SUPERSYMMETRY

and the

STRING DIRAC EQUATION

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CHRONOLOGY

Graduate School: bleeding from many cuts

ICTP Summer 1969: Sugawara and Nuyts

Triple Reggeon Vertex

Sciuto

With Clavelli: NAL 1969-1971

Algebraic Structure of Dual Amplitudes:

$SU(1,1)$ analysis

Koba and Nielsen; Fubini and Veneziano

Gliozzi; Chiu, Matsuda and Rebbi; Thorn

Vector Vertex

Campagna et al.

Summer 1970 in Aspen

Back to Equations

Fall 1970

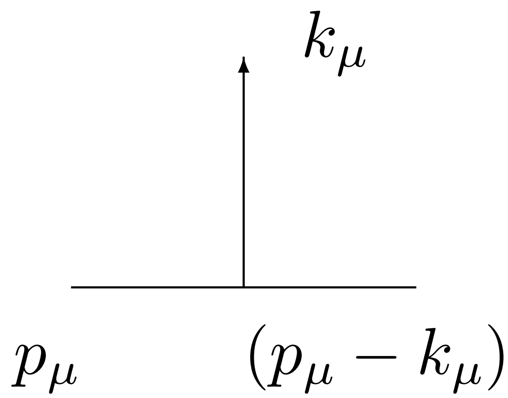
Correspondence Principle

Jan 5, 1971: *Nuovo Cimento* **4A**, 544(1971)

String Dirac Equation

Jan 4, 1971: *Phys. Rev.* **D3**, 2415(1971)

VERTEX



$$V(k_\mu) \; = \; : e^{ik_\mu \, Q_\mu(z)} :$$

Generalized Position

$$Q_\mu(z) \; = \; x_\mu + 2i\alpha' \ln z \, p_\mu + \sqrt{2\alpha'} \sum_{n=0}^{\infty} \left(a_\mu^{(n)} \frac{z^n}{\sqrt{n}} + a_\mu^{(n)\dagger} \frac{z^{-n}}{\sqrt{n}} \right)$$

Nambu, Fubini and Veneziano

$$[\, a_\mu^{(n)} \, , \, a_\nu^{(m)\dagger} \,] \; = \; - \, \eta_{\mu\nu} \delta^{nm}$$

PROPAGATOR

$$p_\mu$$

Generalized Momentum

$$P_\mu \;=\; -\frac{i}{2\alpha'} z \frac{dQ_\mu}{dz}$$

$$P_\mu \;=\; p_\mu + \frac{1}{\sqrt{2\alpha'}} \sum_{n=0}^{\infty} \sqrt{n} \left(a_\mu^{(n)} z^n - a_\mu^{(n)\dagger} z^{-n} \right)$$

$$L_0 \;=\; \alpha' p_\mu p^\mu + \sum_{n=1}^{\infty} n a_\mu^{(n)\dagger} a^{(n)\mu}$$

$$\Delta \;=\; \frac{1}{L_0 + 1}$$

Algebraic Structure

$$SU(1,1): \quad [\, L_m \, , \, L_n \,] \; = \; (m-n) L_{n+m} \, , \quad m, n = 0, \pm 1$$

$$L_1 \; = \; \sqrt{\alpha'} p^\mu a_\mu^{(1)\dagger} + \cdots$$

$$[\, L_n \, , \, Q_\mu(z) \,] \; = \; z^{-n} z \frac{dQ_\mu(z)}{dz} \, , \quad n = 0, \pm 1 \; .$$

$$[\, L_n \, , \, V(k_\mu, z) \,] \; = \; z^{-n} \left(z \frac{d}{dz} - n J_s \right) V(k_\mu, z)$$

$$J_s \; = \; \alpha' \, k^2$$

Vector Vertex

$$V_\rho(k_\mu) \; = \; : P_\rho e^{ik^\mu Q_\mu(z)} :$$

$$[\, L_n \, , \, V_\rho(k,z) \,] \; = \; z^{-n} \left(z \frac{d}{dz} - n J_v \right) V_\rho(k_\mu, z)$$

$$J_v \; = \; \alpha' k^2 + 1$$

Generalize to all n: Virasoro

$$L_n \; = \; \sqrt{\alpha'} p^\mu a_\mu^{(n)\dagger} + \cdots$$

$$[\, L_m \, , \, L_n \,] \; = \; (m-n) L_{n+m} + \frac{D}{12} m(m^2 - 1) \delta_{m,-n}$$

$$-1 \; = \; J_s \; = \; J_v \quad \rightarrow \quad \text{massless spin one particle}$$

$$P_\mu \ = \ p_\mu + \frac{1}{\sqrt{2\alpha'}} \sum_{n=0}^{\infty} \sqrt{n} \left(a_\mu^{(n)} z^n - a_\mu^{(n)\dagger} z^{-n} \right)$$

