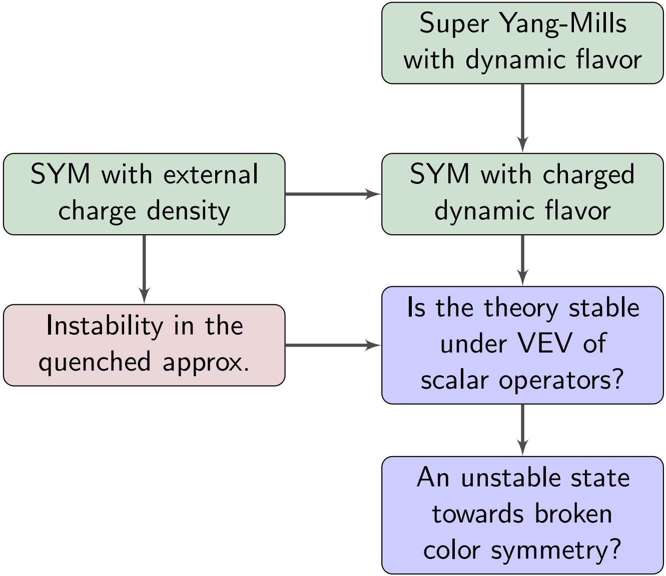
Instabilities of finite density SYM theories from holography

Javier Tarrío
University of Barcelona

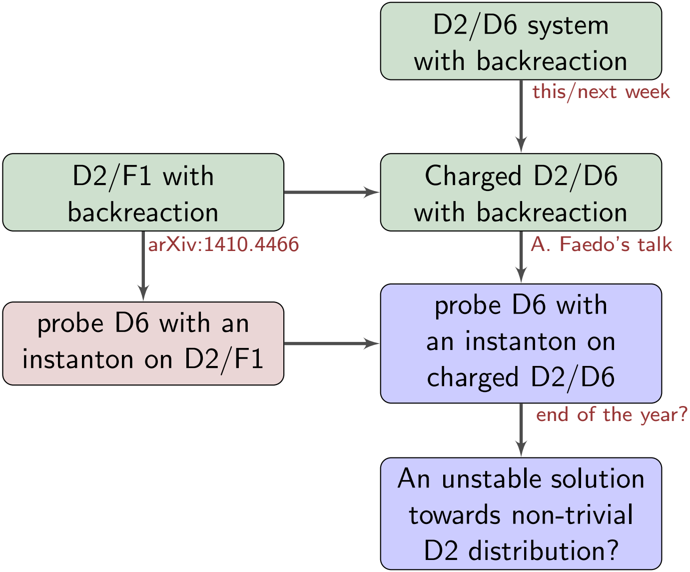
with A. Faedo, A. Kundu, D. Mateos, C. Pantelidou

Florence, April 29th 2015

Context


Super Yang-Mills
with dynamic flavor
SYM with external
charge density
SYM with charged
dynamic flavor
Instability in the
quenched approx.
Is the theory stable
under VEV of
scalar operators?
An unstable state
towards broken
color symmetry?

Context


D2/D6 system
with backreaction
this/next week
D2/F1 with
backreaction
Charged D2/D6
with backreaction
arXiv:1410.4466
A. Faedo's talk
probe D6 with an
instanton on D2/F1
probe D6 with
an instanton on
charged D2/D6
end of the year?
An unstable solution
towards non-trivial
D2 distribution?

Motivation

To understand if

an instanton configuration on the worldvolume of a flavor brane triggers an instability in the dual field theory at finite charge density,

eventually breaking the gauge symmetry group.

Table of Contents

Introduction

⇒ Let me consider for concreteness $\mathcal{N} = 4$ SYM in 3+1.

⇒ In this talk we work in the presence of an external charge density and with fundamental matter in the 't Hooft limit

$$N_c \to \infty \quad \text{with} \quad \frac{n_q}{N_c^2} \quad \text{and} \quad N_f \quad \text{fixed.}$$

Higgs branch in flavored $\mathcal{N} = 4$ [Guralnik et al. '04] [Erdmenger et al. '05]

	$X^{0,1,2,3}$ (Minkowski)	$Y^{4,5,6,7}$ (Φ_1 and Φ_2)	$Z^{8,9}$ (Φ_3)
D3	$\times$	$-$	$-$
D7	$\times$	$\times$	$-$

$\Rightarrow$ The superpotential reads

$$W = \tilde{Q}_i \Phi_3 Q^i + \mathrm{tr}\,[\Phi_1, \Phi_2]\,\Phi_3$$

$\Rightarrow$ Two simple ways to extremize: Coulomb and Higgs branches of moduli spaces. Recall also the condition

$$\tilde{Q}_i Q^i + \mathrm{tr}\,[\Phi_1, \Phi_2] = 0$$

The picture on the worldvolume of the D7

⇒ The action as sum of two parts

$$S = -T_7 \int \mathrm{d}^4 x \, \mathrm{d}^4 y \, e^{-\phi} \, \mathrm{Str} \sqrt{\hat{G} + F} + \frac{T_7}{2} \int \mathrm{Str} \, \hat{C}_4 \wedge F \wedge F$$

with F depending only on the NS-D directions y^M.

