From GGRT to GSO through the Ademollo *et al.* Collaboration

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Plan of the talk

1. From dual models to the relativistic string

2. The Ademollo *et al.* Collaboration
   - 1973: The interacting string and DRM
   - 1974: Unified model for open and closed strings
   - 1975: Soft Dilatons and Scale Renormalization
   - 1975/76: New Superconformal Algebras

3. 1976: A magic spring in Paris

Outline

1. From dual models to the relativistic string

2. The Ademollo *et al.* Collaboration
   - 1973: The interacting string and DRM
   - 1974: Unified model for open and closed strings
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   - 1975/76: New Superconformal Algebras

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Prologue: the string as an analogous mechanical model for DRM spectrum

1969 Nambu, Nielsen and Susskind formulated independently the conjecture that the underlying microscopic structure of the physical states of dual resonance model (DRM) is a vibrating string

Nambu: " ....This equation suggests that the internal energy of a meson is analogous to that of a quantized string of finite length.. "

Susskind: "... a continuum limit of a chain of springs..."

1970 Nambu (unpublished) and Goto wrote a string action proportional to the area swept by the string in the external target space as a function of the string coordinates $x_\mu(\tau, \sigma)$ $(\mu = 1, \ldots, D)$

$$S = -\frac{1}{2\pi\alpha'} \int_{\tau_i}^{\tau_f} d\,\tau \int_0^{\pi} d\,\sigma \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2}$$

The GGRT paper

- The correct treatment and the quantization of the Nambu-Goto action was performed in the seminal paper of Goddard, Goldstone, Rebbi and Thorn (October, 1972)
- They pointed out the fundamental role of the reparametrization invariance of the string action
- The choice of the orthonormal (or conformal) gauge $\dot{x}^2 + x'^2 = \dot{x} \cdot x' = 0$ implied at once
  - the D'Alembert equation of motion $\ddot{x}_\mu - x''_\mu = 0$
  - at the classical level, the vanishing of 2D energy momentum tensor $T_{++} = T_{--} = 0 \quad [T_{\pm\pm} \equiv (\dot{x} \pm x')^2]$
  - at the quantum level, the Virasoro gauge conditions on the physical states:
    $L_n|phys\rangle = (L_o - \alpha_0)|phys\rangle = 0 \;\; [L_n = \frac{1}{2i\pi}\oint dz\, z^{n+1} T_{++} \;, z = e^{-i\tau}]$
  - no Lorentz anomaly and only transverse degrees of freedom for $D = 26$

The Brink & Nielsen mass formula

- The physical states of string models can be written in terms of free Bose (a_n^i) and Fermi (b_r^i, d_n^i) harmonic oscillators acting on a vacuum state $|0\rangle$ $(m, n \in \mathbb{N}; r, s \in \mathbb{Z} - \frac{1}{2})$
$[a_m^\mu, a_n^{\dagger\,\nu}] = \eta^{\mu\nu}\delta_{m\,n}\ ;\ \{b_r^\mu, b_s^{\dagger\,\nu}\} = \eta^{\mu\nu}\delta_{r\,s}\ ;\ \{d_m^\mu, d_n^{\dagger\,\nu}\} = \eta^{\mu\nu}\delta_{m\,n}$
- The free string Hamiltonian (in the transverse gauge)

$$H_{NS} = L_0 - \alpha_0 = -\alpha' M^2 + \sum_{i=1}^{D-2}\left(\sum_{n\in\mathbb{N}} a_n^{\dagger\,i} a_n^i + \sum_{r\in\mathbb{N}+\frac{1}{2}} b_r^{\dagger\,i} b_r^i\right) - \alpha_0$$

suggests interpreting $-\alpha_0$ as the zero point energy of a free vibrating string (Brink & Nielsen 1973):

$$-\alpha_0 = M_0^2 = \frac{D-2}{2}\left(\sum_{n\in\mathbb{N}}{}' n - \sum_{r\in\mathbb{N}+\frac{1}{2}}{}' r\right)$$

