** dynamics
- good at δp small, **but** breaks at δp large or large dissipation

Higher dimensional universal steady state

Conjecture: late time generic CFT connected to asymptotic heat baths



$$\left\{ \frac{P_{SS}}{P_0}, \frac{J_{SS}}{P_0} \right\} = f\left(\frac{\Delta P}{P_0}, d\right)$$

Assumptions : $\nabla_\mu T^{\mu\nu} = 0$ and thermal equilibrium at $x = \pm L$

→ Flow driven steady state

- **What about diffusion?**
- **Which branch is realized?**

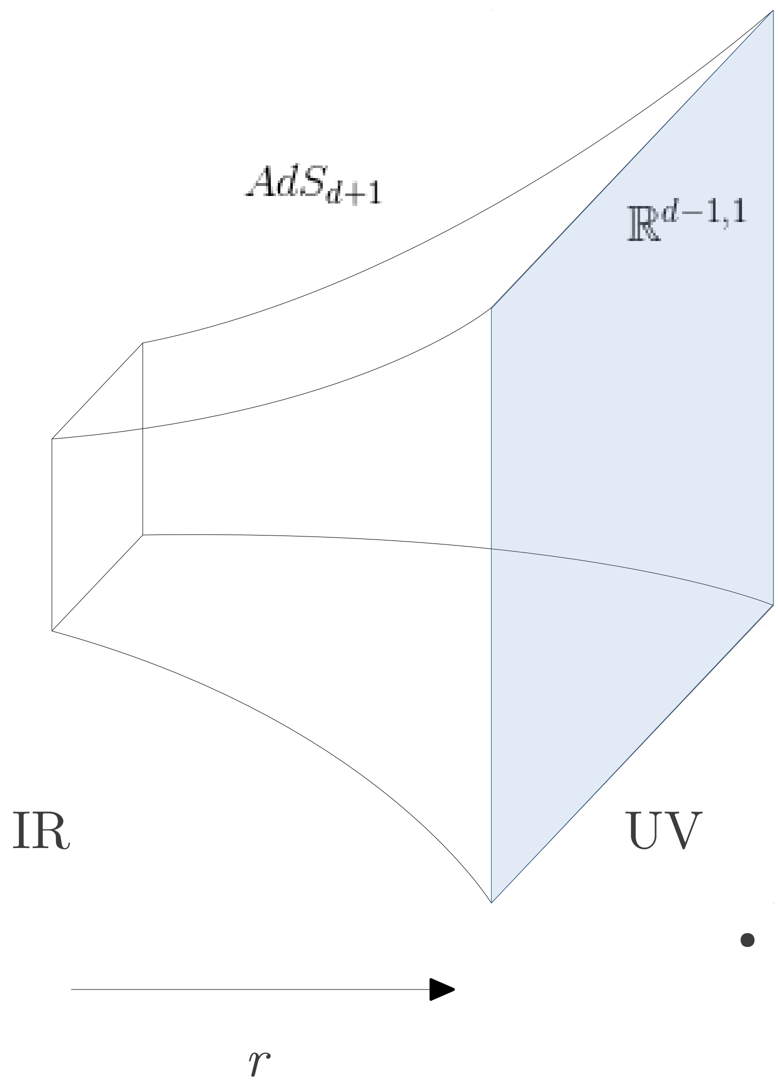
→ $\boxed{\text{Gauge/Gravity duality} \quad \text{- non-equilibrium dynamics}}$

