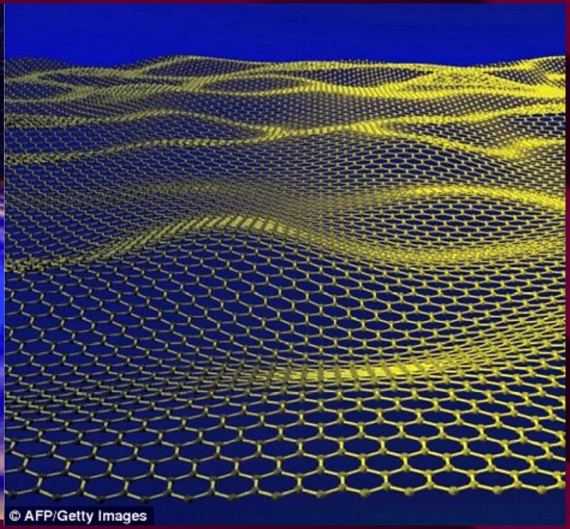
Vacuum Alignment in Holographic Graphene

Nick Evans   University of Southampton

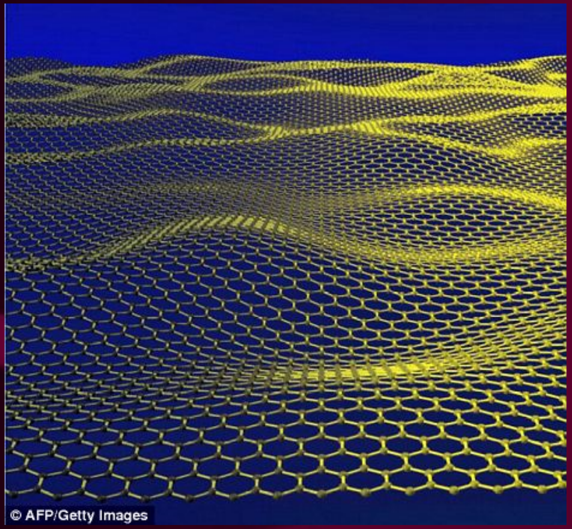
Keun-Young Kim

Peter Jones



Florence, April 2015

Motivation 1 - Graphene



Graphene is a 2+1d surface embedded in a 3+1d space

The low energy effective degrees of freedom on the surface are Dirac fermions

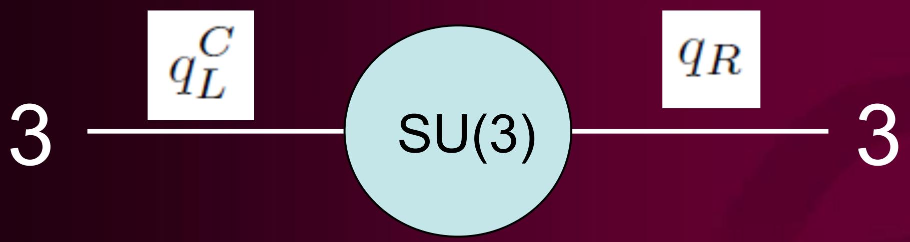
They interact with 3+1d QED but through

$$\alpha_{EM} = \frac{e^2}{4\pi\epsilon_0 \hbar c_{eff}}$$

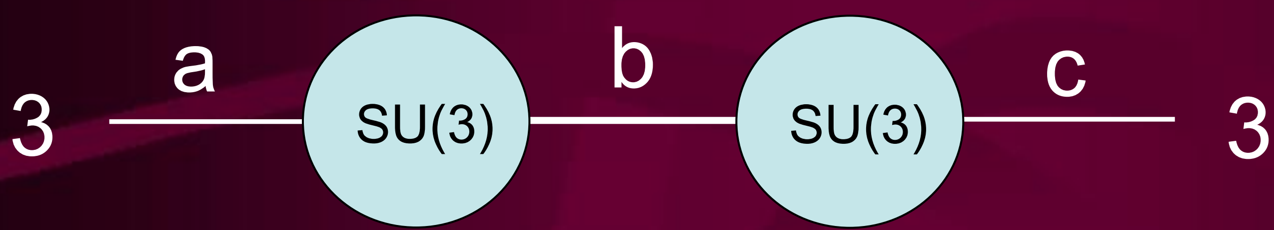
The interactions may be strongish… although the theory is near conformal with no mass gap.

This provides some motivation to study 2+1d probe defects in N=4 SYM in 3+1d using holography… can we throw up any new phenomena that might be experimentally realized?

Motivation 2 – Vacuum Alignment at Strong Coupling



This is a ``moose'' of QCD. The strong interactions generate a condensate $\langle \bar{q}_L q_R \rangle$ which breaks the chiral symmetries to the vector…



But what happens in this extended moose when both gauge groups are at strong coupling? Do a and b condense breaking the b flavour gauge group? b and c condense? Something else?

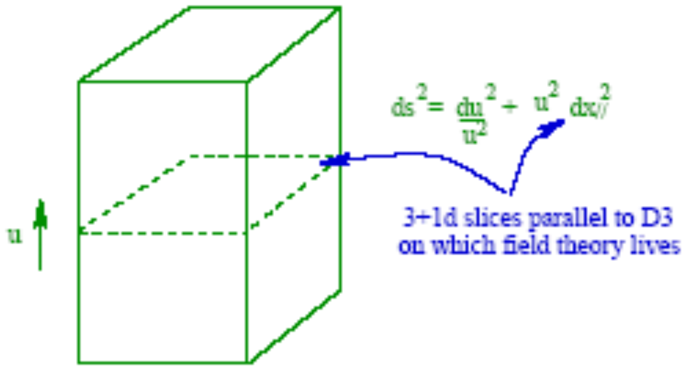
I've long been interested in setting up a holographic competition between two condensation patterns… we will realize something like this…

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$

Pretty well established by this point!



u corresponds to energy (RG) scale in field theory

The SUGRA fields act as sources

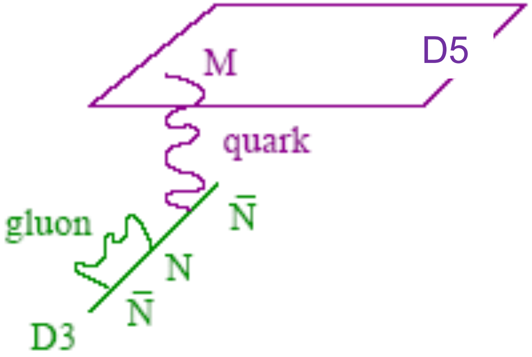
$$\int d^4x \; \Phi_{SUGRA}(u_0)\lambda\lambda$$

eg asymptotic solution ($u \to \infty$) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle\lambda\lambda\rangle}{u^3}$$

Adding Quarks

Karch, Randall + Erdmenger, Guralnik, Kirsch



Quarks can be introduced via D5 branes in AdS

The brane set up is

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D5	-	-	-	•	-	-	-	•	•	•

We will treat D5 as a probe – quenching in the gauge theory

Minimize D5 world volume with DBI action

$$S_{D5} = -T_5 \int d\xi^6 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

The Field Theory

DeWolfe, Freedman, Ooguri

N=4 SYM bulk

$$\begin{aligned} S_4 &= \frac{1}{g^2}\int d^4x[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{i}{2}\bar{\lambda}^a\gamma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a + \frac{\theta}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \\ &+ (D^\mu X^{Aa})^\dagger D_\mu X^{Aa} - \frac{i}{2}\bar{\chi}^{Aa}\gamma^\mu D_\mu\chi^{Aa} + F^{Aa}\bar{F}^{Aa} \\ &+ \sqrt{2}f^{abc}(\bar{X}^{Ab}\bar{\lambda}^a L\chi^{Ac} - \bar{\chi}^{Ab}R\lambda^a X^{Ac}) + if^{abc}\bar{X}^{Ab}D^aX^{Ac} \\ &+ \frac{1}{\sqrt{2}}\epsilon_{ABC}f^{abc}(F^{Aa}X^{Bb}X^{Cc} + \bar{F}^{Aa}\bar{X}^{Bb}\bar{X}^{Cc} - \bar{\chi}^{Aa}(LX^{Cc} + R\bar{X}^{Cc})\chi^{Bb})] \end{aligned}$$

