From lattice model to CFT via fermionic basis

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Based on joint works with

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Plan

- Introduction: $\mathbf{t}^*(\zeta)$
- Fermionic structure of local operators on the lattice
- Fermionic structure in CFT
- Synthesis: relation between lattice and CFT

1. Introduction: $\mathfrak{t}^*(\zeta)$

Consider an infinite chain of spins 1/2:

$$\mathfrak{H}_{\mathbf{S}} = \bigotimes_{j=-\infty}^{\infty} \mathbb{C}^2 \,.$$

We study XXZ (equivalently six-vertex) model, basic object is the R-matrix:

$$R_{1,2}(\zeta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad b(\zeta) = \frac{\zeta - \zeta^{-1}}{\zeta q - \zeta^{-1} q^{-1}}\,, \quad c(\zeta) = \frac{q - q^{-1}}{\zeta q - \zeta^{-1} q^{-1}}\,,$$

Introduce the adjoint R-matrix

$$\mathbb{R}_{i,j}(\zeta)(\bullet) = R_{i,j}(\zeta)\ \bullet\ R_{i,j}(\zeta)^{-1}\,.$$

Define further the following, rather formal, object

$$\mathbb{T}_{a,[-\infty,\infty]}(\zeta) = \prod_{j=-\infty}^{\overset{\curvearrowleft}{\infty}} \mathbb{R}_{a,j}(\zeta)\,.$$

Denote $S(k)=\frac{1}{2}\sum_{j=-\infty}^{k}\sigma_j^3$, and consider operators

$$\mathcal{O}q^{2\alpha S(0)}\,; \quad \mathcal{O} \text{ is local}, \quad q^{2\alpha S(0)} \text{ is called `primary field}'.$$

We want to make sense of the operator

$$\mathbf{t}^*(\zeta)(\mathcal{O}q^{2\alpha S(0)})=\mathrm{Tr}_a\Bigl(\mathbb{T}_{a,[-\infty,\infty]}(\zeta)(\mathcal{O}q^{\alpha(2S(0)+\sigma_a^3)})\Bigr)\,,$$

First observe that due to $[\sigma_a^3+\sigma_j^3, R_{a,j}]=0$ if $\mathcal{O}$ lives on $[1,n]$ we can rewrite

$$\mathbf{t}^*(\zeta)(\mathcal{O}q^{2\alpha S(0)})=\mathrm{Tr}_a\Bigl(\mathbb{T}_{a,[1,\infty]}(\zeta)(\mathcal{O}q^{\alpha(2S(0)+\sigma_a^3)}\Bigr)\,.$$

I shall use

$$\check{\mathbb{R}}_{i,j}(\zeta)=\mathbb{R}_{i,j}(\zeta)\mathbb{P}_{i,j}\,.$$

where $\mathbb{P}_{i,j}$ stands for the adjoint action of the permutation.
We have

$$\check{\mathbb{R}}_{i,j}(\zeta)\Big|_{\zeta^2=1}=I\,,$$

hence

$$\check{\mathbb{R}}_{i,j}(\zeta)=I+(\zeta^2-1)\mathbf{r}_{i,j}(\zeta)\,,$$

where the operator $\mathbf{r}_{i,j}(\zeta)$ is regular at $\zeta^2=1$.

Property:

$$\mathbf{r}_{i,j}(\zeta)(X) = 0$$

if the operator X acts trivially on the i-th and the j-th components of the tensor product.

Then for large but finite l

$$\begin{aligned}
&\mathrm{Tr}_a\left(\mathbb{T}_{a,[1,l]}(\zeta)(\mathcal{O}q^{\alpha(2S(0)+\sigma_a^3)})\right)\\
&= \mathrm{Tr}_a\Big(\check{\mathbb{R}}_{a,l}(\zeta)\check{\mathbb{R}}_{l,l-1}(\zeta)\cdots\check{\mathbb{R}}_{n+2,n+1}(\zeta)\mathbb{T}_{n+1,[1,n]}(\zeta)(\mathcal{O}q^{\alpha(2S(0)+\sigma_{n+1}^3)})\Big)\\
&= \sum_{j=n}^{l-1}(\zeta^2-1)^{j-k}\mathbf{r}_{j+1,j}(\zeta)\cdots\mathbf{r}_{n+2,n+1}(\zeta)\mathbb{T}_{n+1,[1,n]}(\zeta)((\mathcal{O}q^{\alpha(2S(0)+\sigma_{n+1}^3)}))\\
&+ O((\zeta^2-1)^{l-n})\,.
\end{aligned}$$