Gauge/Gravity duality

- **non-equilibrium** dynamics

AdS/CFT correspondence

- Generating functions: $Z_{CFT} \equiv e^{-S_{AdS}}$



Fields in Ads ⟷ Operators in CFT

$$\phi \leftrightarrow \mathcal{O}$$

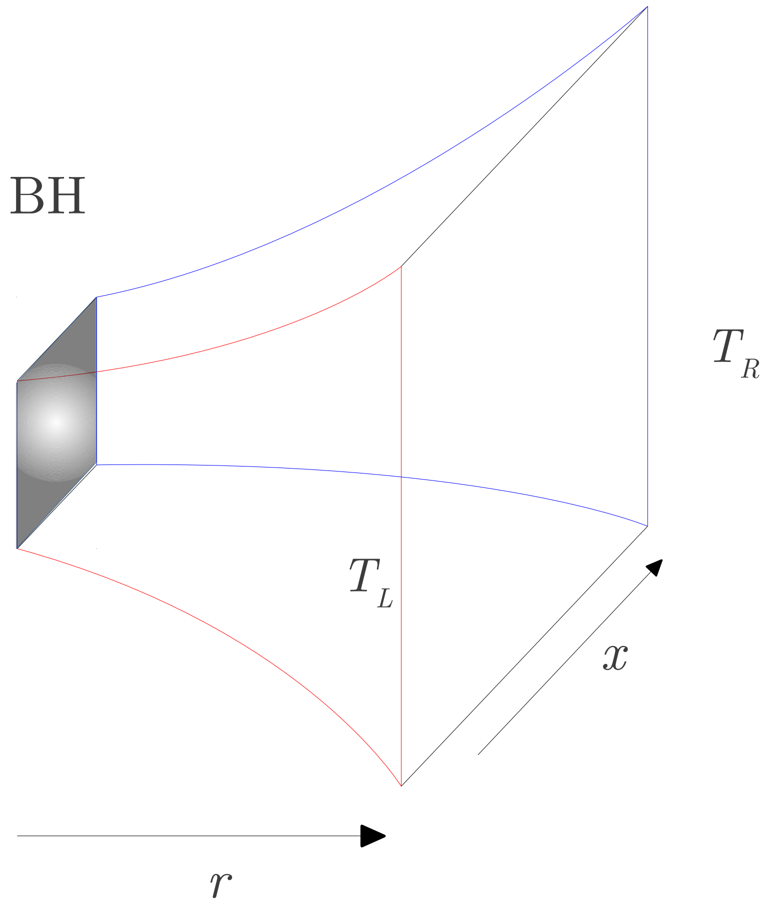
$$g_{\mu\nu} \leftrightarrow T_{\mu\nu}$$