2+1d brane hypermultiplet

$$\begin{aligned} S_3 &= S_{kin} + S_X\,, \\ S_{kin} &= \frac{1}{g^2}\int d^3x\left((D^kq^i)^\dagger D_kq^i - i\bar{\Psi}^i\rho^kD_k\Psi^i + \bar{f}^if^i + i\bar{q}^i\bar{\lambda}_1^aT^a\Psi^i - i\bar{\Psi}^i\lambda_1^aT^aq^i\right) \\ S_X &= \frac{1}{g^2}\int d^3x\,[-\sigma^A_{ij}\bar{\Psi}^iX_V^{Aa}T^a\Psi^j - \sigma^A_{ij}(\bar{q}^i\bar{\chi}_1^{Aa}T^a\Psi^j + \bar{\Psi}^i\chi_1^{Aa}T^aq^j) \\ &+\sigma^A_{ij}(\bar{q}^iX_V^{Aa}T^af^j + \bar{f}^iX_V^{Aa}T^aq^j + \bar{q}^i(F_V^{Aa} - D_6X_H^{Aa})T^aq^j)]\,. \end{aligned}$$

$\mathcal{N} = 4$ supersymmetric

$SO(2,1)$

$SO(3)\times SO(3) \sim SU(2)_H\times SU(2)_V$

A hypermultiplet mass breaks SO(3) → SO(2)

Quarks In AdS

[redacted]

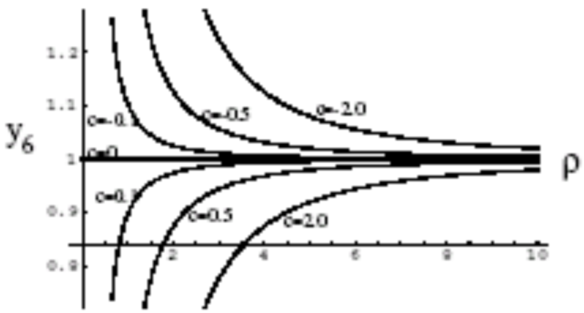
$$S_{D5} = -T_5 \int d^6\xi \, \epsilon_2 \, \rho^2 \sqrt{1 + \frac{R^2 g^{ab}}{\rho^2 + w_5^2 + w_6^2} (\partial_a w_i \partial_b w_i)}$$

EoM is: $\frac{d}{d\rho}\left[\frac{\rho^2}{\sqrt{1+\left(\frac{du_6}{d\rho}\right)^2}} \frac{du_6}{d\rho} \right] = 0$

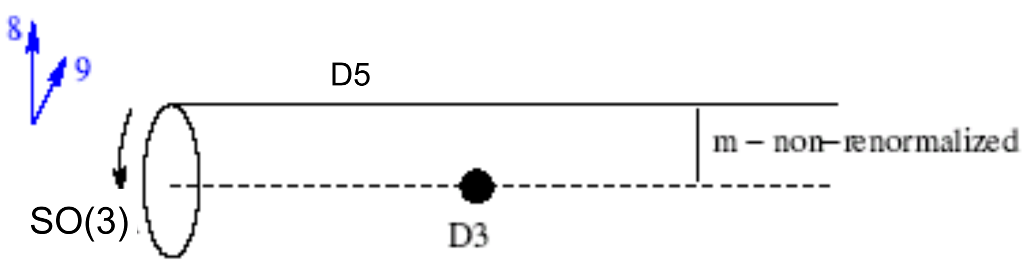
UV asymptotic solution is

$$u_6 = m + \frac{c}{\rho} + \ldots$$

m is the quark mass, c the $\langle \bar{q} q \rangle$ condensate



In AdS regular D5 solution is flat brane



Mesons in AdS_5

Kruczenski, Myers, Mateos, Winters

The D7 lie flat in AdS. We can consider fluctuations that describe R-chargeless mesons

$$w_6 + i w_5 = d + \delta(\rho) e^{ik.x}$$

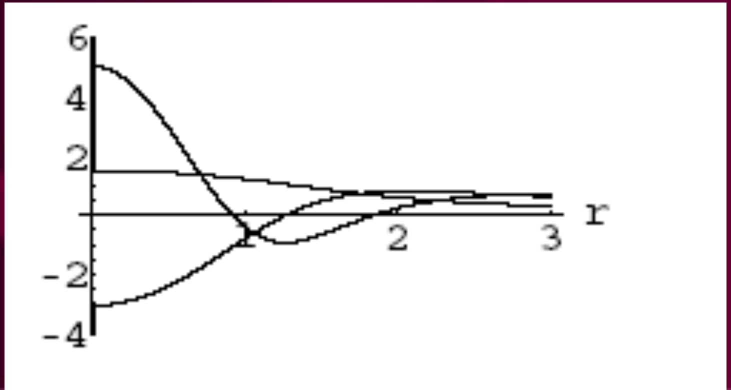
δ satisfies a linearized EoM

$$\partial_z^2 \delta(z) + \frac{\bar{M}^2}{(d^2 z^2 + 1)^2} \delta(z) = 0 \qquad z = 1/\rho$$

and the mass spectrum is

$$M_n = \frac{d}{R^2} \sqrt{(2n+1)(2n+3)}, \qquad n = 0, 1, 2, \ldots$$

Tightly bound - meson masses suppressed relative to quark mass



Orthonormal wave functions

Magnetic Field Induced Symmetry Breaking

$$P[G]+F=\begin{pmatrix} -\frac{r^2}{R^2} & & & & & \\ & -\frac{r^2}{R^2} & B & & & \\ & -B & -\frac{r^2}{R^2} & & & \\ & & & \frac{R^2}{r^2}(1+(\partial_\rho w_6)^2) & & \\ & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & \frac{R^2}{r^2}\rho^2.. \end{pmatrix}$$

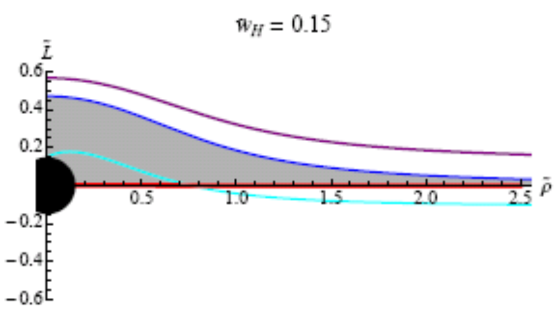
$$\mathcal{L}=\rho^2\sqrt{1+(\partial_\rho w_6)^2}\sqrt{1+\frac{B^2R^4}{r^4}}$$

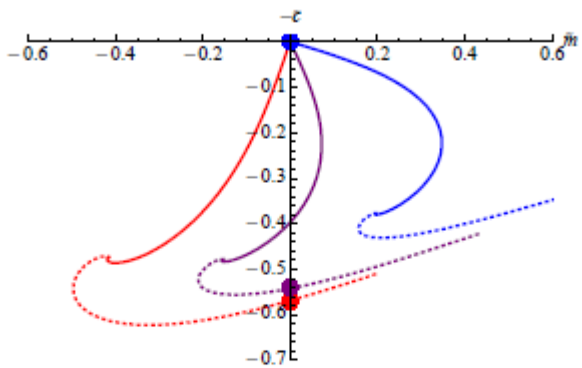
Johnson, Filev, Kundu….

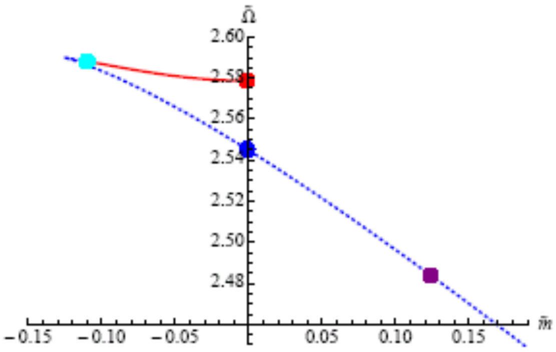
Put in B thorough susy partner of mesons..

$$A^\mu \sim \bar{q}\gamma^\mu q + A^\mu_{\text{background}}$$

Not B of SU(N)..





