Nuclear Reaction Physics

Lectures at GGI FNHP 2016

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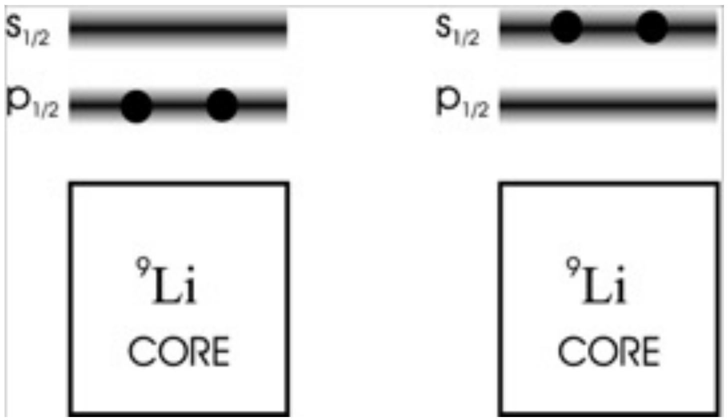
^{11}LI model

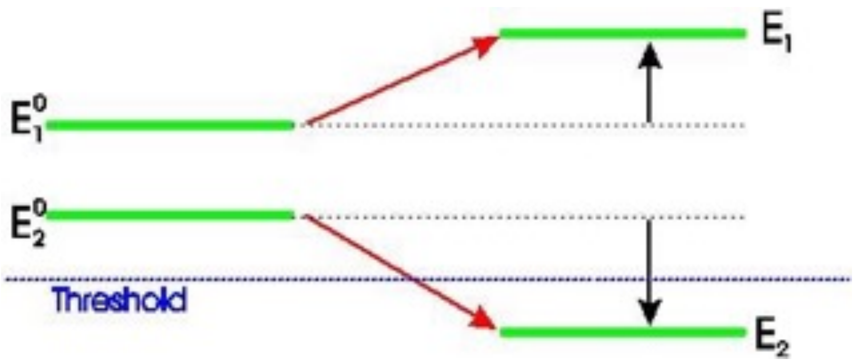
Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$

$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



- Mechanism of binding by Hermitian interaction



Two-level model parameters

- Energy-independent width is not consistent with definitions of threshold

$$A_2^2 = \gamma_2(E) = \alpha\sqrt{E},$$

$$A_1^2 = \gamma_1(E) = \beta E^{3/2}$$

Squeezing of phase-space volume in s and p waves, Threshold E_c=0

Model parameters:
ε_1=100, ε_2=200,
A_1=7.1 A_2=3.1 (red); α=1, β=0.05 (blue)
(in units based on keV)
Upper panel: Energies with A_1=A_2=0 (black)

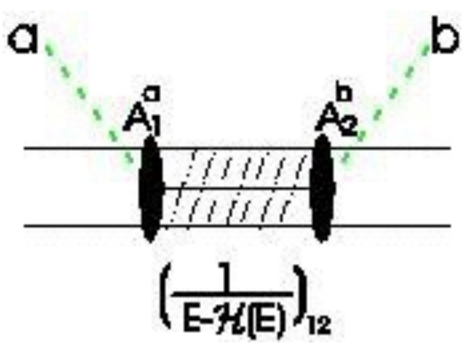
Scattering and cross section

Scattering Matrix

$$S^{ab} = (s^a)^{1/2}(\delta^{ab} - T^{ab})(s^b)^{1/2}$$

where $s^a = \exp(i\delta_a)$
is smooth scattering phase

$$T^{ab} = \sum_{12} A_1^{a*} \left(\frac{1}{E - \mathcal{H}}\right)_{12} A_2^b$$



Solution in two-level model

$$T(E) = \frac{E(\gamma_1 + \gamma_2) - \gamma_1\epsilon_2 - \gamma_2\epsilon_1 - 2vA_1A_2}{(E - \mathcal{E}_+)(E - \mathcal{E}_-)}$$

Cross section

$$\sigma(E) = \frac{\pi}{k^2}|S(E) - 1|^2$$

^{11}Li model

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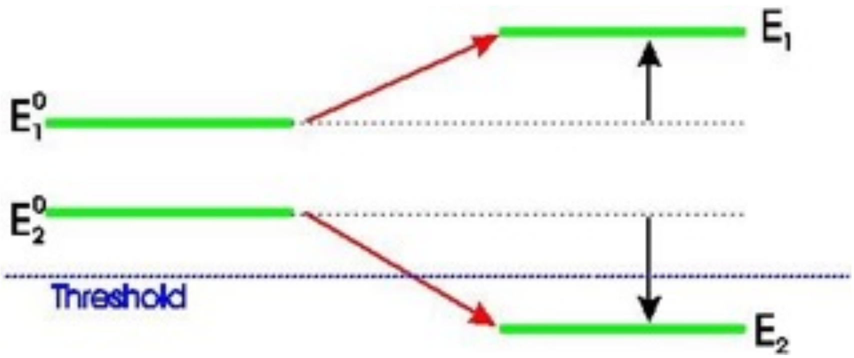
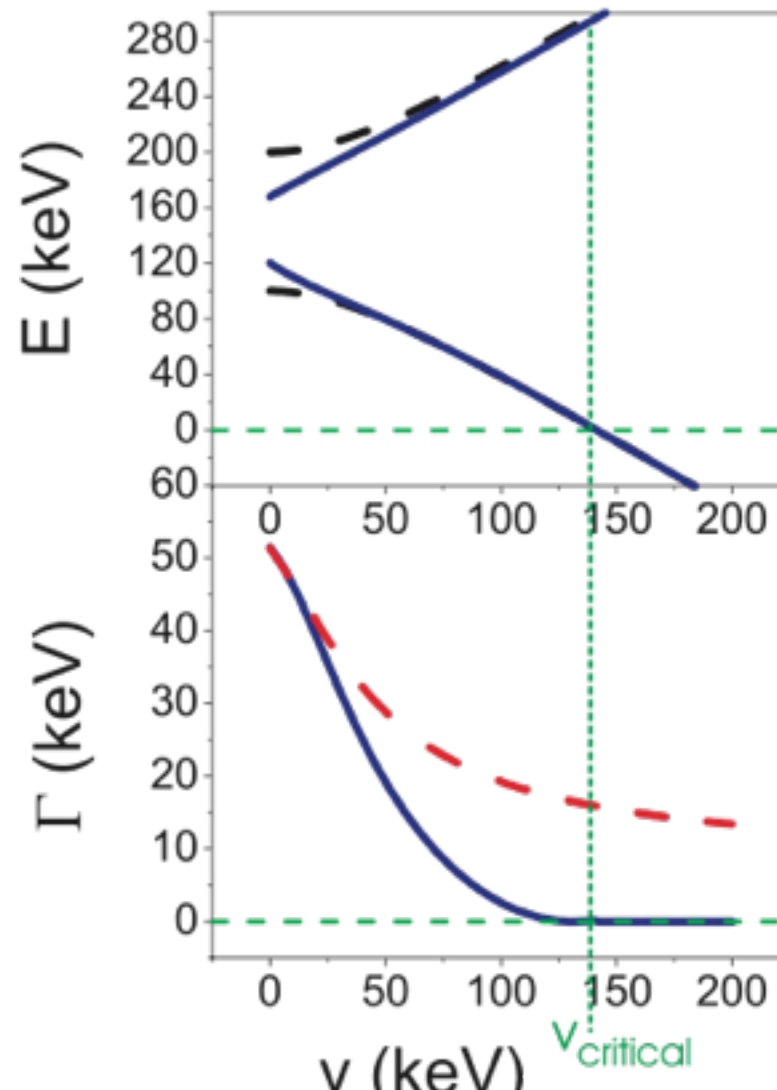


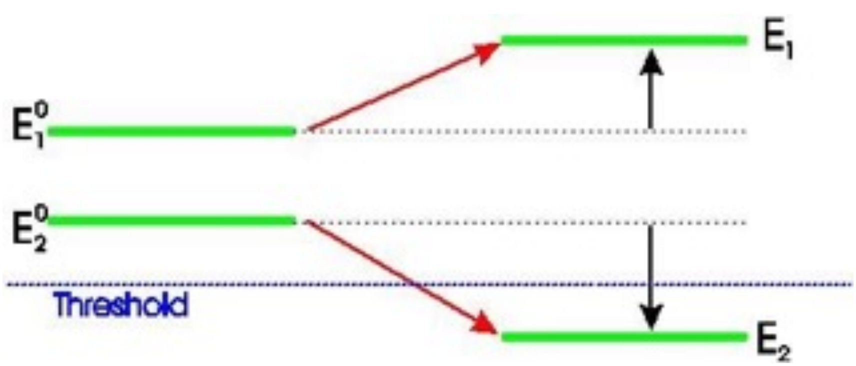


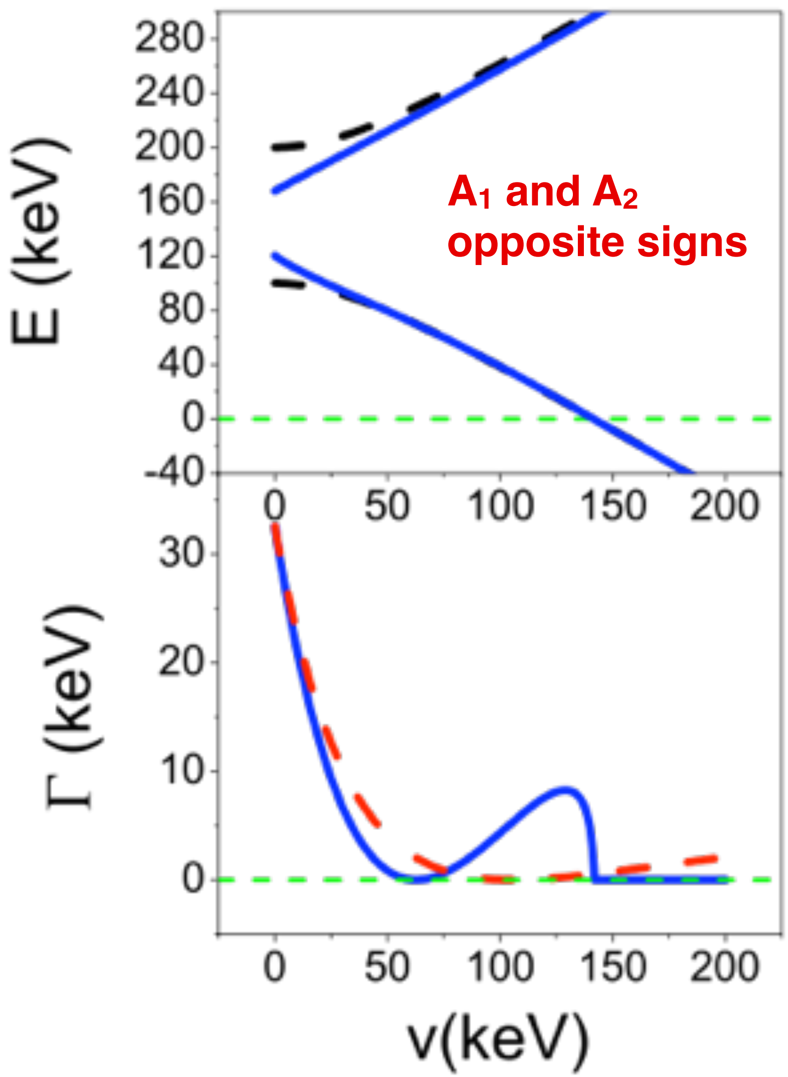
^{11}LI model

Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$

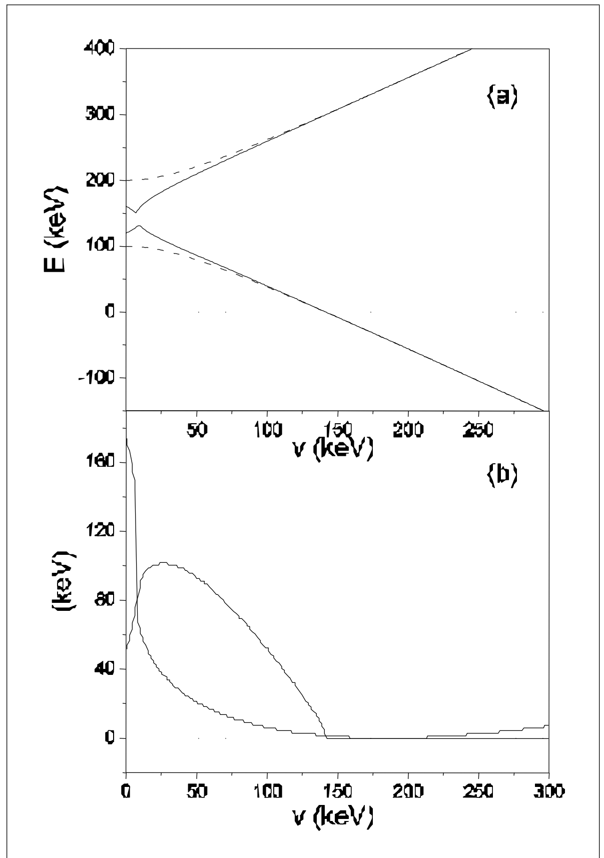
$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1 A_2 \\ v - \frac{i}{2}A_1 A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$



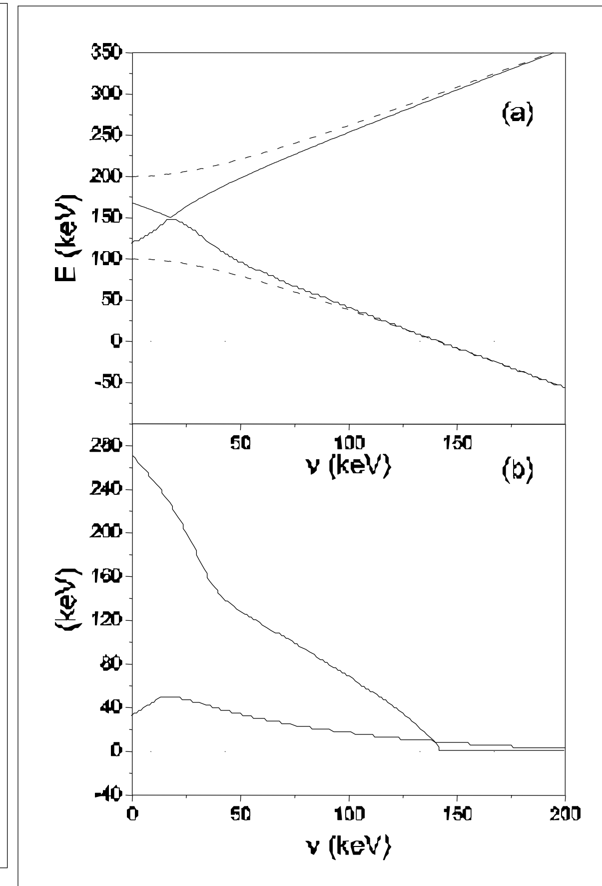


Dynamics of eigenstates in two-level system

Model parameters:
$A_1^2=0.05\ (E)^{3/2}$, $A_2^2=10\ (E)^{1/2}$
$A_1 A_2 <0$; $v>0$



Model parameters:
$A_1^2=0.05\ (E)^{3/2}$, $A_2^2=15\ (E)^{1/2}$
$A_1 A_2 <0$; $v>0$



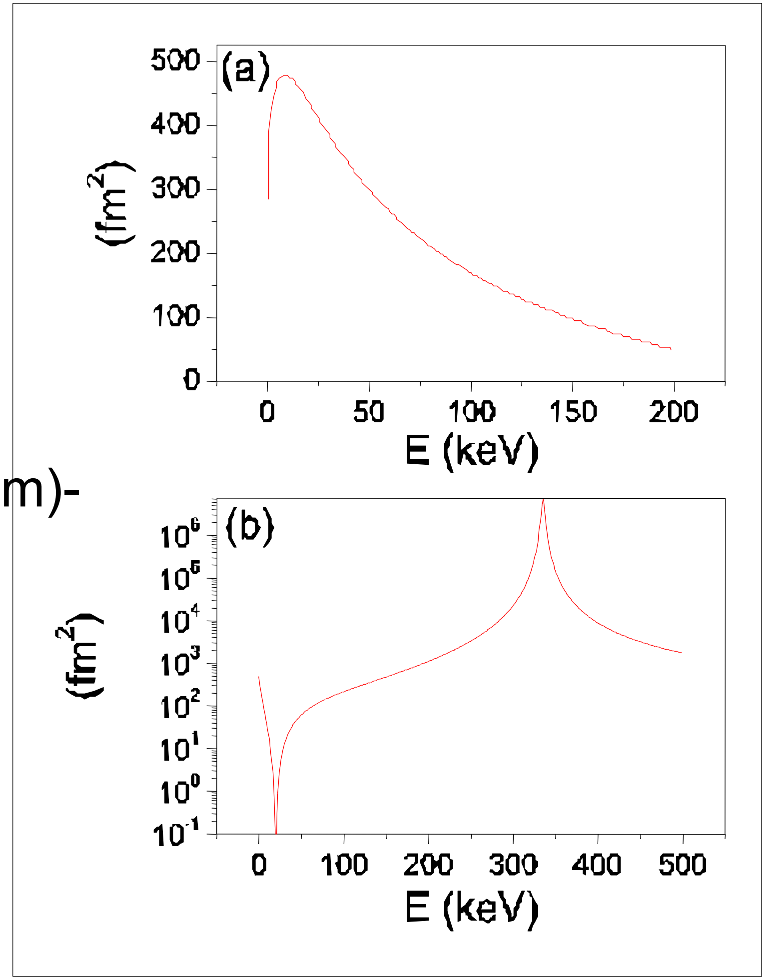
Cross section near threshold

•No direct interaction v=0
Breit-Wigner resonance

$$T(E) = \frac{\gamma_1 + \gamma_2}{E - \epsilon + (i/2)(\gamma_1 + \gamma_2)}$$

•Below critical v (both states in continuum)-
sharp resonances

•Above critical v
•One state is bound-
"attraction" to sub-threshold region
fig (a)
•Second state –resonance, fig (b)



