Holographic Entanglement in Gauss-Bonnet gravity: time and shadows

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Outline

1. Motivation
2. Gauss Bonnet gravity
3. HEE in time dependent GB gravity
4. HEE shadows in GB gravity

Motivation

Bulk reconstruction. Emergence of spacetime.

1. String corrections, finite but large λ_{tHooft}
   - Generic form of higher derivatives corrections is not known
   - Effective five-dimensional gravity theory

$$S = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R - 2\Lambda + L^2(\alpha_1 R^2 + \alpha_2 R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma})\right)$$

   where $\Lambda = -6/L^2$ and we assume $\alpha_i << 1$.

2. Higher derivative theories can lead to interesting physics: *i.e.* $\frac{\eta}{s}$ bound violation for $\alpha_3 > 0$

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 8\alpha_3) + \mathcal{O}(\alpha_i^2)$$

3. Higher derivative theories have $c \neq a$

- Conformal anomaly of 4 dimensional CFT

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4$$

where

$$I_4 = C_{abcd} C^{abcd} = R_{abcd} R^{abcd} - 2 R_{ab} R^{ab} + \frac{1}{3} R^2$$

$$E_4 == R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2$$

- Holographically,

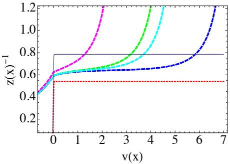
$$\langle T^\mu_\mu \rangle = \frac{\text{something}}{16\pi^2} I_4 - \frac{\text{something}'}{16\pi^2} E_4$$

- comparing both expressions we get

$$\alpha_3 \sim \frac{c - a}{8c}$$

TWO QUESTIONS

1) How deep behind the horizon does the HEE probe in time dependent GB theories?



2) In global AdS $\exists$ regions not probed by minimal surfaces, "shadows". Effect of λ_{GB} on shadows?

Do CFT dual to higher derivative theories "know" more about the bulk?

GAUSS-BONNET GRAVITY

$$S_{grav} = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \frac{\lambda L^2}{2}\mathcal{L}_{(2)}\right),$$
$$\mathcal{L}_{(2)} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

- Exact solutions are known

Black hole solution,

$$ds^2 = -\frac{L^2}{z^2}\frac{f(z)}{f_0}dv^2 + \frac{L^2}{z^2}\left(-\frac{2}{\sqrt{f_0}}dz dv + d\bar{x}^2\right),$$
$$f(z) = \frac{1}{2\lambda}[1 - \sqrt{1 - 4\lambda(1 - mz^4)}].$$

$$dv = dt - \frac{dz}{f(z)}$$

$$f_0 = \frac{1}{2\lambda}\left(1 - \sqrt{1-4\lambda}\right). \tag{1}$$

In Poincarè coordinates

$$ds^2 = -\frac{L^2}{z^2}\frac{f(z)}{f_0}dt^2 + \frac{L^2}{z^2}d\bar{x}^2 + \frac{L^2}{z^2}\frac{dz^2}{f(z)}. \tag{2}$$

Note that:

- Causality bounds:

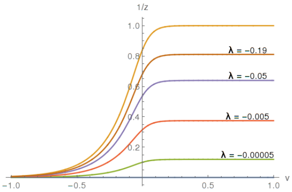
$$-7/36 \leq \lambda \leq 9/100, \tag{3}$$

- Central charges:

$$c = \pi^2 \frac{L^3}{l_p}(1 - 2\lambda f_0), \qquad a = \pi^2 \frac{L^3}{l_p^3}(1 - 6\lambda f_0) \tag{4}$$

- Singularity at finite z for $\lambda < 0$,

$$z_{sing} = \frac{1}{\sqrt{2}m(v)^{1/4}}(-1/\lambda + 4)^{1/4}$$



HEE in time dependent Gauss-Bonnet

$$S = S_{grav} + \kappa S_{ext}$$

where the external source is unespecified

$$ds^2 = -\frac{L^2}{z^2}\frac{f(z,v)}{f_0}dv^2 + \frac{L^2}{z^2}\left(-\frac{2}{\sqrt{f_0}}dzdv + d\bar{x}^2\right),$$

$$\text{where} \qquad f_0 = \frac{1}{2\lambda}(1-\sqrt{1-4\lambda}),$$

$$f(z,v) = \frac{1}{2\lambda}\left[1-\sqrt{1-4\lambda(1-m(v)z^4)}\right]$$

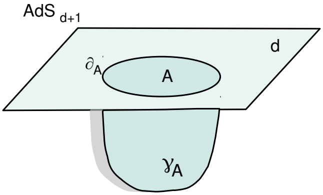
- $m(v)$ is arbitrary
- S_{ext} yields the following energy-momentum tensor

$$(16\pi G_N)\kappa T^{ext}_{\mu\nu} = \frac{3}{2}z^3\frac{dm}{dv}\delta_{\mu v}\delta_{\nu v}.$$

Previous work focused on thermalizarion time (Li, Wu and Yang 2013)

- Apparent horizon $z_{AH} = m(v)^{1/4}$
- Event horizon: $z'_{EH}(v) = -\frac{1}{2\sqrt{f_0}} f(z_{EH}, v)$