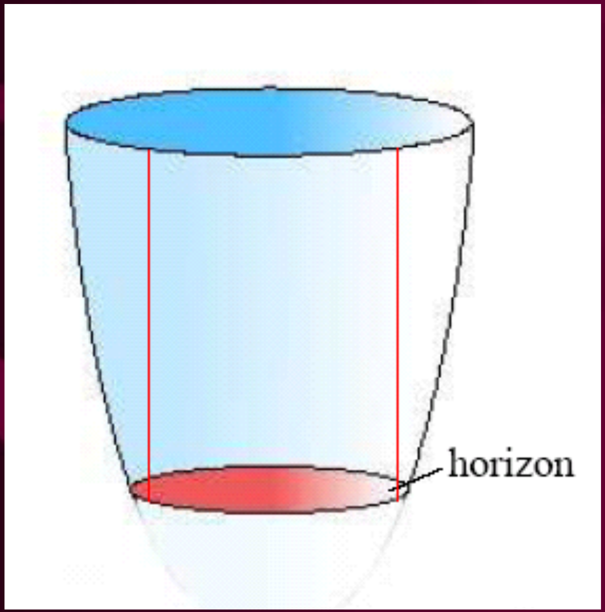


Finite T - AdS-Schwarzschild

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f} dr^2 + R^2 d\Omega_5^2$$

where $R^4 = 4\pi g_s N \alpha'^2$ and

$$f := 1 - \frac{r_H^4}{r^4}\ , \qquad r_H := \pi R^2 T\ .$$



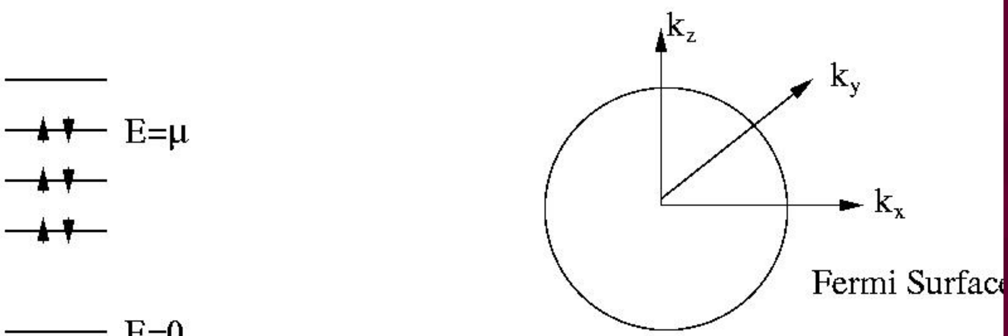
Quarks are screened by plasma

Asymptotically AdS, SO(6) invariant at all scales… horizon swallows information at rH …. Witten interpreted as finite temperature… black hole… has right thermodynamic properties…

Chemical Potential

Kobayashi, Mateos, Myers, Matsuura, Thomson

At finite density the Fermi-sea of quarks fills up to an energy called the chemical potential

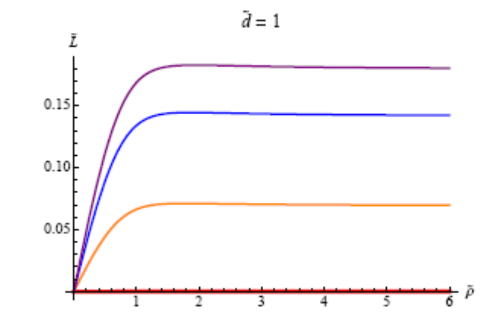


$$\bar{\psi}_i (i \not{D} + \mu \gamma_0) \psi_i$$

$$\bar{\psi} i (-i A^t \gamma_0) \psi \quad \rightarrow \quad \bar{\psi} \mu \gamma_0 \psi$$

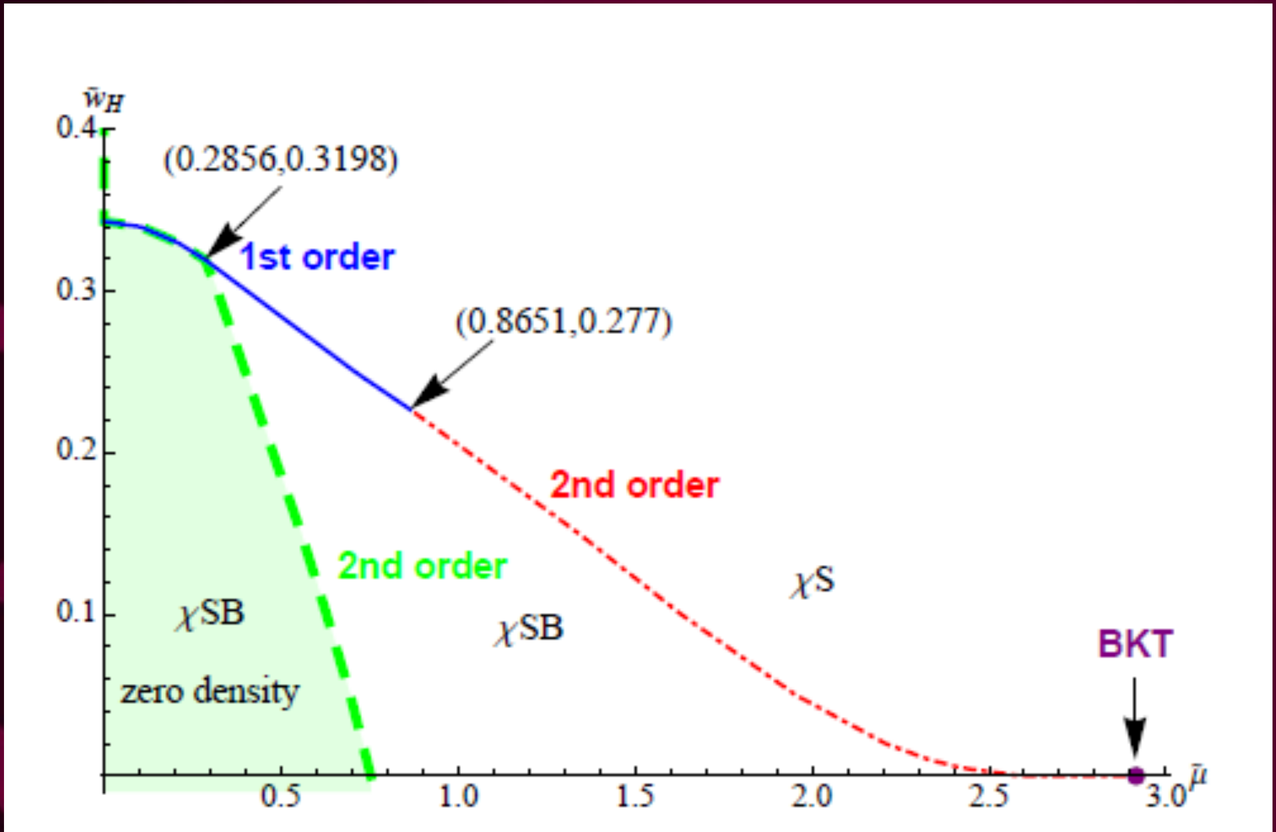
We can think of m as a background vev for the temporal component of the photon…

$$P[G] + F = \begin{pmatrix} -\frac{r^2}{R^2} & & & \partial_\rho A_0 & & \\ & -\frac{r^2}{R^2} & B & & & \\ & -B & -\frac{r^2}{R^2} & & & \\ \partial_\rho A_0 & & & \frac{R^2}{r^2}(1+(\partial_\rho w_6)^2) & & \\ & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & \frac{R^2}{r^2}\rho^2.. \end{pmatrix}$$

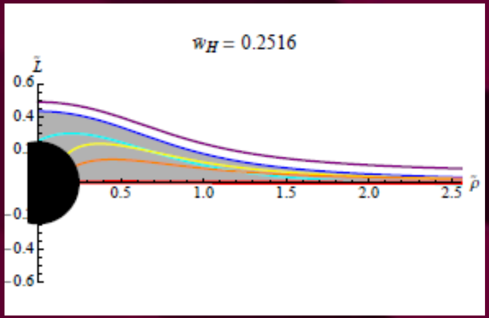


μ induces quarks to fill the vacuum…. ie a spike of strings grows between the D5 and the D3…

Phase Diagram for B Field Theory, m=0



with Keun-Young Kim and Maria Magou **arXiv: 1003.2694**



BH wants to eat…

Density wants to spike

B wants to curve off axis

Quasi-normal modes & meson melting

BEEGK… Sonnenschein… Hoyos…. Myers, Mateos…

Linearized fluctuations in eg the scalars on the D5 brane must now enter the black hole horizon…