Black hole ⟷ Finite temperature

- Real time dynamics in interacting systems

Black brane steady states

- Thermalization: driven steady state

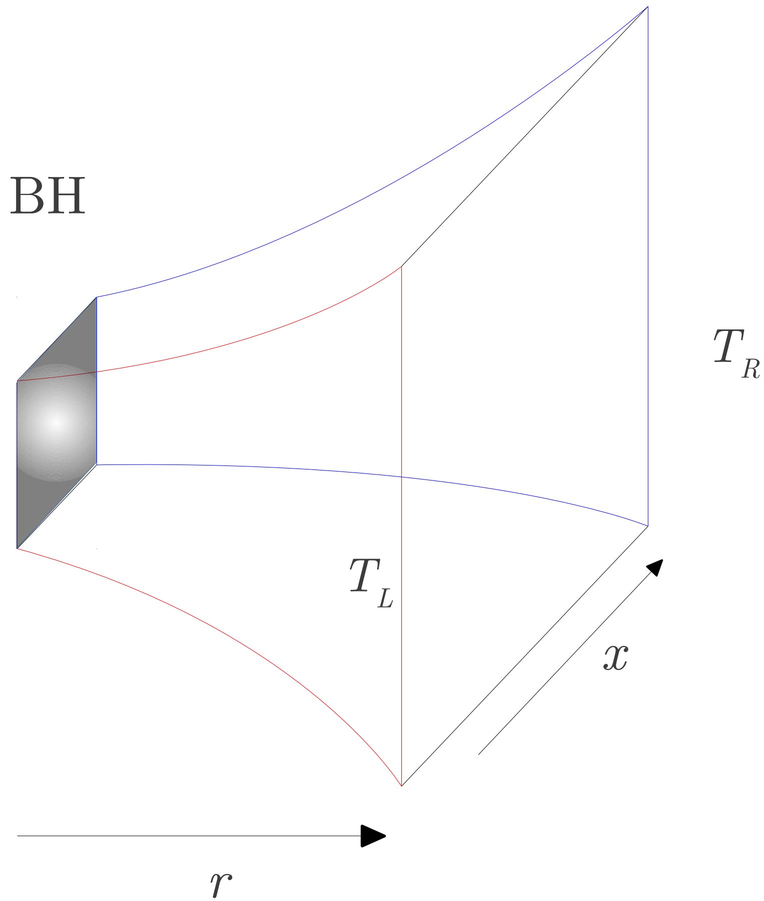


$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left(R + \frac{d(d-1)}{L_{AdS}^2}\right)$$

Black brane steady states

- Thermalization: driven steady state in ABJM (planar, strongly coupled)

2+1 strongly coupled CFT in flat space



$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R + \frac{6}{L_{AdS}^2}\right)$$

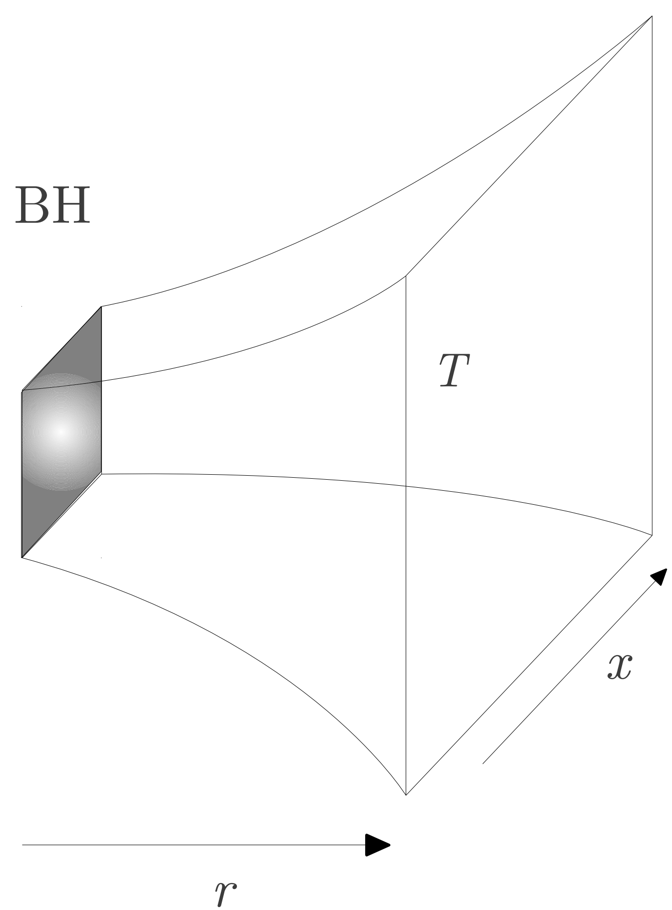
Black brane sols such that dual CFT :

$$Tr(\rho T^{\mu\nu}(t, x_\perp, x))|_{x\to\pm\infty} = \begin{pmatrix} 2P(T_{R/L}) & 0 & 0 \\ 0 & P(T_{R/L}) & 0 \\ 0 & 0 & P(T_{R/L}) \end{pmatrix}$$

Black brane steady states

- Homogeneous black brane

$$ds^2 = 2dtdr - r^2\left(1 - \left(\frac{4\pi T}{3r}\right)^3\right)dt^2 + r^2\left(dx_\perp^2 + dx^2\right)$$



$$Tr(\rho T^{\mu\nu}) = \begin{pmatrix} 2P(T) & 0 & 0 \\ 0 & P(T) & 0 \\ 0 & 0 & P(T) \end{pmatrix}$$

$$P(T) = p_0\left(\frac{4\pi T}{3}\right)^3 \qquad \left(\text{ ABJM: } \; p_0 = \frac{2N^2}{9\sqrt{2\lambda}}\right)$$

