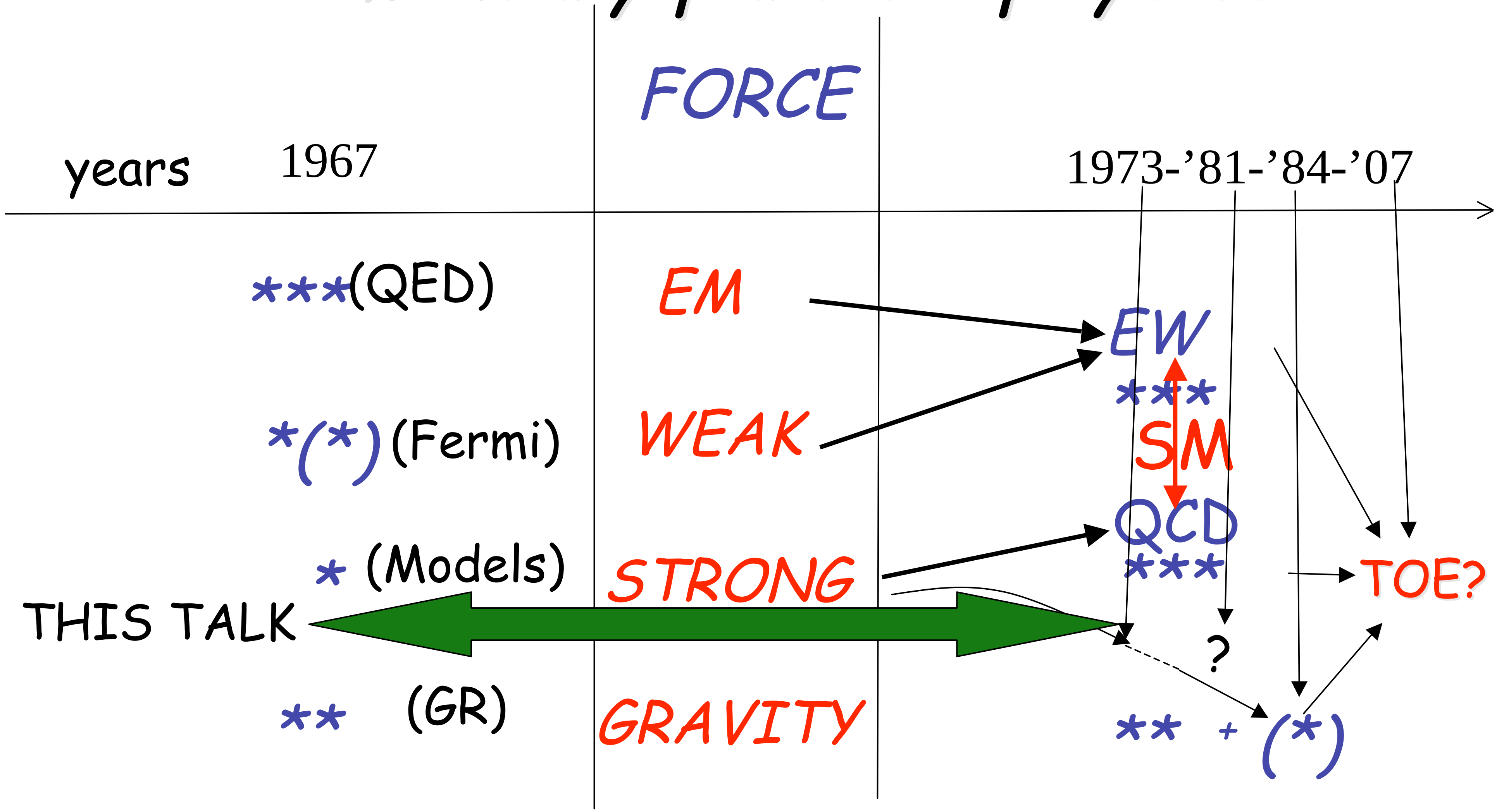
THE BIRTH OF STRING THEORY
(Florence, 18-19 May 2007)

Rise and Fall of the Hadronic String
(Gabriele Veneziano)

An evolving "Michelin guide" of elementary particle physics



Outline

- Prehistory
- Dual Resonance Models
- Hints of a string
- Good and bad news
- QCD takes over

With apologies for some inevitable overlap with previous & next talk!

Prehistory (see also previous talk)

STRONG INTERACTIONS ~ 1967

No Theory, rather:

A handful of models capturing one or another aspect of hadronic physics e.g.

- Short range i.e. no massless particles
- Symmetries, conservation laws (P, C, T, I, SU(3),…)
- Many metastable states (resonances) extending to large J: an ever increasing zoo?

Why did we take the (a posteriori) wrong way?

A QFT approach looked hopeless:

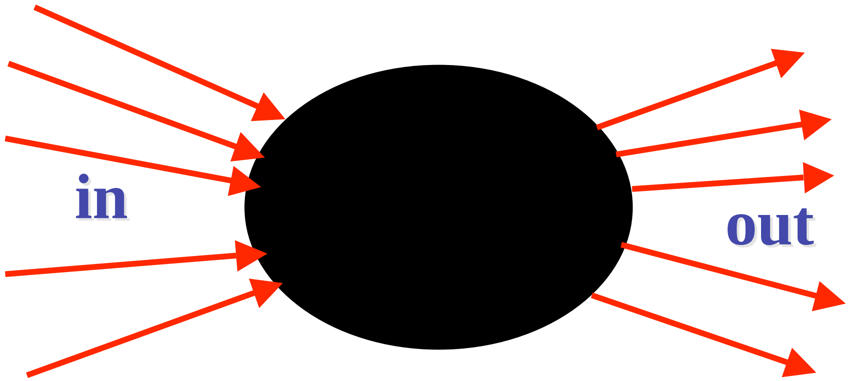
1. Too many d.o.f. => too many fields
2. High-J QFT's are pathological

An S-matrix approach looked more promising:

The S-Matrix (Heisenberg 1943)

$$\langle out|S|in\rangle = S(in \to out) = complex\ number$$

$$|S(in \to out)|^2 = Prob.\ for: in \to out$$



- Symmetries: easy to implement on S
- Causality => analyticity, dispersion relations
- Conservation of Prob => Unitarity constraint: $SS^\dagger = 1$