Quasi-normal modes are those modes that near the horizon have only in-falling pieces…

The mass of the bound states become complex – they decay into the thermal bath…

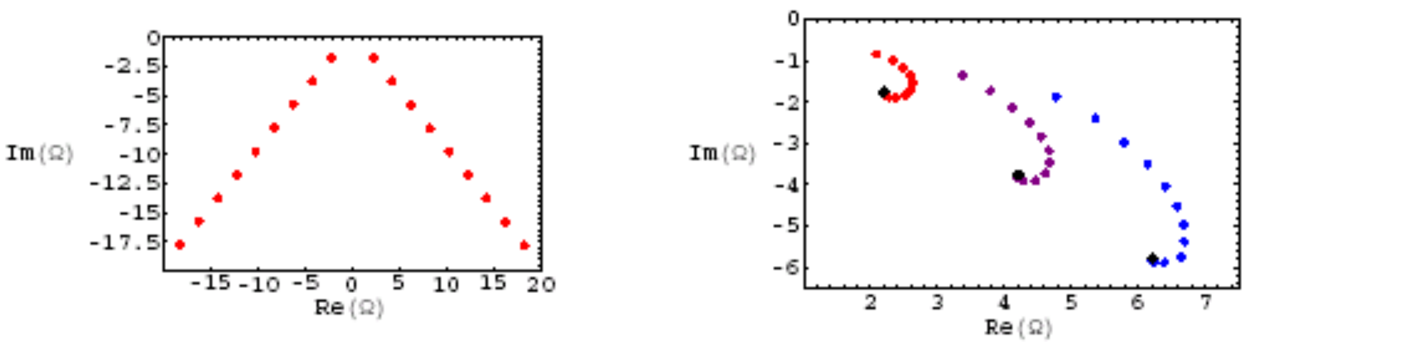


Figure 7.4: The lowest quasi-normal modes for $m_q = 0$ on the left and the three lowest quasi-normal modes for increasing m_q on the right. The black points on the right show the limiting values for $m_q = 0$.

Second Order Mean Field Behaviour

A mean field second order transition is just an effective Landau - Ginsberg (Higgs) Model

$$V_{eff}(\phi) = \alpha_2(O_c - O)\phi^2 + \alpha_4\phi^4$$

$$\phi \sim \sqrt{O - O_c}$$

Holographic Berezinskii-Kosterlitz-Thouless Transitions with Kristan Jensen

$$S_5 = \int d\rho\, \rho^2 \sqrt{1 + L'^2 - A_0'^2}\sqrt{1 + \frac{B^2}{w^4}}$$

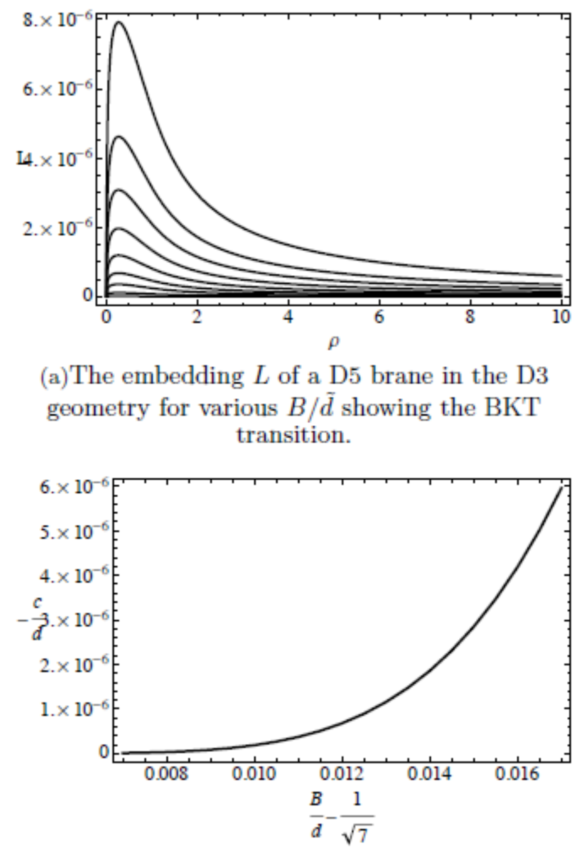
T=0 transition changes…

K. Jensen, A. Karch, D. T. Son, and E. G. Thompson, Phys. Rev. Lett. **105**, 041601 (2010), 1002.3159.

Exponential scaling of order parameter away from the transition…

D. B. Kaplan, J.-W. Lee, D. T. Son, and M. A. Stephanov, Phys. Rev. **D80**, 125005 (2009), 0905.4752.

Key is in D3/D5 system d and B have same dimension…

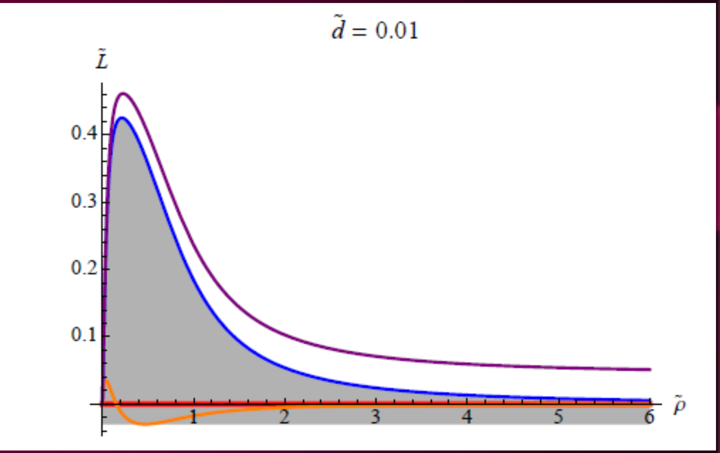


(a)The embedding L of a D5 brane in the D3 geometry for various $B/\tilde{d}$ showing the BKT transition.

(b)A plot of the quark condensate c versus B across the D3/D5 BKT transition.

Instability of flat embedding

$$\tilde{\mathcal{L}}_5 \sim -\frac{\mathcal{N}}{2}\sqrt{\tilde{d}^2 + B^2 + \rho^4}L'^2 + \frac{\mathcal{N}B^2L^2}{\rho^2\sqrt{\tilde{d}^2 + B^2 + \rho^4}}$$



Small rho limit solutions:

$$\frac{L}{\rho} \sim \left(\frac{1}{\rho}\right)^{\Delta}$$

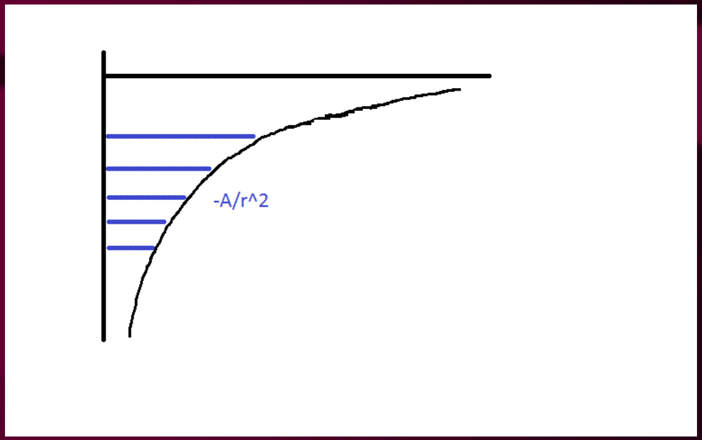
$$\Delta_{\pm} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 + m^2}$$

$$m^2 = -2B^2/(\tilde{d}^2 + B^2)$$

$$\Delta_{\rm IR} = \frac{1 + \sqrt{\frac{\tilde{d}^2 - 7B^2}{\tilde{d}^2 + B^2}}}{2}$$

B and d enter on same footing because same dimension…. For fixed d raising B triggers complex D - an instability that correctly predicts the transition point…

The Schroedinger well becomes unstable (A > 1/4) with an infinite number of negative energy states growing from zero… leading to exponential behaviour…



Breitenlohner-Freedman (BF)

In our analysis we use the results for a scalar in AdS_{p+1}: The solution of the equation of motion is

$$\frac{L}{\rho} \sim \left(\frac{1}{\rho}\right)^{\Delta} \tag{11}$$

$$\Delta_{\pm} = \frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 + m^2}\ . \tag{12}$$

and the Breitenlohner-Freedman (BF) bound [65] is given by $-p^2/4$

$\mathrm{AdS}_2,\ m^2_{\mathrm{BF}} = -1/4$

0+1d theory rules IR?