Hence $\mathbf{t}^*(\zeta)(\mathcal{O}q^{2\alpha S(0)})$ is well defined as Taylor series.

In other words if we understand $\mathbf{t}^*(\zeta)(\mathcal{O}q^{2\alpha S(0)})$ as power series in ζ^2-1,

it is well-defined.

2. Fermionic structure of local operators on the lattice

We shall consider more sophisticated auxiliary space, q-oscillators by Bazhanov-Lukyanov-Zamolodchikov:

$$q^{D}\mathbf{a}\,q^{-D}=q^{-1}\mathbf{a},\quad q^{D}\mathbf{a}^{*}q^{-D}=q\,\mathbf{a}^{*}\,,\quad \mathbf{a}\,\mathbf{a}^{*}=1-q^{2D+2},\quad \mathbf{a}^{*}\mathbf{a}=1-q^{2D}\,.$$

The we define

$$R_{A,j}(\zeta)=\begin{pmatrix}1-\zeta^{2}q^{2D_A+2} & -\zeta\mathbf{a}_A\\ -\zeta\mathbf{a}^{*}_A & 1\end{pmatrix}_j\begin{pmatrix}q^{-D_A} & 0\\ 0 & q^{D_A}\end{pmatrix}_j\,.$$

We want to define some new operators using the q-oscillators, but we have to be careful about spin of our local operators. Define

$$\mathbb{S}(\bullet)=[S(\infty),\bullet]\,,$$

and consider

$$X=\mathcal{O}q^{2(\alpha-s)S(0)}\quad \mathbb{S}(\mathcal{O})=s\mathcal{O}\,.$$

Now I shall make some magic tricks.
First, take finite l, restrict X to $\mathcal{O}q^{2(\alpha-s)S_{[-l+1,0]}}$, and define

$$\mathbf{k}(\zeta)(X_{[-l+1,l]}) = \zeta^{\alpha-2s-1}\mathrm{Tr}_{a,A}\Big\{\sigma_a^+\mathbb{T}_{a,[-l+1,l]}(\zeta) \\ \times \mathbb{T}_{A,[-l+1,l]}(\zeta)\Big(q^{(\alpha-s-1)(2D_A+\sigma_a^3)-2S_{[-l+1,l]}}X_{[-l+1,l]}\Big)\Big\}\,.$$

Once again it is obvious that $[-l+1,l]$ can be changed to $[1,l]$. But the right reduction does not hold. In order to reach it we proceed as follows. Let $\Delta f(\zeta)=f(\zeta q)-f(\zeta q^{-1})$, and define the 'primitive' function

$$\mathbf{f}(\zeta) = \Delta_\zeta^{-1}\mathbf{k}(\zeta)(X) \\ = \int_\Gamma \Delta_\zeta^{-1}\psi(\zeta/\xi,\alpha)\cdot\mathbf{k}(\xi)(X)\frac{d\xi^2}{2\pi i\xi^2}\,,\quad \psi(\zeta,\alpha)=\zeta^\alpha\frac{\zeta^2+1}{2(\zeta^2-1)}\,,$$

poles of $\Delta_\zeta^{-1}\psi(\zeta/\xi,\alpha)$ are outside and the poles of $\mathbf{k}(\xi)(X)$ are inside Γ.
Then, for some obscure reason, apply to it the Baxter operator:

$$\mathbf{b}^*(\zeta)=\mathbf{f}(\zeta q)+\mathbf{f}(\zeta q^{-1})+\mathbf{t}^*(\zeta)\mathbf{f}(\zeta)\,.$$

This is a magic definition.