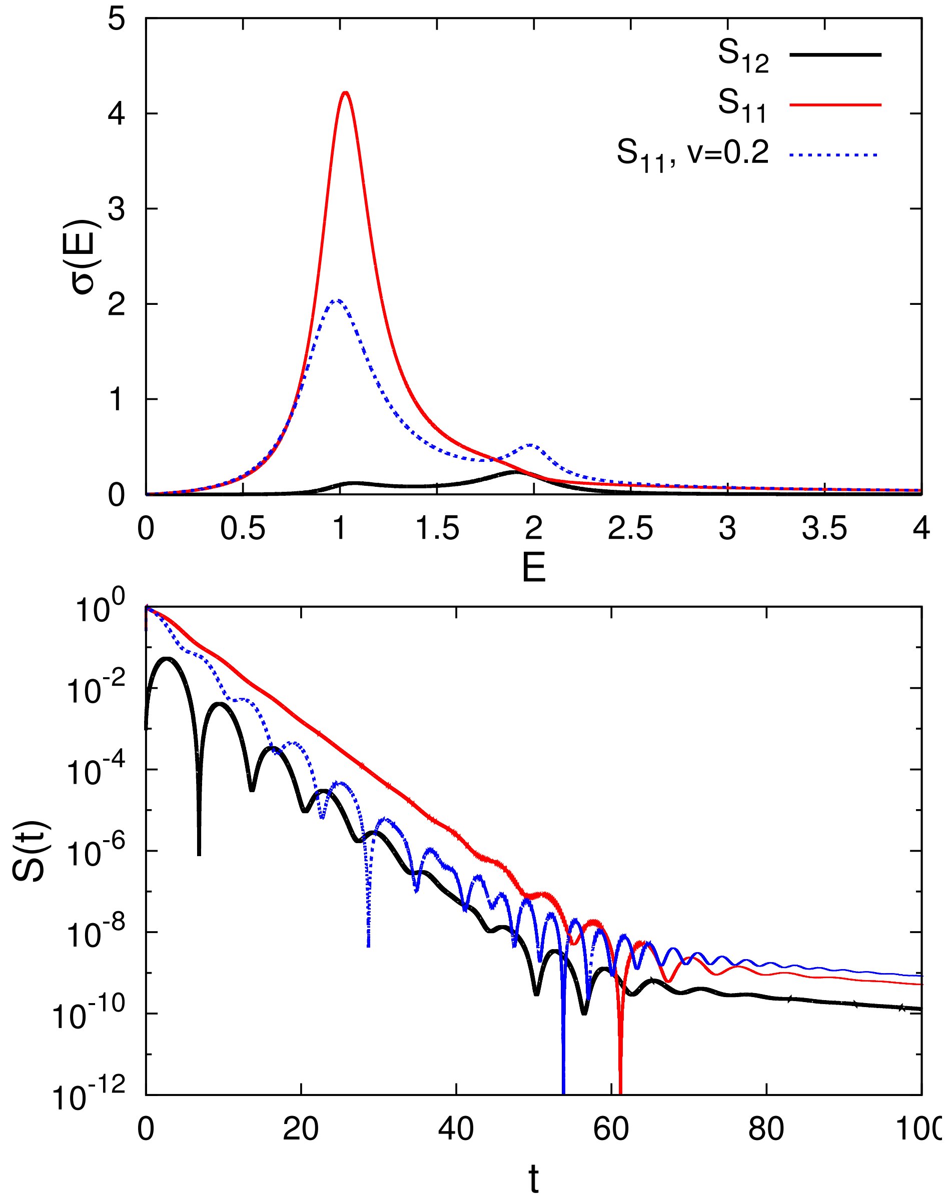
Model parameters:
ε_1=100, ε_2=200, v=180 (keV)
A_1^2=0.05 $(E)^{3/2}$, A_2^2=15 $(E)^{1/2}$

Two-level system

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)\Gamma_1 & v - (i/2)A_1A_2 \\ v - (i/2)A_1A_2 & \epsilon_2 - (i/2)\Gamma_2 \end{pmatrix}$$

$$\Gamma_1 = A_1^2, \quad \Gamma_2 = A_2^2,$$

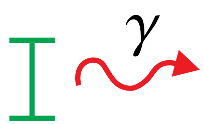
$$S(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2$$



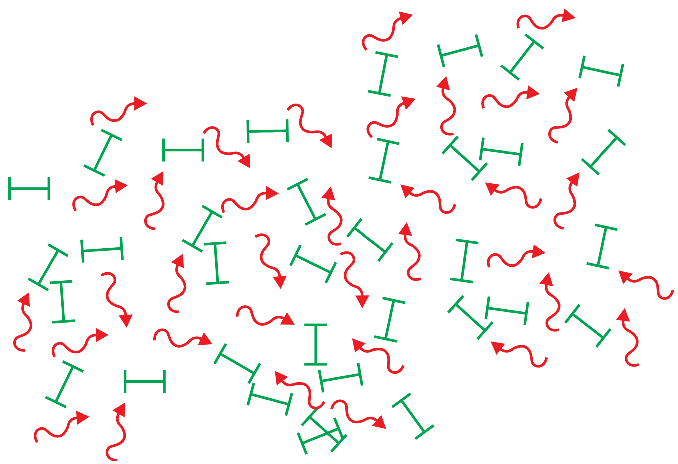
Superradiance, collectivization by decay

Dicke coherent state
N identical two-level atoms
coupled via common radiation

Single atom γ

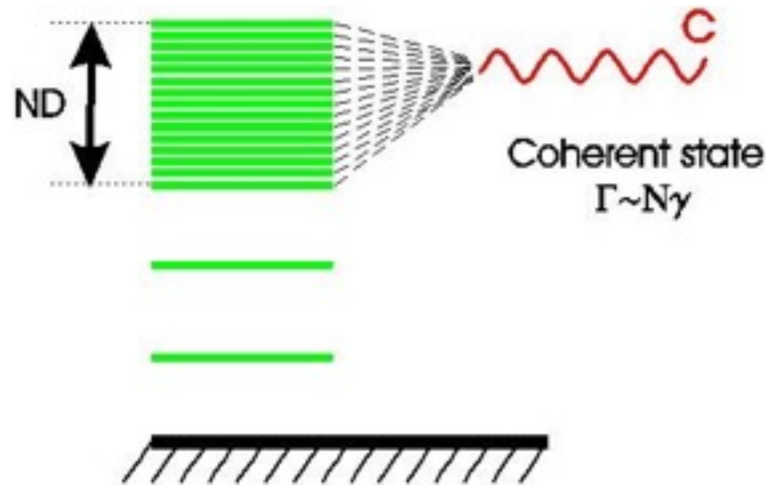


Coherent state $\Gamma \sim N\gamma$



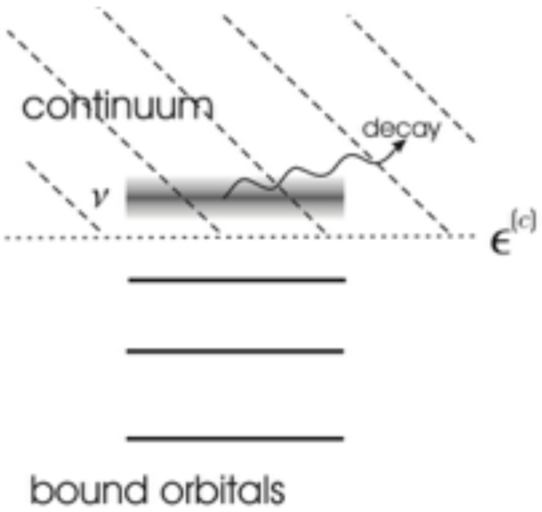
Volume $\ll \lambda^3$

Analog in nuclei
Interaction via continuum
Trapped states) self-organization

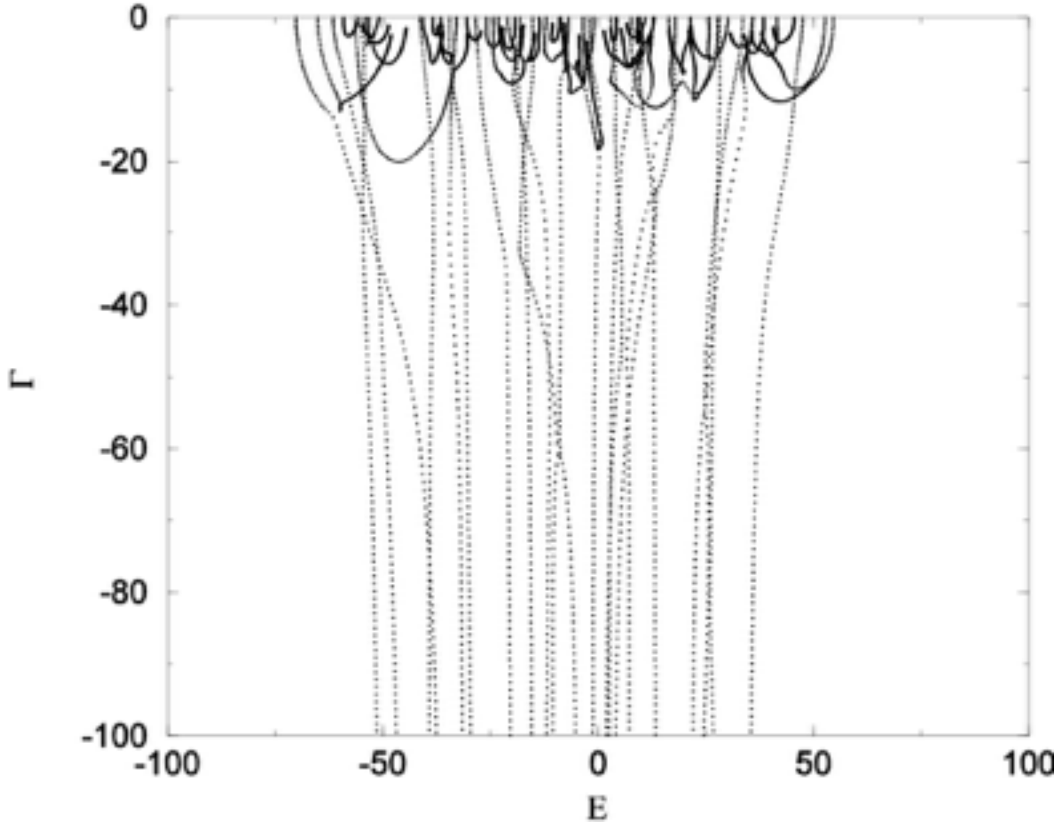


g ~ D and few channels
- Nuclei far from stability
- High level density (states of same symmetry)
- Far from thresholds

Single-particle decay in many-body system



Evolution of complex energies E=E-i $\Gamma/2$ as a function of γ



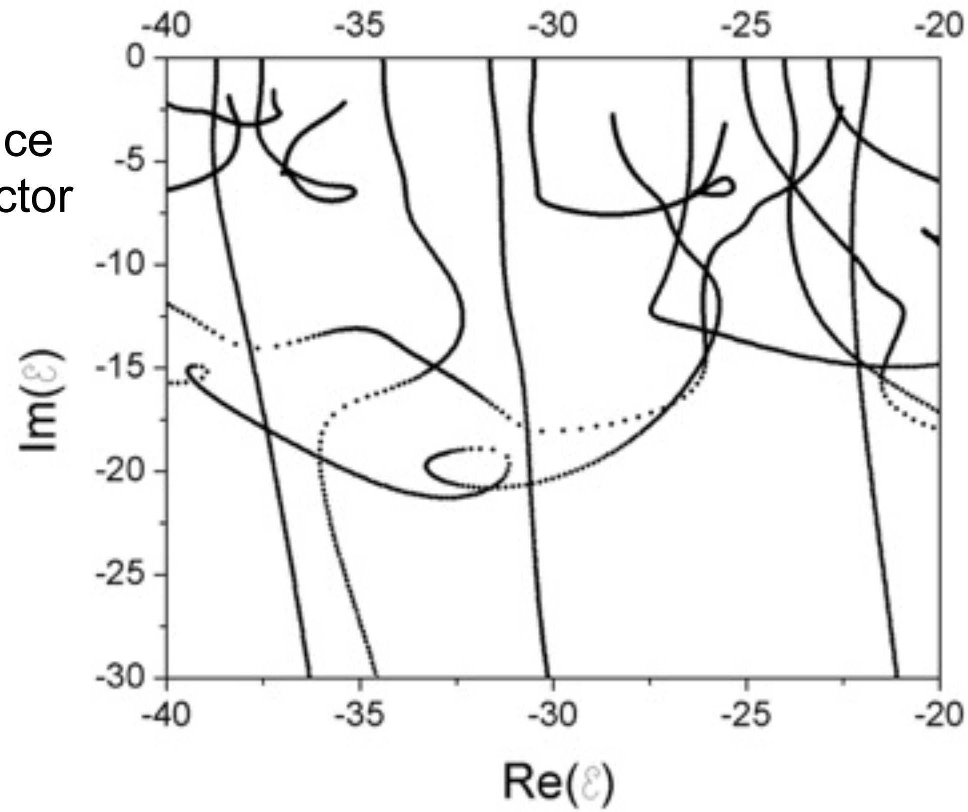
- Assume energy independent W
- Assume one channel $\gamma=A^2$
- System 8 s.p. levels, 3 particles
- One s.p. level in continuum $e=\varepsilon - i\gamma/2$

Total states 8!/(3! 5!)=56; states that decay fast 7!/(2! 5!)=21

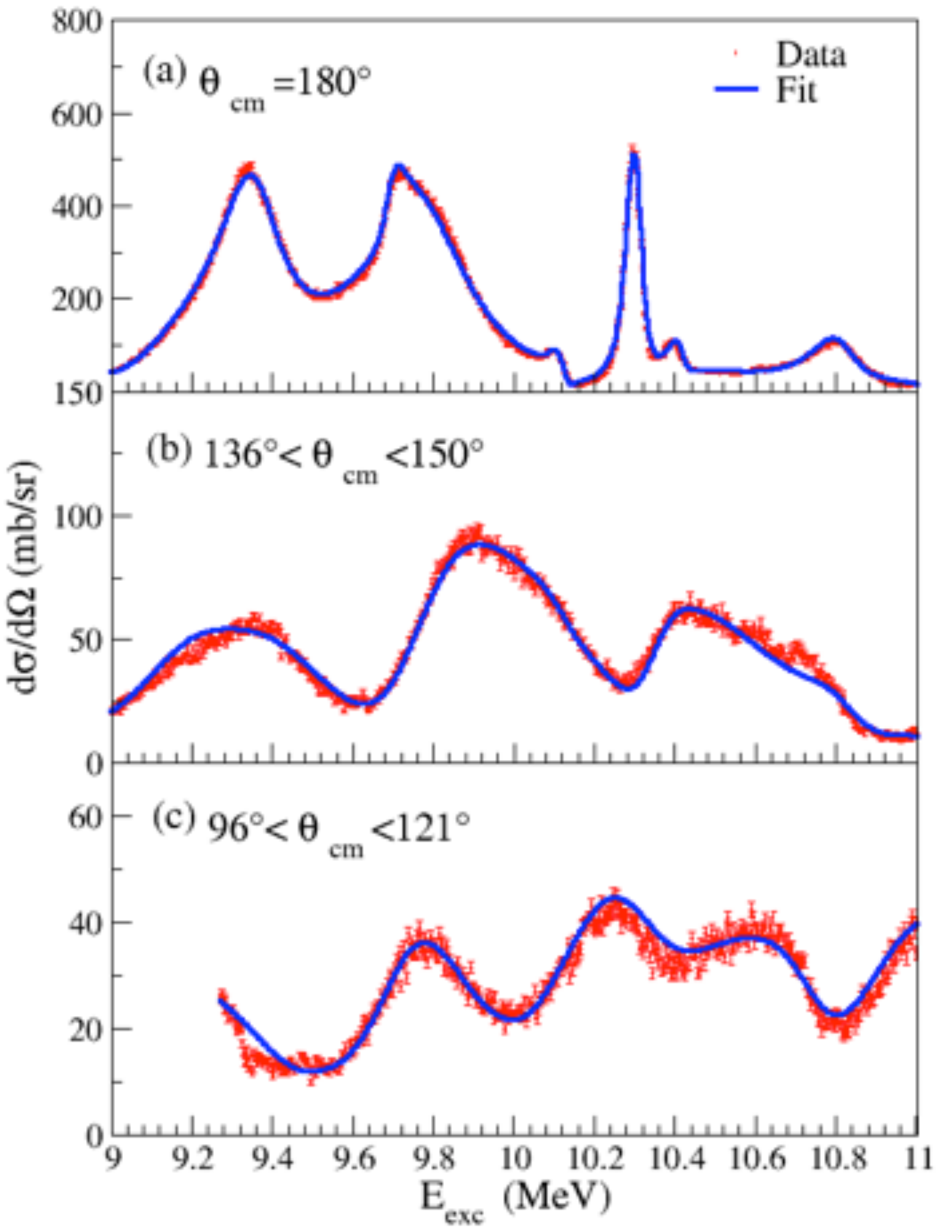
Evolution of eigenstates in the complex plane

As γ increases dynamics changes

- Shell model limit
- Weak, non-overlapping resonance
  $\Gamma_\Phi = \gamma\, n_\Phi$ n_Φ –spectroscopic factor
- Intermediate regime
- Superradiant regime



Resonances in ^{18}O observed via ^{14}C+α



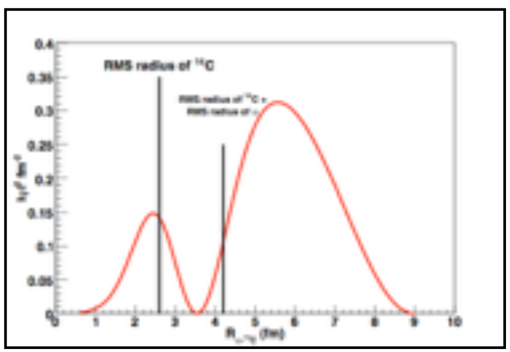
E_{exc}(MeV)	J^{π}	Γ_{tot}(keV)	Γ_{α}(keV)	θ_{α}^{2}
8.04	1^-	2	2	0.02
8.21	2^+	1	1	<0.01
8.29	3^-	8	2	0.09
8.78	2^+	70	1	<0.01
8.98	2^+	60	4	0.01
9.17	1^-	240	205	0.24
9.36	2^+	24	1	<0.01
9.39	3^-	155	103	0.47
9.69	3^-	56	0.1	<0.01
9.79	2^+	263	167	0.20
9.76	1^-	740	658	0.48
9.90	0^+	2100	2100	1.20
10.10	3^-	17	12	0.02
10.30	4^+	23	16	0.08
10.34	2^+	111	20	0.02
10.40	3^-	48	17	0.02

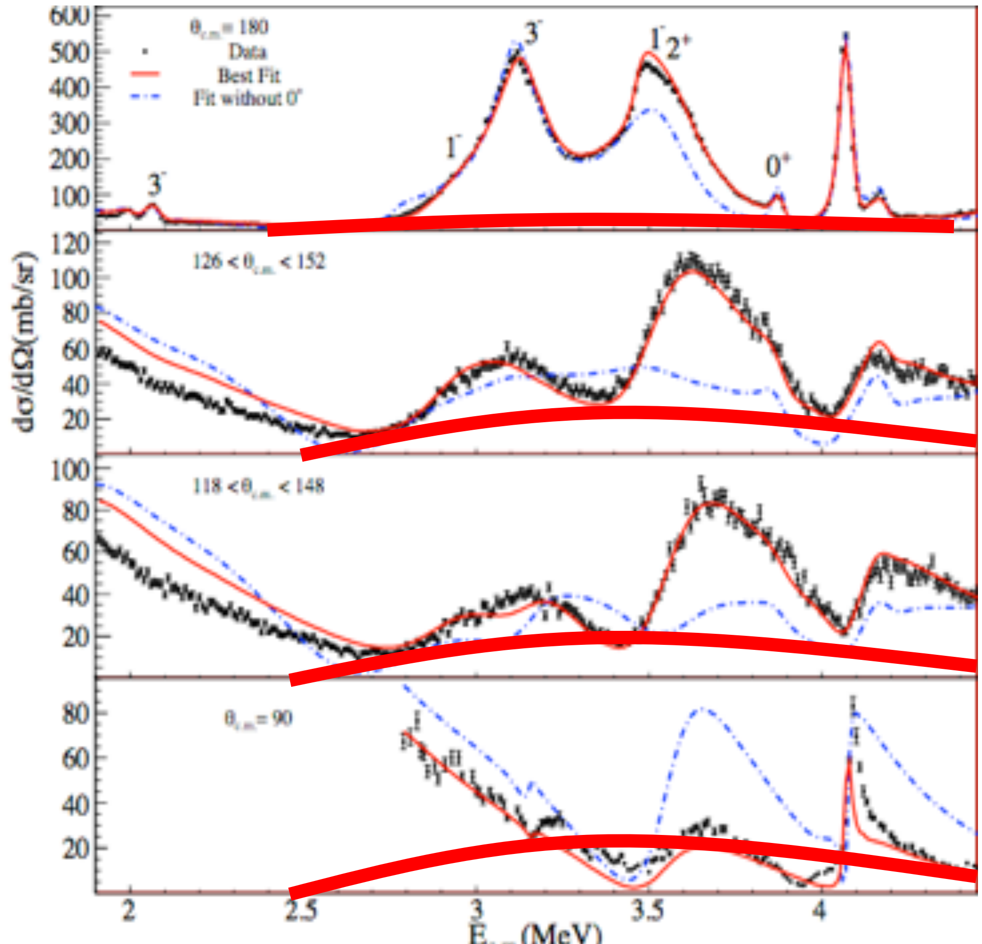
0^+ state at excitation energy of 9.9 MeV

Very broad $\Gamma \approx 2$ MeV
0^+ state at 10 MeV excitation energy was observed.
It has purely α-cluster configuration.
Formally $\theta_\alpha^2 = 2.6$ (at 5.2 fm)

$\theta_\alpha^2 = 1.5$ (at 6.5 fm)

for the 0^+ state.