Small rho limit mass

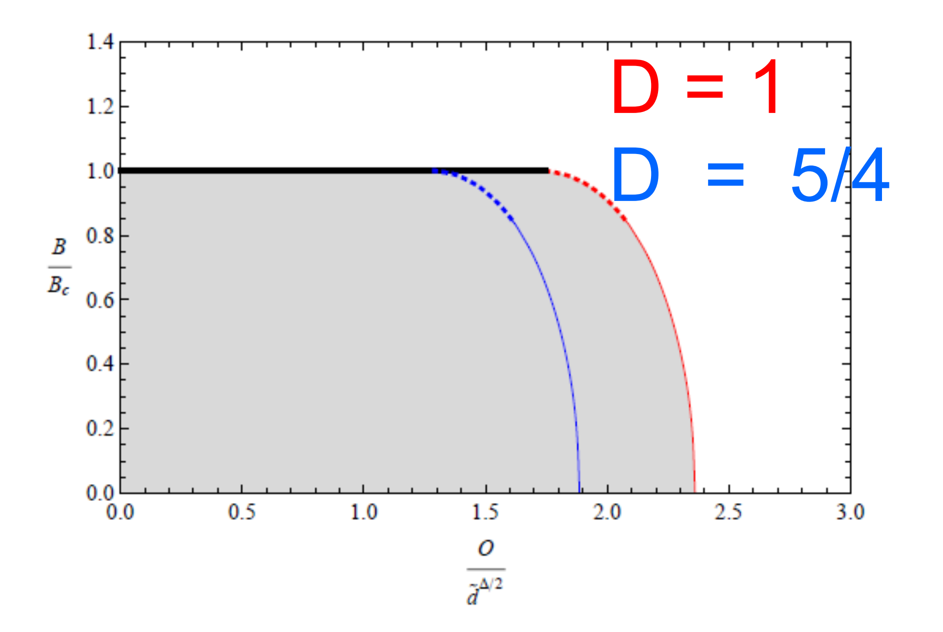
$m^2 = -2B^2/(\tilde{d}^2 + B^2)$

From Mean-Field 2nd Order to BKT

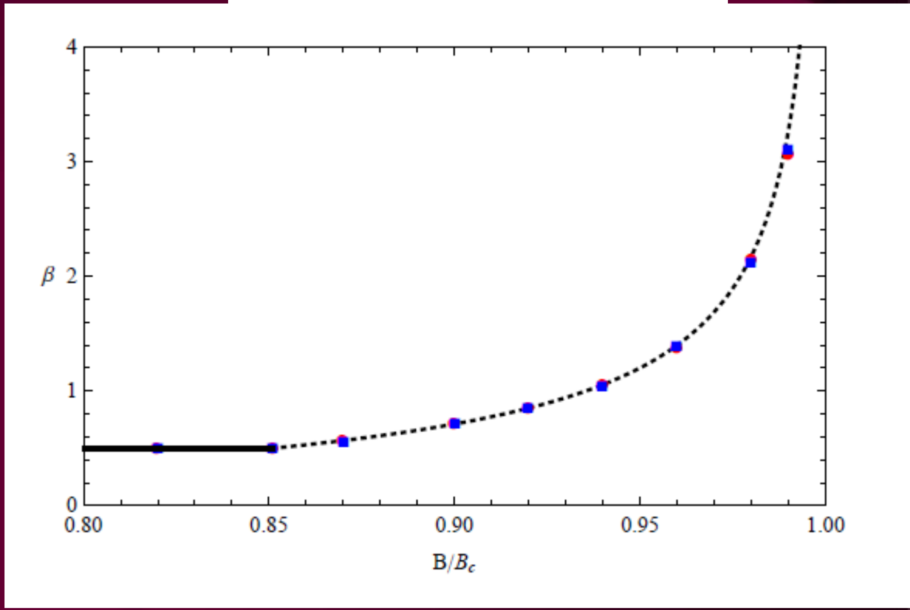
With Keun-Young Kim and Kristan Jensen arXiv:1008.1889

$$\tilde{S}_5 = -\mathcal{N}\int d\rho\,\sqrt{1+L'^2}\sqrt{\tilde{d}^2+\rho^4\left(1+\frac{B^2}{w^4}+\frac{O^2}{w^{2\Delta}}\right)}$$

If we add a phenomenological operator O that causes symmetry breaking but is not dim 2… B+d triggers BKT…. O +d is second order mean-field… what about O+B+d:



$$c \sim (B - B_c)^{\beta}$$



Faulkner, Horowitz,Roberts - **arXiv:1008.1581** [hep-th]

Bilayer Exciton Condensation

Now consider two separated D5/ graphene sheets (Karch…, Skenderis, Taylor…)

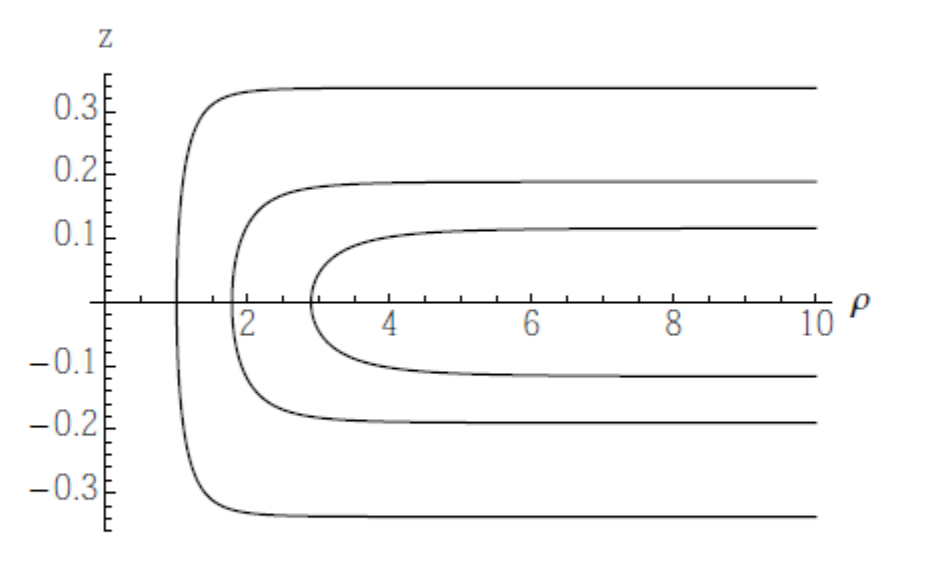
Semenoff…

(NB we are not making graphite which has a gap, but aligning sheets to keep two sets of massless fermions – you might stick them to the sides of some substrate…)

$$S \sim T \int d^6\xi e^{\phi} \sqrt{-\mathrm{det} G}$$
$$\sim \int d\rho\, e^{\phi} \rho^2 \sqrt{1 + L'^2 + (\rho^2 + L^2)^2 z'^2}\,,$$

$$\partial_\rho \left[\frac{\rho^6 z'}{\sqrt{1+\rho^4 z'^2}} \right] = 0\,.$$

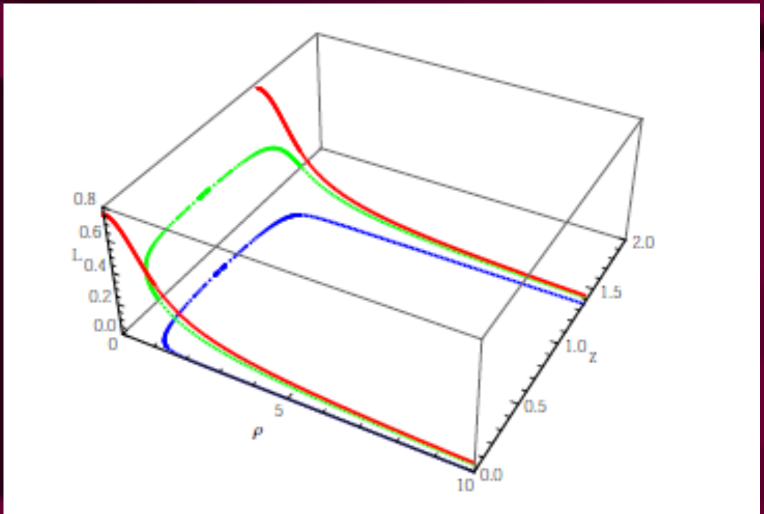
$$\Pi_z = \frac{\rho^6 z'}{\sqrt{1+\rho^4 z'^2}}\,.$$