For this $\mathbf{b}^*(\zeta)$ we have

$$\mathbf{b}^*(X) = \mathrm{Tr}_c\left(\mathbb{T}_{c,[n+1,l]}(\zeta)\mathbf{g}_c(\zeta,\alpha)(X)\right)\ ,$$

where $\mathbf{g}_c(\zeta,\alpha)$ is certain operator such that $\mathbf{g}_c(\zeta,\alpha)(X)$ has support $[1,n]$, and is regular at $\zeta^2=1$.
Hence once again $\mathbf{b}^*(X)$ is well defined as series in ζ^2-1 for $l=\infty$.
Finally we define

$$\mathbf{c}^*(\zeta)(X) = -q^{-\alpha+2s-2}(1-q^{2(\alpha-2s+1)})\times\left\{\Big(\mathbb{J}\circ\mathbf{b}^*(\zeta)\circ\mathbb{J}\Big)(X)\right\}\Big|_{\alpha\to-\alpha+2s}\ ,$$

where $\mathbb{J}$ is the spin reflection.
All these operators act in $W_\alpha=\oplus_{s\in\mathbb{Z}}W_{\alpha-s,s}$ as

$$\mathbf{t}^*:\ W_{\alpha-s,s}\to W_{\alpha-s,s}\ ,\ \mathbf{b}^*:\ W_{\alpha-s,s}\to W_{\alpha-1,s+1}\ ,\ \mathbf{c}^*:\ W_{\alpha-s,s}\to W_{\alpha-s+1,s-1}\ .$$

Fermionic basis is the basis of

$$W_\alpha^{\mathrm{quo}} = W_\alpha/\mathbf{t}^*W_\alpha\ .$$

The lattice model made a huge step towards CFT.

3. Fermionic structure in CFT

Consider CFT with $c = 1 + 6Q^2, \quad Q = b + 1/b$, i.e. the Liouville model:

$$\mathcal{A}^{\mathrm{Liouv}} = \int \Big[\frac{1}{4\pi}\partial_z\varphi(z,\bar{z})\partial_{\bar{z}}\varphi(z,\bar{z}) + \frac{\boldsymbol{\mu}^2}{\sin \pi b^2} e^{b\varphi(z,\bar{z})}\Big]\frac{idz\wedge d\bar{z}}{2}\,.$$

We consider primary fields $\Phi_a = e^{a\varphi}$ of dimension $\Delta_a = a(Q-a)$.
The Liouville model has the reflection: $a \to Q - a$. It is interesting to consider its perturbation by

$$\frac{\boldsymbol{\mu}^2}{\sin \pi b^2} e^{-b\varphi(z,\bar{z})}\,,$$

which corresponds to sinh(sine)-Gordon model.

After the perturbation we have the symmetry $\varphi \to -\varphi$, and hence $a \to -a$.

How to implement both reflections?

Zamolodchikov's integrals of motion.

We consider the Verma module V_a, Virasoro generators are defined by $\mathbf{l}_k$.

As preparation for future perturbation Zamolodchikov introduces the action integrals of motion (KdV Hamiltonian vector fields in classics):

$$\mathbf{i}_1 = \mathbf{l}_{-1}\,, \qquad \mathbf{i}_3 = 2\sum_{k=-1}^{\infty} \mathbf{l}_{-3-k}\mathbf{l}_k\,,$$

$$\mathbf{i}_5 = 3\Big(\sum_{k=-1}^{\infty}\sum_{l=-1}^{\infty} \mathbf{l}_{-5-k-l}\mathbf{l}_l\mathbf{l}_k + \sum_{k=-\infty}^{-2}\sum_{l=-\infty}^{-2} \mathbf{l}_l\mathbf{l}_k\mathbf{l}_{-5-k-l}\Big)$$

$$+\frac{c+2}{6}\sum_{k=-1}^{\infty}(k+2)(k+3)\mathbf{l}_{-5-k}\mathbf{l}_k\,.$$

What is the relation to our reflections?

First act by $\mathbf{i}_{2j-1}$ on any element of V_a. The coefficients in the Virasoro basis depend on Δ only, so, they are invariant under $a \to Q - a$.

