Problems for Nuclear Clustering

1. In the lecture it was stated without proof that a product of N antisymmetrised Gaussians, each centered around a different spot in space, can go over to a Slater determinant of harmonic oscillator wave functions for N particles in the limit where the centers of all Gaussians converge to the origine.

Let us verify this with a simple example. In one dimension (1D), one considers the following normalised and antisymmetrised product of two Gaussians

$$\Psi(x_1, x_2) = N_{12}\mathcal{A}\left[e^{-(x_1-S)^2/b^2}e^{-(x_1+S)^2/b^2}\right] \tag{1}$$

where N_{12} is the normalisation constant and $\mathcal{A}$ the antisymmetriser.

Show that in the limit $S \to 0$, the two particle wave function $\Psi(x_1, x_2)$ goes over into an antisymmetrised product of the two lowest harmonic oscillator wave functions with b the oscillator constant. Discuss the extrapolation to the situation in 3D for a nucleus with $2N$ neutrons and $2Z$ protons for the ground state and an eventual α gas state.

2. In the lecture it was stated that there is a strong difference between pair condensation and quartet condensation for the case where the chemical potential passes from negative to positive values. Show that the in-medium two particle level density for a pair of free particles at rest with the two momenta above the Fermi level is finite for chemical potential μ positive.

Further show that that the four fermion level density with the c.o.m. momentum of the four fermions at zero goes to zero for energy close to $E \to 4\mu$ for μ positive.
In both cases one considers a homogeneous non-interacting Fermi gas with plane wave states.
Shortly discuss the influence of finite temperature on the α condensation scenario.

Hint: definition of multi-particle level densities above the Fermi level

$$g(E)_n = \int d^3k_1\Theta(e_{k_1} - \mu) \int d^3k_2\Theta(e_{k_2} - \mu)... \int d^3k_n\Theta(e_{k_n} - \mu)\delta(E - e_{k_1} - e_{k_2}... - e_{k_n}) \tag{2}$$

with $e_k = k^2/(2m)$, the kinetic energy, $\Theta(x)$ the Heaviside step function, equal one for x positive and zero elsewise. $\delta(x)$ is the Dirac delta function.