with $\sum'$ regularised sum

- zeta-function regularization (FG 1976)
  $\sum'_{n\geq 0}(n+a) = \zeta(-1) + \frac{a(a-1)}{2} = -\frac{1}{12} + \frac{a(a-1)}{2}$
- Open bosonic string: $\alpha_0 = \frac{D-2}{24}$
- Lorentz invariance of the “photon” state $a_1^{\dagger\, i}|0\rangle$ with mass $\alpha' M_1^2 \equiv 1 - \alpha_0$ requires $M_1 = 0 \quad \Rightarrow \quad \alpha_0 = 1$ i.e. $D_{crit} = 26$
- Open NS string: $\alpha_0 = \frac{D-2}{24} + \frac{D-2}{48}$
- Lorentz invariance of the “photon” state $b_{\frac{1}{2}}^{\dagger\, i}|0\rangle$ with mass $\alpha' M_1^2 \equiv \frac{1}{2} - \alpha_0$ requires $M_1 = 0 \quad \Rightarrow \quad \alpha_0 = \frac{1}{2}$ i.e. $D_{crit} = 10$

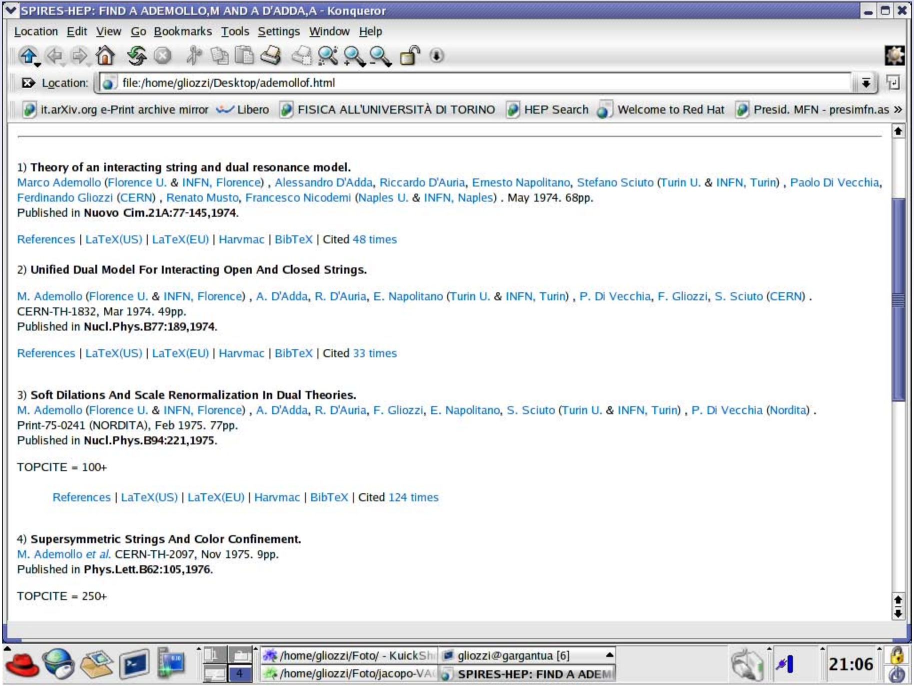
Outline

An unusual collaboration

Shortly after the appearance of the GGRT paper (30 October 1972) a group of former students and young collaborators of Sergio Fubini in Florence,Naples, Rome and Turin decided to join their efforts to understand the dual resonance model in the light of this new mechanical model.

There was no recognised leader inside the group and the ideas circulated freely (by ordinary mail and/or extemporaneous meetings) without any care of priority questions (May 1968 was not too far!)

Ideally, they prosecuted the line of thought of the Fubini-Veneziano collaboration which was concluded that year, combining it with the new physical insight coming from the string picture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1) **Theory of an interacting string and dual resonance model.**
Marco Ademollo (Florence U. & INFN, Florence) , Alessandro D'Adda, Riccardo D'Auria, Ernesto Napolitano, Stefano Sciuto (Turin U. & INFN, Turin) , Paolo Di Vecchia, Ferdinando Gliozzi (CERN) , Renato Musto, Francesco Nicodemi (Naples U. & INFN, Naples) . May 1974. 68pp.
Published in **Nuovo Cim.21A:77-145,1974.**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Cited 48 times

2) **Unified Dual Model For Interacting Open And Closed Strings.**

M. Ademollo (Florence U. & INFN, Florence) , A. D'Adda, R. D'Auria, E. Napolitano (Turin U. & INFN, Turin) , P. Di Vecchia, F. Gliozzi, S. Sciuto (CERN) .
CERN-TH-1832, Mar 1974. 49pp.
Published in **Nucl.Phys.B77:189,1974.**