E.D. Johnson, et al.,

EPJA, 42 135 (2009)

$^{14}C+\alpha$ wave-function that reproduces the observed width of 0^+

Basic Theory

$|1\rangle$ - set of "internal" A-nucleon many-body states (P-space)

$|c;E\rangle$ set of "external" many-body continuum states (Q-space)

Solve problem:

$$H|\Psi\rangle = E|\Psi\rangle$$

where

$$|\Psi\rangle = \sum_1 x_1|1\rangle + \sum_c \int dE' \, \chi^c(E')|c;E'\rangle$$

For structure physics solve for internal coefficients x_1

$$\sum_2 \left[\underbrace{\langle 1|H|2\rangle + \sum_c \int dE' \frac{\langle 1|H|c;E'\rangle\langle c;E'|H|2\rangle}{E - E' + i0}}_{\mathcal{H}_{12}(E)} - \delta_{12}E \right] x_2 = 0$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

$\langle 1|H|2\rangle$ Usual shell-model Hamiltonian involving intrinsic states

$$\langle 1|H|2\rangle = H^{\circ}_{12} + V_{12}$$

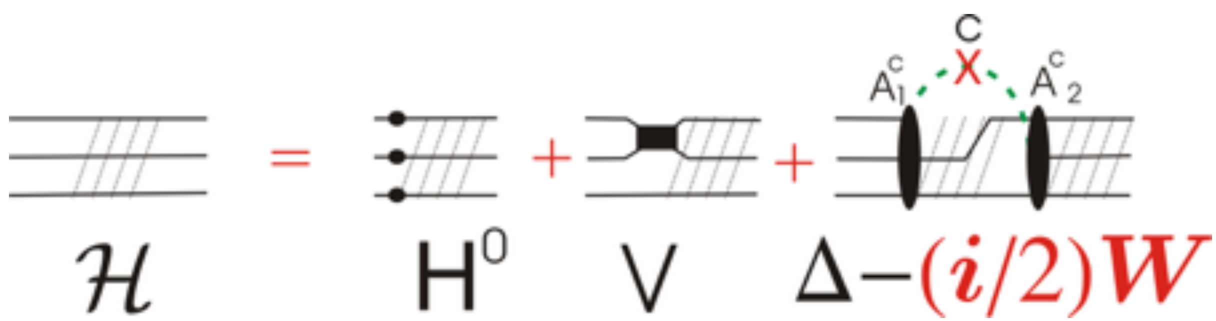
$A^c_1(E') = \langle 1|H|c; E'\rangle$ decay amplitude

$$\sum_c \int dE' \, \frac{A^c_1 A^{c\,*}_2}{E - E'} = \underbrace{\sum_{c\,(\mathrm{all})} P \int dE' \, \frac{A^c_1 A^{c\,*}_2}{E - E'}}_{\Delta(E)} - i\,\underbrace{\pi \sum_{c\,(\mathrm{open})} A^c_1 A^{c\,*}_2}_{W(E)/2}$$

$$\mathcal{H}(E) = H^{\circ} + V + \Delta(E) - \frac{i}{2} W(E)$$

H° s.p energies — V residual inteaction

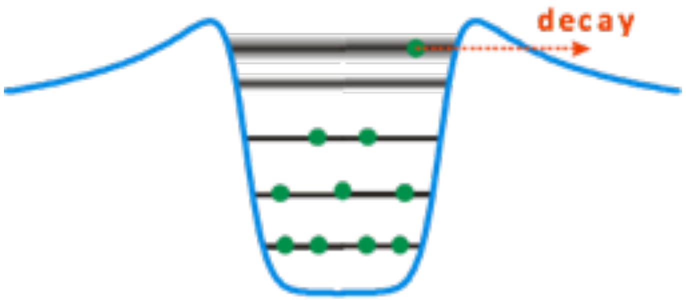
Δ interaction via continuum — W non-Hermitian - decay



The nuclear many-body problem

Traditional shell-model

- Single-particles state (particle in the well)
- Many-body states (slater determinants)
- Hamiltonian and Hamiltonian matrix
- Matrix diagonalization



Continuum physics

- Effective non-hermitian energy-dependent Hamiltonian
- Channels (parent-daughter structure)
- Bound states and resonances
- Matrix inversion at all energies (time dependent approach)

Formally exact approach
Limit of the traditional shell model
Unitarity of the scattering matrix

Effective Hamiltonian Formulation

The Hamiltonian in P is: $$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector: $$|A^c(E)\rangle = H_{QP}|c;E\rangle$$

Self-energy: $$\Delta(E) = \frac{1}{2\pi} \int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E-E'}$$

Irreversible decay to the excluded space: $$W(E) = \sum_{c(\text{open})} |A^c(E)\rangle\langle A^c(E)|$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, Amsterdam 1969
[2] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005).
[3] A. Volya, Phys. Rev. C **79**, 044308 (2009).

Scattering matrix and reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left(\frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$

$$\mathbf{S}_{cc'}(E) = \exp(i\xi_c)\ \{\delta_{cc'} - i\,\mathbf{T}_{cc'}(E)\} \exp(i\xi_{c'})$$

Cross section:

$$\sigma = \frac{\pi}{k'^2} \sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

Additional topics:

- Angular (Blatt-Biedenharn) decomposition
- Coulomb cross sections, Coulomb phase shifts, and interference
- Phase shifts from remote resonances.

Structure of channel vectors and traditional shell model limit

$$|A^c(E)\rangle = a^c(E)\,|c\rangle$$

Channel amplitude

Energy-independent channel vector: structure of spectator components

Perturbative limit in traditional Shell Model: $H|\alpha\rangle = E_\alpha|\alpha\rangle$

$$\Gamma_\alpha = \langle\alpha|W(E_\alpha)|\alpha\rangle \quad \Gamma_\alpha = \sum_c \Gamma_\alpha^c \qquad \Gamma_\alpha^c = \gamma_c(E_\alpha)\,|\langle c|\alpha\rangle|^2$$

Single-particle decay width

$$\gamma_c(E) = |a^c(E)|^2$$

Spectroscopic factor or transition rate

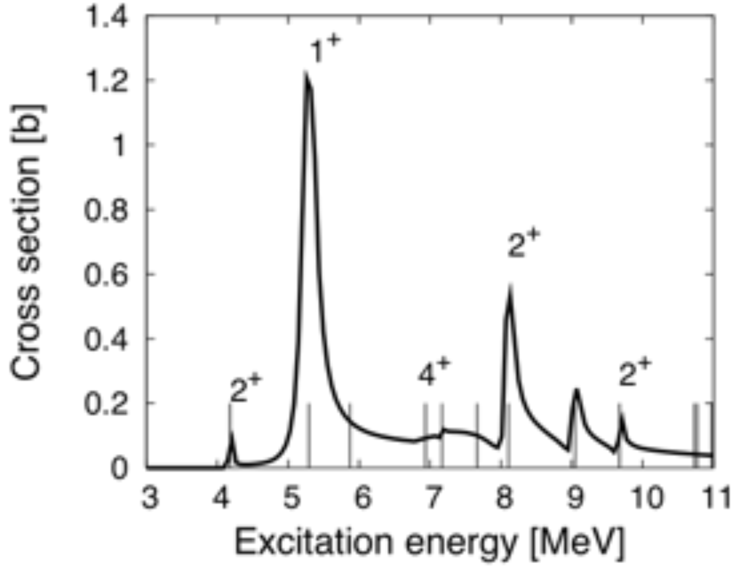
$$C^2S = |\langle c|\alpha\rangle|^2$$

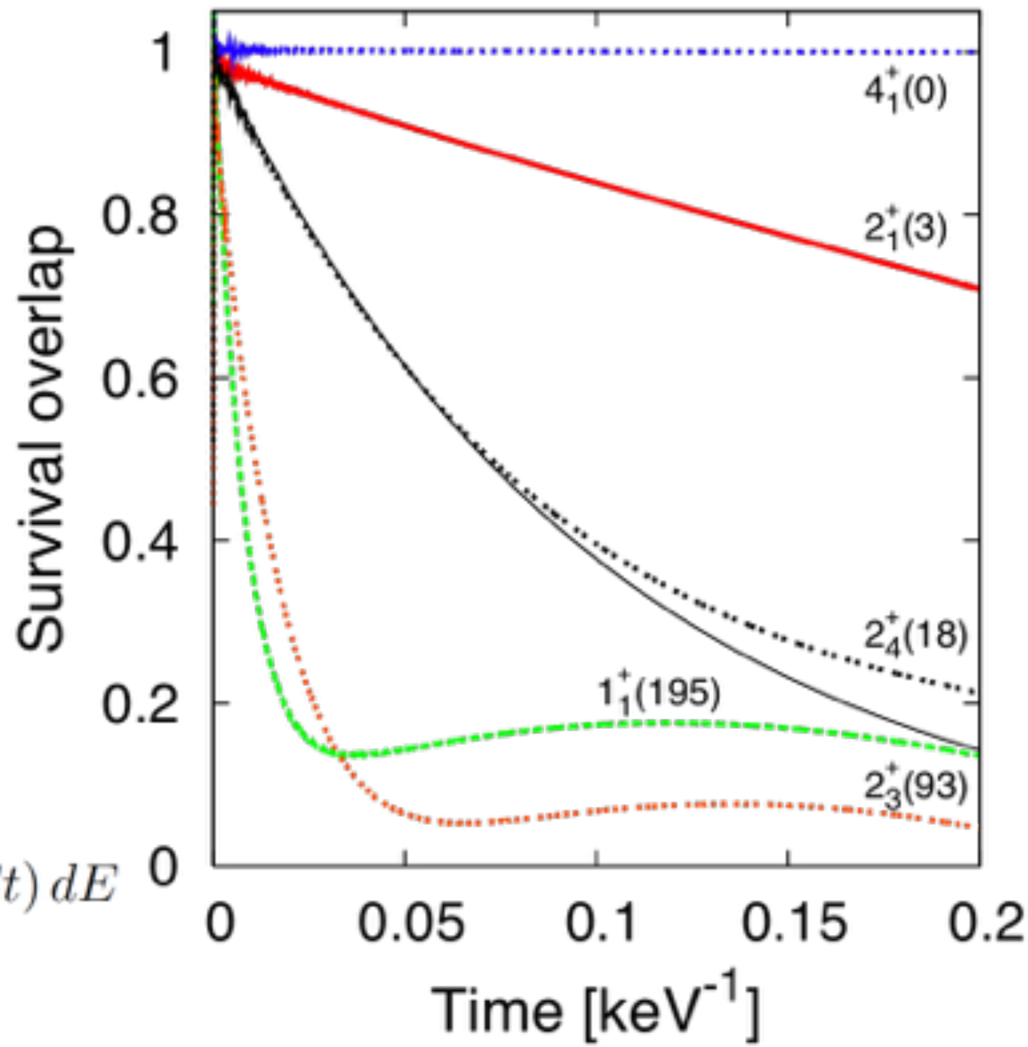
$$B(\mathrm{EM}) = |\langle c|\alpha\rangle|^2$$

Time-dependent approach

- Reflects time-dependent physics of unstable systems
- Direct relation to observables
- Linearity of QM equations maintained
- No matrix diagonalization
- Powerful many-body numerical techniques
- Stability for broad and narrow resonances
- Ability to work with experimental data

Time evolution of decaying states





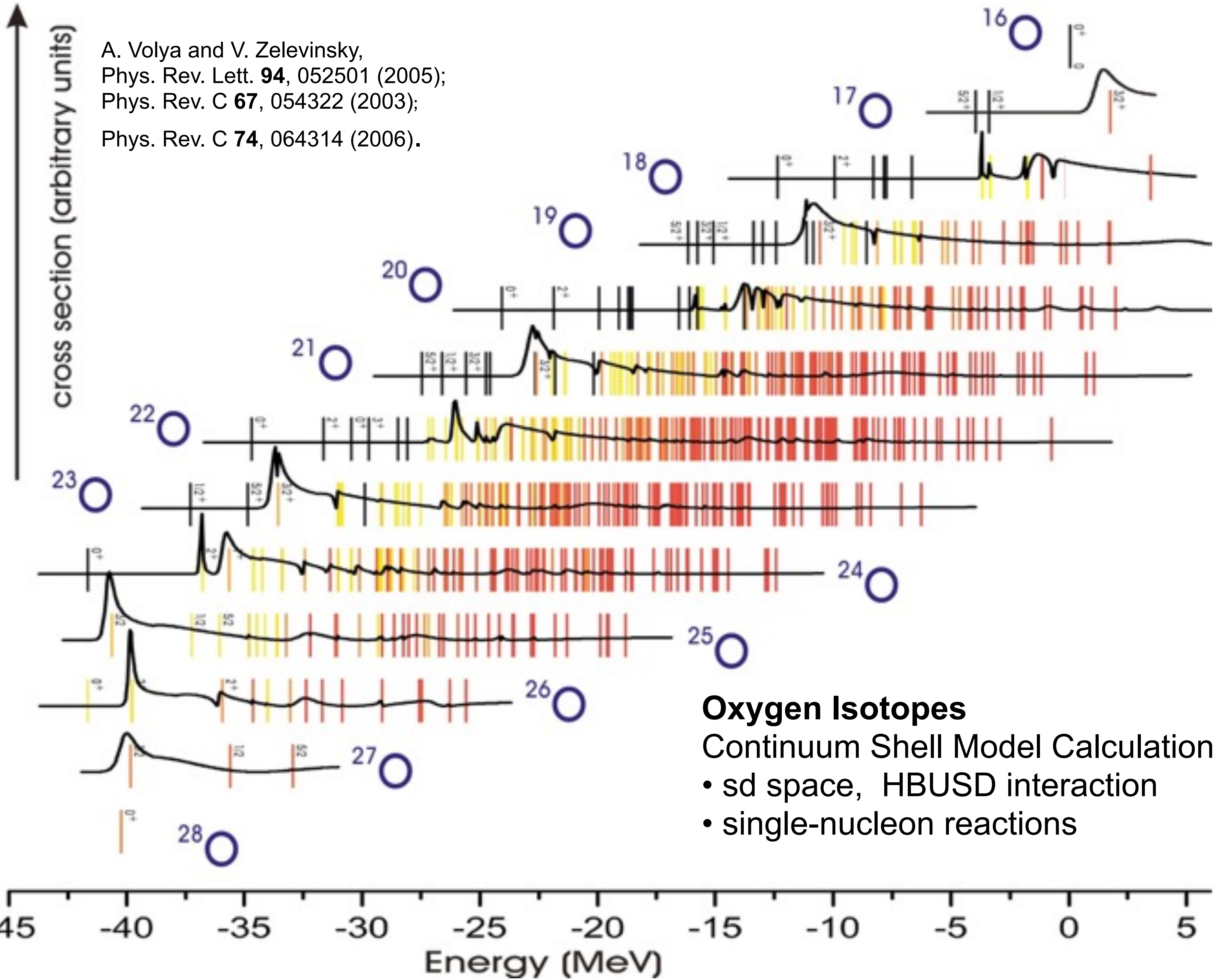
Time evolution of several SM states in ^{24}O. The absolute value of the survival overlap is shown $|\langle\alpha|\mathcal{U}(t)|\alpha\rangle|$

$$\mathcal{U}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{\infty}\mathcal{G}(E)\,\exp(-iEt)\,dE$$

For an isolated narrow resonance

$$|\langle\alpha|\exp(-i\mathcal{E}_\alpha t)|\alpha\rangle| = \exp(-\Gamma_\alpha t/2)$$

A. Volya and V. Zelevinsky,
Phys. Rev. Lett. **94**, 052501 (2005);
Phys. Rev. C **67**, 054322 (2003);
Phys. Rev. C **74**, 064314 (2006).



