NONDECOUPLING OF MAXIMAL SUPERGRAVITY FROM THE SUPERSTRING

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Introduction

This talk is based on arXiv:0704.0777 [hep-th] by Michael Green, Hirosi Ooguri, and JHS.

Recently, there has been some speculation that four-dimensional $\mathcal{N} = 8$ supergravity might be ultraviolet finite to all orders in perturbation theory (Green et al., Bern et al.). If true, this would raise the question of whether $\mathcal{N} = 8$ supergravity might be a consistent theory that is decoupled from its string theory extension.

A related question is whether $\mathcal{N} = 8$ supergravity can be obtained as a well-defined limit of superstring theory. Here we argue that such a supergravity limit of string theory does not exist in four or more dimensions, irrespective of whether or not the perturbative approximation is free of ultraviolet divergences.

We will study limits of Type IIA superstring theory on T^{10-d} for various d.

The analysis is analogous to the study of the decoupling limit on Dp-branes, where field theories on branes decouple from closed string modes in the bulk (Sen, Seiberg).

The decoupling limit on Dp-branes exists for $p \leq 5$. However, for $p \geq 6$ infinitely many new world-volume degrees of freedom appear in the limit. This has been regarded as a sign that a field theory decoupled from the bulk does not exist for $p \geq 6$.

We will find similar subtleties for Type IIA theory on $T^{10-d} \times \mathbb{R}^d$ for $d \geq 4$.

Perturbative spectrum

It will be sufficient to consider the torus to be the product of $(10-d)$ circles, each of which has radius R. Numerical factors, such as powers of 2π, are irrelevant to the discussion and therefore will be dropped.

In ten dimensions, Newton's constant is given by

$$G_{10} = g^2 \ell_{\rm s}^8,$$

where $\ell_{\rm s}$ is the string scale and g is the string coupling constant. Thus, the effective Newton constant in d dimensions is given by

$$G_d \equiv \ell_d^{d-2} = \frac{G_{10}}{R^{10-d}} = \frac{g^2 \ell_{\rm s}^8}{R^{10-d}},$$

where ℓ_d is the d-dimensional Planck length, so that

$$g = \frac{R^{5-\frac{d}{2}}}{\ell_{\rm s}^4} \cdot \ell_d^{\frac{d}{2}-1}.$$

We are interested in whether there is a limit of string theory that reduces to maximal supergravity, which is defined purely in terms of the dynamics of the 256 states in the massless supermultiplet.

In other words, we require the decoupling of all excited string states, together with the Kaluza–Klein excitations and string winding states associated with the $(10-d)$-torus.

A necessary condition for this decoupling is that these states are all infinitely massive compared to the Planck scale ℓ_d. This is achieved by taking

$$\frac{1}{R}, \ \frac{1}{\ell_{\rm s}}, \ \text{ and } \ \frac{R}{\ell_{\rm s}^2} \gg \frac{1}{\ell_d}\,,$$

with ℓ_d fixed.

Then the surviving perturbative states are the 256 massless states of maximal supergravity, which is $\mathcal{N} = 8$ supergravity when $d = 4$.

Nonperturbative spectrum

Let us now consider the spectrum of half-BPS nonperturbative superstring excitations in this limit. First consider a Dp-brane wrapping a p cycle of the torus. The mass of such a state in d dimensions is

$$M_p = \frac{R^p}{g\ell_s^{p+1}} = \frac{R^{p+\frac{d}{2}-5}}{\ell_s^{p-3}} \cdot \ell_d^{1-\frac{d}{2}}.$$

When $d \leq 5$, we also need to consider a NS5-brane wrapping a 5 cycle. This has a mass given by

$$M_{\mathrm{NS5}} = \frac{R^5}{g^2\ell_s^6} = \frac{\ell_s^2}{R^{5-d}} \cdot \ell_d^{2-d}.$$

In order to obtain the pure supergravity theory with 32 supercharges in d dimensions, these nonperturbative states also need to decouple, so their masses must satisfy

$$M_p, M_{\mathrm{NS5}} \gg 1/\ell_d.$$

In the case of $d = 4$ the nonperturbative BPS particle spectrum also includes Kaluza–Klein monopoles, which will be discussed soon.

A Kaluza–Klein momentum state and a wrapped string state have masses $1/R$ and R/ℓ_{s}^2, respectively, and they are half-BPS objects that carry a single unit of a conserved charge. In d-dimensions, their magnetic duals are $(d-4)$-branes.