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Cited 33 times

3) **Soft Dilations And Scale Renormalization In Dual Theories.**
M. Ademollo (Florence U. & INFN, Florence) , A. D'Adda, R. D'Auria, F. Gliozzi, E. Napolitano, S. Sciuto (Turin U. & INFN, Turin) , P. Di Vecchia (Nordita) .
Print-75-0241 (NORDITA), Feb 1975. 77pp.
Published in **Nucl.Phys.B94:221,1975.**

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4) **Supersymmetric Strings And Color Confinement.**
M. Ademollo *et al.* CERN-TH-2097, Nov 1975. 9pp.
Published in **Phys.Lett.B62:105,1976.**

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Print-75-0241 (NORDITA), Feb 1975. 77pp.
Published in **Nucl.Phys.B94:221,1975**.

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References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Cited 124 times

4) **Supersymmetric Strings And Color Confinement.**
M. Ademollo *et al.* CERN-TH-2097, Nov 1975. 9pp.
Published in **Phys.Lett.B62:105,1976**.

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References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 287 times

5) **Dual String With U(1) Color Symmetry.**
M. Ademollo *et al.* IFTT-304, Apr 1976. 59pp.
Published in **Nucl.Phys.B111:77-110,1976**.

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6) **Dual String Models With Nonabelian Color And Flavor Symmetries.**
M. Ademollo *et al.* NORDITA-76/20, Jun 1976. 44pp.
Published in **Nucl.Phys.B114:297,1976**.

TOPCITE = 100+

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 145 times

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Paper 1 to 6 of 6

Nuclear Physics B111 (1976) 77–110

DUAL STRING WITH U(1) COLOUR SYMMETRY

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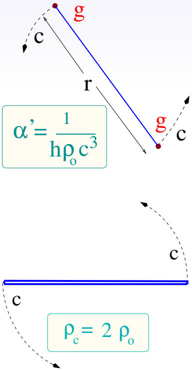
1973-The interacting bosonic string and DRM

1973 : The interacting string

- “.. the relativistic string theory is more than an analogue model for the spectrum of DRM, it can be used to obtain informations on the couplings.. ”



- Leading Regge trajectory of the open string

$$J = \frac{\pi}{2} c \rho_o r^2 = \frac{c}{2\rho_o \pi} m^2 = \hbar \alpha' m^2 c^2 + \alpha_0$$

$m = \pi \rho_o r$ (ρ_o mass density in the string frame)

- The gyromagnetic ratio is G=2, like in the coupling of the “strong photon ” in DRM
- α' of the open string is twice that of the closed string, according to the spectrum of the “Pomeron ” sector calculated by Olive and Scherk (1973)
- $\alpha_o^P = 2\alpha_o^R$

the open string in an external electromagnetic field

- $S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma\, \mathcal{L}(x, \dot{x}, x')$; $\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$
- $\mathcal{L}_{int} = \frac{1}{c}\rho(\sigma)\dot{x}_\mu A^\mu(x)$; $\rho(\sigma) = g_o\delta(\sigma) + g_\pi\delta(\sigma)$
- $A_\mu(x) = \epsilon_\mu\, e^{ik\cdot x}$
- Reparametrization invariance (RI) of the string world-sheet required

$$k^2 = \epsilon \cdot k = 0$$

  i.e. the external field had to be the massless photon state of the open string
- Under these conditions it turned out that the interacting open string had the same mass spectrum of the free case

- The probability amplitude for the emission of a number of photons from an initial string state to a final one coincided exactly with the corresponding N-point DRM amplitude
- This argument was extended also to excited external fields: in the conformal gauge RI implied the conformal invariance of $\mathcal{L}_{int} = \int d\tau\, V(x, x^{(r)})$:

$$i[L_f, V] = \frac{d}{d\tau}\{f(\tau)\, V\}$$

- this established in turn a one-to-one correspondence between the excited vertices and the open string spectrum at D=26. (this was explicitly verified at the level N=2)

Dictionary from DRM to string theory

- All relevant quantities of DRM can be constructed out of the two Fubini- Veneziano operators $Q_\mu(z)$ and $P_\mu(z)$
- Koba-Nielsen circle $z \leftrightarrow \exp(-i\tau)$
- $\sqrt{2}\,Q_\mu(z) \leftrightarrow x_\mu(\tau, 0)$
- $-i\sqrt{2}\,P_\mu(z) \leftrightarrow \dot{x}_\mu(\tau, 0)$