- Steady state black brane asymptotes at $x \to \pm\infty$ w/ $T_{R/L}$

Black brane steady states

Following [Chesler,Yaffe '13]

- Metric ansatz :

$$ds^2 = 2dt\left(dr - A(t,x,r)dt - F(t,x,r)dx\right) + \Sigma^2(t,x,r)\left(e^{B(t,x,r)}dx_\perp^2 + e^{-B(t,x,r)}dx^2\right)$$

- Nested eoms :

$$4\partial_r^2\Sigma + \Sigma\left(\partial_r B\right)^2 = 0$$
$$-\Sigma^2\partial_r^2 F - \Sigma^2\left(\partial_r B\right)\partial_r F + C_F[B,\,\Sigma]F = S_F[\Sigma,\,B]$$
$$4\Sigma^3\partial_r\dot{\Sigma} + 4\Sigma^2\partial_r\Sigma\dot{\Sigma} = S_{\dot{\Sigma}}[\Sigma,\,B,\,F]$$
$$4\Sigma^4\partial_r\dot{B} + 4\Sigma^3\partial_r\Sigma\dot{B} = S_{\dot{B}}[\Sigma,\,B,\,F,\,\dot{\Sigma}]$$
$$2\Sigma^4\partial_r^2 A = S_A[\Sigma,\,B,\,F,\,\dot{\Sigma},\,\dot{B}]$$
$$\ddot{\Sigma} = Q_{\ddot{\Sigma}}[\Sigma,\,B,\,F,\,A]$$
$$\partial_z\dot{F} = Q_{\partial_z\dot{F}}[\Sigma,\,B,\,F,\,A]$$

- $\dot{X} = \partial_t X + A\partial_r X$

- C and S depend only on spatial derivatives

- Q depends on spatial and time derivatives

- UV boundary conditions ($r \to \infty$) : $ds^2 = 2dtdr + r^2\left(-dt^2 + dx_\perp^2 + dx^2\right)$

$$A = \frac{1}{2}\left(r + \xi(t,x)\right)^2 - \partial_t \xi(t,x) + \frac{a_1(t,x)}{r + \xi(t,x)} + \mathcal{O}\left(r^{-2}\right)$$

$$F = -\partial_x \xi(t,x) + \frac{f_1(t,x)}{r + \xi(t,x)} + \frac{3\partial_x b_3(t,x)}{4(r + \xi(t,x))^2} + \mathcal{O}\left(r^{-3}\right)$$

$$\Sigma = r + \xi(t,x) - \frac{3b_3(t,x)}{40(r + \xi(t,x))^5} + \mathcal{O}\left(r^{-6}\right)$$

$$B = \frac{b_3(t,x)}{(r + \xi(t,x))^3} + \mathcal{O}\left(r^{-4}\right) .$$

- Stress tensor of dual CFT: $\mathrm{Tr}\left(\rho T^{\mu\nu}\right) = p_0 \begin{pmatrix} -2a_1 & \frac{3}{2}f_1 & 0 \\ \frac{3}{2}f_1 & -a_1 - \frac{3}{2}b_3 & 0 \\ 0 & 0 & -a_1 + \frac{3}{2}b_3 \end{pmatrix} ,$