Organizing the hadronic zoo

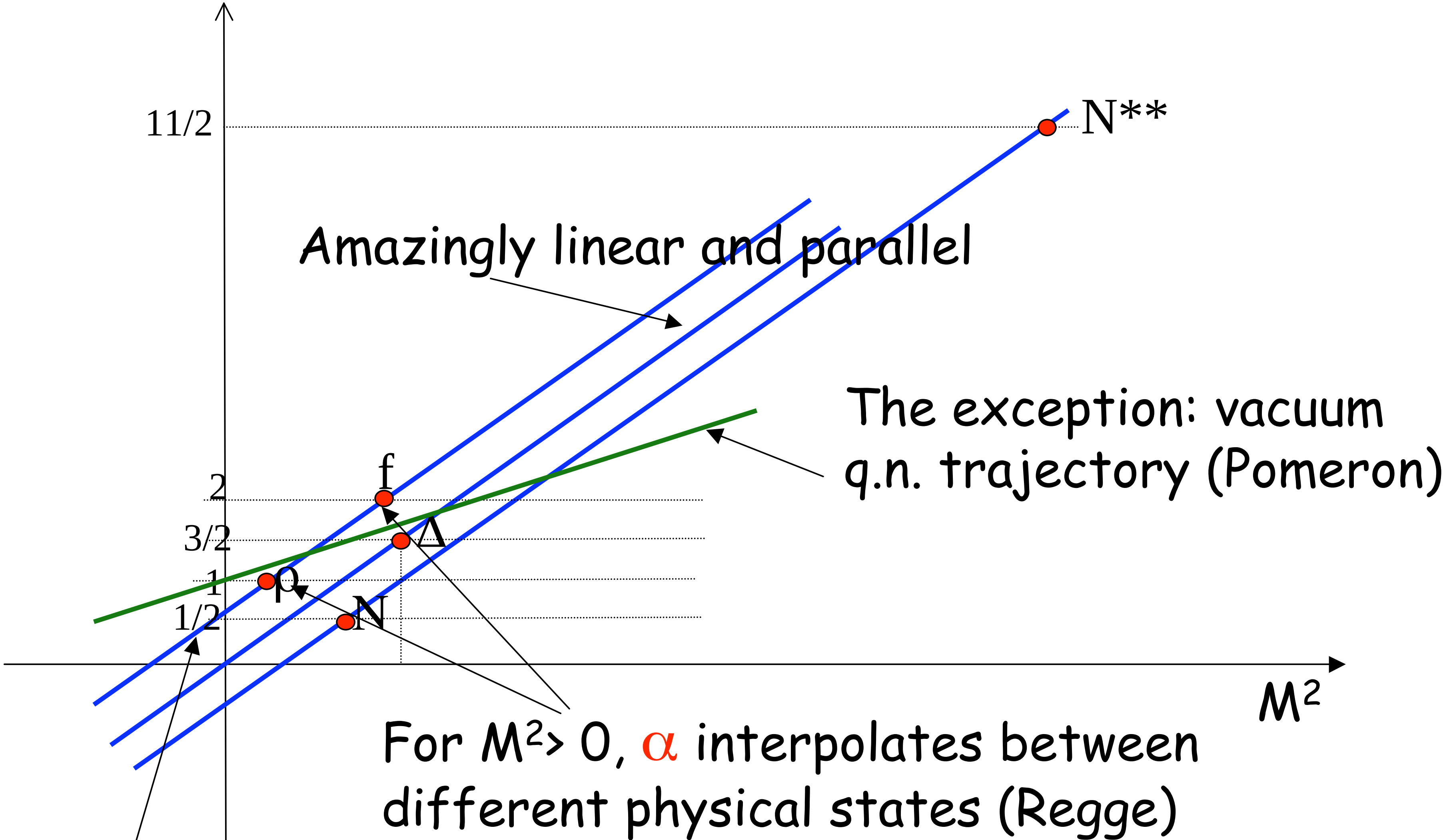
A) Group theory:

- $SU(2)_I$, $SU(3)_{F,}$ same-J particles
- SU(6).. combining $\Delta J \leq 1$ particles

B) Regge theory of complex J

- For combining different-J particles (Regge)
- For describing high-energy scattering (Chew-Mandelstam)

$J/\hbar = \alpha(M^2)$ = Regge trajectory



For $M^2 < 0$, α controls high-energy scattering at momentum transfer ~ |M| (Chew-Mandelstam)

Chew's "expensive" bootstrap…

Add to the general constraints of symmetry, causality, unitarity that of Nuclear Democracy

"All hadrons lie on Regge trajectories @ $M^2>0$;

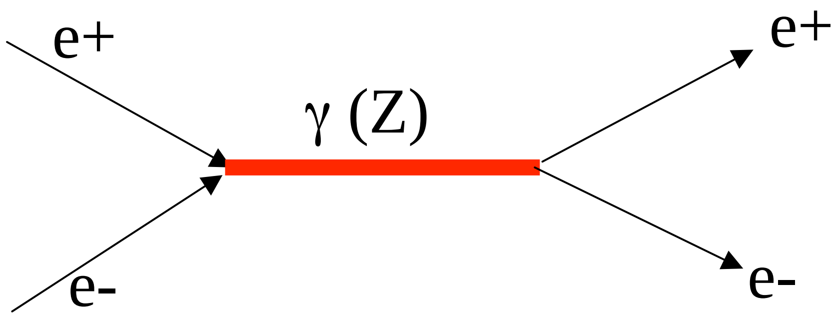
All asymptotics fixed by same trajectories @ $M^2<0$"

Will this give a unique S-matrix?

The S-matrix knew about Regge-Chew-Maldestam…twice:

$$S = S_{s\text{-channel}} + S_{t\text{-channel}}$$

Cf. QED: $e^+ e^- \dashrightarrow e^+ e^-$ is given (to lowest order in α) by the coherent sum of two Feynman diagrams



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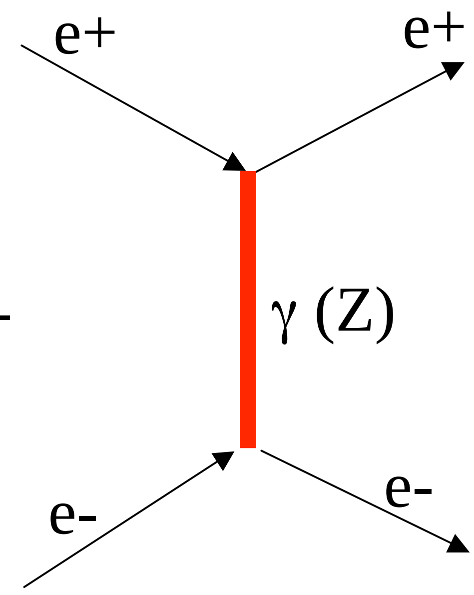
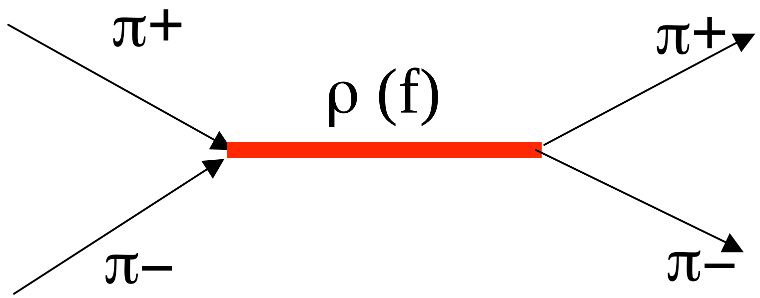