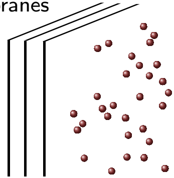
⇒ If the field strength is self-dual $F = *F$ then eom satisfied and

$$S = -T_7 \, N_f \int \mathrm{d}^4 x \, \mathrm{d}^4 y$$

Microscopic interpretation [Arean et al. '07]

⇒ The D7-branes carry some D3-charge on them

$$S_{WZ} = \frac{T_7}{2} \int \mathrm{d}^4x\, \mathrm{d}^4y\, \hat{C}_4 \wedge F \wedge F = T_3\, k \int \mathrm{d}^4x\, \hat{C}_4$$



⇒ Indeed, the description of D7-branes with instanton is equivalent to the study of k dielectric D3-branes (Myers effect)

Key points in the calculation

⇒ In field theory a moduli space exists only for non-abelian flavor group. This is seen also in the holographic dual (regularity of the solution).

⇒ In the gravity side one can solve the linear self-duality condition, which allows to find the solutions.

⇒ The latter point is fortunate, since we do not know non-abelian DBI

$$S \simeq -\frac{T_7}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\hat{C}_4 \wedge \mathrm{Str}\,(F \wedge *F - F \wedge F) + \cdots$$

Abelian case [Ammon et al. '12]

⇒ A moduli space arises if one allows finite charge density

$$A = A_0(y^M) + A_M(y^M)$$

where the field strength must be self-dual $F = *F$ w.r.t. an effective metric that includes the charge density d, where

$$\partial_r A_0 = \frac{d}{\sqrt{r^6 + d^2}} \quad ; \quad A_M = \sum_{\ell=1}^{\infty} K_\ell \left(r^3 + \sqrt{r^6 + d^2} \right)^{-\frac{\ell+1}{3}} \mathcal{Y}_M^{\ell,-}$$

⇒ The charge density regularizes the solution, and furthermore it still allows to solve a linear equation!

⇒ Unfortunately the probe approximation breaks down near the origin. Some backreaction needed.

$\Rightarrow$ Thus: is there a moduli space in a system with backreacted charge density for dynamic fundamental matter? (Nope)

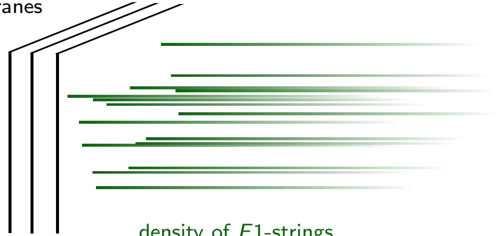
$\Rightarrow$ But is there a calculation in a system with backreacted charge density for dynamic fundamental matter at all? (Nope, this is work in progress)

$\Rightarrow$ I introduce now preliminary results for the case in which the fundamental matter is non-dynamic.

Table of Contents

The external charge case [Kumar '12] [Faedo et al. '14]

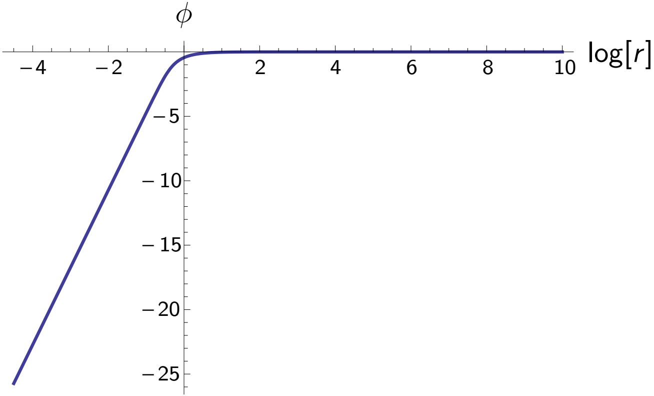
Charge with non-dynamic quarks → only strings in the holographic description.