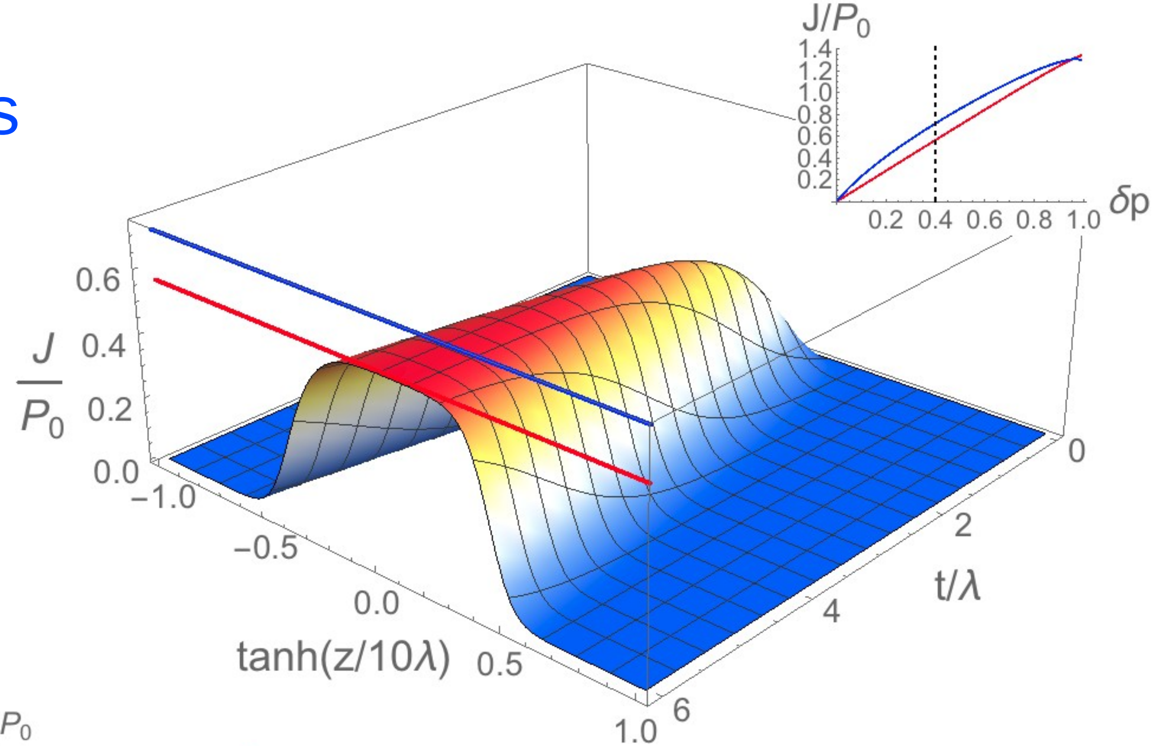
- To generate steady state impose:

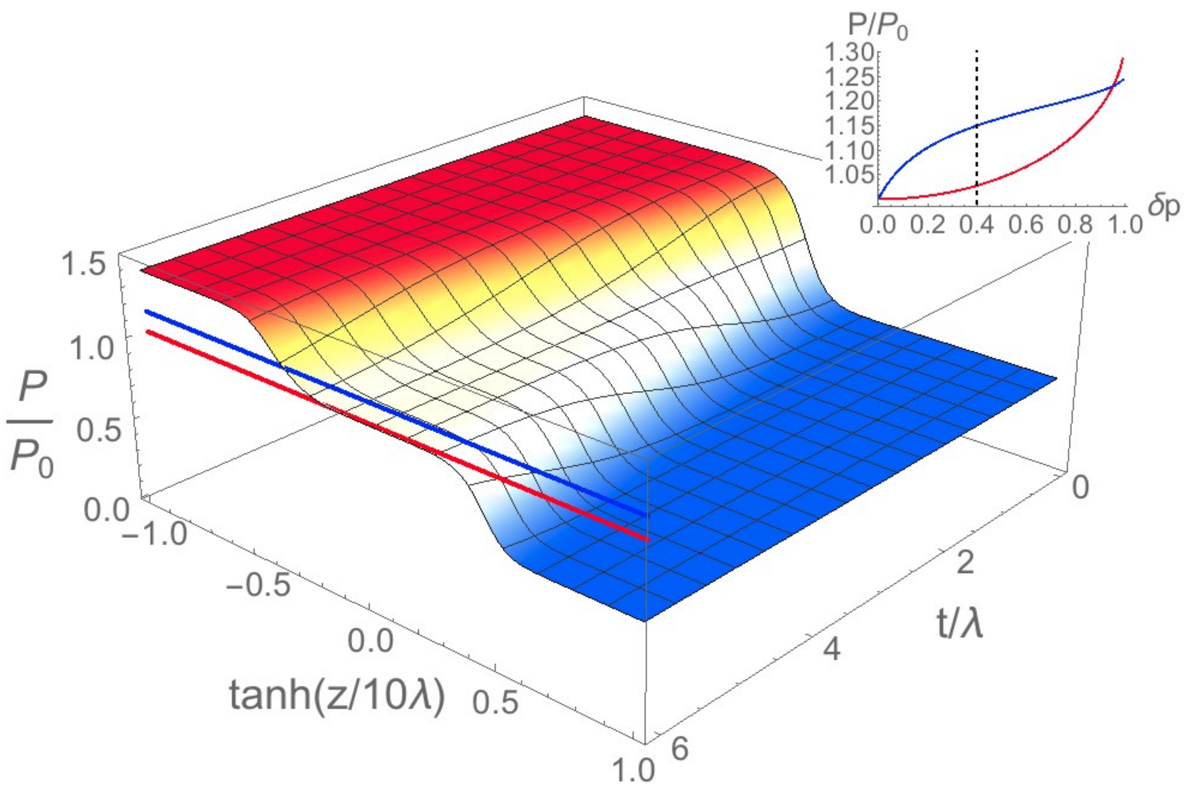
$$\boxed{\begin{array}{l} p_0 a_1(t, x \to \pm\infty) = -P(T_{R/L}) \\[2em] f_1(t=0, x \to \pm\infty) = 0 \qquad b_3(t=0, x \to \pm\infty) = 0 \end{array}}$$

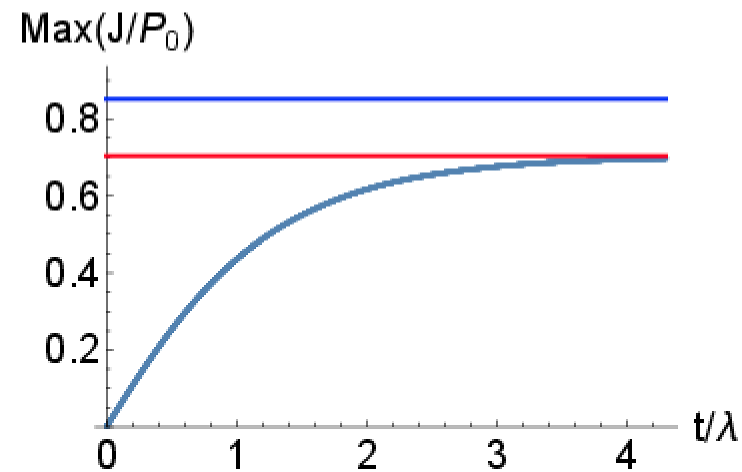
$$t = 0 \quad \to \quad B = 0 \qquad f_1 = 0 \qquad a_1 = -A_0\left(1 - \alpha \tanh\left(\beta \tanh\left(\frac{x}{\lambda}\right)\right)\right)$$

Numerical Results

- $\delta p = 0.4$







Conclusions

- 1+1 CFT steady state is universal. 1+D is conjectured to be too.
- Far from equilibrium CFT generates late time steady state
- Good agreement with the predicted universal result for $\delta p < 0.7$
- Very large pressure difference? Transition to the other branch?
- Extension to non-CFTs, add conserved currents
- Experimentally testable...
- Gauge/gravity: insight on far from equilibrium dynamics