Likewise…



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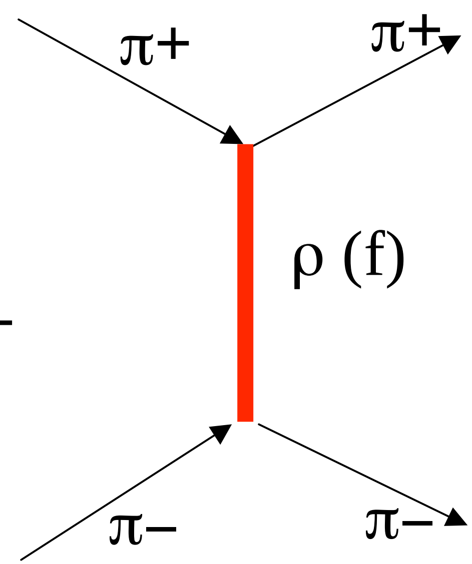
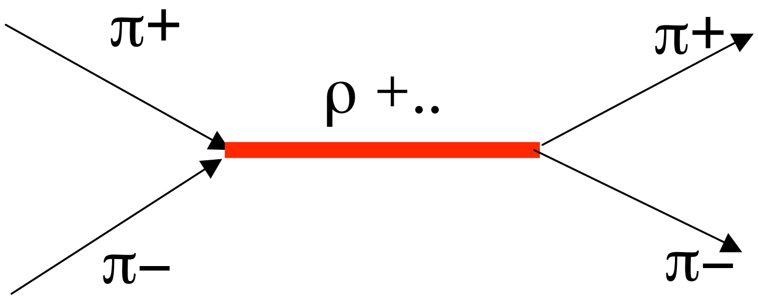


…and a cheap one

Erice, 1967: Gell Mann bringing news from Caltech:

Dolen-Horn-Schmit duality: s-and t-channel descriptions are roughly equivalent, complementary, DUAL (Cf. QM)

Adding them = double counting!



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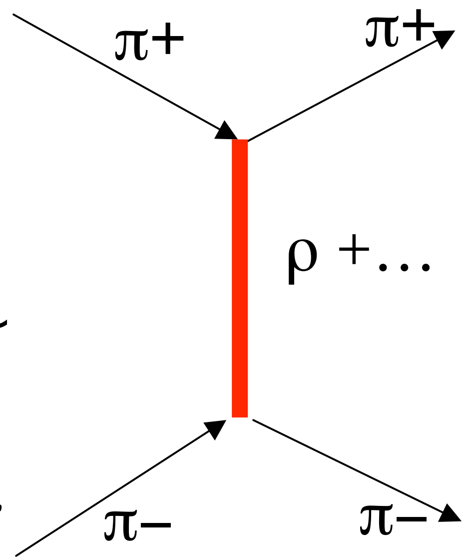
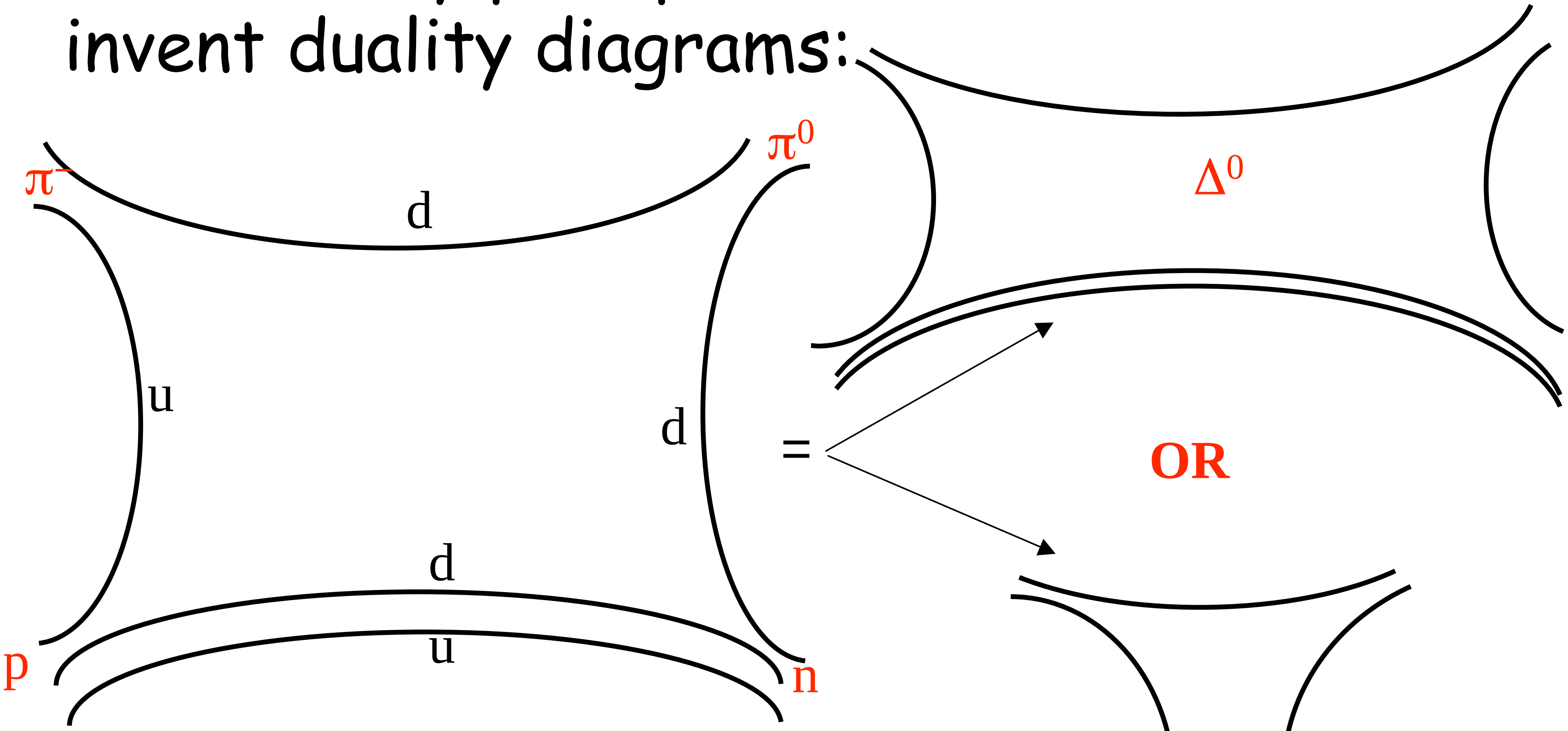
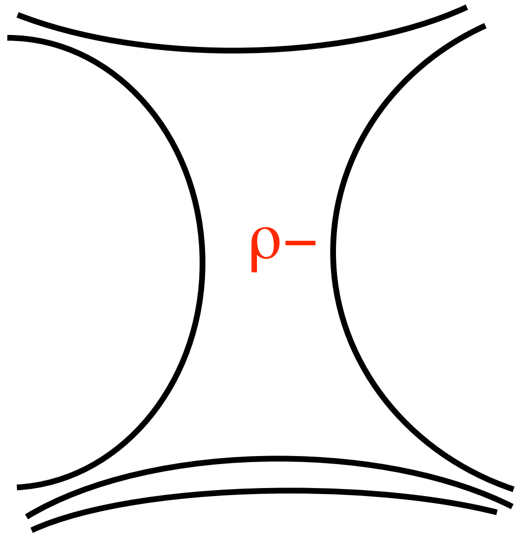