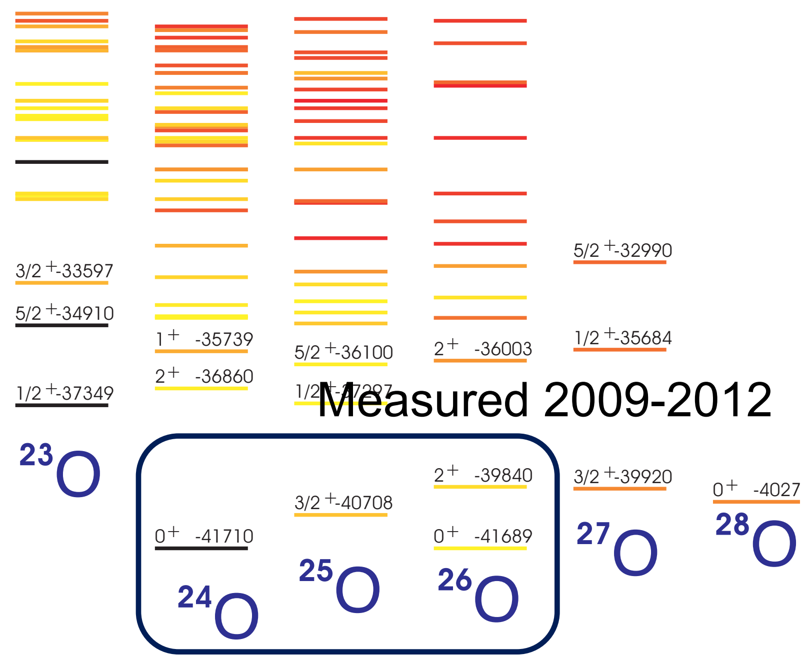
Oxygen Isotopes

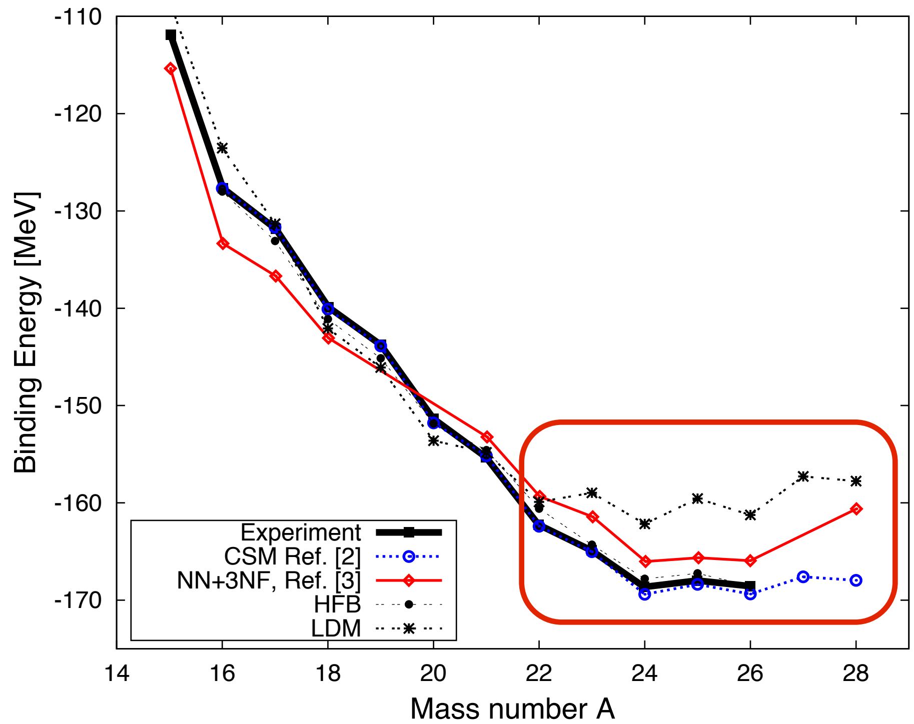
Continuum Shell Model Calculation

- sd space, HBUSD interaction
- single-nucleon reactions

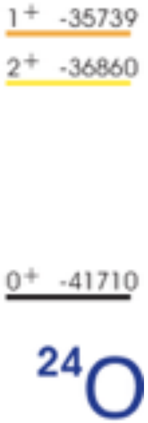
Predictive power of theory

Continuum Shell Model prediction 2003-2006



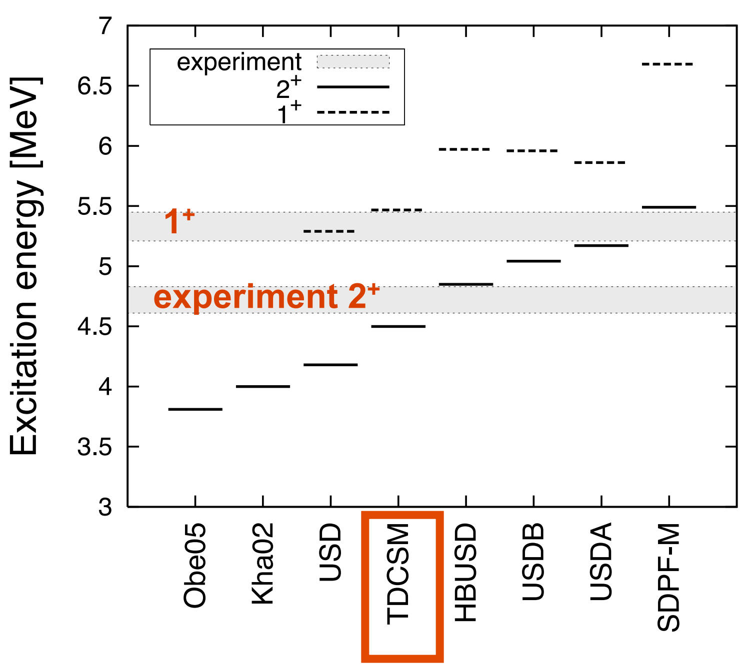


[1] C. R. Hoffman et al., Phys. Lett. B **672**, 17 (2009); Phys.Rev.Lett.**102**,152501(2009); Phys.Rev.C **83**,031303(R)(2011); E. Lunderberg et al., Phys. Rev. Lett. **108**, 142503 (2012).
[2] A.V. and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005); Phys. Rev. C **67**, 054322 (2003); **74**, 064314 (2006).
[3] G. Hagen et.al Phys. Rev. Lett. **108**, 242501 (2012)



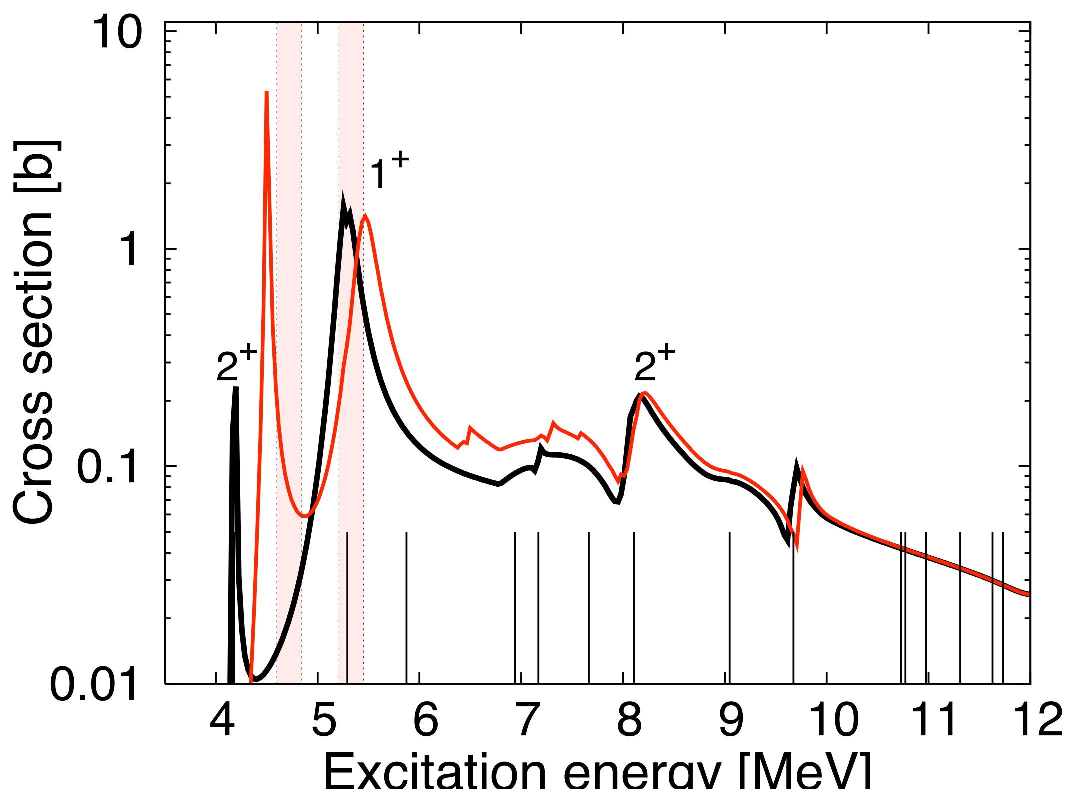
Virtual excitations into continuum

Figure: Theory predictions for states in ^{24}O



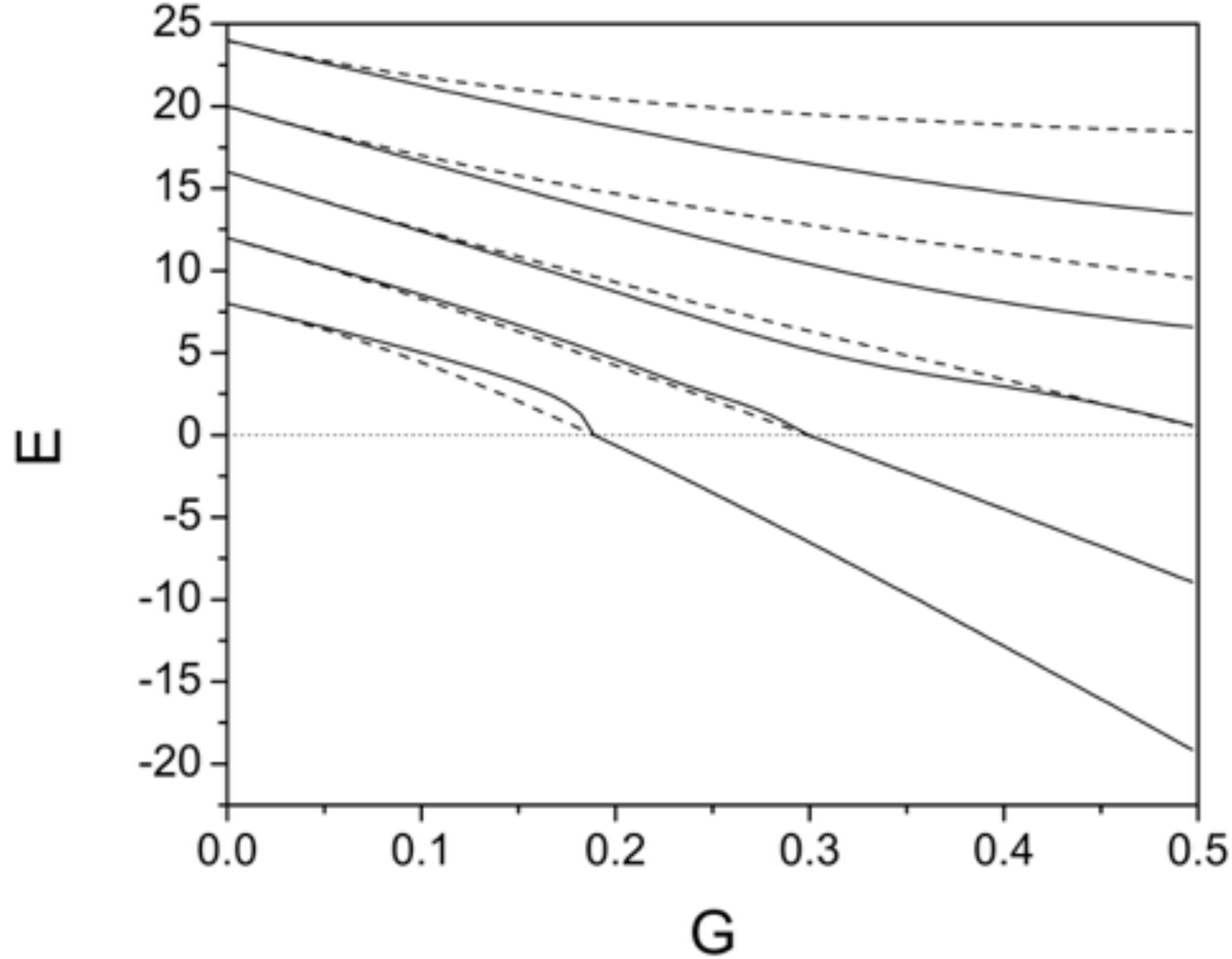
Experimental data from:
C. Hoffman, et.al. Phys. Lett. **B672**, 17 (2009)

Figure: $^{23}O(n,n)^{23}O$ Effect of self-energy term (red curve). Shaded areas show experimental values with uncertainties.



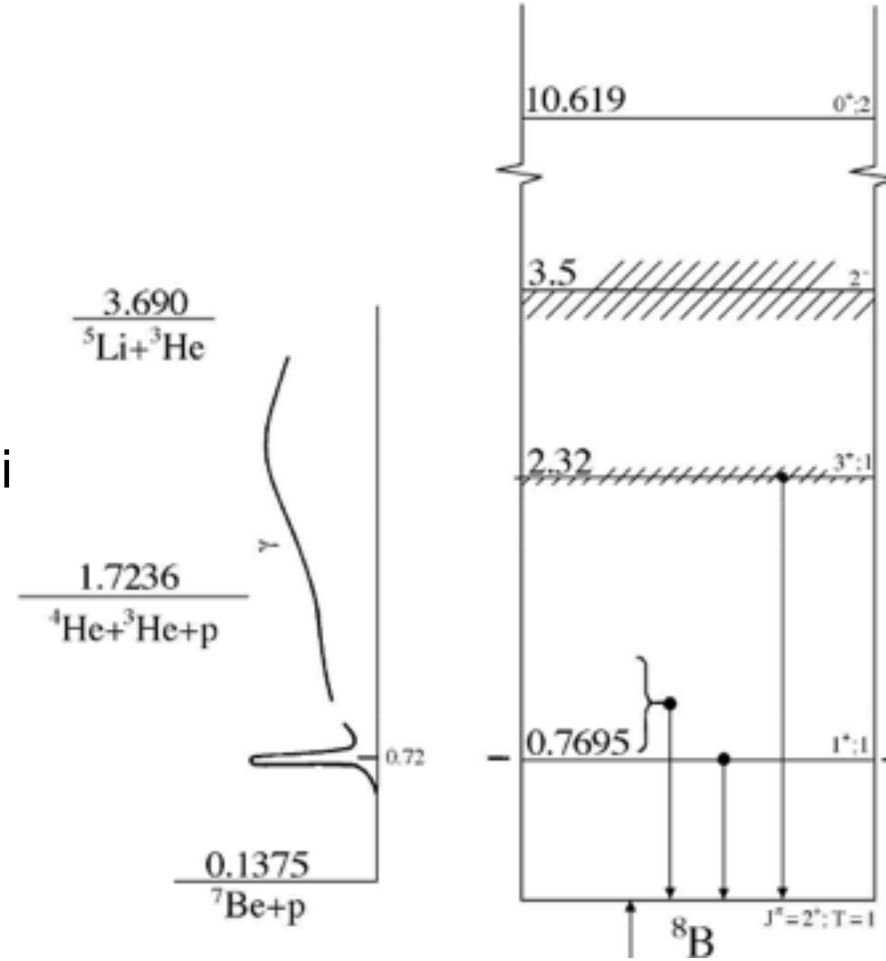
Two-level model, many-body system with pairing

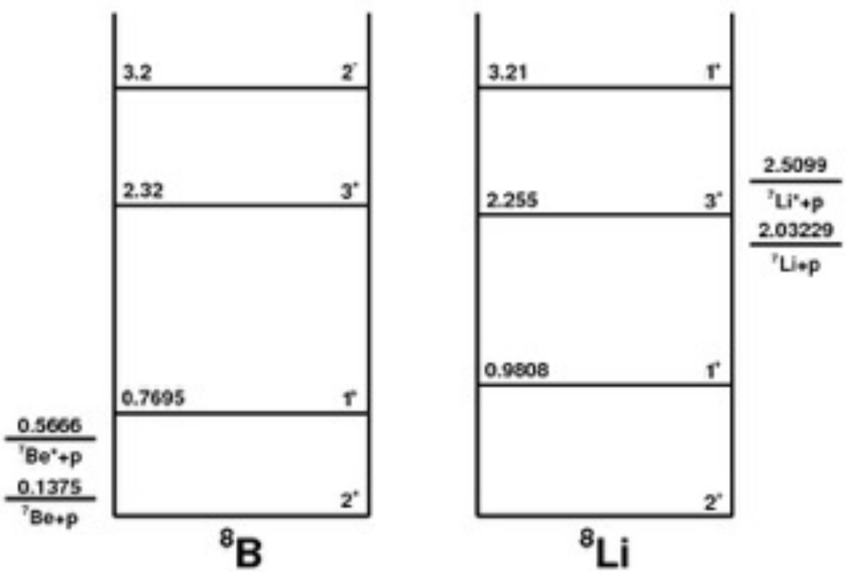
$j_1=j_2=9/2$, 10 particles
$\varepsilon_1=1$, $\varepsilon_2=3$
Constant pairing G
Coupling to decay $e_j=\varepsilon-i/2\ \alpha_j\ E^{1/2}$



States in ^8B

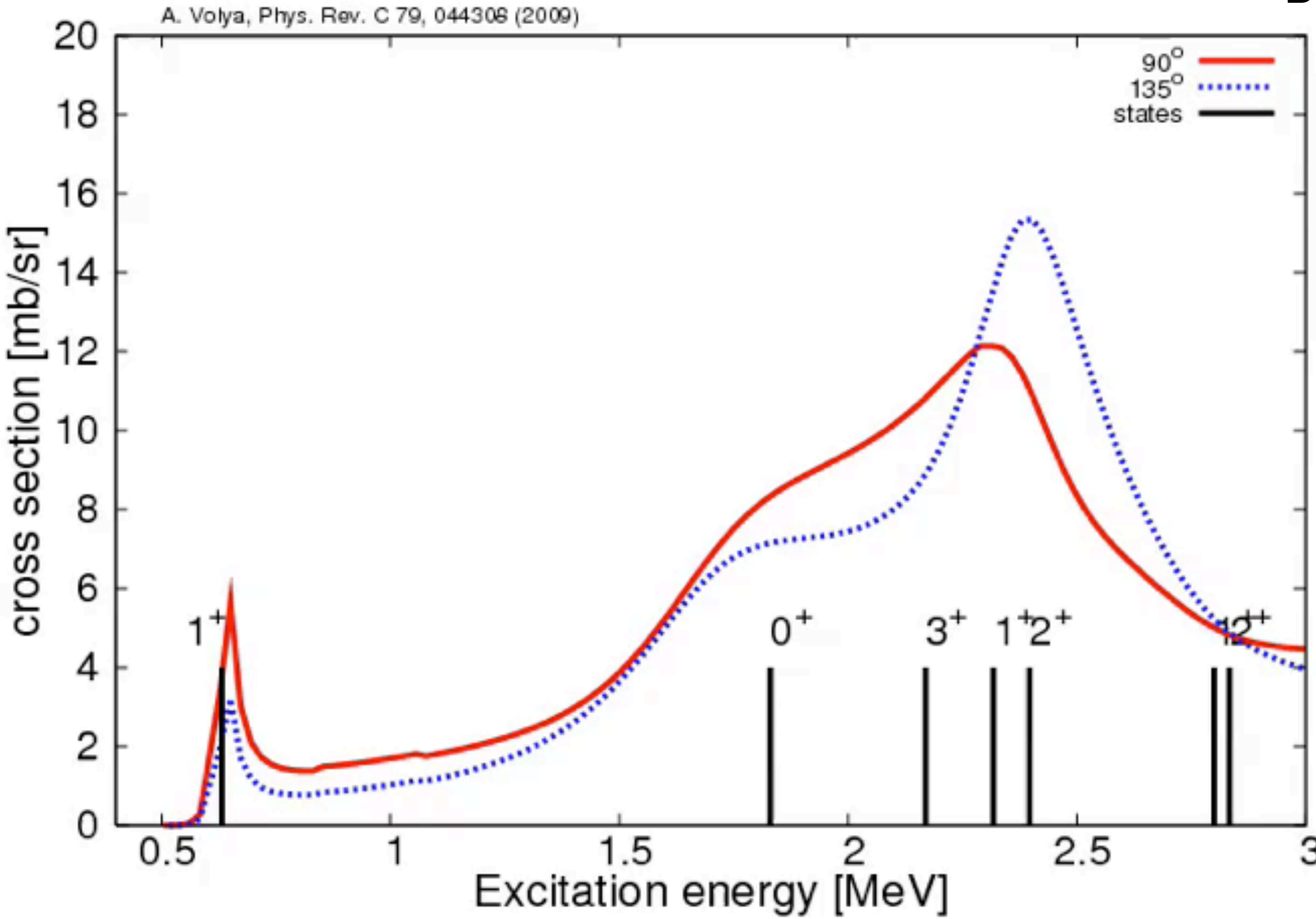
- Ab-initio and no core theoretical models predict low-lying 2^+, 0^+, and 1^+ states
- Recoil-Corrected CSM suggests low-lying states
- Traditional SM mixed results
- These states were not seen in ^8B and in ^8Li