In SUGRA we have the RR forms

$$F_5 = 4L^4(1 + *)\omega_5 \quad ; \quad F_3 \sim \lambda \frac{n_q}{N_c^2}\, \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

(Part of the) numeric solution



Crossover at a scale $n_q^{1/3}$.

IR geometry [Azeyanagi et al. '09] [Kumar '12]

There is an exact solution with a dimensionally reduced metric

$$\mathrm{d}s^2 = -r^{2z}\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + \frac{1}{r^2}\mathrm{d}r^2 \ ,$$

with $z = 7$, which means

$$t \to \Lambda^7 \, t \ , \qquad x^i \to \Lambda \, x^i \ ,$$

and running dilaton

$$e^{\phi} \sim n_q^{-2} \, r^6,$$

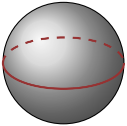
Next...

In this numeric background solution we introduce $N_f \ll N_c$ probe $D7$-branes...

...and the quenched approximation is parametrically valid for all radii!

Embedding the probe and ansatz [Karch et al. '02]

	x^0	$x^{1,2,3}$	r	$S^3 \subset S^5$	θ, ϕ
D3	$\times$	$\times$	$-$	$-$	$-$
F1	$\times$	$-$	$\times$	$-$	$-$
D7	$\times$	$\times$	$\times$	$\times$	$-$



We pick an ansatz like the one at the beginning of the talk

$$A = A_0(y)\mathrm{d}t + A_M(y)\mathrm{d}y^M$$

and the system is described by the action

$$S = -T_7 \int \mathrm{d}^4x\, \mathrm{d}^4y\, e^{-\phi} \sqrt{\hat{G} + F} + \frac{T_7}{2} \int \hat{C}_4 \wedge F \wedge F + \frac{T_7}{3!} \int A \wedge \hat{F}_3 \wedge F \wedge F$$

(No exact linearization of the problem is possible.)

Taming those nasty angles

$\Rightarrow$ Nightmarish action leading to pde's with four coupled fields

$\Rightarrow$ Invoke group theory arguments to reduce to one tractable ordinary differential equation: $SO(4) \simeq SU(2)_L \times SU(2)_R$

Harmonic	transformation	quantum number
$\mathcal{Y}^\ell$	$\left(\frac{\ell}{2}, \frac{\ell}{2}\right)$	$\ell \geq 0$
$\nabla_i \mathcal{Y}^\ell$	$\left(\frac{\ell}{2}, \frac{\ell}{2}\right)$	$\ell \geq 1$
$\mathcal{Y}_i^{\ell,\pm}$	$\left(\frac{\ell \mp 1}{2}, \frac{\ell \pm 1}{2}\right)$	$\ell \geq 1$

$\Rightarrow$ Consider the $SU(2)_R$ singlets

$$A = A_0(y)\mathrm{d}t + A_M(y)\mathrm{d}y^M = A_0(r)\mathrm{d}t + \Psi(r)\mathcal{Y}_i^{1,-}\mathrm{d}y^i$$

Numeric strategy

⇒ Take

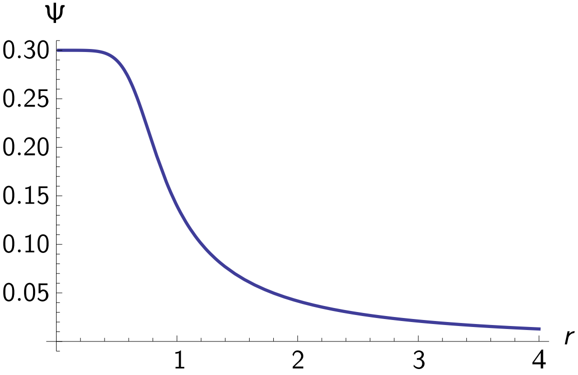
$$A = A_0(r)\mathrm{d}t + \Psi(r)\,\mathcal{Y}_i^{1,-}\mathrm{d}y^i = A_0(r)\mathrm{d}t + \Psi(r)\,\alpha_i w^i$$

$$\left(\mathrm{d}w^i = \frac{1}{2}\epsilon^{ijk} w^j \wedge w^k\right)$$

⇒ A_0' can be solved for and plugged in Ψ's equation, this introduces parameter d