Now perform the bosonization:

$$\mathbf{l}_k = \frac{1}{4}\sum_{j\neq 0,k} a_j a_{k-j} + (i(k+1)Q/2 + \pi_0)a_k\,, \quad k \neq 0\,,$$

$$\mathbf{l}_0 = \frac{1}{2}\sum_{j=1}^{\infty} a_{-j}a_j + \pi_0(\pi_0 + iQ)\,,$$

where the Heisenberg generators satisfy

$$[a_k, a_l] = 2k\delta_{k,-l}\,,$$

and zero-mode is canonical: $\pi_0 = \frac{\partial}{i\partial\phi_0}$.
Take any element of V_a created by the Heisenberg generators, apply to it $\mathbf{i}_{2j-1}$, and consider coefficients in the Heisenberg basis. They are invariant under the reflection

$$a \to -a, \qquad a_k \to -a_k\,.$$

Thinking of this having in mind the perturbation to sinh(sine)-Gordon, one understands what Zamolodchikov integrals are all about.

Similarly to the lattice model we are interested in the quotient space:

$$\mathcal{V}_a^{\mathrm{quo}} = \mathcal{V}_a \ / \ \sum_{k=1}^{\infty} \mathbf{i}_{2k-1}\mathcal{V}_a \, .$$

We shall use $\equiv$ for equality in this space. Since Zamolodchikov's integrals respect both reflections it makes sense to ask whether there is a basis respecting them in the quotient. The following statement has been checked up to level 10 (general prove is absent).

Proposition. There are fermionic creation operators $\boldsymbol{\beta}^*_{2j-1}$, $\boldsymbol{\gamma}^*_{2j-1}$, which create the basis of $\mathcal{V}_a^{\mathrm{quo}}$

$$\boldsymbol{\beta}^*_{I^+}\boldsymbol{\gamma}^*_{I^-}\Phi_a \, ,$$

where $I^{\pm}$ are multiindices with odd positive entries , $\#(I^+) = \#(I^-)$,

$\boldsymbol{\beta}^*_{I^+} = \prod_{p\in I^+}\boldsymbol{\beta}^*_p$, $\boldsymbol{\gamma}^*_{I^+} = \prod_{p\in I^+}\boldsymbol{\gamma}^*_p$. This basis possesses two properties.

Virasoro basis of $\mathcal{V}_a^{\text{quo}}$ is generated by $\mathbf{l}_{-2k}$, while the Heisenberg basis is created by some monomials (there is no *a priori*) choice of even degrees in a_{-k}.

- On Virasoro side

$$\boldsymbol{\beta}^*_{I^+}\boldsymbol{\gamma}^*_{I^-}\Phi_a \equiv C_{I^+,I^-}\prod_{p\in I^+} D_p(a)\prod_{p\in I^+} D_p(Q-a)$$
$$\times\Big(P^{\text{even}}_{I^+,I^-}(\{\mathbf{l}_{-2k}\},\Delta,c)+d\cdot P^{\text{odd}}_{I^+,I^-}(\{\mathbf{l}_{-2k}\},\Delta,c)\Big)\Phi_a\,,$$

where $d=(b-b^{-1})(Q-2a)\,.$

- On Heisenberg side

$$\boldsymbol{\beta}^*_{I^+}\boldsymbol{\gamma}^*_{I^-}\Phi_a \equiv C_{I^+,I^-}\prod_{p\in I^+} \widetilde{D}_p(a)\prod_{p\in I^+} \widetilde{D}_p(-a)$$
$$\times\Big(Q^{\text{even}}_{I^+,I^-}(\{a_{-k}\},a^2,c)+\tilde{d}\cdot Q^{\text{odd}}_{I^+,I^-}(\{a_{-k}\},a^2,c)\Big)\Phi_a\,,$$

where $\tilde{d}=a(b-b^{-1})$.