Interference between resonances

^{8}Be

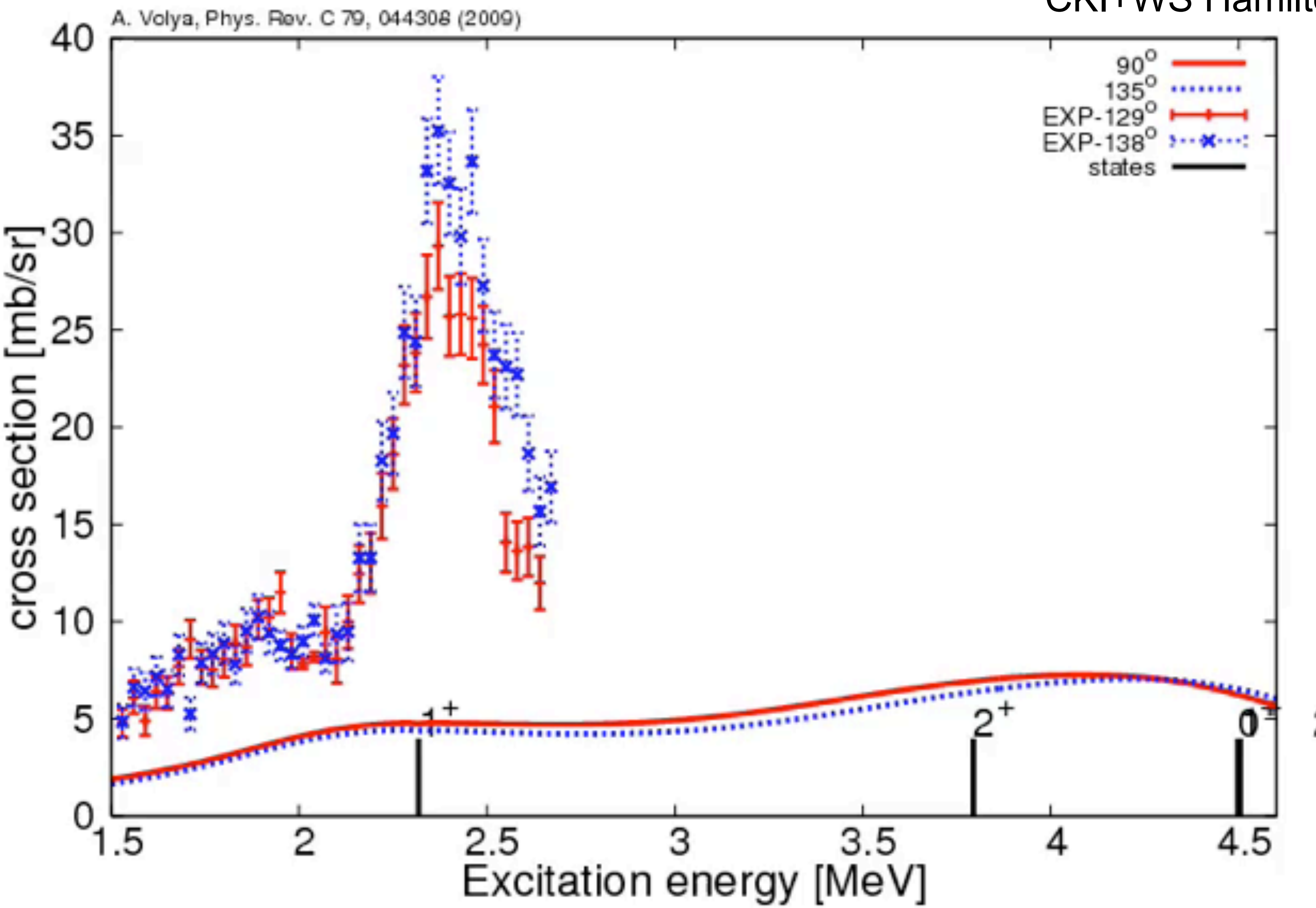


See animation at www.volya.net

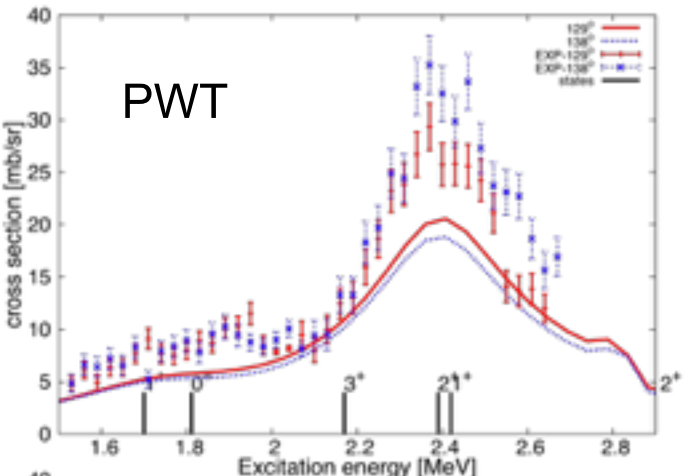
Understanding observables and cross sections

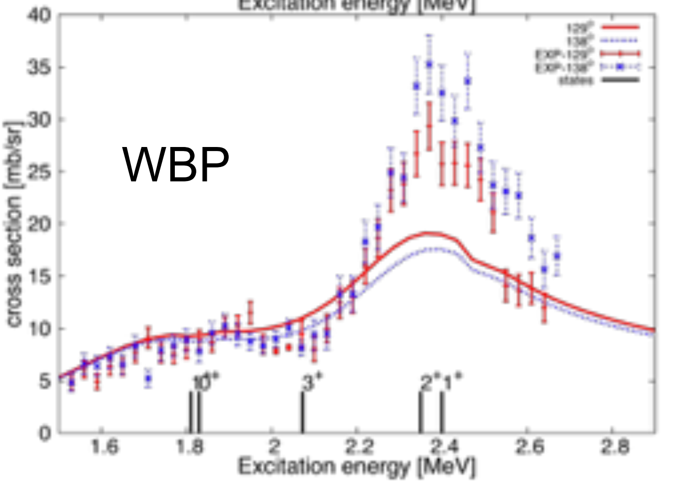
$^7Be(p,p')^7Be$

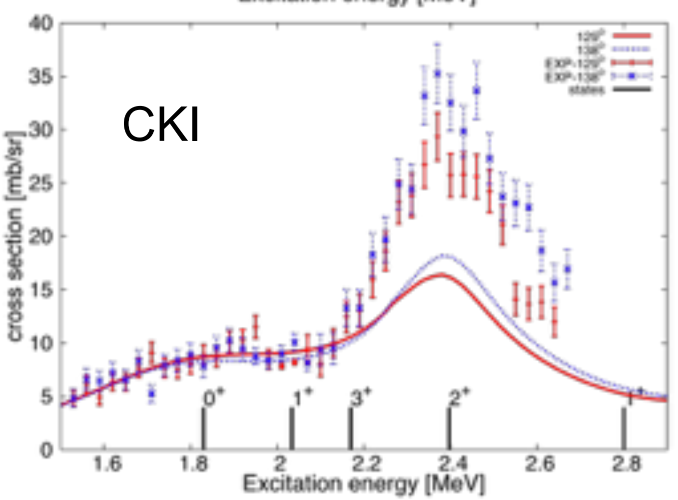
CKI+WS Hamiltonian



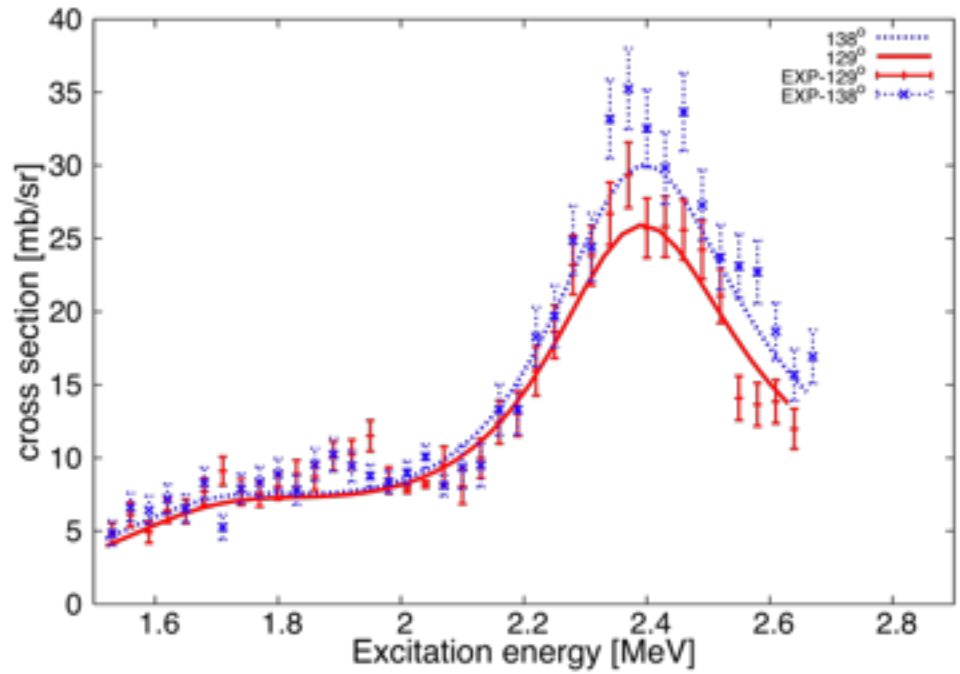
See animation at www.volya.net







R-matrix fit and TDCSM for $^{7}Be(p,p)^{7}Be$



Channel Amplitudes from TDCSM and final best fit

	J^{π}	$p_{1/2}$, I=3/2	$p_{3/2}$, I=3/2	$p_{1/2}$, I=1/2	$p_{3/2}$, I=1/2
FIT	2^{+}	-0.293	0.293		0.534
CKI	2^{+}	-0.168	0.164		0.521
FIT	1^{+}	-0.821	-0.612	0.375	0.175
CKI	1^{+}	-0.840	-0.617	0.332	0.178

Unitarity and flux conservation

Take: $\mathbf{W} = \mathbf{a}\mathbf{a}^\dagger$

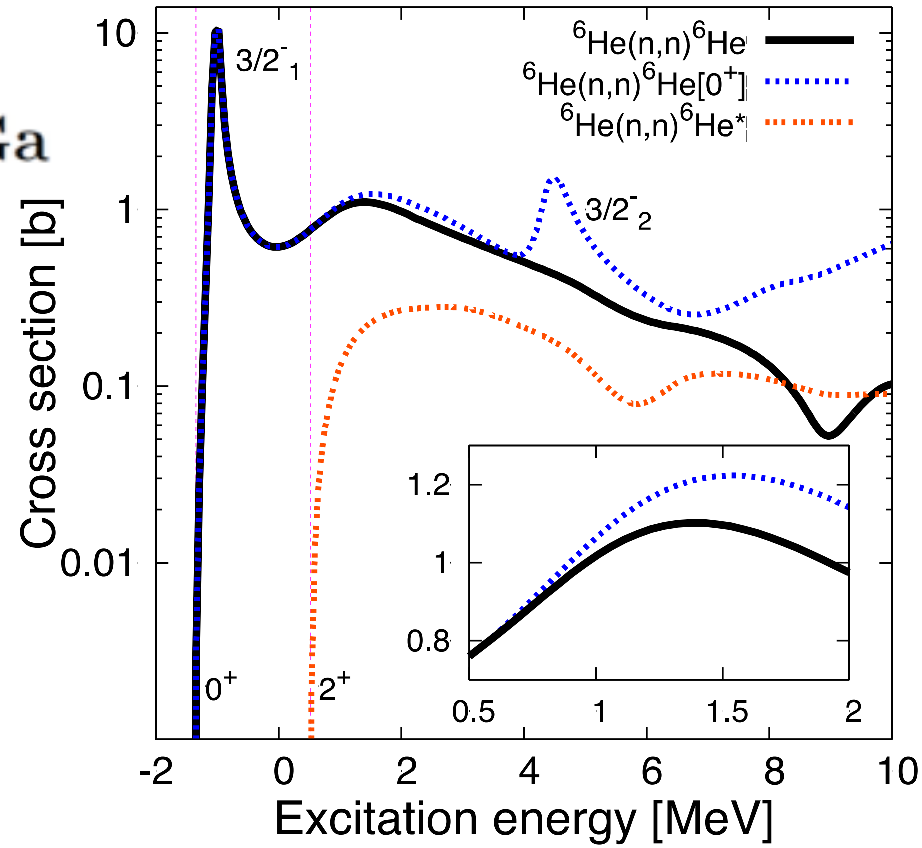
Exact relation:

$$\mathbf{S} = \frac{1 - i/2\,\mathbf{K}}{1 + i/2\,\mathbf{K}} \qquad \mathbf{K} = \mathbf{a}^\dagger \mathbf{G} \mathbf{a}$$

$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{S}^\dagger\mathbf{S} = \mathbf{1}$$

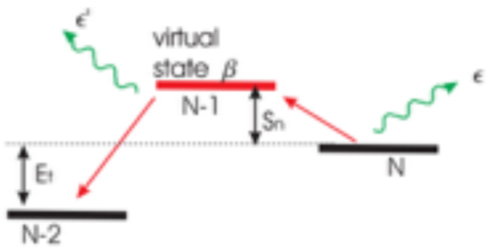
- Cross section has a cusp when inelastic channels open
- The cross section is reduced due to loss of flux
- The p-wave discontinuity $E^{3/2}$

Figure: ^{6}He(n,n) cross section
- Solid curve-full cross section
- Dashed (blue) only g.s. channel
- Dotted (red) inelastic channel



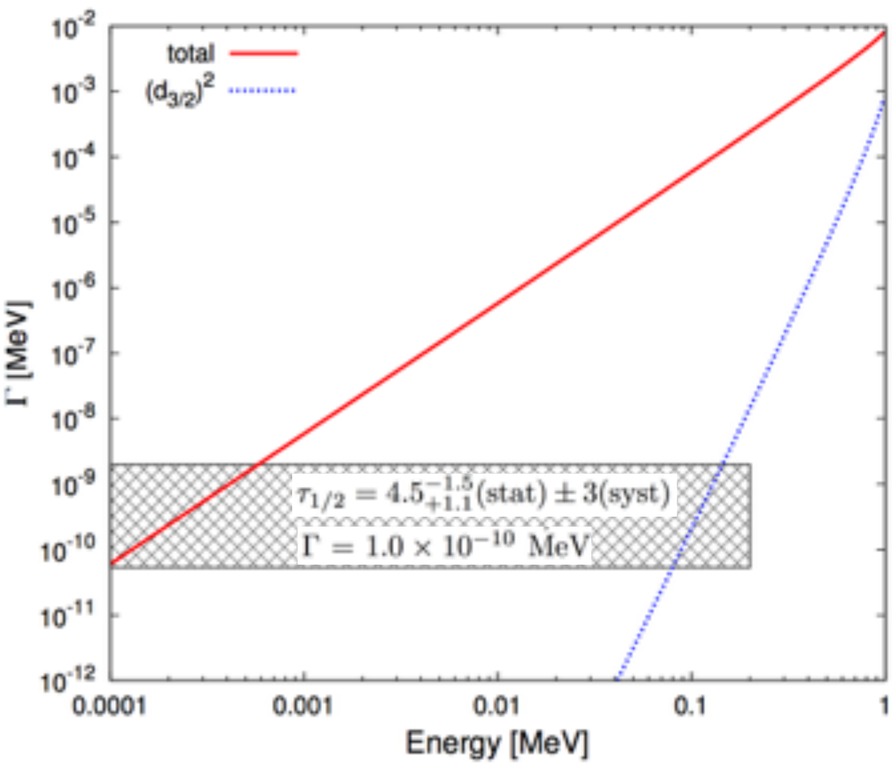
Two-neutron sequential decay of ^{26}O

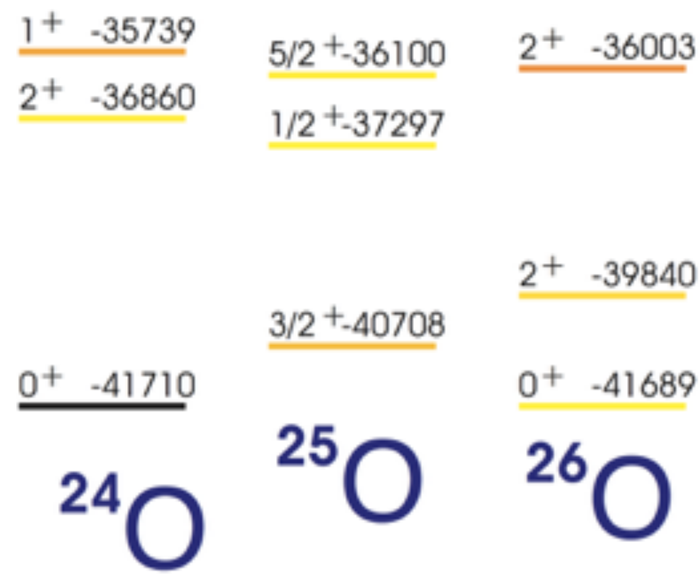
A. Volya and V. Zelevinsky, *Continuum shell model,* Phys. Rev. C **74**, 064314 (2006).