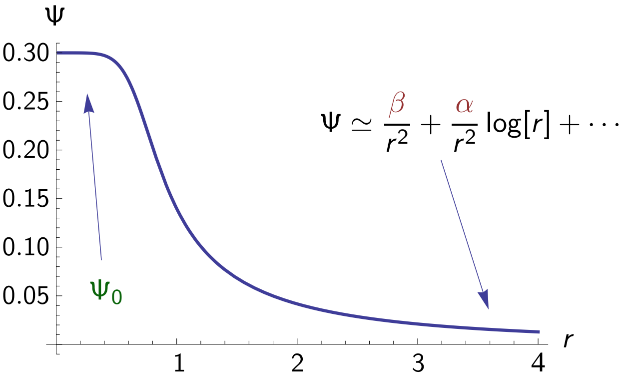
⇒ Impose regularity at the horizon and integrate numerically for given value of d

Numeric strategy



Integration with fixed d

Numeric strategy



Integration with fixed d

Table of Contents

Holographic dual [Kruczenski '03]

$$\Psi \simeq \frac{\beta}{r^2} + \frac{\alpha}{r^2}\log[r] + \cdots \quad \Leftrightarrow \quad m^2 L^2 = -4$$

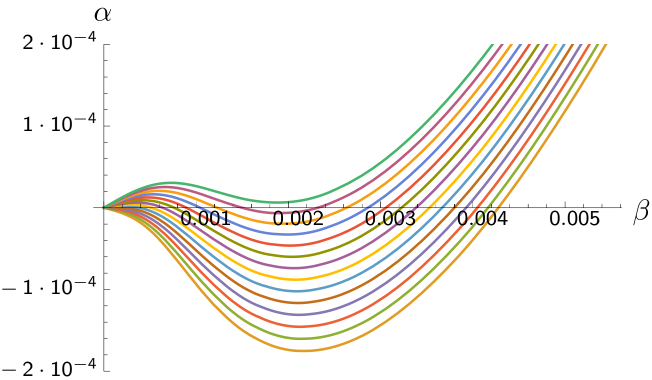
$\Rightarrow$ This tells us that $\Psi\,\alpha_i$ is dual to operator with $\Delta = 2$

$$\mathcal{O}^i \sim Q^\dagger \sigma^i Q$$

$\Rightarrow$ The mode β dual to VEV and α to source [Bianchi et al. '01] [Karch et al. '05]

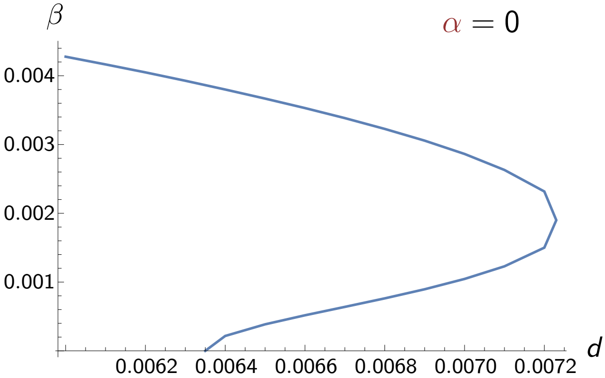
$$\frac{\delta \mathcal{F}}{\delta \alpha} = -\frac{8\pi^2 T_7}{L^4}\beta$$

Unsourced operator

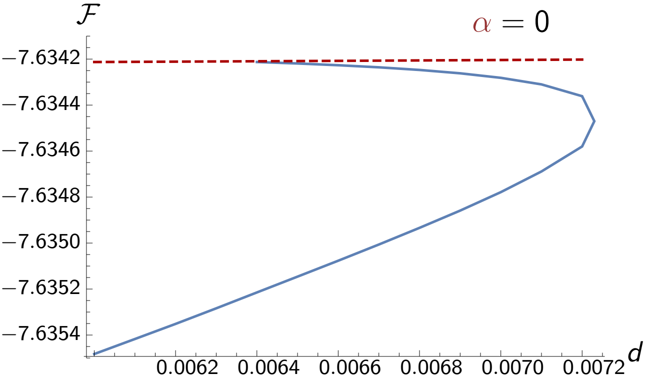


The mode β dual to VEV and α to source

Unsourced operator



Unsourced operator



Double-trace deformation

$$\Psi \simeq \frac{\beta}{r^2} + \frac{\alpha}{r^2}\log[r] + \cdots \quad \Leftrightarrow \quad m^2 L^2 = -4$$