* Alternative approaches to the interacting string based on functional integration were proposed by Mandelstam (1973) and Gervais & Sakita(1973)

The bosonic string in an external gravitational field

- This game was extended also to the case of external gravitons by coupling Nambu-Goto action to a target-space metric $g_{\mu\nu}$
- Reparametrization Invariance yielded the right vertex operator of the “strong graviton” i.e. the massless closed string state
- as in the case of the photon, the external gravitational field was required to be on shell
- this appears to be a precursor of the equations of motion obtained much later requiring the vanishing of the β-function in the σ-model formulation of the string action (Lovelace,1984)

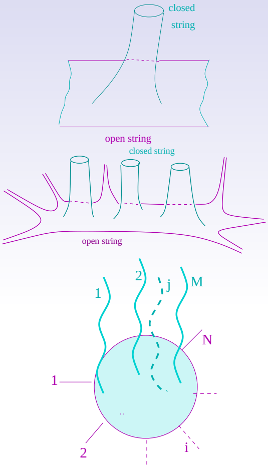
Ademollo *et al.* 1973:

"... The problem is whether the gravitational interaction may apply to an open string as well as to a closed one. In principle there is no difficulty in solving this problem and it would be interesting ... to have some information on the couplings between an open string and the strong graviton which is a particular state of a closed string."

Ademollo *et al.* 1974:

"We are led .. to a geometrical formulation of duality: all the physical laws which describe the dynamics of the string must be covariant under the reparametrization group of their world surface"

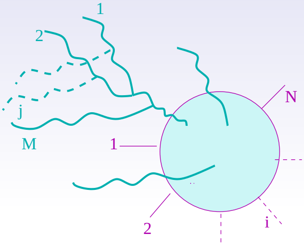
1974:Unified dual model for interacting open and closed strings

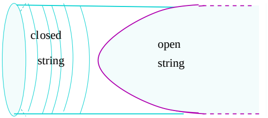


- Covariant vertex of emission of a closed string from an open one
  $\mathcal{W}_\beta(z,\bar z;k) = \mathcal{V}_\alpha(z,\tfrac{1}{2}k)\,\mathcal{V}_{\bar\alpha}(\bar z,\tfrac{1}{2}k)$
  - z = world-sheet coordinate
  - $\mathcal{V}_\alpha(z,p)$ = covariant open string vertex of momentum p
  - α = internal quantum numbers
- $dV = \prod_{i=1}^N \prod_{j=1}^M \frac{dx_i}{x_i}\frac{d^2 z_j}{|z_j|^2}\theta(x_{i+1}-x_i)$
- T^* ordering prescription according to the moduli of z_j and x_i

$$A(M,N) = \int \frac{dV}{dV_{abc}} \langle 0|T^*\left(\prod_{i=1}^N \prod_{j=1}^M \mathcal{V}_{\alpha_i}(x_i,p_i)\mathcal{W}_{\beta_j}(z_j,\bar z_j;k_j)\right)|0\rangle$$

Factorization in a closed string channel

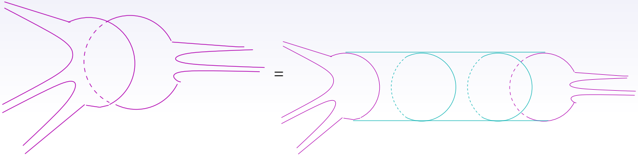




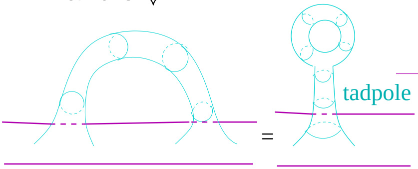
- The mixed amplitude can also be factorized in a closed string channel (here is the first example of what nowadays is called boundary state formalism)
- The closed string factor coincides with the Shapiro-Virasoro model
- A new feature: closed-open string transition represented by a double pole whenever this transition is kinematically possible

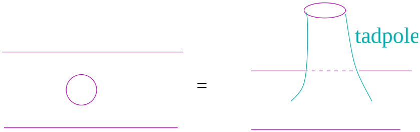
1975-Soft Dilatons and Scale Renormalization in Dual Theories

- Unitarity corrections to string amplitudes= sum over topologically inequivalent surfaces
- The topology of a (orientable) surface is determined by the number of holes and handles