Predicted Q-value: 21 keV

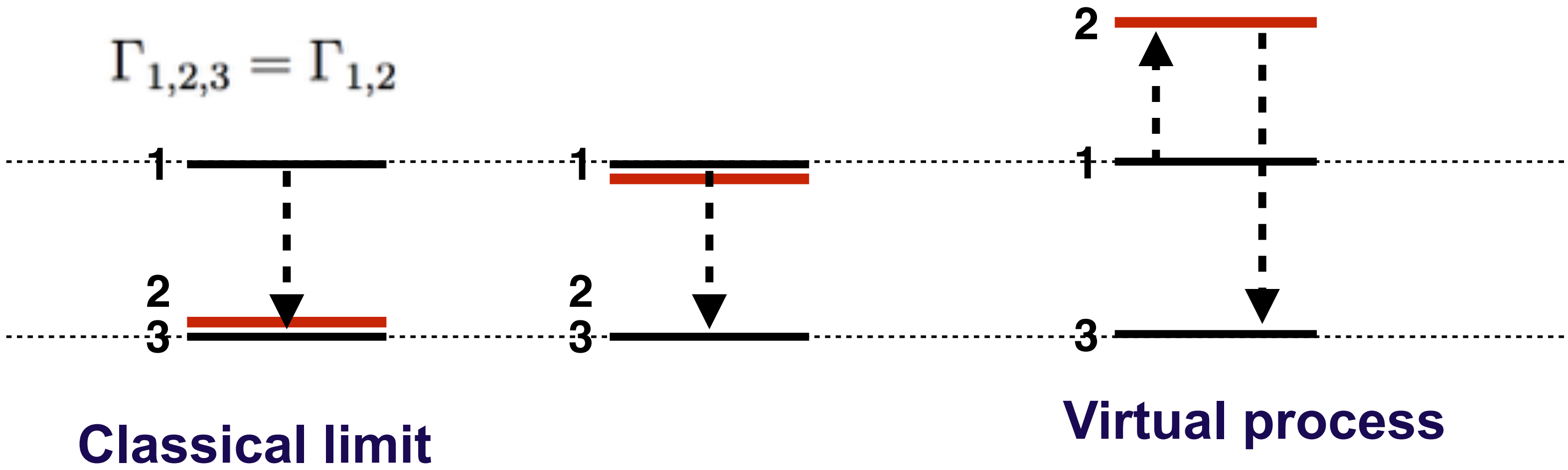
	E[MeV]	$\langle ^{26}O\vert j, ^{25}O\rangle$	$\langle ^{25}O\vert j, ^{24}O\rangle$	Γ[MeV]
$s_{1/2}$	4.41	1.36	0.14	2.72×10^{-6}
$d_{3/2}$	1.00	1.42	0.96	2.00×10^{-14}
$d_{5/2}$	5.61	-0.53	6.67×10^{-4}	4.18×10^{-23}

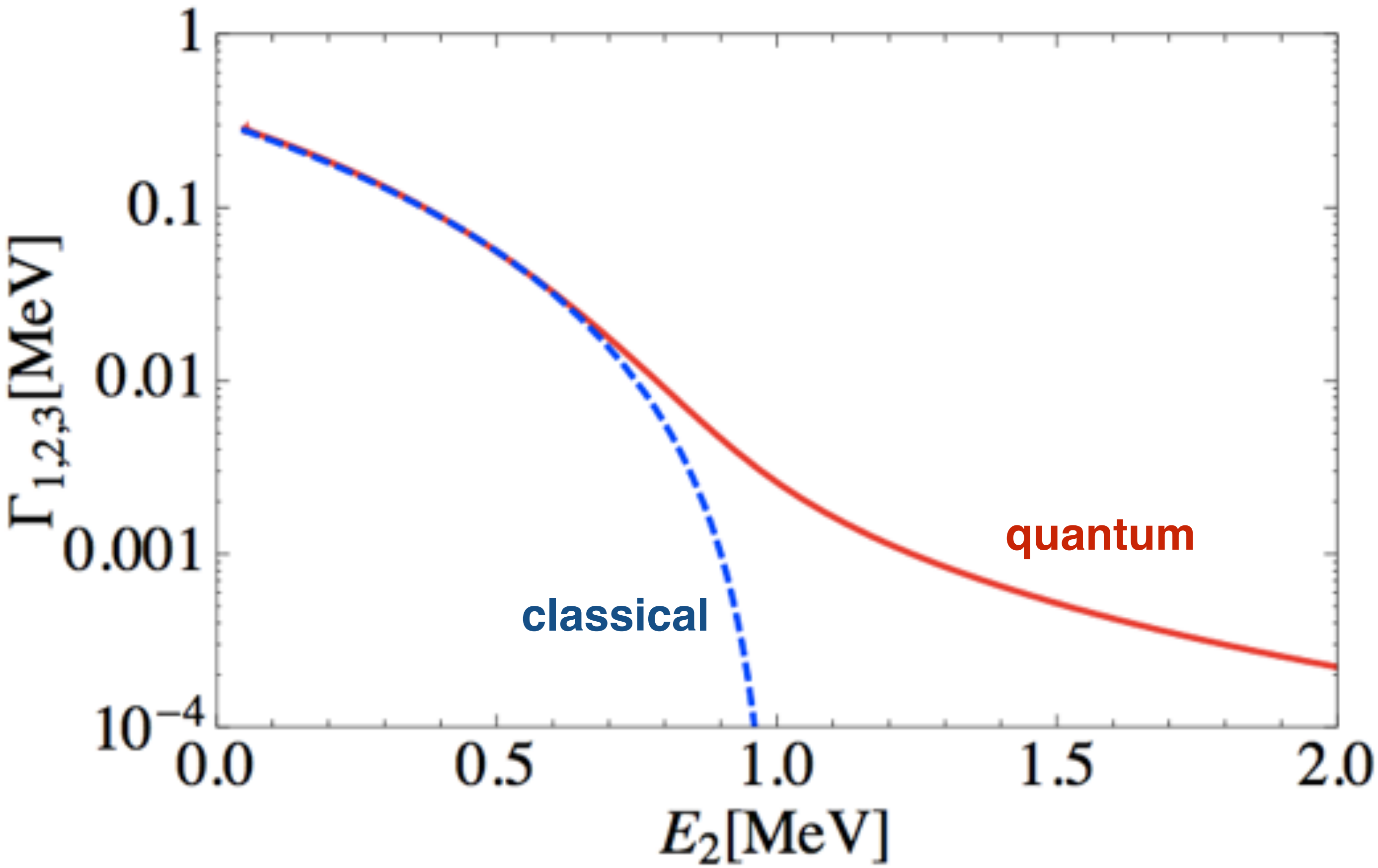




Z. Kohley, et.al PRL 110, 152501 (2013) (experiment)

Neutron pair decay, sequential mechanism





Neutron pair decay, sequential mechanism

Formalism

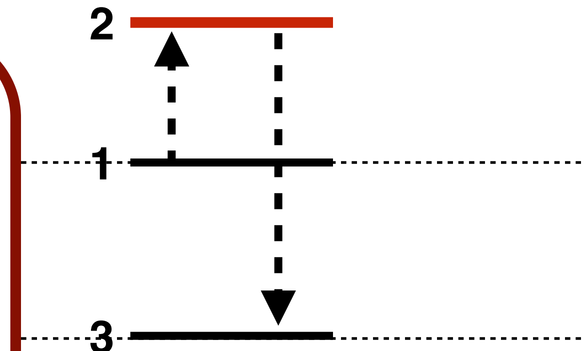
$$A_{1,2,3}(\epsilon_1, \epsilon_2) = \frac{A_1(\epsilon_1)\, A_2(\epsilon_2)}{\epsilon_2 - \left(E_2 - \frac{i}{2}\Gamma_{2,3}(\epsilon_2)\right)}$$

Note well-known 2s-1s photon decay

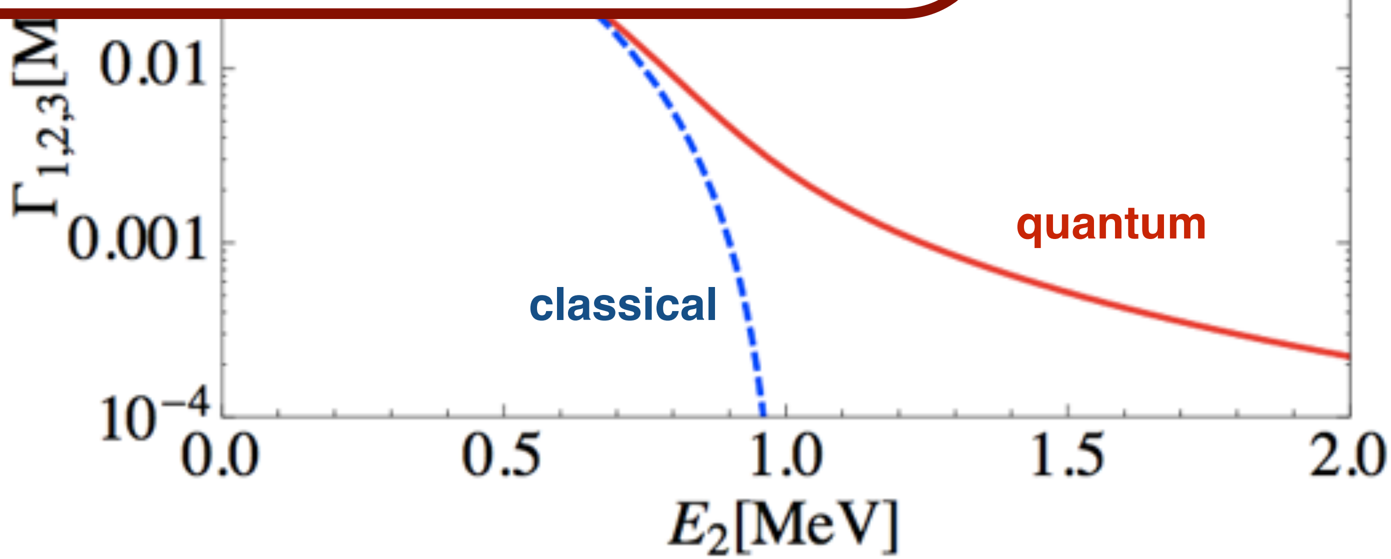
$$\frac{d\Gamma(E)}{d\epsilon_1 d\epsilon_2} = 2\pi\delta(E - \epsilon_1 - \epsilon_2)\, |A_T(\epsilon_1, \epsilon_2)|^2$$

Classical limit $\Gamma/|E - i\Gamma/2|^2 \approx 2\pi\delta(E)$

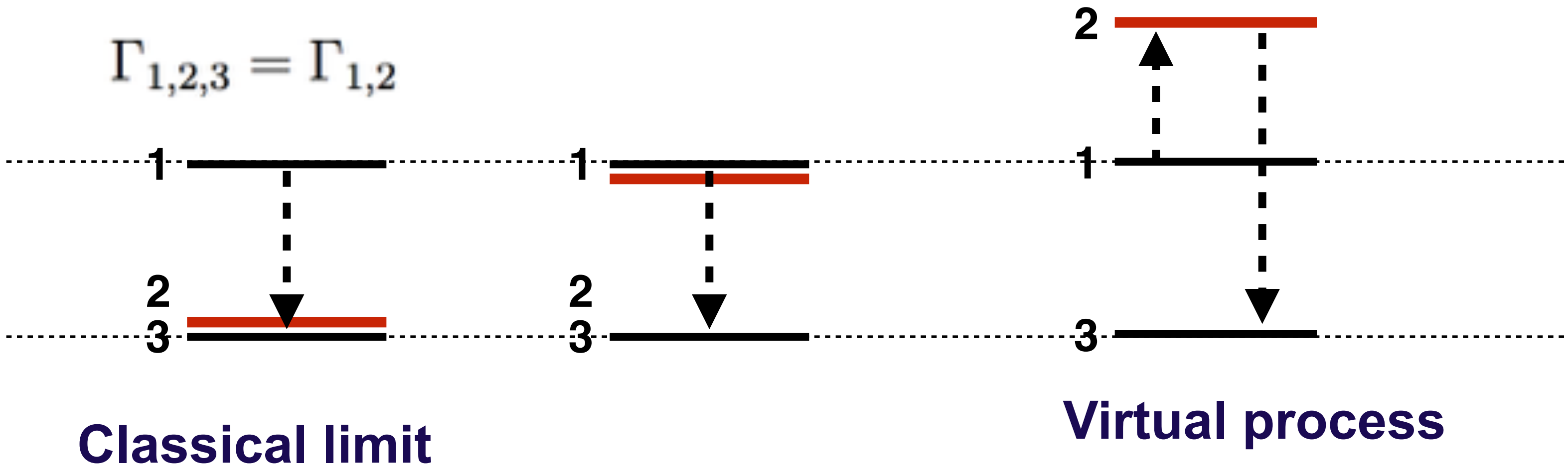
$$\Gamma_{1,2,3} = \Gamma_{1,2}$$

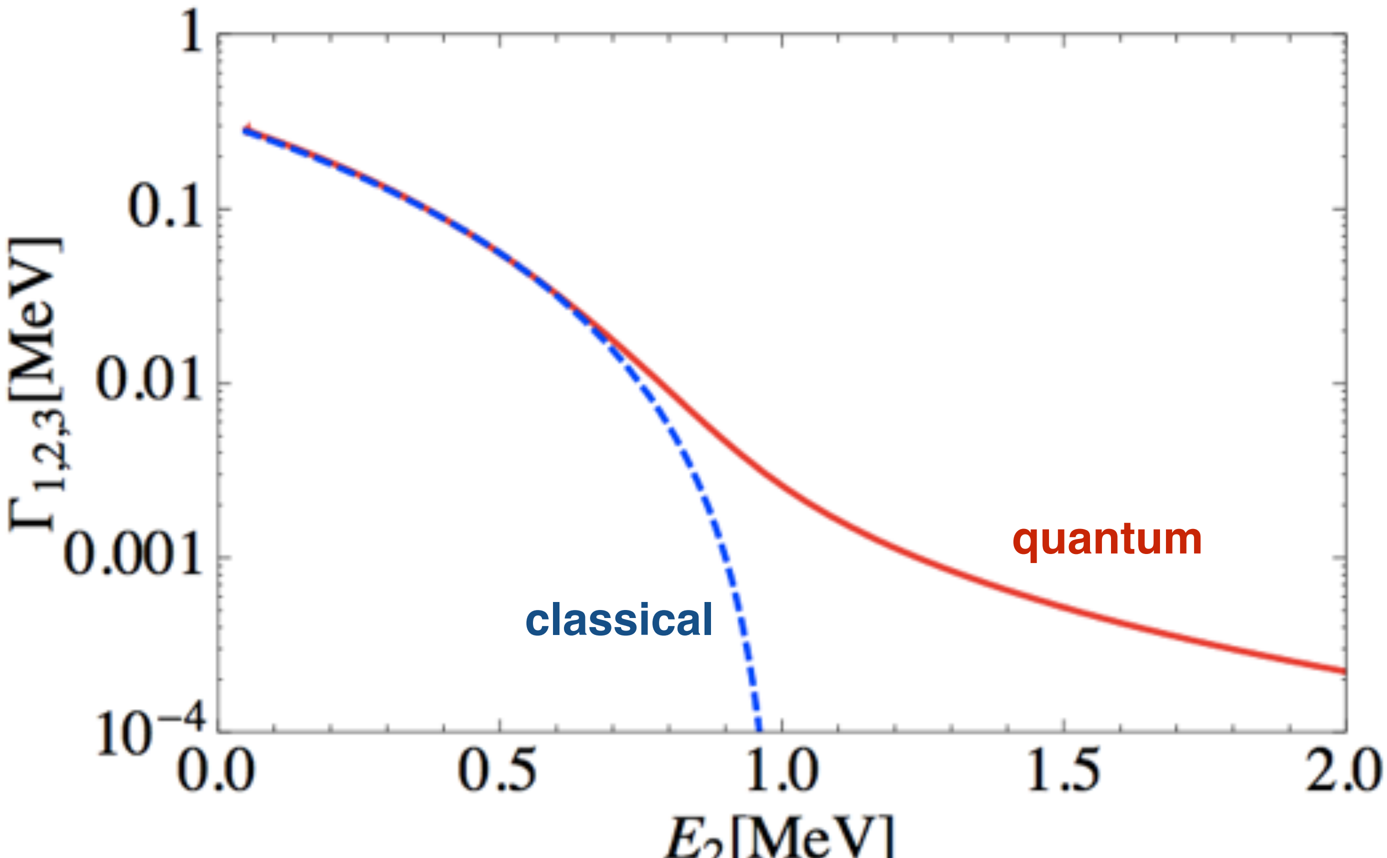


Virtual process

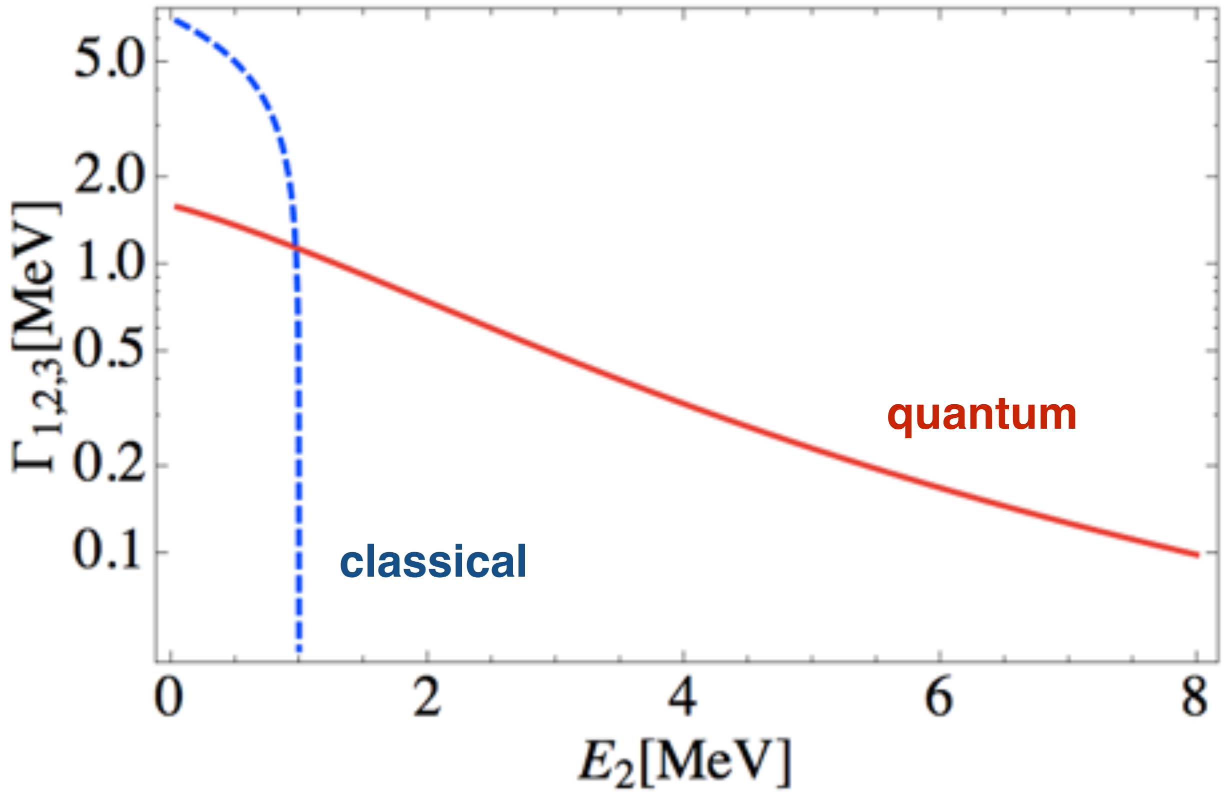


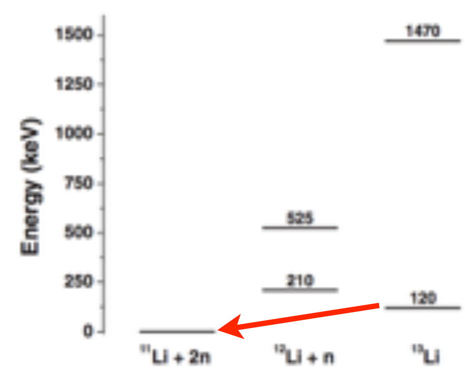
Neutron pair decay, sequential mechanism