A non-trivial yet LINEAR relation…

- DHS duality prompted Harari and Rosner to invent duality diagrams:





NB: Quarks were just a mnemonics for QN's in those days

N.B. The Pomeron was dual to BKGND

π N scattering looked too complicated
We* decided to consider a simpler case:

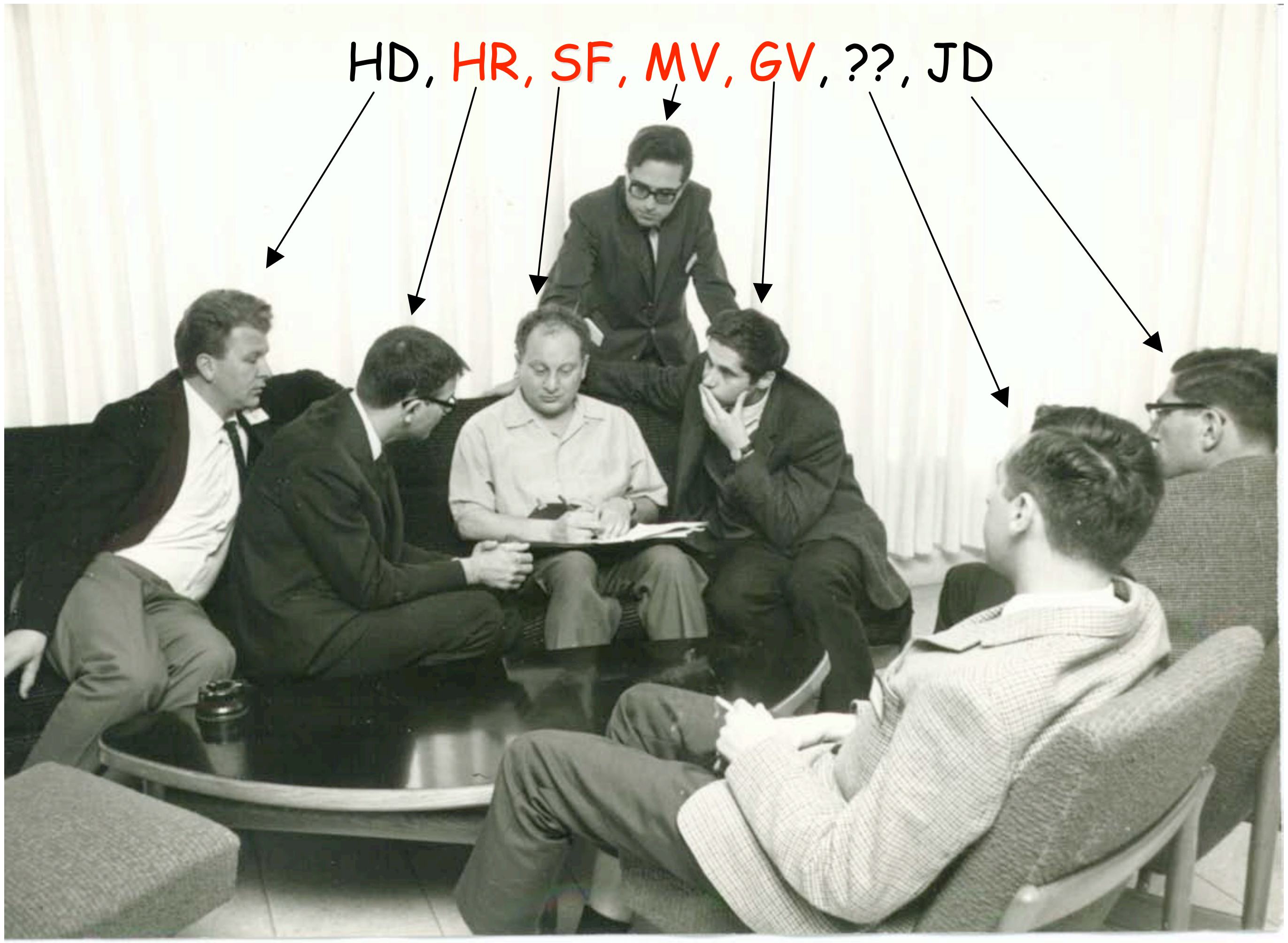
$\pi\ \pi \to \pi\ \omega$

Very symmetric & very selective in QN's (ρ, ρ*..)

Between the fall of 1967 and the summer of 1968 we made much progress in finding solutions to this "Easy Bootstrap".

*) Ademollo, Rubinstein, Virasoro, GV (+Bishari & Schwimmer) with advice and encouragement of Sergio Fubini

Weizmann Institute, 1967



A cheap solution to a cheap bootstrap

The ARVV ansatz that worked amazingly well for the DHS bootstrap in $\pi \pi \to \pi \omega$ was simply:

$$Im\, A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))} (\alpha' s)^{\alpha(t)-1} (1 + O(1/s))$$

with: $$\beta(t) \sim const.,\ \alpha(t) = \alpha_0 + \alpha' t$$

i.e. a linear leading Regge trajectory accompanied by parallel "daughter" trajectories. The latter, if suitably tuned, were improving the agreement in an increasingly large range of t

Which was the road that led from the above ansatz to an "exact solution"? Three main ingredients (besides the boat trip..) were used:

1. Look at A rather than at Im A (A = analytic function)
2. Impose exact crossing symmetry : A(s,t) = A(t,s)
3. Emphasize resonances over Regge (A ~ meromorphic)

1. Easy to show that $$Im\, A(s,t) = \frac{\beta(t)}{\Gamma(\alpha(t))}(\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

corresponds to $$A(s,t) = \beta(t)\Gamma(1-\alpha(t))(-\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