Covariant prescription



Hubeny, Rangamani, Takayanagi 07

$$S_A = \frac{Area_{extrm}(\gamma_A)}{G_N^{d+1}}$$

Codimension 2 surface
Homology condition
$\gamma_A \sim A$
$\exists$ bulk region r s.t. $\delta r = \gamma_A \cup A$

Entanglement entropy in GB

(Hung, Myers, Solkin, 2011)

$$S_{EE} = \frac{1}{4G_N}\int_{\Sigma} d^3\xi\sqrt{\gamma}(1+\lambda L^2 R_{\Sigma}) + \frac{1}{2G_N}\int_{\partial\Sigma} d^2\xi\sqrt{h}\,\lambda\, K$$

- R_{Σ} : Ricci scalar for intrinsic geometri on Σ
- K : trace of extrinsic curvature on $\partial\Sigma$

- Study “rectangular strip" for the time-dependent case.
- $z(x), v(x)$
- Induced metric on the co-dimension two surface is

$$ds^2 = \frac{L^2}{z^2}(dx_2^2 + dx_3^2) + \frac{L^2}{z^2}\left(1 - \frac{f}{f_0}v'^2 - \frac{2}{\sqrt{f_0}}v'z'\right)dx^2, \quad (5)$$

Thus,

$$\sqrt{\gamma} = \frac{L^3}{\sqrt{f_0}}\frac{1}{z^3}\left(f_0 - f v'^2 - 2\sqrt{f_0}v'z'\right)^{1/2} \;, \tag{6}$$

$$\lambda L^2\sqrt{\gamma}R_\Sigma = (2L^3\lambda\sqrt{f_0})\frac{z'^2}{z^3\left(f_0 - f v'^2 - 2\sqrt{f_0}v'z'\right)^{1/2}} + \frac{dF}{dz}\,, \tag{7}$$

where,

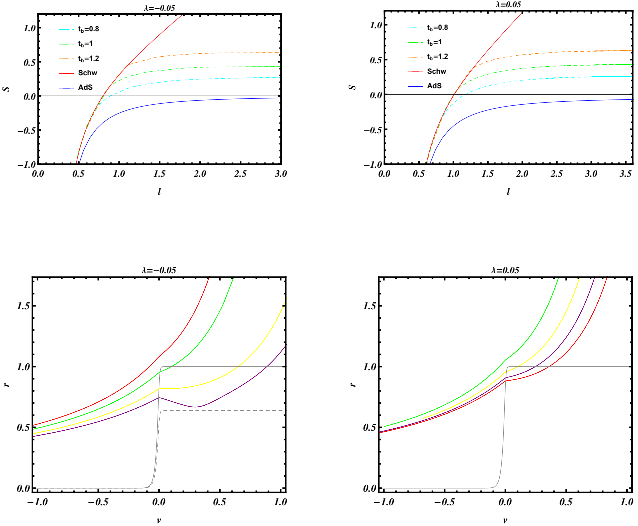
$$F(x) = (4L^3\lambda\sqrt{f_0})\frac{z'}{z^2\left(f_0 - f v'^2 - 2\sqrt{f_0}v'z'\right)^{1/2}} \tag{8}$$

Finally, action to be extremized is,

$$S_{\text{eff}} = \frac{L^3}{4G_N\sqrt{f_0}} \int \frac{dz}{z^3} \left[\left(f_0 - f v'^2 - 2\sqrt{f_0} v' z' \right)^{1/2} + \frac{2\lambda f_0 z'^2}{\left(f_0 - f v'^2 - 2\sqrt{f_0} v' z' \right)^{1/2}} \right]$$

- Time dependent, minimal surfaces penetrate the horizon, but do not reach singularity
- How does this change with λ
- in other words , the region accesible to the holographic probes increases or decreases with λ?

Results



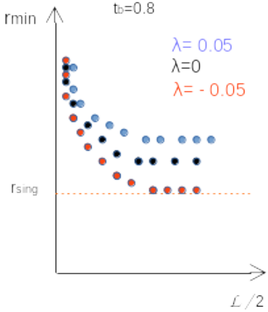


Figure : illustration of r_{min} vs $\ell/2$ (numerics in progress)

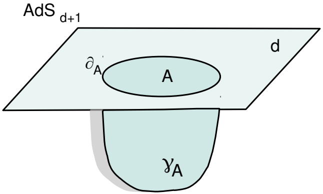
Theories with $\lambda < 0$ probe deeper behind the horizon than Einstein gravity . Theories with $\lambda > 0$ explore less

For $\lambda < 0$ and large ℓ the entanglement probes can reach arbitrarily close to the singularity

HEE Shadows in GB

Global, static.