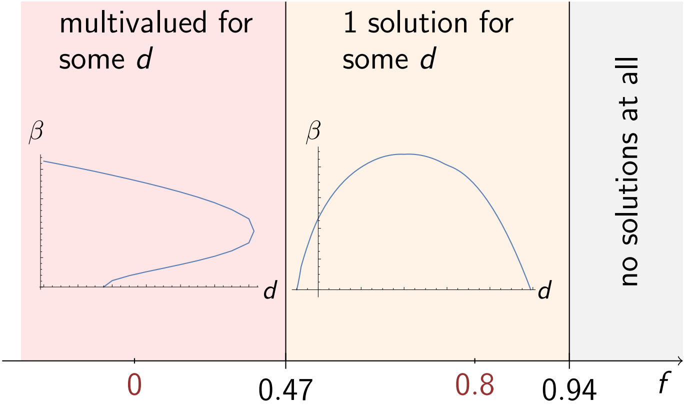
$\Rightarrow$ Alternative quantization possible. If $\alpha = f\,\beta$ double trace deformation [Witten '01]

$$S \to S + \frac{f}{2}\int \mathrm{d}^4x\,\mathcal{O}_\Psi^2$$

$\Rightarrow$ The mode α dual to VEV and $\beta - \frac{\alpha}{f}$ to source [Papadimitriou '07] and Ioannis' talk

$$\frac{\delta\mathcal{F}}{\delta\left(\beta - \frac{\alpha}{f}\right)} = \frac{8\pi^2 T_7}{L^4}\alpha$$

Double-trace deformation



Going to $2+1$ dimensions

⇒ Similar construction possible for 2+1 dimensional SYM with external charge density, $z=5$ and $\theta=1$

⇒ Instanton dual to $\Delta=1$ operator in alternative quantization

$$\Psi\,\alpha_i \simeq \left(\frac{\alpha}{r^2}+\frac{\beta}{r^3}+\cdots\right)\alpha_i \quad \leftrightarrow \quad \mathcal{O}^i \sim Q^\dagger \sigma^i\, Q$$

⇒ Double trace deformation also possible

$$\frac{\delta\mathcal{F}}{\delta\left(\beta - f\alpha\right)} = \frac{8\pi^2\,T_6}{L^5}\alpha$$

Table of Contents

Conclusions

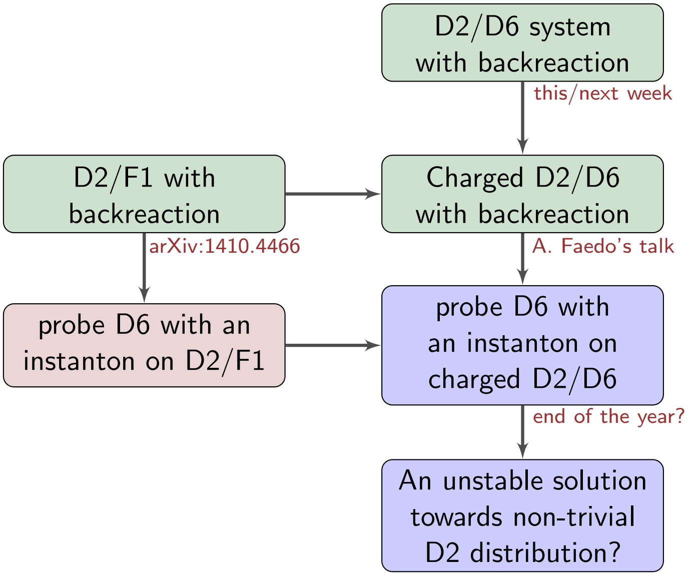
$\Rightarrow$ Things we have shown in this talk

- Included a massless flavor in a setup with an external charge density (∞-ly massive flavors).
- The setup seems thermodynamically unstable towards condensation of $\mathcal{O}^I \sim Q^\dagger \sigma^I Q$.

$\Rightarrow$ Things we have not shown in this talk

- We have not singled out a charged massless flavor of the background.
- We have not shown that a theory with charged dynamic quarks is unstable.

Outlook


D2/D6 system with backreaction
this/next week
D2/F1 with backreaction
Charged D2/D6 with backreaction
arXiv:1410.4466
A. Faedo's talk
probe D6 with an instanton on D2/F1
probe D6 with an instanton on charged D2/D6
end of the year?
An unstable solution towards non-trivial D2 distribution?

Outlook

$\Rightarrow$ We want to repeat the calculation in a system with dynamic flavor [Work in progress]

$\Rightarrow$ In the background supergravity solution new RR fluxes turned on.

$\Rightarrow$ This reflects in an effective charge density on the brane

$$d \longrightarrow d + \mathcal{B}(r)$$

Thank you

Work supported by the Juan de la Cierva program of the Spanish Ministry of Economy and by ERC StG HoloLHC - 306605