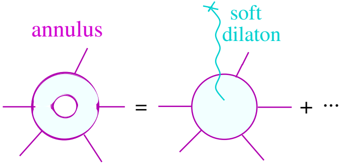
- Two kinds of holes $\Rightarrow$
- One kind of handle $\Downarrow$





Ademollo *et al.* 1975:

- The divergent part of the tadpole-like diagrams can be thought of as the contribution of the on-shell soft dilaton decaying into the vacuum
- The tadpole contribution to string amplitudes can be decomposed into a sum of a divergent part and a remainder in a Lorentz covariant and reparametrization covariant way
- For the planar tadpole (Cremmer & Scherk '72; Clavelli & Shapiro '73) such a decomposition can be explicitly performed by factorization, through the use of the IR $k \to 0$ limit of the on-shell covariant dilaton vertex of the unified open-closed string model (the tachyon divergence is cancelled by analytic continuation)



- The soft dilaton limit of the amplitudes is particularly simple when all the physical states of open and/or closed strings are massless:
$\lim_{k\to 0} T(k,p_1,p_2,\ldots,p_n) =$
$\pi g_c \alpha'^{\frac{d-2}{2}} \left[\sqrt{\alpha'}\frac{\partial}{\partial\sqrt{\alpha'}} - \frac{d-2}{2}\left(\frac{1}{2}g\frac{\partial}{\partial g} + g_c\frac{\partial}{\partial g_c}\right)\right] T(p_1,p_2,\ldots,p_n)$
- The sum over all the possible soft dilaton insertions can be explicitly performed
- the net effect is simply a Renormalization of the slope α' and the open string (g) and closed string (g_c) couplings
- $\alpha'_{\mathcal{R}} = Z\alpha'$; $g_{\mathcal{R}} = Z^{\frac{2-d}{8}} g$; $g_{c\mathcal{R}} = Z^{\frac{2-d}{4}} g_c$

1975/76: New superconformal algebras

A feedback from Supersymmetry

- 1971: Gervais & Sakita reformulate the NS-R algebra as a supersymmetry in 2D field theory
- 1973: Wess& Zumino define supersymmetry transformations in D=4 space-time
- 1974: Salam & Strathdee introduce the notion of superfield and extended supersymmetry
- 1975: Ademollo *et al.* extend the NS-R algebra to $\mathcal{N} = 2$ and $\mathcal{N} = 4$ superconformal algebras using D=2 free superfields (fully appreciated only after the first String Revolution)

Virasoro $[L_m, L_n] = (m-n) L_{m+n} + \frac{2b+f}{24} D n(n^2-1)\delta_{m,-n}$

N S–R $\{G^j_r, G^j_s\} = 2 L_{r+s} + \frac{(b+f)D}{16}(4r^2-1)\,\delta_{r,-s}$

$[L_n, G^j_r] = (\frac{n}{2} - r) G^j_{n+r}$

$\{G^o_r, G^j_s\} = 2i(r-s) T^j_{r+s}$

U(1) $[L_m, T^j_n] = -n\, T^j_{m+n}$

***N*=2** $[T^j_m, T^j_n] = \frac{(b+f)D}{16}\delta_{m,-n}$

$[T^j_m, G^o_r] = \frac{i}{2} G^j_{m+r}$; $[T^j_m, G^j_r] = \frac{-i}{2} G^o_{m+r}$

	b	f
VM	1	0
NS–R	1	1
***N*=2**	2	2
***N*=4**	4	4

SU(2) $\{G^i_r, G^j_s\} = 2i\,\epsilon_{ijk}(r-s) T^k_{r+s}$

***N*=4** $\boxed{[T^i_m, T^j_n] = i\,\epsilon_{ijk} T^k_{m+n}}$ Kac–Moody ; $[T^i_m, G^j_r] = \frac{i}{2}\epsilon_{ijk} G^k_{m+r} - \frac{i}{2}\delta^{ij} G^o_{m+r}$

- 1976: the $\mathcal{N} = 2$ string turned out to have $D_{crit} = 2$ and only one physical state; the $\mathcal{N} = 4$ string contained ghosts for any $D \Rightarrow$ no room for a realistic string theory of strong interactions
- The extra dimensions of the bosonic and NS-R strings appeared rather embarrassing for a theory of the hadrons
- QCD was emerging as a sound field theory of strong interactions
- The connection between strings and Yang-Mills-Einstein theory in the $\alpha' \to 0$ limit (Neveu & Scherk '72, Yoneya '73) led to a re-interpretation of the dual models being a short-distance modified theory of fundamental forces rather than a hadron theory with the wrong spectrum (Scherk & Schwarz, 1975)

Outline

The first theory of supergravity was proposed in March 1976 by Ferrara, Freedman and van Nieuvenhuizen at ENS (which I witnessed from a very short distance) and soon after by Deser and Zumino at CERN.