The functions $D_p(a)$, $\widetilde{D}_p(a)$ are expressed through γ-functions:

$$D_p(a) = (-1)^{(p+1)/2} C^p \frac{\Gamma\left(\frac{2a+pb^{-1}}{2Q}\right)\Gamma\left(\frac{2(Q-a)+pb}{2Q}\right)}{((p-1)/2)!}\,,$$

$$\widetilde{D}_p(a) = (-1)^{(p+1)/2} C^p \frac{\Gamma\left(1+\frac{pb^{-1}+2a}{2Q}\right)\Gamma\left(1+\frac{pb-2a}{2Q}\right)}{((p-1)/2)!}\,.$$

So, under both reflections $\boldsymbol{\beta}^*_{2j-1} \leftrightarrow \boldsymbol{\gamma}^*_{2j-1}$. As a bonus the same is true for the duality $b \to 1/b$.

Simplest examples:

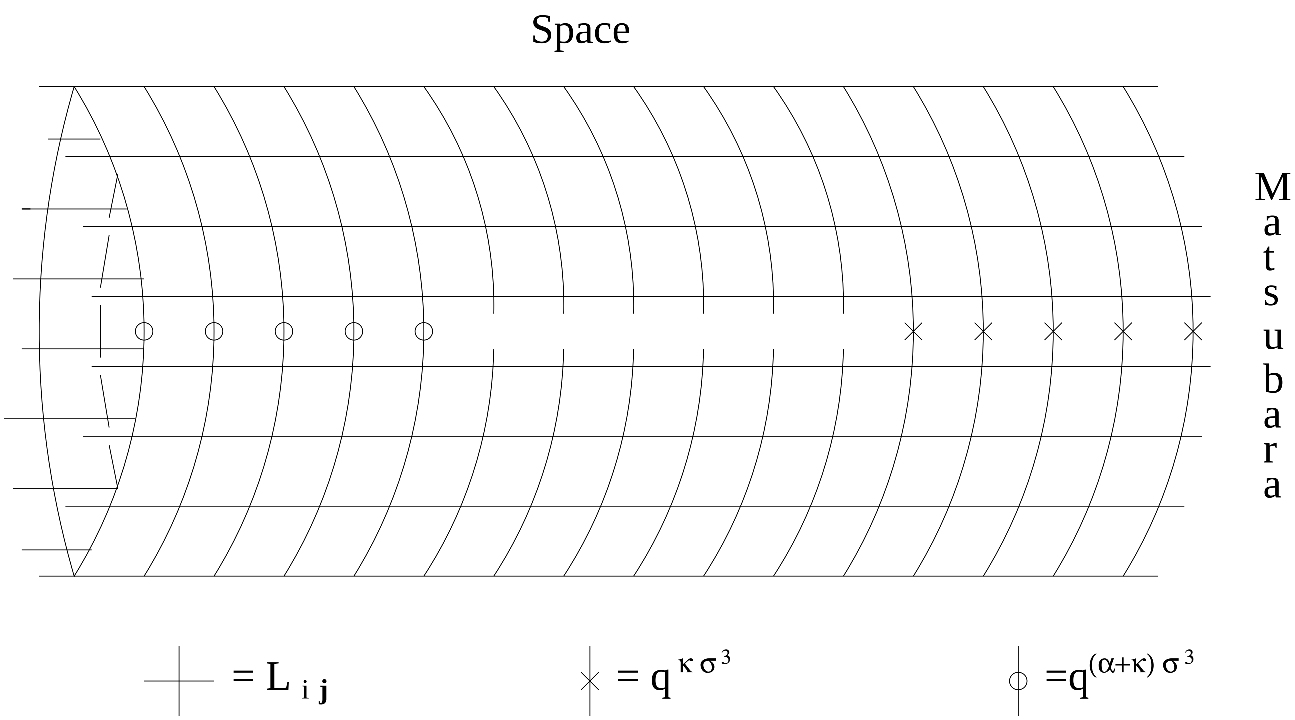
$$P^{\text{even}}_{\{1\},\{1\}} = \mathbf{l}_{-2}\,,$$

$$P^{\text{even}}_{\{1\},\{3\}} = \mathbf{l}^2_{-2} + \frac{2c-32}{9}\mathbf{l}_{-4}\,, \quad P^{\text{odd}}_{\{1\},\{3\}} = \frac{2}{3}\mathbf{l}_{-4}\,.$$

4. Synthesis: relation between lattice and CFT

General wisdom: in QFT the convergence is always weak. So, it makes sense to compare expectation values, matrix elements, *etc.*

Consider the partition function given by the picture



where $L_{i,\mathbf{j}} = \sigma^3_i R_{i,\mathbf{j}}(q^{-1/2})\sigma^3_{\mathbf{j}}$.