Holographic Entanglement entropy



Ryu, Tagayanagi 06

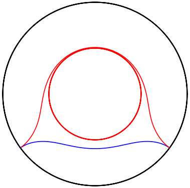
$$S_A = \frac{Area_{min}(\gamma_A)}{G_N^{d+1}}$$

Codimension 2 surface
Homology condition

$\gamma_A \sim A$

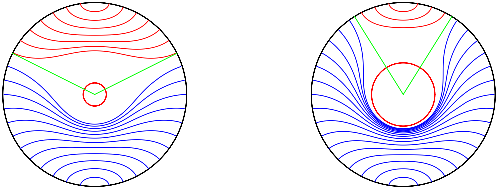
$\exists$ bulk region r s.t. $\delta r = \gamma_A \cup A$

Minimal surfaces in global BTZ



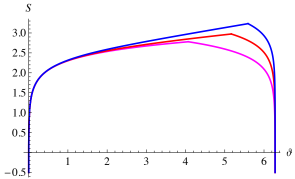
$$S_A(\vartheta) = \begin{cases} \frac{c}{3}\log\left(\frac{2\,r_\infty}{r_+}\sinh(r_+\vartheta/2)\right), & \vartheta \le \vartheta^{\mathcal{X}} \\ \frac{c}{3}\pi\,r_+ + \frac{c}{3}\log\left(\frac{2\,r_\infty}{r_+}\sinh(r_+(2\pi-\vartheta)/2)\right), & \vartheta \ge \vartheta^{\mathcal{X}} \end{cases}$$

$$\vartheta^{\mathcal{X}}(r_+) = \frac{2}{r_+}\coth^{-1}\left(2\coth(\pi\,r_+) - 1\right) .$$



SHADOWS

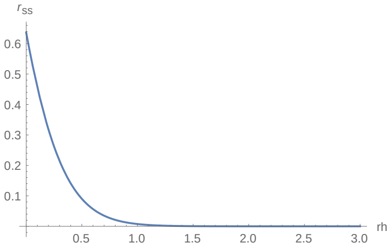
- Entanglement shadow: regions of the bulk not reached by any HEE probe *i.e.* maximum depth among all boundary regions. Balasubramanian et. al. 2014
- Behavior associated with phase transition



Entanglement Shadow

Freivogel et. al 14.12.5175

$$\Delta = r_* - r_h = \frac{2r_H e^{-\pi r_H}}{\sinh(\pi r_H)}$$



- $\Delta \sim \# r_H + ...$ for $r_H << \ell_{AdS}$
- $\Delta \sim r_H^2 e^{-\# r_H} + ...$ for $r_H >> \ell_{AdS}$

Similar limiting behaviour in AdS_5.

Entanglement Shadows in Gauss-Bonnett

- Black hole in global AdS_5, small λ
- Assume the 3-dimensional boundary region of interest is $O(3)$ symmetric, $r(\theta)$

$$ds^2 = -\frac{f(r)}{f_\infty}dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2(\theta)d\Omega_2)$$

$$f(r) = 1 + \frac{r^2}{2\lambda}(1 - \sqrt{1 + 4\lambda((rh^2 + rh^4 + \lambda))/r^4 - 4\lambda]})$$

where f_∞ is a convenient normalization factor, $f_\infty = \frac{1-\sqrt{1-4\lambda}}{2\lambda}$

HEE, prescription for higher derivatives.
Action to minimize,

$$\mathcal{L} = \sqrt{\frac{r'(\theta)^2}{f(r)} + r(\theta)^2}\left(r(\theta)^2 \sin(\theta)^2 + 2\lambda\right) + \\ 2\lambda \frac{r(\theta)^2 \cos(\theta)^2 + \sin(2\theta) r(\theta) r'(\theta) + \sin(\theta)^2 r'(\theta)^2}{\sqrt{\frac{r'(\theta)^2}{f(r)} + r(\theta)^2}}$$

Study shadows numerically –in progress.

Following Frievogel et al 14125175, approximate solution near the horizon

- expand eom close to the horizon
- assume $r'(\theta)$ is small

$$r''(\theta) + 2\cot(\theta) r'(\theta) + r(\theta) H(rh, \lambda) + \tilde{H}(rh, \lambda)$$

Can be solved with $r(0) = r_*, \;\; r'(0) \sim 0$.
For small λ,

$$r(\theta) = \frac{1}{k^3 rh^2} \csc(\theta)(k^3 rh^3 \sin(\theta) + (rh - r_s)(6k(1 + 2rh^2)\theta\lambda\cosh(k\theta) - (6\lambda + rh^2(k^2 + 12\lambda))\sinh(k\theta))$$

where $k = \sqrt{5 + rh^2}$

Shadow size: $r_* - r_h \equiv \Delta$,

- Large black holes, similar behaviour as $\lambda = 0$

$$\Delta \sim rh^2 e^{-\# rh}$$

- Small black holes

$$\Delta \sim \# rh + \lambda \, p(rh)$$

$$\text{where} \;\; p(rh) > 0$$

$\rightarrow$ For $\lambda < 0$ shadow is smaller

Conclusions:

- In time dependent case, EE can explore arbitrarily close to the singularity.
- In static global case, shadow size is smaller for$\lambda_{GB} < 0$

Theories with $\lambda_{GB} < 0$ "know more" of the bulk than $\lambda \geq 0$.

- Bulk reconstruction. What CFT observables access regions in entanglement shadow? *i.e* what is the right probe?
- How generic is the entanglement shadow region?
- Nonlocality?