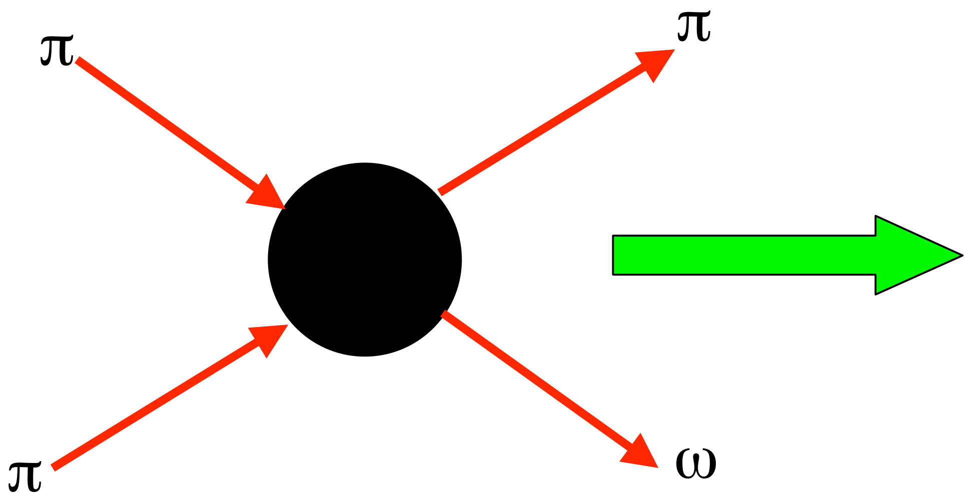
3. A(s,t) already exhibits resonances (poles) in the t-channel but still only a smooth Regge behaviour in s: However, using

$$\frac{\Gamma(1-\alpha(s))}{\Gamma(2-\alpha(s)-\alpha(t))} \to (-\alpha' s)^{\alpha(t)-1}(1+O(1/s))$$

We can satify both 2. and 3. by simply writing:

$$A(s,t) = \beta\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} = \beta\, B(1-\alpha(s), 1-\alpha(t))$$

Exact DHS duality is implied by analyticity, resonance dominance (=> duality between two infinite sets of resonances in different channels!), and good (Regge) asymptotics!



$$A(\pi\pi \to \pi\omega) =$$

$$g^2 \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} +$$

$$(s \leftrightarrow u) + (t \leftrightarrow u) =$$

$$g^2\, B(1-\alpha(s), 1-\alpha(t)) +$$

$$(s \leftrightarrow u) + (t \leftrightarrow u)$$

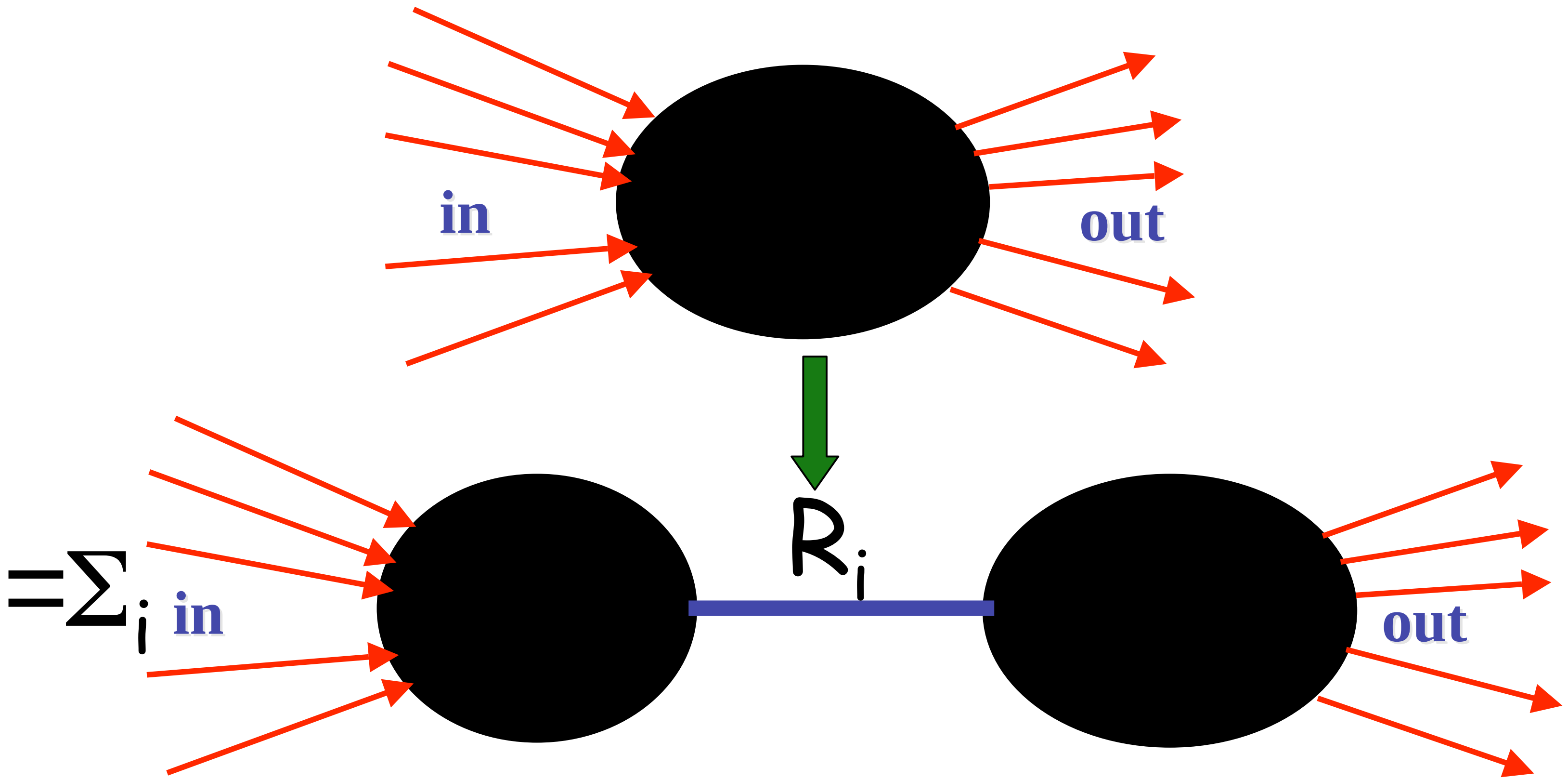
Dual Resonance Models
(see also next talk)

Counting states

- There was a big worry based on previous experience: possibly, in order to satisfy all the constraints, the model had to contain "ghosts", states produced with negative probability. If so the model would have been inconsistent.
- To answer that question one had to identify first all the states. The way to do so was via a property of S, known as factorization. It is basically what unitarity reduces to in the single-particle-exchange approximation

Factorization

Q: How many terms are needed (in the sum over i) in order to have, for all in and out states,



- This could not be done using just the Beta function, but, after a short while, in the fall of 1968, several people (BR, V, GS, CT, CP, KN) had found its (pretty unique) generalization to multi-particle initial and final states.
- The result of the counting of states (FV, BM, 1969) turned out to be very surprising.
- Because of the parallel daughters, we were expecting a mild degeneracy (increasing, say, like a power of M). Instead, the number of states grew much faster, like exp(b M), with b some constant (with dimensions 1/mass and of order $(\alpha')^{1/2}$).