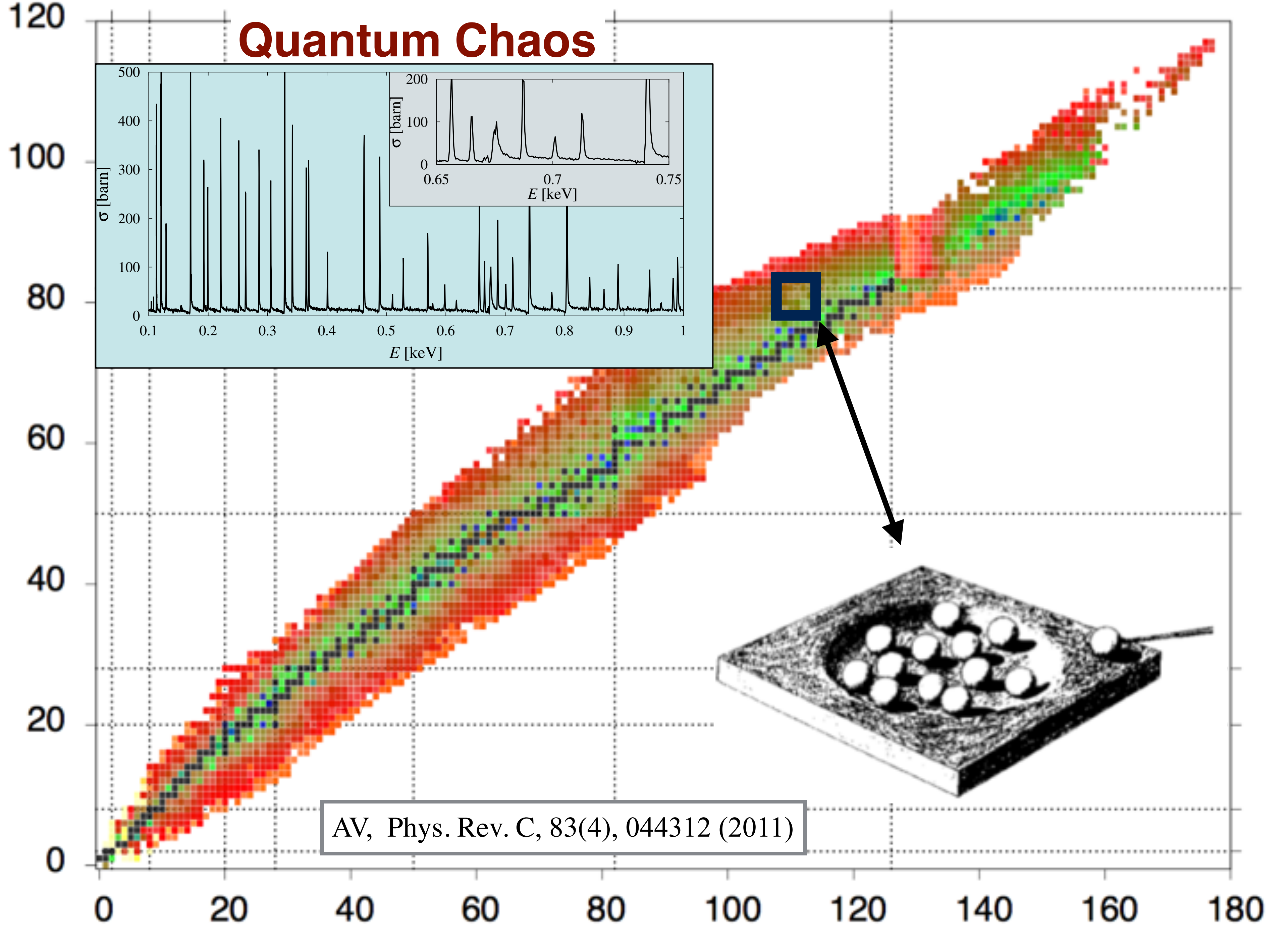
Low energy s-wave sequential decay (neutral particles)





- Classical and one-body decay limit is never reached
- n-n scattering length.
- Sequential decay is slower
- Different phase space volume

Z. Kohley, E. Lunderberg, P. A. DeYoung, A. Volya, T. Baumann, et. al Phys. Rev. C **87,** 011304(R) (2013)
A. Volya, EPJ **38**, 03003 (2012).


Quantum Chaos
σ [barn]
E [keV]
AV, Phys. Rev. C, 83(4), 044312 (2011)

Violation of PTD?

P. E. Koehler, et.al Phys. Rev. Lett**. 105,** 072502 (2010)
P. E. Koehler, et.al, *Phys. Rev. C 76 (2007)***.**
J. F. Shriner, *Phys. Rev. C* ***32,*** *694 (1985)***.**
R. R. Whitehead, et.al*, Phys. Lett. B* ***76****, 149 (1978).*

Too many narrow states!

Relative to what? How to quantify

- Fit to PTD, effective $v<1$
- The distribution is too peaked, relative to the normal (normality test)
- Moments, correlations etc…

Published online 24 August 2010 | *Nature* **466**, 1034 (2010) |
doi:10.1038/4661034a

News

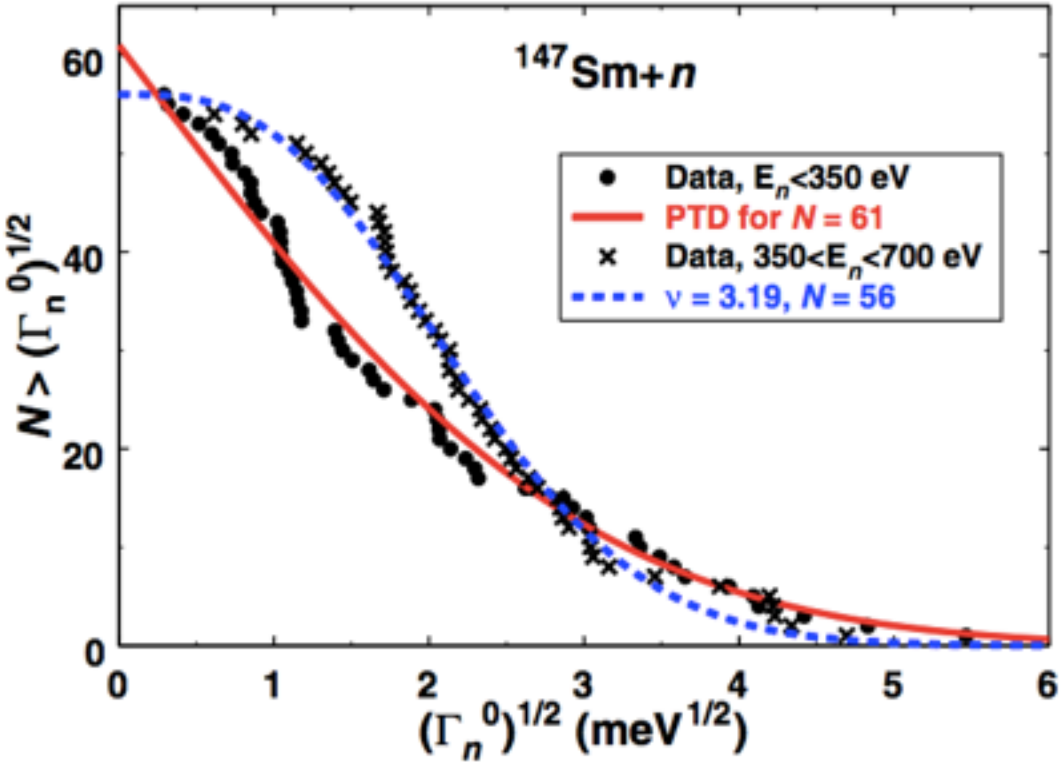
Nuclear theory nudged

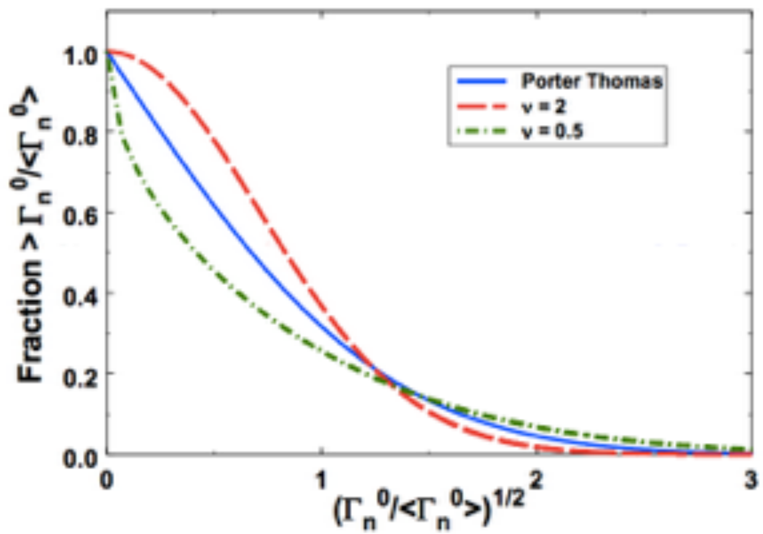
Results from mothballed facility challenge established theory.

Nuclear theory nudged?
Violation of Porter-Thomas Distribution

Random matrix theory is rejected with 99.997% probability [Koehler, et. al. Phys. Rev. Lett. 105, 072502 (2010)] In platinum $\nu = 0.5$

Implications:
Capture rates, astrophysical reactions, nuclear reactors, critical mass, shielding...





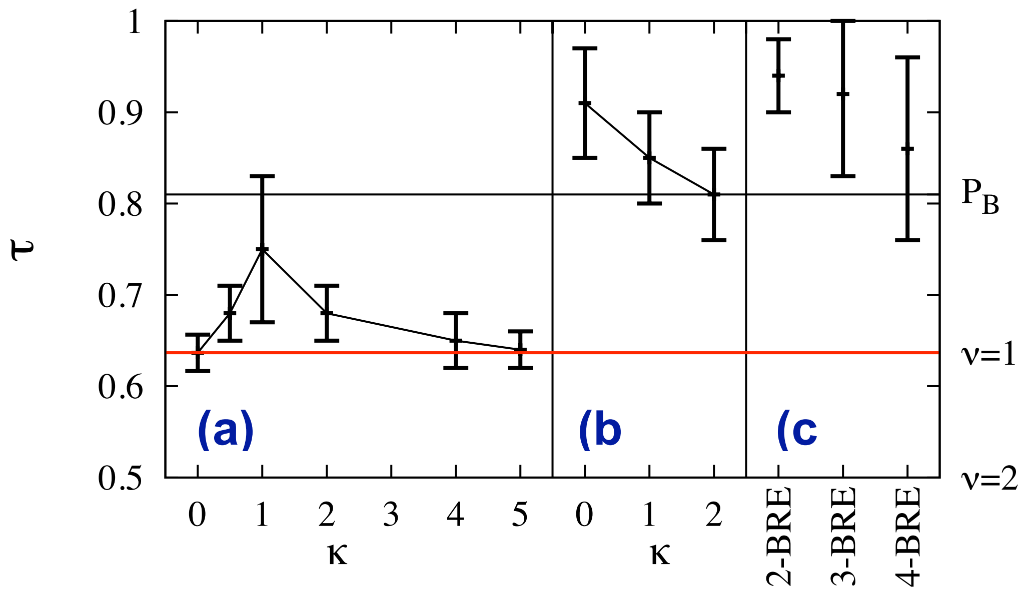
Nuclear theory nudged?
Violation of Porter-Thomas Distribution

Interaction with continuum [1]

(a) Overlapping resonances
(b) Memory effect and overlapping resonances (2-body interactions)
(c) Many-body interactions

the two-body or other low-rank Hamiltonian does not lead to dynamical mixing of states strong enough for the decaying system to lose all memory of its creation.

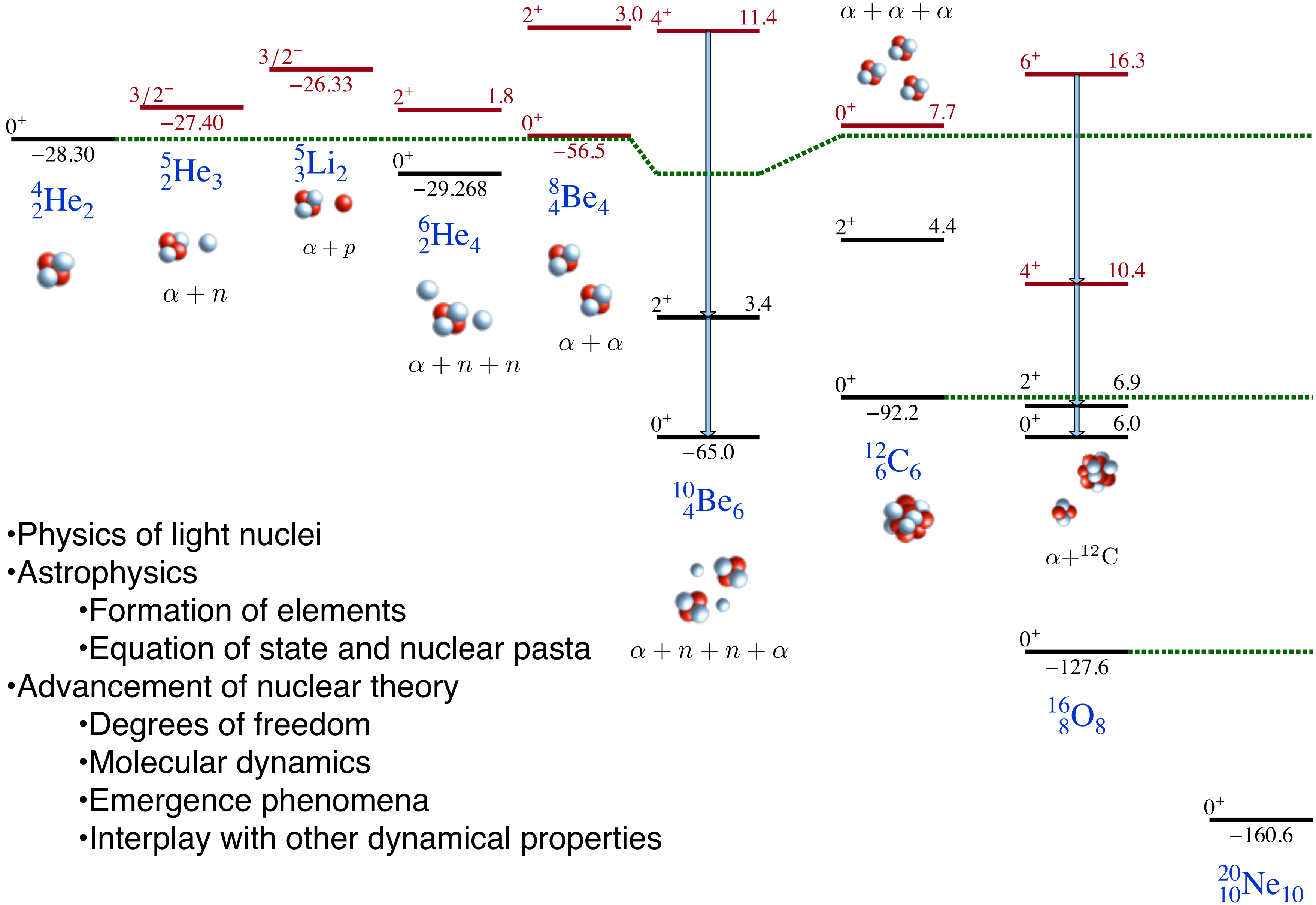
Coefficient of variation
Statistical normality test



Nuclear shape and chaos [2]

[1]A. Volya, Phys. Rev. C **83**, 044312 (2011).
[2] V. Abramkina and A. Volya, Phys. Rev. C **84**, 024322 (2011).