There is a Sakai-Sugimoto like condensation – it is condensation between fermions on one sheet and those on the other

Bilayer Condensation vs Monolayer Condensation

With Keun-Young Kim ArXiv:1311.0149



A B field generates a condensate within a layer…. The N=4 field generate a condensation between layers… which wins? Can both condensates exist at one time?

$$S \sim \int d\rho\, \rho^2 \sqrt{1 + \frac{1}{(\rho^2+L^2)^2}} \sqrt{1 + L'^2 + (\rho^2+L^2)^2 z'^2}\,.$$

$$\Pi_z = \rho^2 \sqrt{1 + \frac{1}{(\rho^2+L^2)^2}} \frac{(\rho^2+L^2)^2 z'}{\sqrt{1 + L'^2 + (\rho^2+L^2)^2 z'^2}}\,.$$

The Legendre transformed action is

$$S_{LT} \simeq \int d\rho\, \sqrt{1+L'^2} \frac{\sqrt{\rho^4(1+(\rho^2+L^2)^2) - \Pi_z^2}}{\rho^2+L^2}\,.$$

$$z'^2 = \frac{\Pi_z^2(1+L'^2)}{\rho^4(\rho^2+L^2)^2(1-\Pi_z^2+(\rho^2+L^2)^2)}\,.$$

Use Pz conserved quantity to reduce the problem to a single ODE.

Pick Pz

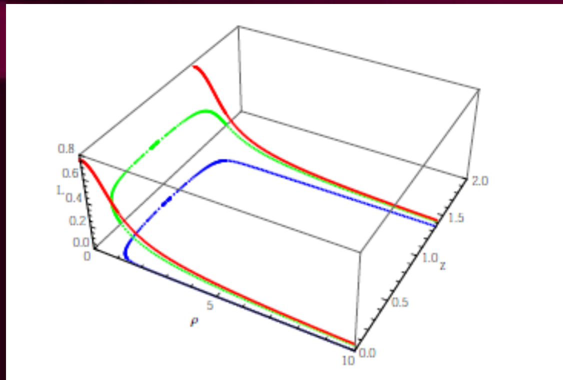
Solve for L subject to L'(r_min)=0

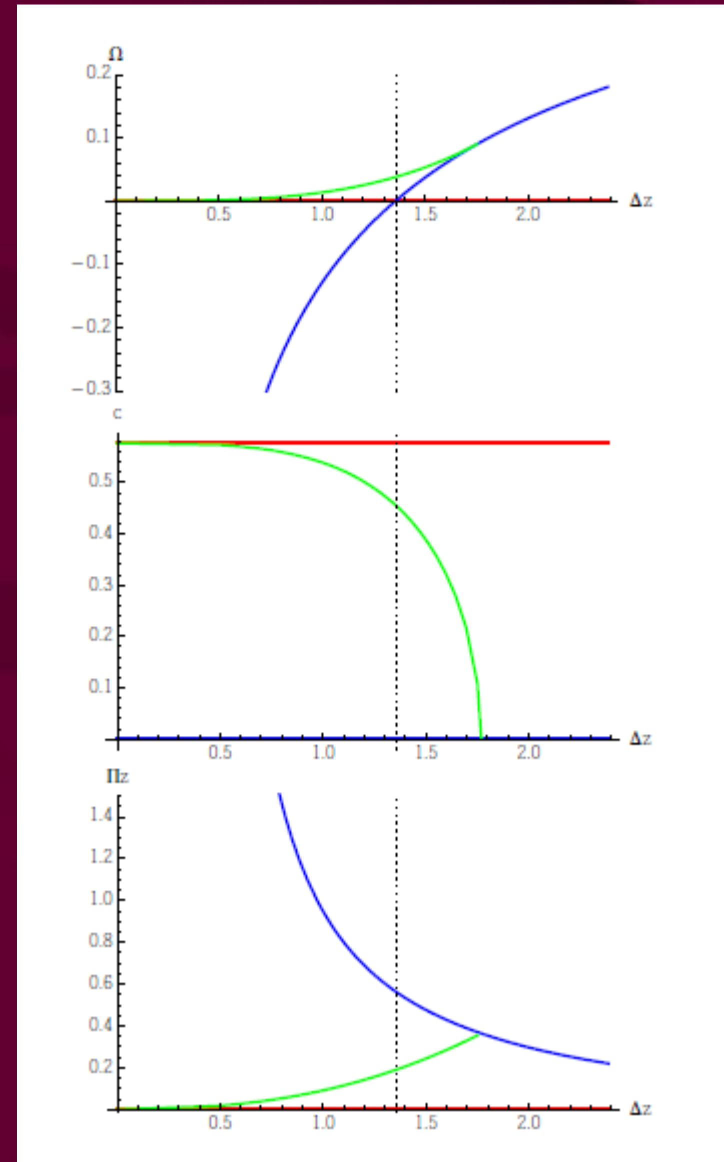
Now solve for z.. Is z' infinite at r_min?

Try a new r_min until a smooth embedding is found

Try a new Pz to get a new separation

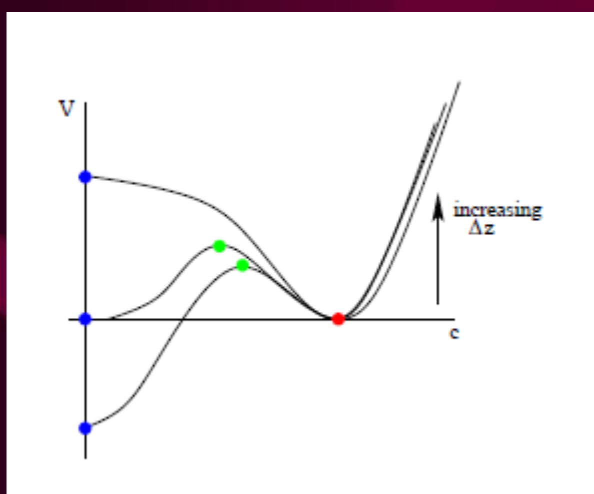
Bilayer Condensation vs Monolayer Condensation





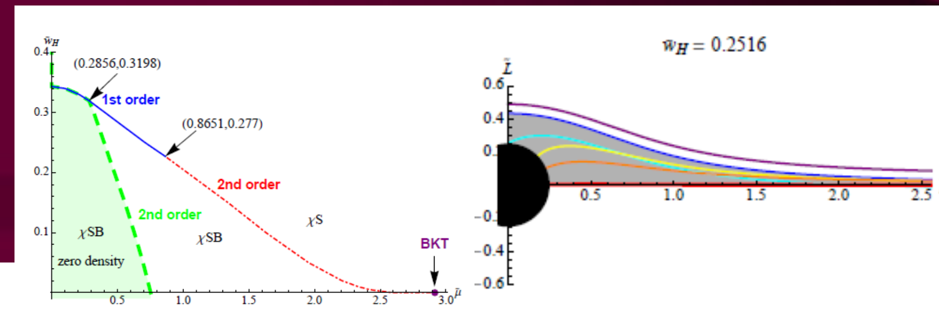
A first order transition between single and bilayer condensates as Dz is decreased…