- Although unexpected, this was just the behaviour postulated by Hagedorn a few years earlier (~1965) on more phenomenological basis (e.g. a Boltzmann factor in final particle spectra)
- Taken at face value, such a density of states leads to a limiting (maximal, Hagedorn) temperature T_H given by

$$k_B T_H = c^2/b \ (\sim 150 \text{ MeV})$$

- And, sure enough, there were some ghosts!
- The FV-BM factorization procedure was cumbersome. It was soon replaced by a much more handy operator formalism (FGV, Nambu)

- In that formalism a sufficient set of states consisted of the energy levels of an infinite set of decoupled harmonic oscillators with quantized frequencies:

$$|N_{n,\mu}\rangle \sim \prod_{n,\mu} (a^{\dagger}_{n,\mu})^{N_{n,\mu}} |0\rangle \ , (n = 1, 2, \ldots ; \mu = 0, 1, 2, 3)$$

$$\alpha' M^2 = \sum_{n,\mu} n \, a^{\dagger}_{n,\mu} \, a_{n,\mu} \equiv L_0 - p^2$$

$$[a_{n,\mu} \, , \, a^{\dagger}_{m,\nu}] = \delta_{n,m} \eta_{\mu\nu} \ , \ \eta_{\mu\nu} = diag(-1, 1, 1, 1)$$

Because of the "wrong" sign of the timelike c.r., states created by an odd number of timelike operators were ghosts. Was the DRM doomed? Well, almost.
One (tiny?) hope remained: all those states were sufficient but perhaps only a (ghost-free) subset was necessary

In FV's original paper the following (so-called "spurious") states were found to be unnecessary

$$L_{-1}|X\rangle \equiv \left(p\cdot a_1^{\dagger} + \sum_n \sqrt{n(n+1)}\, a_{n+1}^{\dagger}\cdot a_n \right)|X\rangle$$

(with |X> any state)

This was probably sufficient to eliminate the ghosts created by the time component of a_1. But what about all others? The situation looked almost desperate...until Virasoro (1969) made a crucial discovery. Iff $\alpha(0) = 1$ one could enlarge enormously the space of "spurious" states to:

$$L_{-m}|X\rangle \equiv \left(p\cdot a_m^{\dagger} + \sum_n \sqrt{n(n+m)}\, a_{n+m}^{\dagger}\cdot a_n \right)|X\rangle$$

(with m=1,2,..)

=> for $\alpha(0) = 1$, there was a chance to eliminate all the ghosts!!
$\alpha(0) = 1$ gives a massless J=1 state but people kept hoping...

Formal developments

Between the summer of 1969 and the spring of 1970 several developments took place within the operator formalism:

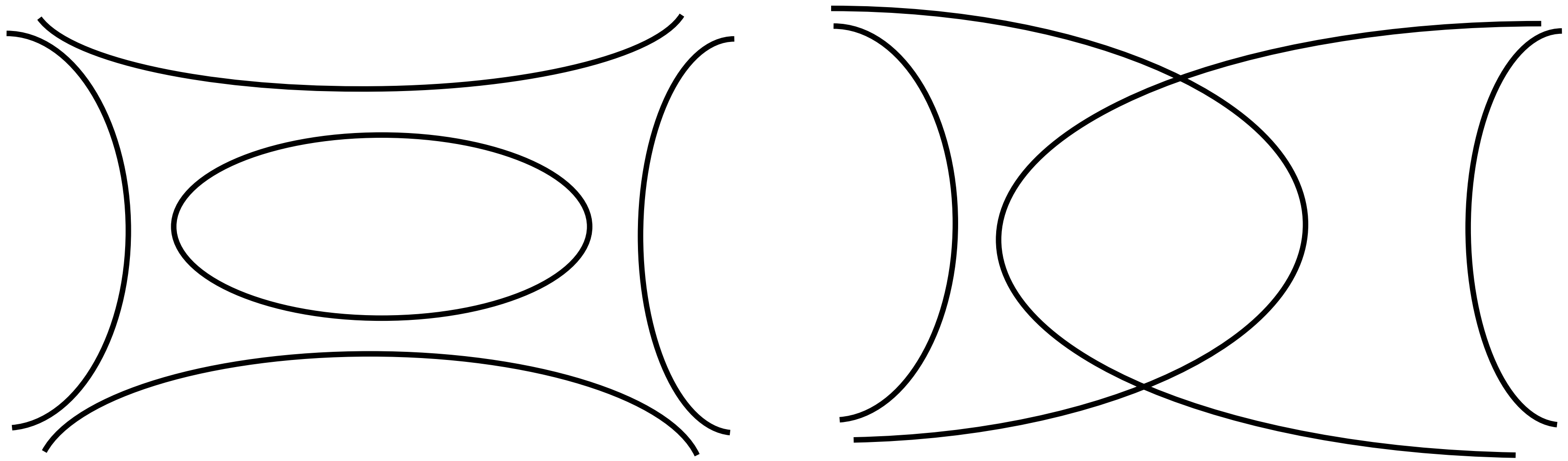
1. Sciuto's vertex and the Caneschi-Schwimmer-V twist
2. Discovery (Gliozzi & Chiu-Matsuda-Rebbi) that (L_0 , $L_{\pm 1}$) satisfy an SU(1,1) algebra.
3. Construction (FV and Gervais, 1969) of fields (Q(z)) and «Vertex Operators», V(k); their correlators, SU(1,1) action on them, as a result:
4. Duality, factorization and spurious/physical-state conditions all came out algebraically
5. After Virasoro's work, FV (1970) extended all this to the whole set of L_n and «quickly» guessed their algebra… missing the crucial «central charge», soon discovered by J. Weis (Cf. FV's NAIP => Virasoro algebra)