At that time I began to discuss with Joel Scherk the possibility to have a similar structure in the NS-R string theory.

We realized that in the closed string sector, besides the bosonic states already studied by Clavelli and Shapiro (1973), there was also a fermionic sector (with Neveu-Schwarz left movers and Ramond right movers) with a massless gravitino, hence in the $\alpha' \to 0$ limit the massless sector had to yield a D=10 version of supergravity.

We used this fact as a secret tool to extract information on the matter couplings in D=4 supergravity.

1) **Locally Supersymmetric Maxwell-Einstein Theory.**
S. Ferrara, Joel Scherk (Ecole Normale Superieure) , P. van Nieuwenhuizen (SUNY, Stony Brook) .
PTENS 76/17, Aug 1976. 9pp.
Published in **Phys.Rev.Lett.37:1035,1976.**
TOPCITE = 50+

References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 51 times

2) **Scalar Multiplet Coupled To Supergravity.**
S. Ferrara, D.Z. Freedman, P. van Nieuwenhuizen (SUNY, Stony Brook) , P. Breitenlohner (Munich, Max Planck Inst.) , F. Gliozzi, Joel Scherk (Ecole Normale Superieure) . ITP-SB-76-46, Sep 1976. 20pp.
Published in **Phys.Rev.D15:1013,1977.**

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3) **Matter Couplings In Supergravity Theory.**
S. Ferrara, F. Gliozzi, Joel Scherk (Ecole Normale Superieure) , P. Van Nieuwenhuizen (SUNY, Stony Brook) . PTENS-76-19, Sep 1976. 52pp.
Published in **Nucl.Phys.B117:333,1976.**

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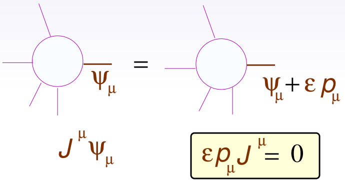
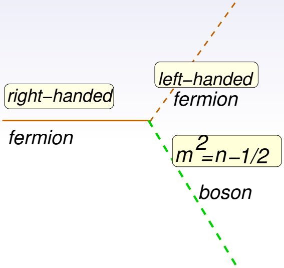
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 79 times

- The Lorentz invariance of the NS-R model implies that the gravitino ψ_μ is massless
- The two-dimensional reparametrization invariance requires the on-shell gauge invariance $\delta\psi_\mu = p_\mu\,\epsilon$ ($\epsilon =$ arbitrary spinor) hence the conservation of a supercharge
- the gravitino can be consistently coupled only in a supersymmetric theory at any order in α'
- this would imply (global) supersymmetry of the open string sector of NS-R model
- ? why the tachyon does not have a supersymmetric partner?
- I started to study this problem with David Olive and Joel Scherk



- The Neveu-Schwarz sector having $\alpha' \mathrm{mass}^2 = n - \frac{1}{2}$ (like the tachyon) has no fermionic counterpart in the Ramond sector
- This sector transforms a right-handed fermion into a left-handed fermion
- It decouples altogether if one imposes the Weyl condition $\psi_{\mathrm{left}} = 0$ on the ground state spinor of the Ramond sector ($m = 0$)
- the fermion-fermion and the fermion-antifermion have the same spectra of bosonic bound states
- to avoid infinite degeneracy of the bosonic spectrum one is led to require that the fermions satisfy the Majorana condition (possible only if D is 2 or 4 modulo 8)