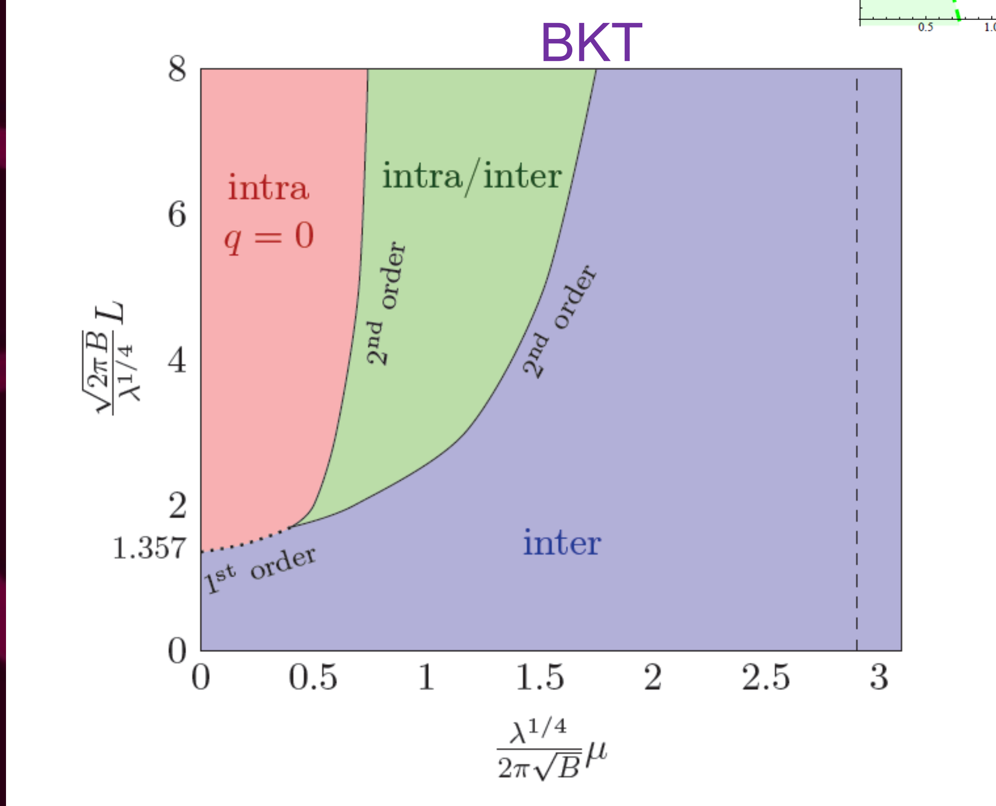
Mixed condensate configurations exist… but are always local potential maxima…



Holographic D3-probe-D5 Model of a Double Layer Dirac Semimetal

Gianluca Grignani,[a] Namshik Kim,[b] Andrea Marini,[a] Gordon W. Semenoff[b]





At finite density the dual condensation mechanisms do co-exist!

Finite T phase diagram under investigation…

Graphene in a Cavity

With Peter Jones arXiv:1407.3097

"Graphene is probably not strongly coupled but close to it… one way to change the effective coupling of QED is to place it in a cavity between mirrors…"

$$\int d^3x dz \frac{1}{e^2} F^2 = \int d^3x \frac{L}{e^2} F^2$$

N=4 on a Compact Space

Use the AdS soliton…

$$ds^2 = \frac{R^2}{r^2} h^{-1}(r) dr^2 + \frac{r^2}{R^2}\left(dx_{2+1}^2 + h(r) dz^2\right) + d\Omega_5^2$$

with

$$h(r) = 1 - \left(\frac{r_0}{r}\right)^4$$

the circumference of the z direction is $R^2\pi/r_0$.

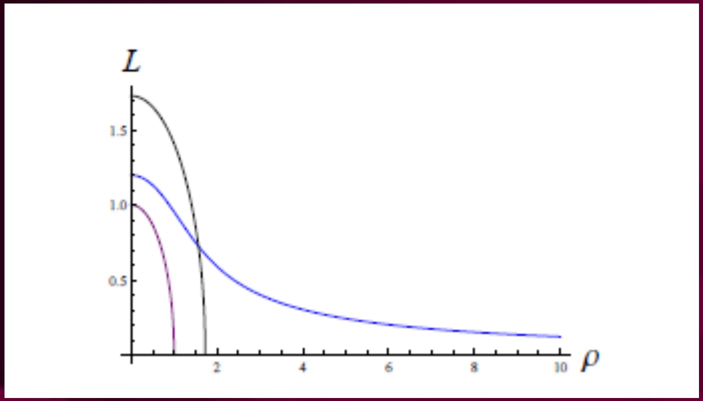
$$w = \left(r^2 + (r^4 - r_0^4)^{1/2}\right)^{1/2}$$

$$ds^2 = \frac{w^2}{R^2}\left(g_x dx_{2+1}^2 + g_z dz^2\right) + \frac{R^2}{w^2}\left(dw^2 + w^2 d\Omega_5^2\right)$$

$$g_x = \left(\frac{w^4 + r_0^4}{2w^4}\right)$$

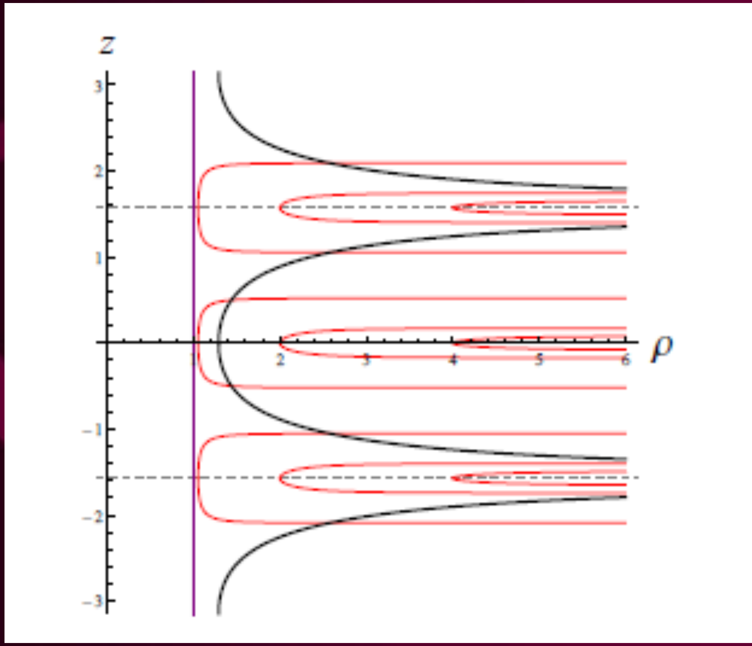
$$g_z = \frac{(w^4 - r_0^4)^2}{2w^4(w^4 + r_0^4)}$$

Probe D5 in Compact N=4 SYM



A monolayer – the blue embedding closes off before the geometry does at r0=1

Chiral symmetry breaking.

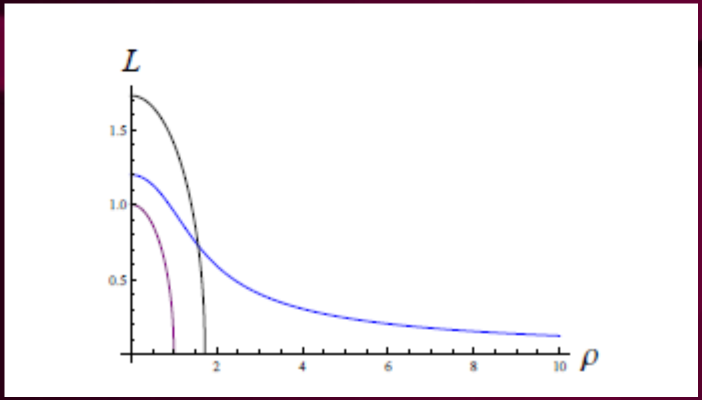


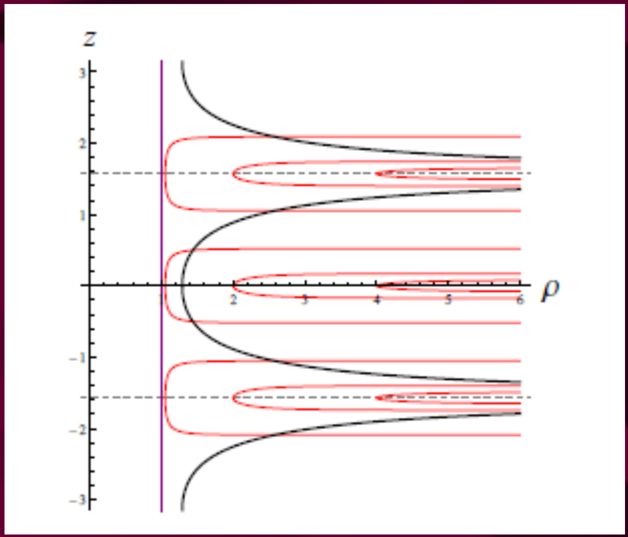
Bilayers – red linked solutions

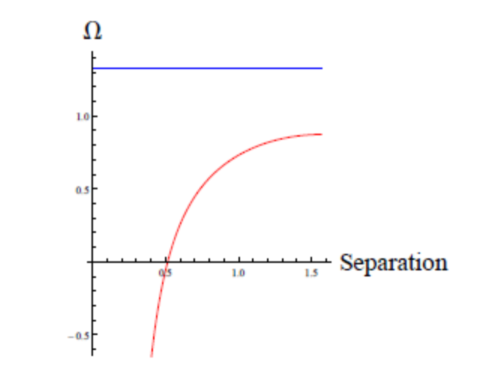
Those that dip down to r0 are precisely half the width of the circle apart… you can wrap both ways…