Define Averaging

$$< \cdots >_n \ = \ \frac{1}{2\pi} \int_{-\pi}^{\pi} d\tau e^{in\tau} \cdots \ . \qquad (z \ = \ e^{i\tau})$$

$$< P_\mu >_0 \ = \ p_\mu \ ; \qquad < Q_\mu >_0 \ \approx \ x_\mu$$

Field Theory Structure: Klein-Gordon Equation

$$0 \ = \ p^2 + m^2 \ = \ < P^\mu >_0 < P_\mu >_0 + m^2$$

Correspondence Principle

$$< A >< B > \quad \rightarrow \quad < A B >$$

$$0 \;=\; p^2 + m^2 \quad \rightarrow \quad < P^\mu P_\mu > + m^2 \;=\; L_0 + m^2$$

Gupta-Bleuler Conditions

$$0 \;=\; p \cdot a^{(n)\dagger} \quad \rightarrow \quad < P^\mu P_\mu >_n \;=\; L_n \approx 0 \;\; (n > 0)$$

Lorentz Generators

$$(x_\mu p_\nu - x_\nu p_\mu) \quad \rightarrow \quad < Q_\mu P_\nu - Q_\nu P_\mu >$$

Dirac Equation

$$0 \;=\; \gamma_\mu p^\mu + m$$

View Dirac Matrices as dynamical variables

$$< \Gamma_\mu > \;=\; \gamma_\mu \;\;\rightarrow\;\; \Gamma_\mu(\tau)$$

$$< \Gamma_\mu > \;=\; \gamma_\mu$$

$$\Gamma_\mu \;=\; \gamma_\mu + i\gamma_5 \sum_{n=0}^{\infty} \left(b_\mu^{(n)} z^n + b_\mu^{(n)\dagger} z^{-n} \right)$$

$$\{ \Gamma_\mu(\tau) \,,\, \Gamma_\nu(\tau') \} \;=\; g_{\mu\nu}\delta(\tau - \tau')$$

$$\{ b_\mu^{(n)} \,,\, b_\nu^{(m)\dagger} \} \;=\; -g_{\mu\nu}\delta^{n,m}$$

Fermionic oscillators as spacetime vectors!!(lucky)

New Dirac Operator

$$< \Gamma_\mu >< P^\mu > +m \quad \rightarrow \quad < \Gamma_\mu P^\mu > +m$$

$$F_n \; = \; < \Gamma_\mu P^\mu >_n$$

New Kind of Algebra

$$\{ F_n \, , \, F_m \} \; = \; 2L_{n+m}$$

$$[L_n \, , \, L_m] \; = \; (m-n)L_{m+n}$$

$$[L_n \, , \, F_m] \; = \; (2m-n)F_{m+n}$$

$$L_n \; = \; < \left(P_\mu P^\mu - \frac{i}{4} \Gamma^\mu \frac{d\Gamma_\mu}{d\tau} \right) >_n$$

Fermion Propagator

$$S = \frac{1}{< \Gamma \cdot P > - m}$$

Fermion Gupta-Bleuler Conditions

$$F_n = \gamma_5 \, p^\mu b_\mu^{(n)\dagger} + \cdots \approx 0 (n > 0)$$

Lorentz Generators (Spinor Part)

$$\gamma_{[\mu}\gamma_{\nu]} \rightarrow < \Gamma_{[\mu}\Gamma_{\nu]} >$$

Tried (wrong) Gauge Invariant Vertex : $\Gamma_\mu e^{ik \cdot Q(z)}$:

Correct Vertex : $\Gamma_5 e^{ik \cdot Q(z)}$:

Neveu and Schwarz; Thorn

Epilogue

Following Dirac is a good thing:

found a New Type of Symmetry

(but, did not know about Grassmann numbers!)

Do Not Ask Mathematicians!

Following Feynman is also good:

Natural String Potential $B_{\mu\nu}$ (with Kalb, 1973)

Covariant String Field Theory (with Marshall, 1973-4)

Left Dual Models to Survive

with a genetic contribution to the Heterotic String