Counting the physical open string states

$$\alpha' m^2 = n, \quad (n = 0, 1, \dots)$$

$$[a^\mu_m, a^{\dagger\,\nu}_n] = \eta^{\mu\,\nu}\delta_{m\,n}\ ;\ \{b^\mu_r, b^{\dagger\,\nu}_s\} = \eta^{\mu\,\nu}\delta_{r\,s}\ ;\ \{d^\mu_m, d^{\dagger\,\nu}_n\} = \eta^{\mu\,\nu}\delta_{m\,n}$$

n	Fermi sector (Ramond)	multiplicity	Bose sector (NS)
0	$\|0\rangle\psi$ $\psi =$ Majorana-Weyl spinor	8	$b^{\dagger\,\mu}_{\frac{1}{2}}\|0\rangle$ $(\mu{=}1,2,\dots D-2)$ gauge boson
1	$d^{\dagger\,\mu}_1\|0\rangle\psi$ $a^{\dagger\,\mu}_1\|0\rangle\psi$	128	$b^{\dagger\,\mu}_{\frac{3}{2}}\|0\rangle$ $a^{\dagger\,\mu}_1 b^{\dagger\,\nu}_{\frac{1}{2}}\|0\rangle$ $b^{\dagger\,\mu}_{\frac{1}{2}} b^{\dagger\,\nu}_{\frac{1}{2}} b^{\dagger\,\rho}_{\frac{1}{2}}\|0\rangle$
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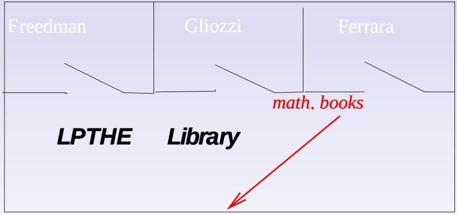

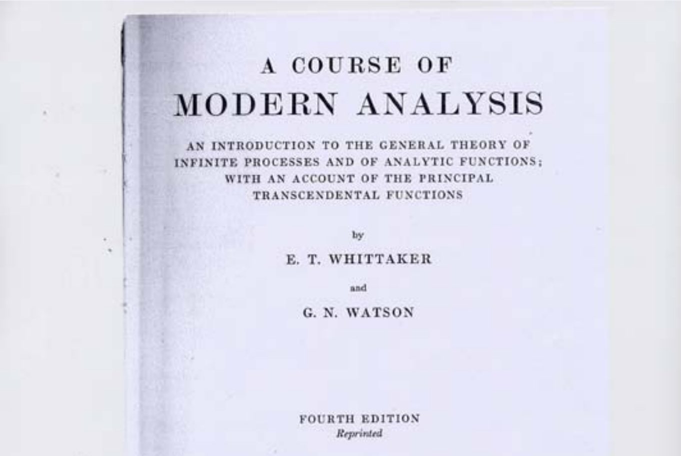
A COURSE OF

MODERN ANALYSIS

AN INTRODUCTION TO THE GENERAL THEORY OF INFINITE PROCESSES AND OF ANALYTIC FUNCTIONS; WITH AN ACCOUNT OF THE PRINCIPAL TRANSCENDENTAL FUNCTIONS

by

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and

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Reprinted

Example. Shew that*

$$\left\{\prod_{n=1}^{\infty}(1-q^{2n-1})\right\}^8+16q\left\{\prod_{n=1}^{\infty}(1+q^{2n})\right\}^8=\left\{\prod_{n=1}^{\infty}(1+q^{2n-1})\right\}^8.$$

(Jacobi.)

21·4. *The differential equation satisfied by the Theta-functions.*

We may regard $\vartheta_3(z\mid\tau)$ as a function of two independent variables z and τ; and it is permissible to differentiate the series for $\vartheta_3(z\mid\tau)$ any number of times with regard to z or τ, on account of the uniformity of convergence of the resulting series (§ 4·7 corollary); in particular

$$\frac{\partial^2\vartheta_3(z\mid\tau)}{\partial z^2}=-4\sum_{n=-\infty}^{\infty}n^2\exp(n^2\pi i\tau+2niz)$$

$$=-\frac{4}{\pi i}\frac{\partial\vartheta_3(z\mid\tau)}{\partial\tau}.$$

Consequently, the function $\vartheta_3(z\mid\tau)$ satisfies the partial differential equation

$$\frac{1}{4}\pi i\frac{\partial^2 y}{\partial z^2}+\frac{\partial y}{\partial\tau}=0.$$

The reader will readily prove that the other three Theta-functions also satisfy this equation.

21·41. *A relation between Theta-functions of zero argument.*

The remarkable result that

$$\vartheta_1'(0)=\vartheta_2(0)\,\vartheta_3(0)\,\vartheta_4(0)$$

will now be established†. It is first necessary to obtain some formulae for differential coefficients of all the Theta-functions.