It defines a linear functional Z^κ on $W_{\alpha,0}$.

Important property of our fermionic basis is

$$Z^{\kappa}\Big(\prod \mathbf{t}^*(\zeta_j^0)\prod \mathbf{b}^*(\zeta_j^+)\mathbf{c}^*(\zeta_j^-)(q^{2\alpha S(0)})\Big)=\prod \rho(\zeta_j^{(0)})\,\det\Big(\omega(\zeta_i^{(+)},\zeta_j^{(-)})\Big)\,.$$

$\rho(\zeta)$ and $\omega(\zeta,\xi)$ are two functions which are defined only by Matsubara data.

One more piece of wisdom: in QFT one must clearly separate UV and IR problems, in other words local properties and environment.

There are two boundary conditions described by Matsubara transfer-matrices with twists κ and $\kappa'=\kappa+\alpha$. We want to emancipate κ'. Already on the lattice we can perform a kind of Feigin -Fuchs -Dotsenko -Fateev which results in $\kappa'\to\kappa'-2\frac{1-\nu}{\nu}s$. This requires changing the boundary conditions and introducing some ‘screening operators’ cooked of fermions $\mathbf{c}^*(\zeta)$ for $\zeta^2\to 0$ instead of usual $\zeta^2\to 1$.

The relation between lattice and CFT constants

$$q=e^{\pi i\nu}\,,\quad \nu=1+b^2,\quad \frac{2a}{Q}=\alpha\,,\quad \frac{2P}{Q}=\kappa\,,\quad \frac{2P'}{Q}=\kappa'\,.$$

Then Z^{κ} is identified with

$$\langle Q/2-P'\,|\mathrm{descendant}(\Phi_a)|\;Q/2+P\rangle\,.$$

Comparing this expression with the tree-point functions in CFT for $P=P'$ we conclude that

$$\mathbf{b}^*((Ca)^{\nu}\lambda)\to\boldsymbol{\beta}^*(\lambda)\,,\quad \mathbf{c}^*((Ca)^{\nu}\lambda)\to\boldsymbol{\gamma}^*(\lambda)\,,$$

where

$$\boldsymbol{\beta}^*(\lambda)=\sum_{j=1}^{\infty}\boldsymbol{\beta}^*_{2j-1}\lambda^{-\frac{2j-1}{\nu}}\,,\quad \boldsymbol{\gamma}^*(\lambda)=\sum_{j=1}^{\infty}\boldsymbol{\gamma}^*_{2j-1}\lambda^{-\frac{2j-1}{\nu}}\,,$$

a is the lattice spacing introduced in natural way: $2\pi R=\mathbf{n}a$,

$$C=\frac{\Gamma\left(\frac{1-\nu}{2\nu}\right)}{2\sqrt{\pi}\,\Gamma\left(\frac{1}{2\nu}\right)}\Gamma(\nu)^{\frac{1}{\nu}}\,.$$

Some fun. Set

$$D_V^{(10)}(\Delta,c) = \Big(\Delta+6\Big)\Big(-23794+2905c+(-2285+983c)\Delta + (1447+71c)\Delta^2+(149+c)\Delta^3+3\Delta^4\Big)\,,$$