The BPS saturation condition together with the Dirac quantization condition implies quite generally that the mass m of a BPS particle and the tension $\mathcal{T}$ of its magnetic dual $(d-4)$-brane are related by

$$m\mathcal{T} \sim \frac{1}{G_d} = \frac{1}{\ell_d^{d-2}}.$$

Applying this to $d = 4$, we see that there is no limit in four dimensions where we can keep all BPS particles heavier than the Planck scale. In particular, magnetic duals of Kaluza–Klein excitations, which are the Kaluza–Klein monopoles, are BPS states with masses $\sim R/\ell_4^2 \to 0$.

Similarly, magnetic duals of wrapped strings are NS5-branes wrapping 5-cycles of T^6, and their masses go as $\ell_{\mathrm{s}}^2/R\ell_4^2 \to 0$.

Two dimensions

In two dimensions, there are no magnetic duals of BPS particles, and we expect that there is a smooth limit where all BPS particles become infinitely massive.

When $d = 2$, the conditions we want to impose are

$$M_p = \frac{1}{R}\left(\frac{\ell_{\mathrm{s}}}{R}\right)^{3-p} \quad \text{and} \quad M_{\mathrm{NS5}} = \frac{1}{R}\left(\frac{\ell_{\mathrm{s}}}{R}\right)^{2} \longrightarrow \infty.$$

This is achieved by letting $R \longrightarrow 0$ with $g = (R/\ell_{\mathrm{s}})^4$ held finite. In this limit, all particle masses are much higher than the Planck mass, except for the massless $\mathcal{N} = 16$ supergravity states.

Dp-brane and NS5-brane instantons wrapping cycles of the T^8 have Euclidean actions proportional to

$$(\ell_{\mathrm{s}}/R)^{3-p} \sim g^{\frac{p-3}{8}} \quad \text{and} \quad (\ell_{\mathrm{s}}/R)^2 \sim g^{-\frac{1}{4}}.$$

All these actions remain finite and non-zero in the limit, but their effects are not uniformly suppressed for small g.

Three dimensions

In three dimensions it is possible to define a limit where all BPS particles become infinitely massive simultaneously. In this case, magnetic duals of BPS particles are (-1)-branes, namely instantons, and their Euclidean actions vanish in the limit. Thus, one would expect nonperturbative effects to be very large in three dimensions even though no singularity is apparent from the spectrum.

In three dimensions, the relation with Chern-Simons gauge theory suggests that pure Einstein gravity is finite to all orders in perturbation theory. However, this theory has no propagating degrees of freedom.

It is not known whether there is a finite quantum gravity theory in three dimensions that includes propagating (scalar or spin-1/2) degrees of freedom. The fact that we find limits of string theory compactifications with a finite number of such propagating degrees of freedom in these dimensions may be encouraging, though the implications of the nonperturbative instanton contributions need to be understood.

Four dimensions

When $d = 4$, the conditions for the wrapped nonperturbative branes to have infinite masses are

$$M_p = \frac{1}{\ell_4}\left(\frac{\ell_{\rm s}}{R}\right)^{3-p} \quad \text{and} \quad M_{\rm NS5} = \frac{\ell_{\rm s}^2}{R\ell_4^2} \to \infty.$$

This cannot be realized simultaneously for all $p = 0, 2, 4, 6$.

This is in accord with the general argument given earlier, since a wrapped Dp-brane and a wrapped D$(6-p)$-brane are electric–magnetic duals.

Similarly, the magnetic duals of Kaluza–Klein excitations and wrapped strings are Kaluza–Klein monopoles and wrapped NS5-branes, whose masses behave as R/ℓ_4^2 and $\ell_{\mathrm{s}}^2/R\ell_4^2$, respectively. There are infinitely many such states since they have arbitrary integer charges.

In the limit $R, \ell_{\mathrm{s}}^2/R \to 0$, there is no mass gap and the spectrum becomes continuous.

The reappearance of ten dimensions

To understand the implications of these infinitely many light states, we note that among the elements of the four-dimensional U-duality group $E_7(\mathbb{Z})$ is the four-dimensional S-duality transformation that interchanges the 28 types of electric charge with the corresponding magnetic charges.

This duality is described by the following transformations of the moduli,

$$\mathrm{S}: \; R \rightarrow \tilde{R} = \frac{\ell_4^2}{R} \;\text{ and }\; \ell_{\mathrm{s}} \rightarrow \tilde{\ell}_{\mathrm{s}} = \frac{\ell_4^2}{\ell_{\mathrm{s}}}.$$

Note that this transformation inverts the radius R in four-dimensional Planck units (in contrast to T-duality, which inverts R in string units). It implies that

$$g \to \tilde{g} = 1/g,$$

which maps BPS states into each other.