The no-ghost theorem

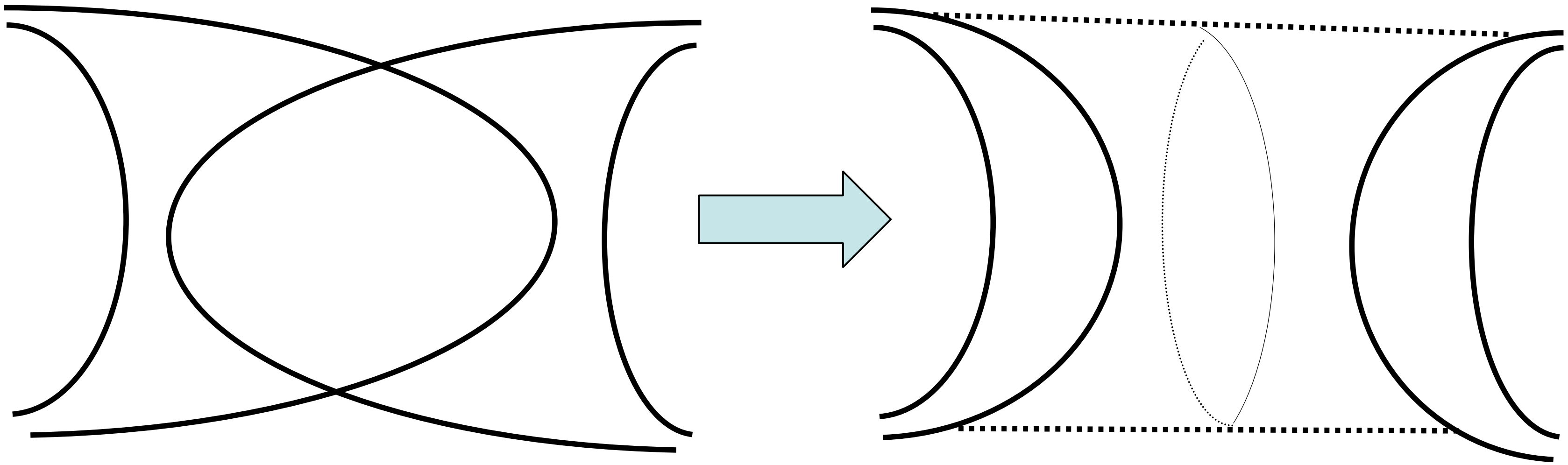
- At this point the machinery was almost ready for a final assault to the ghost-killing program
- An essential step turned out to be the construction of the DDF (Di Vecchia, Del Giudice, Fubini) positive-norm states. They were in one-to-one correspondence with (D-2) sets of harmonic oscillators (D = dimensionality of spacetime = 4?)
- A talk to the MIT mathematicians: no proof came out of them, but Kac-Moody algebras etc.
- The no-ghost theorem was proven instead by R. Brower and by P. Goddard & C.Thorn
- It only worked for $\alpha(0) = 1$ and $D \leq 26$! At D=26 the DDF states were both necessary and sufficient. At D<26 some other positive norm states were needed. At D>26 ghosts were still present among the physical states.

Loops

- The DRM was the analogue of the tree-level approximation of a QFT. In order to implement fully unitarity (e.g. give a finite widths to the resonances) loops had to be added.
- Having identified the physical states, this was (almost) a technical problem. One had just to be careful not letting ghosts circulate in the loops.
- Planar and non-planar loops were needed:



The non-planar loop reserved some surprise..

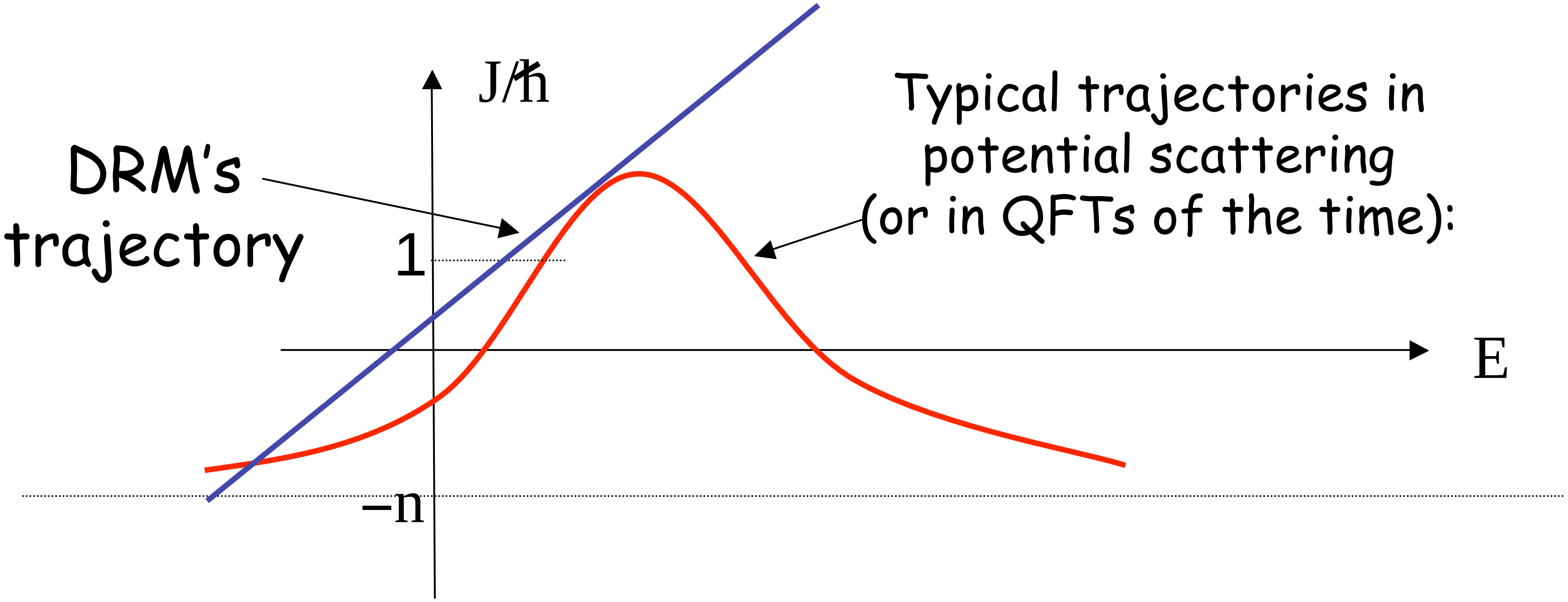


Lovelace (1970) discovered that, for D≠ 26, this loop gave a non-sensical singularity in the vacuum channel. For D= 26 it gave new positive-norm physical states with vacuum QN. Those were just the (already known) states of the Shapiro-Virasoro DRM (later interpreted as a closed-string).
For a theory of hadrons this was a candidate Pomeron trajectory but its intercept was wrong, again by a factor 2!
Actually the first time a critical D (alas ≠ 4) was found!

(Partly missed) hints of a string?