Exciton cendensation

Probe D5 in Compact N=4 SYM

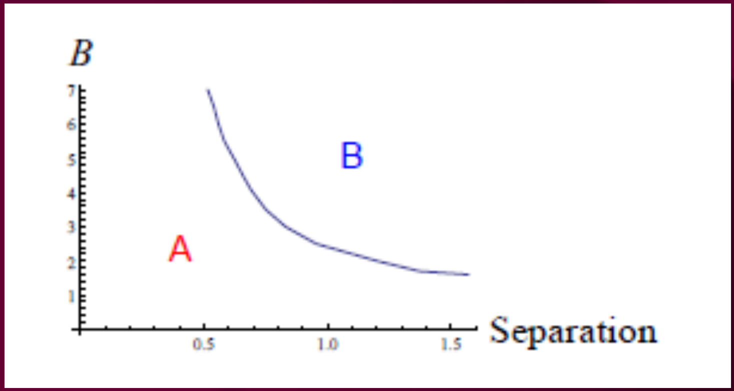






Linked solutions are always energetically favoured.

Unless you add B…



N=4 SYM + Probes Inbetween Mirrors

Takayanagi proposed that to put N=4 between mirrors should use the soliton… treat the boundaries as surfaces of constant tension… arXiv:1108.5152

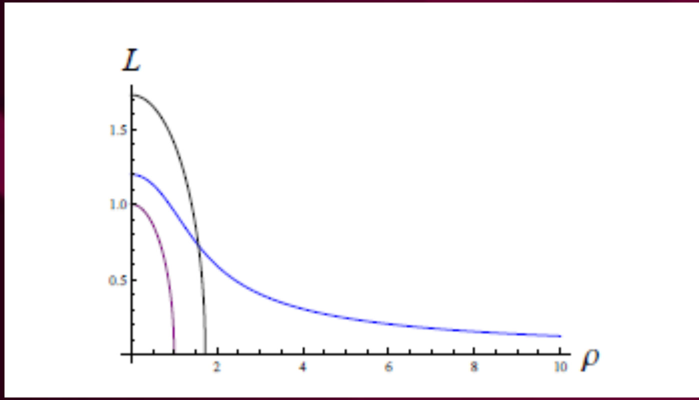
$$I = \frac{1}{16\pi G_N}\int_{\text{bulk}}\sqrt{-g}(R-2\Lambda) + \frac{1}{8\pi G_N}\int_{\text{bound}}\sqrt{-h}(K-\mathcal{T})$$

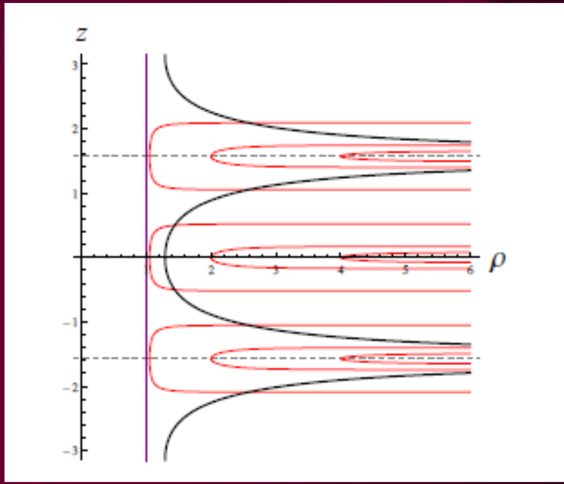
Require tensions match at all r

$$K_{ab} = (K-\mathcal{T})h_{ab}$$

$$z'(r) = \pm\frac{R\mathcal{T}}{r^2 h(r)\sqrt{4h(r)-R^2\mathcal{T}^2}}$$

This produces the black edge to the space… so the D5 embeddings then don't make sense…



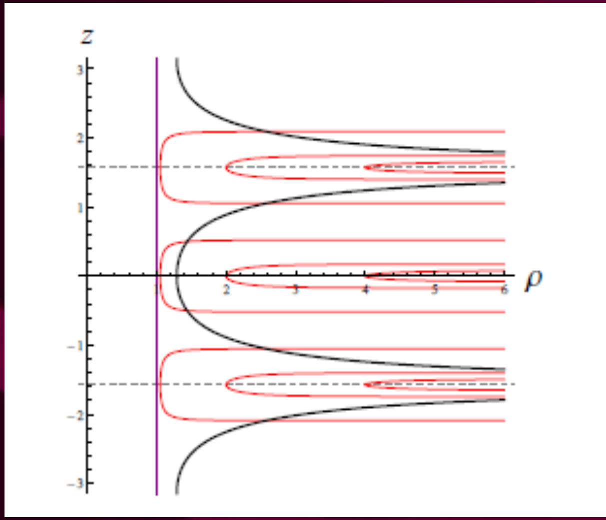


N=4 SYM + Probes Inbetween Mirrors

Takayanagi proposal looks flawed… it may just not be consistent to have a boundary in N=4 SYM (how do you build the mirror?)…

Or do we need boundary interactions with D5?

Simplest fudge is just to take the soliton and impose mirror reflection on probe sources in space – we're assuming the N=4 vacuum is local or at least only knows about the scale of the mirror separation…



Amusingly there is then exciton condensation with the mirror reflection of the probe…

N=4 SYM + Bilayers Inbetween Mirrors

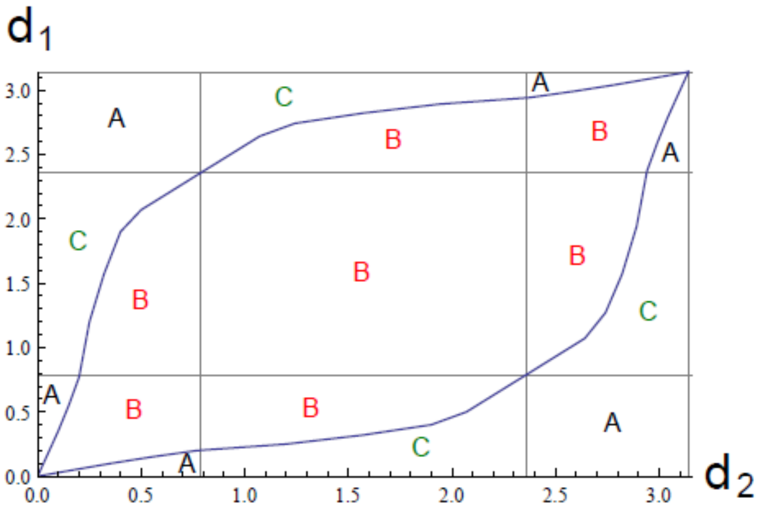


FIG. 5: The phase diagram of the bilayer theory in an interval between two mirrors of separation π. d_1 and d_2 measure the distance from one mirror to the first and second defect respectively. We have marked the lines $d_{1/2} = \pi/4, 3\pi/4$ because these are the separations within which condensation with the mirror image are possible. In phase A both D5s condense with their mirror images. In phase B the two D5s form a U-shaped configuration. In phase C the probe nearest the mirror displays exciton condensation with its mirror partner whilst the other probe takes up the lone configuration of Fig 1.

Summary

Lot's of fun with probe D5s in AdS:

* mu-T phase diagram of probe D5s with B

* non-mean field transitions (BKT)

* exciton condensation between bilayers

* vacuum alignment issues in bilayers with B field

* and in a cavity…