For example, a wrapped Dp-brane is interchanged with a wrapped D$(6-p)$-brane. Similarly, a Kaluza–Klein excitation is interchanged with a Kaluza–Klein monopole (whereas T-duality would relate it to a wrapped F-string). Thus, in the dual frame in which the compactification scale $\tilde{R} \to \infty$, the six-torus is decompactified.

The fact that an infinite set of states from the nonperturbative sector become massless shows that the limit of interest does not result in pure $\mathcal{N} = 8$ supergravity in four dimensions. Rather, it results in 10-dimensional decompactified string theory with the string coupling constant inverted.

These results illustrate the conjectures of Vafa and Ooguri about the geometry of continuous moduli parameterizing the string landscape. The conjectures concern consistent quantum gravity theories with finite Planck scale in four or more dimensions.

Among the conjectures are the statements that, if a theory has continuous moduli, there are points in the moduli space that are infinitely far away from each other, and an infinite tower of modes becomes massless as a point at infinity is approached.

Since the limit considered here corresponds to a point in the moduli space of string compactifications at infinite distance from a generic point in the middle of moduli space, the conjectures predict than an infinite number of particles become massless in the limit. For $d = 4$, we have found that these include the Kaluza–Klein monopoles.

The moduli space of pure $\mathcal{N} = 8$ supergravity also contains infinite distance points, but it does not take account of the light particles that appear near these points. If these particles were included, one would have string theory and not $\mathcal{N} = 8$ supergravity. Thus, $\mathcal{N} = 8$ supergravity is in the Swampland.

Similarly, there are many superstring compactifications with $\mathcal{N} < 8$ supersymmetry. Discarding stringy states in these compactifications results in further supergravity theories in the Swampland.

Scattering Amplitudes

It is interesting to see how scattering amplitudes behave in the limit under consideration. Consider a four-dimensional graviton scattering amplitude where the graviton momenta are below the four-dimensional Planck scale.

According to formulas given earlier the ten-dimensional Planck length, ℓ_{10}, is given by

$$\ell_{10} = g^{1/4}\ell_{\mathrm{s}} = R^{3/4}\ell_4^{1/4}.$$

After the S-duality transformation, the limit $R \to 0$ turns into $\tilde{R} \to \infty$, and $\tilde{\ell}_{10} = \tilde{R}^{3/4}\ell_4^{1/4} \to \infty$ in ten dimensions.

Since $\tilde{\ell}_{10} \ll \tilde{R}$, the extra dimensions decompactify and the theory is effectively ten-dimensional. Furthermore, if we take this limit keeping the graviton momenta fixed (in units of the four-dimensional Planck mass), the scattering process becomes trans-Planckian. Generically, we expect that it will involve formation and evaporation of virtual black holes in ten dimensions.

Conclusion

The original motivation of this work was to investigate the relation between superstring theory and $\mathcal{N} = 8$ supergravity and to explore under what conditions supergravity might be ultraviolet finite.

We found that for $d \geq 4$ there is no limit of toroidally compactified superstring theory in which the stringy effects decouple and only the 256 massless supergravity fields survive below the four-dimensional Planck scale.

This is true whether or not there are ultraviolet divergences in supergravity perturbation theory. We would like to make specific deductions about that, but so far this has not been achieved.

It might be instructive to compare the situation to that of the conifold limit of Calabi–Yau compactified type II superstring theory studied by Strominger. In that case, certain terms in the low-energy effective theory that are independent of the string coupling constant g, due to the decoupling of vector and hypermultiplet fields, can be computed in string perturbation theory.

One can estimate the singularity of these terms using the fact that a brane wrapping a vanishing cycle describes a nonperturbative BPS particle that becomes massless in the conifold limit.

If one could identify analogous terms in $\mathcal{N} = 8$ supergravity, one could transform the Feynman diagram computation in four-dimensional supergravity into a corresponding computation in ten dimensions, which might give insight into the question of ultraviolet finiteness.

The situation is qualitatively different in two and three dimensions ($d = 2, 3$), where all non-supergravity states develop masses larger than the Planck scale in the limit and therefore they can decouple. In these cases only the 256 massless supergravity states survive, and a self-contained quantum gravity theory decoupled from string theory might exist.

However, in the $d = 3$ case there are instantons with zero action, which give rise to large nonperturbative contributions.

In the $d = 2$ case the instanton actions do not vanish in the limit, but some of them vanish when $g \longrightarrow 0$. Therefore, the amplitudes might have large nonperturbative contributions in this case, too.