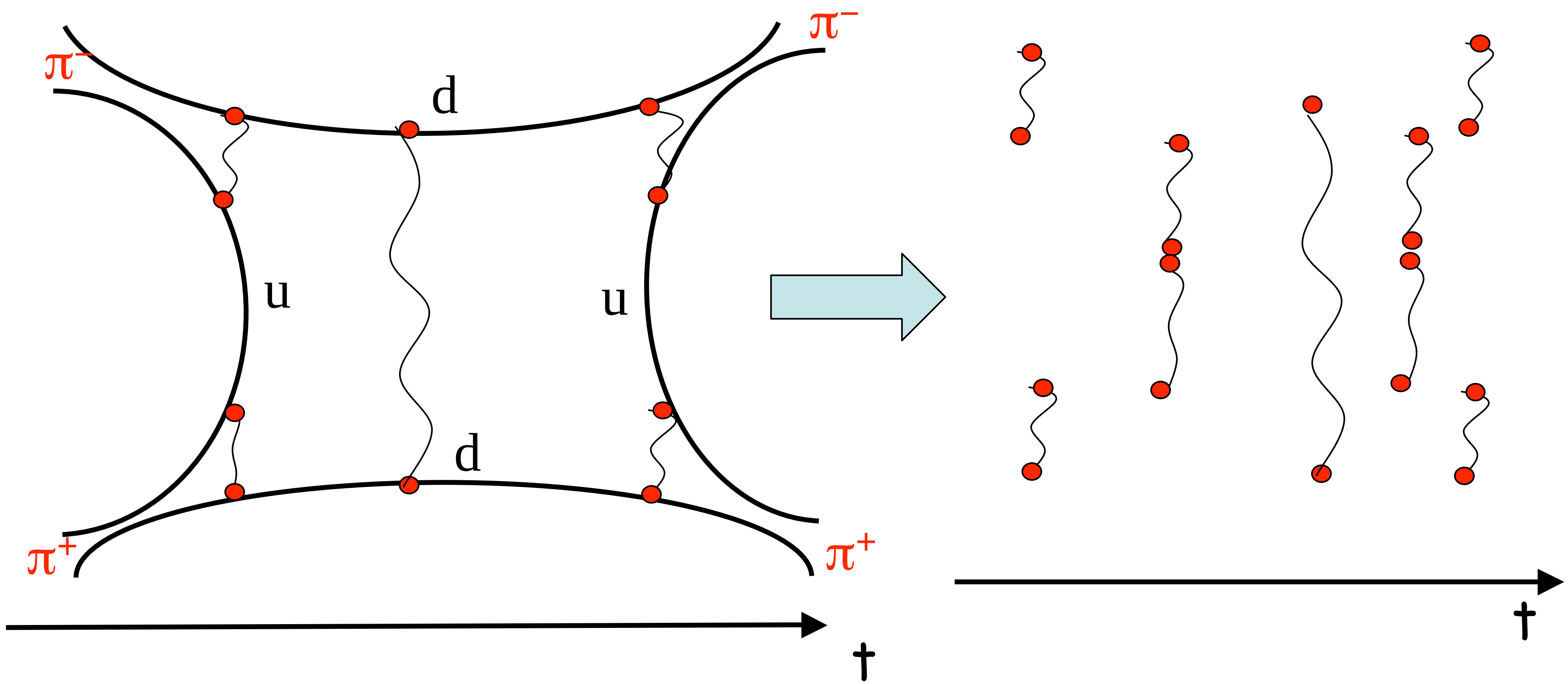
1. From linear Regge trajectories
2. From duality and duality diagrams
3. From the harmonic oscillators
4. From Q(z) and its correlators
5. From DDF «transverse» states

A first missed hint?



α' = dJ/dM^2 ~ 10^{-13} cm/GeV ~ cnst.
Its inverse, T = 10^{13} GeV/cm has dimensions of a string tension (NB, c=1 but no ħ needed)!

A second missed hint?



It's a string…but not the right one!

- The remaining hints were not missed (Nambu, Susskind, Nielsen,..) but identification remained qualitative for sometime
- Eventually, around 1972, the connection with strings was established on solid grounds, in particular through the work of GGRT (deriving, once more, the $\alpha(0) = 1$ and D=26 constraints)
- Paradoxically, now that the DRM had been raised to the level of a Theory, it became apparent that it was not the right one for strong interactions

Good and bad news

1. The good (theoretical) news (see talks by AN, PR, FG,MBG)

   NS and R extensions,

   GSO projection and tachyon elimination (1977)

   ⇒ Fully consistent superstring theories did exist!

   GS (1984) Consistent realistic superstring theories may exist!

2. The bad (phenomenological) news (for the hadronic string)

   $D \neq 4$

   m=0 states with J = 0, ..2

   Softness, whereas…

   - Scaling in $R = \sigma(e^+ e^- \to \text{hadrons})/\sigma(e^+ e^- \to \mu^+ \mu^-)$
   - Bj scaling
   - Large p_t at the ISR

   were all showing evidence for point-like structure in the hadrons

Competition from QFT

QCD came about with its

1. Proven ultraviolet freedom
2. Conjectured infrared slavery i.e. confinement

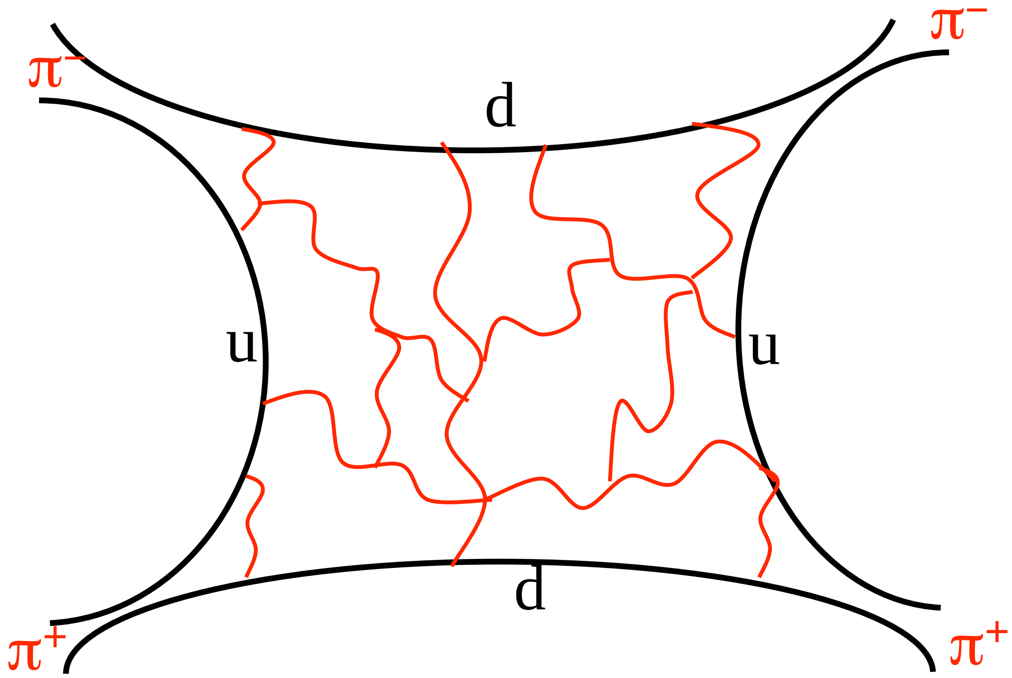
Not the kind of QFT we had discarded..

I kept trying some phenomenology with string theory using its topological structure very unlike that of any QFT

(planar Reggeon vs. non-planar Pomeron)

I gave up in 1974, when, 't Hooft showed that even topology comes out of QCD, provided one considers a 1/N expansion….

- In large-N QCD duality diagrams take up a precise meaning, they are planar Feynmann diagrams bounded by quark propagators & filled with gluons



- They give naturally the narrow-resonance approximation we had been using all the time..
- At sub-leading order the non-planar diagrams give new bound states, the glueballs, and presumably the Pomeron as the Regge trajectory they lie on
- The Hagedorn temperature is re-interpreted as a deconfining temperature for quarks and gluons
- It all seems to fall beautifully into place…
- Except that we still do not know which is the string theory that Nature used to deceive us (a question that has become once more fashionable)

I hope we'll find out one day…