then

$$P^{\text{even}}_{\{5\},\{5\}}(\{\mathbf{l}_{-2k}\}) = (\mathbf{l}_{-2})^5 + \frac{20(22+2c+(c-16)\Delta)}{9(6+\Delta)}\mathbf{l}_{-4}(\mathbf{l}_{-2})^3 + \frac{1}{6804\; D_V^{(10)}(\Delta,c)}$$
$$\times\Big\{6\Big(-2394125160+328307580c-11439180c^2+2245740c^3+ (4571783552-642113226c+9291216c^2+1626898c^3)\Delta$$
$$+(283889270-184441506c+1485447c^2+487564c^3)\Delta^2$$
$$+(-306733490-17698098c+377931c^2+59192c^3)\Delta^3$$
$$+(-32577650-3648594c+199578c^2+1106c^3)\Delta^4$$
$$+(-4856082+80724c+4998c^2)\Delta^5+(-126000+5040c)\Delta^6\Big)(\mathbf{l}_{-4})^2\mathbf{l}_{-2}$$

$$+\,72\Big(306222000-173805840c+10920960c^2+353640c^3$$
$$+\,(381614464-23068800c-1839477c^2+394058c^3)\Delta$$
$$+\,(-105570444+28836996c-2363925c^2+120078c^3)\Delta^2$$
$$+\,(-5062960+1902948c-186516c^2+10948c^3)\Delta^3$$
$$+\,(6142752-591276c+16296c^2+168c^3)\Delta^4$$
$$+\,(183204-17388c+504c^2)\Delta^5\Big)\mathbf{l}_{-6}(\mathbf{l}_{-2})^2$$
$$+\,3\Big(-36240157632+6121778448c-402247260c^2+15838734c^3+980700c^4$$
$$+\,(61259894752-7807807432c+120911226c^2-916009c^3+946078c^4)\Delta$$
$$+\,(-7496632304+562374632c-138115254c^2+2579783c^3+269878c^4)\Delta^2$$
$$+\,(-2902569880+343253716c-42063144c^2+978190c^3+31388c^4)\Delta^3$$
$$+\,(611052008-52433468c-1301100c^2+80872c^3+1568c^4)\Delta^4$$
$$+\,(38386992-1678896c-154944c^2+7008c^3)\Delta^5$$
$$+\,(3894912-324864c+6912c^2)\Delta^6\Big)\mathbf{l}_{-6}\mathbf{l}_{-4}$$

$$
\begin{aligned}
&+ 18\Big(84650153280 - 14906569500c + 601240950c^2 - 1997070c^3 + 598500c^4\\
&+ (-63120449168 + 11108354394c - 726265569c^2 + 16981463c^3 + 370230c^4)\Delta\\
&+ (-4980065552 + 1173915830c - 54554649c^2 - 1312234c^3 + 171640c^4)\Delta^2\\
&+ (2427198620 - 271665042c + 16272864c^2 - 804287c^3 + 26670c^4)\Delta^3\\
&+ (11156180 + 22230214c - 2038338c^2 + 42124c^3 + 560c^4)\Delta^4\\
&+ (9021768 + 97848c - 64800c^2 + 2064c^3)\Delta^5\\
&+ (649152 - 54144c + 1152c^2)\Delta^6\Big)\mathbf{l}_{-8}\mathbf{l}_{-2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \Big(2402050721280 - 453732439584c - 1008508824c^2 + 736353804c^3 \\
&+ 10413480c^4 + 2116800c^5 + (-750420745088 + 104186820112c \\
&+ 5982020544c^2 - 1576485004c^3 + 55452740c^4 + 1381800c^5)\Delta \\
&+ (472993701600 - 34597963440c + 5014768290c^2 - 448750215c^3 \\
&+ 5613636c^4 + 585060c^5)\Delta^2 + (141065264032 - 14296085648c \\
&+ 1417241010c^2 - 80355379c^3 + 222908c^4 + 91140c^5)\Delta^3 \\
&+ (-36292325160 + 7660662252c - 400215072c^2 + 5783664c^3 \\
&- 80556c^4 \ + 5880c^5)\Delta^4 + (3111074008 - 286403588c + 23527848c^2 \\
&- 1241092c^3 + 23912c^4)\Delta^5 + (5295360 + 16262592c - 1153344c^2 \\
&+ 20352c^3)\Delta^6 + (4612608 - 297216c + 4608c^2)\Delta^7\Big)\mathbf{l}_{-10}\Big\} \,.
\end{aligned}
$$