* Jacobi describes this result (*Fund. Nova*, p. 90) as 'aequatio identica satis abstrusa.'

† Several proofs of this important proposition have been given, but none are simple. Jacobi's original proof

Ut a typographorum mendis, quantum fieri potuit, mundus aderet liber, Cl. Scherk curare voluit, cui ea de re valde me strictum esse profiteor. Quae emendanda restant, ad calcem iecta sunt.

Scribebam m. Febr. a. 1829
ad Univ. Regiom.

$$11)\quad \frac{2k'K}{\pi} = \left\{\frac{(1-q)(1-q^2)(1-q^3)\ldots}{(1+q)(1+q^2)(1+q^3)\ldots}\right\}^2$$

$$12)\quad \frac{2\sqrt{k}.K}{\pi} = 2\sqrt[4]{q}\left\{\frac{(1-q^2)(1-q^4)(1-q^6)\ldots}{(1-q)(1-q^3)(1-q^5)\ldots}\right\}^2$$

$$13)\quad \frac{2\sqrt{k'}K}{\pi} = \left\{\frac{(1-q^2)(1-q^4)(1-q^6)\ldots}{(1+q^2)(1+q^4)(1+q^6)\ldots}\right\}^2.$$

E formulis 7), 8) sequitur aequatio identica satis abstrusa:

$$14)\quad \{(1-q)(1-q^3)(1-q^5)\ldots\}^8 + 16q\{(1+q^2)(1+q^4)(1+q^6)\ldots\}^8 = \{(1+q)(1+q^3)(1+q^5)\ldots\}^8.$$

Supergravity and the spinor dual model.

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Abstract: We find that spinor dual model is locally supersymmetric not only in the two dimensional surface spanned by the string, but also with respect to the embedding space-time.

of gravity of [redacted] in interaction with the other massless fields present in the theory, namely the Yang-Mills fields themselves, a scalar field $A(x)$ and one antisymmetric tensor field $A_{\mu\nu}(x)$. [15-26]

So the scalar dual model is not only invariant under coordinate transformations in 2 dimensions, but also under coordinate transformations in critical dimension D ($D = 26$!), because of the existence of a graviton-like particle in its spectrum.

The connection which was established between the dual scalar model and the Yang-Mills-Einstein theory has led to a reinterpretation [16,27,28] of the dual models as being a short distance modified theory of fundamental forces rather than a hadronic theory with the wrong spectrum. As in such a model κ (the gravitational constant) and g (the gauge coupling constant) are related through α' [16], one can convince oneself that these modifications occur only at distances of the order of Planck's length (10^{-33} cm).

We would like to study if the Neveu-Schwarz-Ramond [29] model has similar properties. It is well-known that the model has a bigger invariance than the Veneziano model. In addition to the position degrees of freedom of the string, a spinor field is attached to the string [30,31]. Local supersymmetric transformations, first discovered by Gervais and Sakita [32] mix these two degrees of freedom. Historically it is the study of these transformations which led Wess and Zumino [33] to introduce and develop the concept of global supersymmetry in four dimensions. The locally supersymmetric counterpart of the Nambu action which describes the spinning string, was found by Zumino [34] in the superspace formalism by Brink, Di Vecchia and Howe [35] and by Deser and Zumino [36] in the two-dimensional space

$$f_B(q) = \prod_{n=1}^{\infty}(1-q^{2n})^{-8}\frac{1}{2}\left[\prod_{n=1}^{\infty}(1+q^{2n-1})^8 - \prod_{n=1}^{\infty}(1-q^{2n-1})^8\right]$$

$$f_F(q) = 8\,q\prod_{n=1}^{\infty}(1-q^{2n})^{-8}\prod_{n=1}^{\infty}(1+q^{2n})^8$$

- The “Aequatio identica satis abstrusa”

$$f_B(q) = f_F(q)$$

is a necessary condition for supersymmetry in the target space

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SUPERSYMMETRY, SUPERGRAVITY THEORIES AND THE DUAL SPINOR MODEL

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We study the connection between the dual spinor model and supersymmetry. We show that in the low-energy region, the dual spinor model yields a supersymmetric Yang-Mills theory with O(4) internal symmetry and a supergravity theory with O(4) internal symmetry.

1. Introduction