Clustering in light nuclei



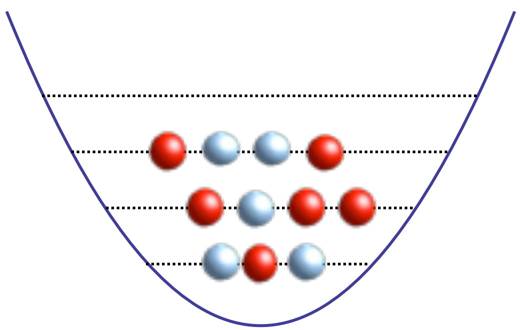
- Physics of light nuclei
- Astrophysics
    - Formation of elements
    - Equation of state and nuclear pasta
- Advancement of nuclear theory
    - Degrees of freedom
    - Molecular dynamics
    - Emergence phenomena
    - Interplay with other dynamical properties

Cluster-nucleon configuration interaction approach

Traditional shell model configuration

m-scheme

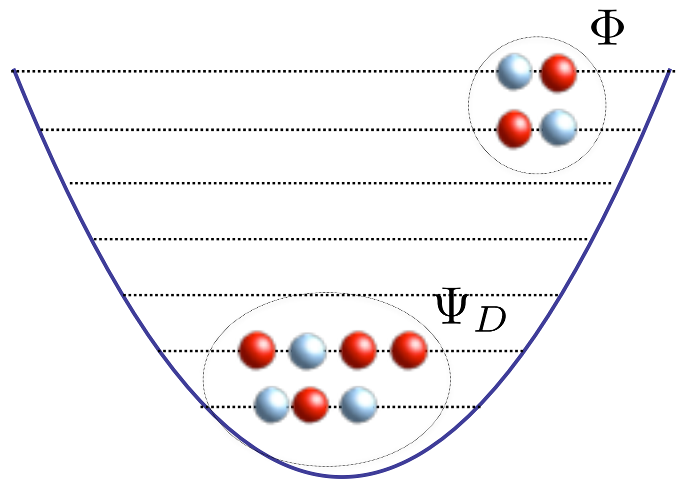
$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \dots a_A^\dagger|0\rangle$$



Cluster configuration

SU(3)-symmetry basis

$$|\text{channel}\rangle \; = \; |\mathcal{A}\{\Phi\,\Psi_D\} \equiv \Phi^\dagger\Psi^\dagger|0\rangle \equiv \Phi^\dagger|\Psi_D\rangle$$

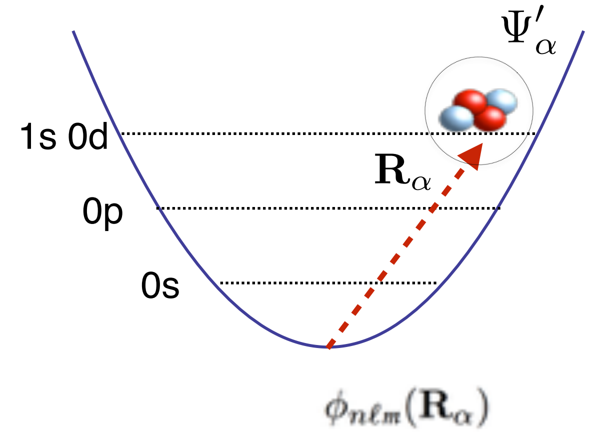


+

$$|\Psi\rangle \quad + \quad \Phi^\dagger|\Psi_D\rangle$$

- **m-scheme and SU(3) basis**
- **Construction and classification of cluster configurations**
- **Center of mass and translational invariance**
- **Non-orthogonality and bosonic principle**

Cluster configurations



Example: alpha decay with ℓ=0 from sd shell
21 way to make L=0 T=0 4-nucleon combination
Each nucleon has 2 oscillator quanta, 8 quanta total
In oscillator basis excitation quanta are conserved
We model alpha as 4-nucleons on s-shell $(0s)^4$

Make single SU(3) operator with quantum numbers (8,0) $\Phi^{\eta}_{(8,0):\ell m}$

Cluster coefficient is known analytically $X^{\eta}_{n'\ell}$

$$\underbrace{\phi_{n\ell m}(1)\phi_{n\ell m}(2)\phi_{n\ell m}(3)\phi_{n\ell m}(4)}_{4\times 2=8\,\text{quanta}} \leftrightarrow \sum_{\eta} X^{\eta}_{n'\ell}\Phi^{\eta}_{(8,0):\ell m} = \underbrace{\phi_{n'\ell' m'}(\mathbf{R}_\alpha)}_{8\ \text{quanta}} \underbrace{\Psi'_\alpha}_{0\ \text{quanta}}$$

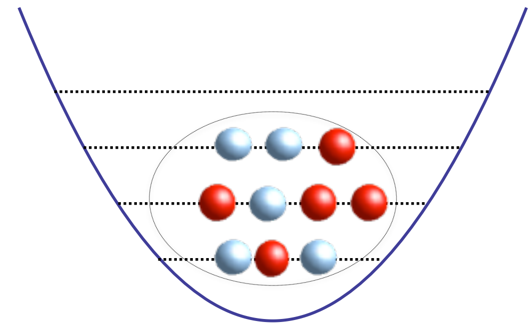
m-scheme state

SU(3) symmetry state

motion of alpha

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Translational invariance



Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state

Center-of-mass vibration

Intrinsic state

Factorizing center of mass in overlap integral

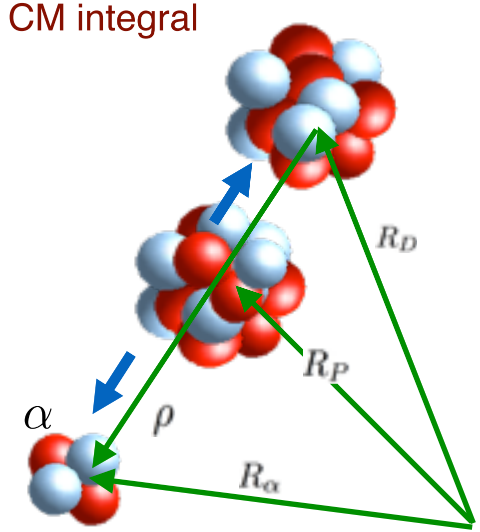
$$\langle \Psi_P | \hat{\mathcal{A}}\{\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha\,\Psi_D\}\rangle = \langle \Psi'_P | \hat{\mathcal{A}}\{\phi_{n\ell m}(\boldsymbol{\rho})\Psi'_\alpha\,\Psi'_D\}\rangle \times \langle \phi_{000}(\mathbf{R}_P)\,\phi_{n\ell m}(\boldsymbol{\rho}) | \phi_{n\ell m}(\mathbf{R}_\alpha)\,\phi_{000}(\mathbf{R}_D)\rangle$$

SM overlap integral (FPC) — Translationally invariant part — Spurious CM integral

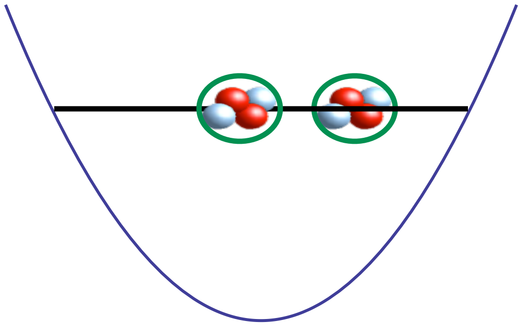
Recoil factor (inverse of Talmi-Moshinsky coefficient)

$$\mathbf{R}_P = \frac{m_D\mathbf{R}_D + m_\alpha\mathbf{R}_\alpha}{m_D + m_\alpha}, \qquad \boldsymbol{\rho} = \mathbf{R}_D - \mathbf{R}_\alpha$$

$$\mathcal{R}_{n\ell} \equiv \left(\langle 00, n\ell : \ell | \{n\ell\}_{m_\alpha}, \{00\}_{m_D} : \ell\rangle\right)^{-1} = (-1)^n \left(\frac{m_D + m_\alpha}{m_D}\right)^{n/2}$$



Bosonic nature of 4-nucleon operators non-orgothogonality



If $\Phi^\dagger$ is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$

$|\Psi_D\rangle = |\Phi\rangle \quad \langle\Phi_D|\hat{\Phi}\hat{\Phi}^\dagger|\Psi_D\rangle = \langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle = 2$

$L = S = T = 0$

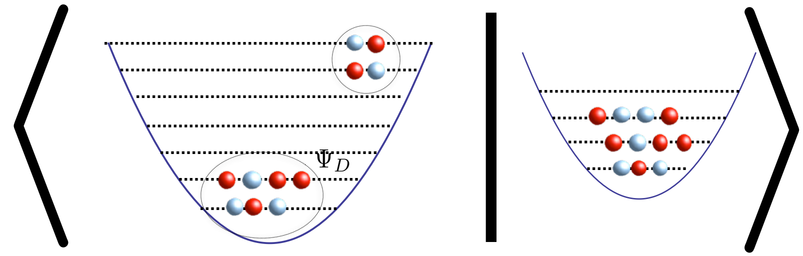
Φ	Ψ_P	$\left|\langle\Psi_P|\hat{\Phi}^\dagger|\Psi_D\rangle\right|^2$	$\langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle$
$(p)^4\,(4,0)$	$(p)^8\,(0,4)$	1.42222*	1.42222
$(sd)^4\,(8,0)$	$(sd)^8\,(8,4)$	0.487903	1.20213
$(fp)^4\,(12,0)$	$(fp)^8\,(16,4)$	0.292411	1.41503
$(sdg)^4\,(16,0)$	$(sdg)^8\,(24,4)$	0.209525	1.5278

* For p-shell the result is known analytically 64/45

Effective operators (alphas) are not ideal bosons
Cluster configurations are not orthogonal and not normalized

Traditional Cluster Spectroscopic Characteristics

$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} | \Psi'_P \rangle =$$



$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \mathcal{R}_{n\ell} \sum_\eta X^\eta_{n\ell} \, \mathcal{F}^\eta_{n\ell}$$

Recoil Factor   Cluster Coefficient   Fractional Parentage Coefficient

Traditional “old” spectroscopic factors

$$\varphi_\ell(\rho) = \sum_n \langle \phi_{n\ell} | \varphi_\ell \rangle \, \phi_{n\ell}(\rho)$$

Expand radial motion in HO wave functions

$$\mathcal{S}^{(\mathrm{old})}_\ell = \langle \varphi_\ell | \varphi_\ell \rangle = \int \rho^2 d\rho \, |\varphi_\ell(\rho)|^2 = \sum_n |\langle \phi_{n\ell} | \varphi_\ell \rangle|^2$$

Orthogonality condition model, new SF

- Non-orthogonal set of channels (over-complete set of configurations)
- Pauli exclusion principle
- Matching procedure, asymptotic normalization, connection to observables
- No agreement with experiment on absolute scale

Resonating group method

$$\hat{\mathcal{H}}_\ell \, f_\ell(\rho) = E \hat{\mathcal{N}}_\ell f_\ell(\rho) \qquad \hat{\mathcal{N}}_\ell^{-1/2} \hat{\mathcal{H}}_\ell \hat{\mathcal{N}}_\ell^{-1/2} F_\ell(\rho) = E F_\ell(\rho)$$

New spectroscopic factor

$$\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2} \varphi_\ell(\rho)$$

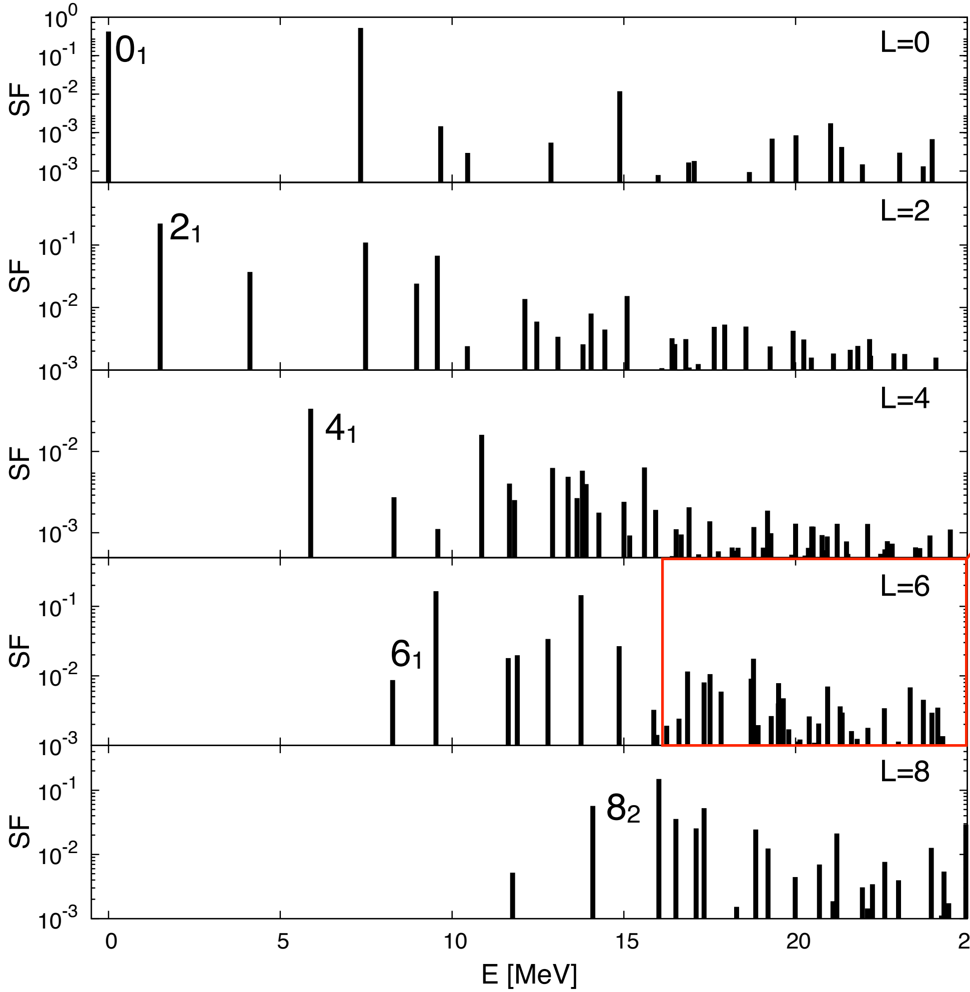
$$S_\ell^{(\mathrm{new})} \equiv \langle \psi_\ell | \psi_\ell \rangle = \int \rho^2 d\rho \, |\psi_\ell(\rho)|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)
S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.
T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

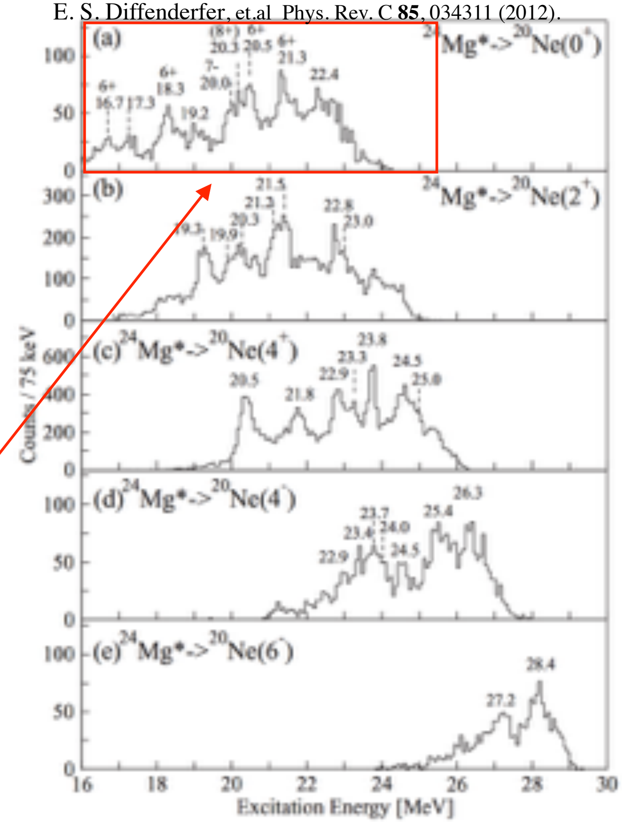
Alpha cluster spectroscopic factors in ^{24}Mg

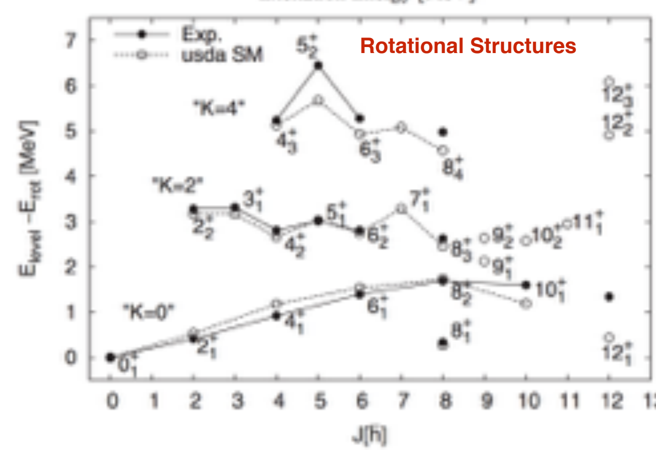
Theoretical calculations in SD shell



Experimental results

E. S. Diffenderfer, et.al Phys. Rev. C **85**, 034311 (2012).

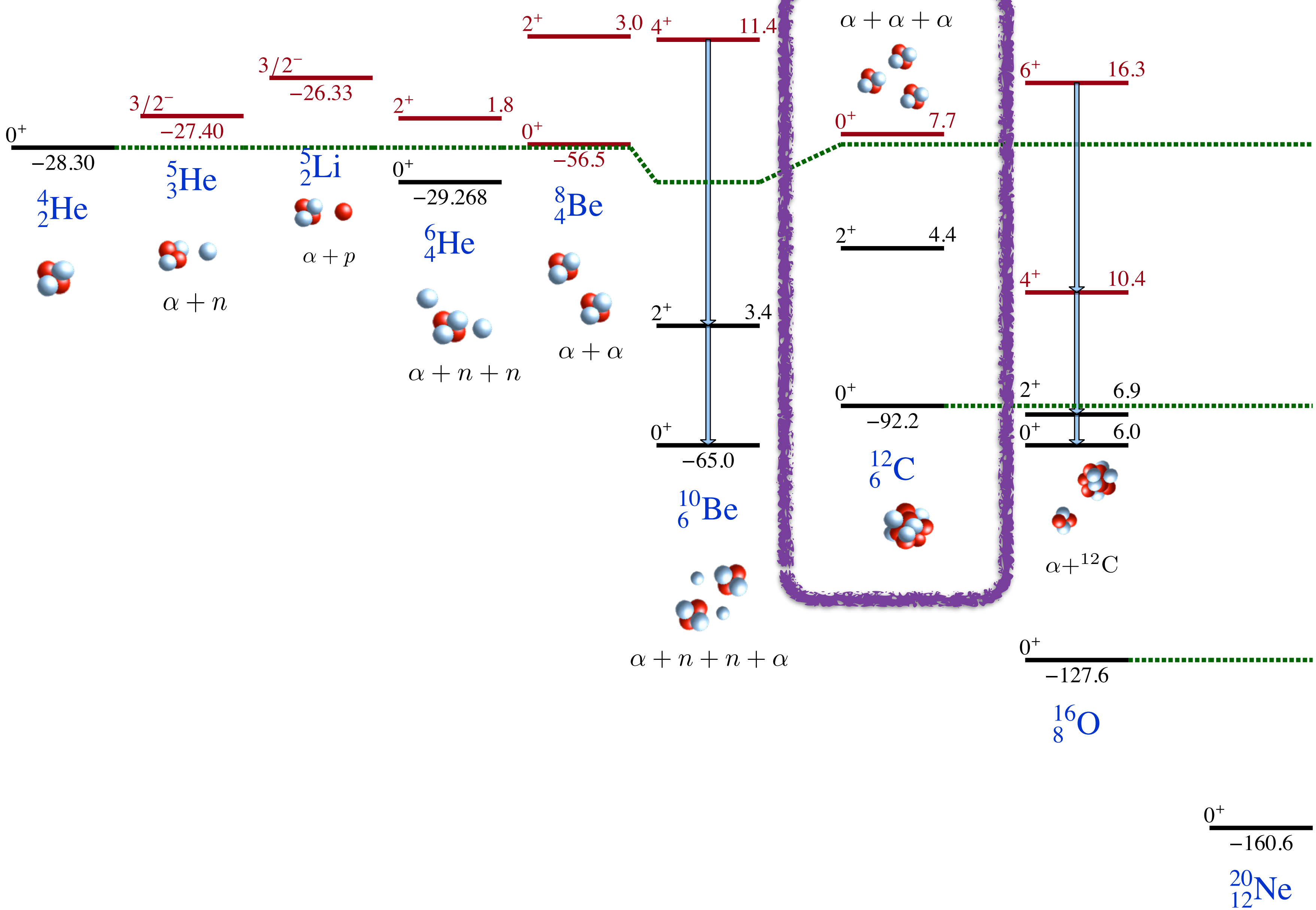


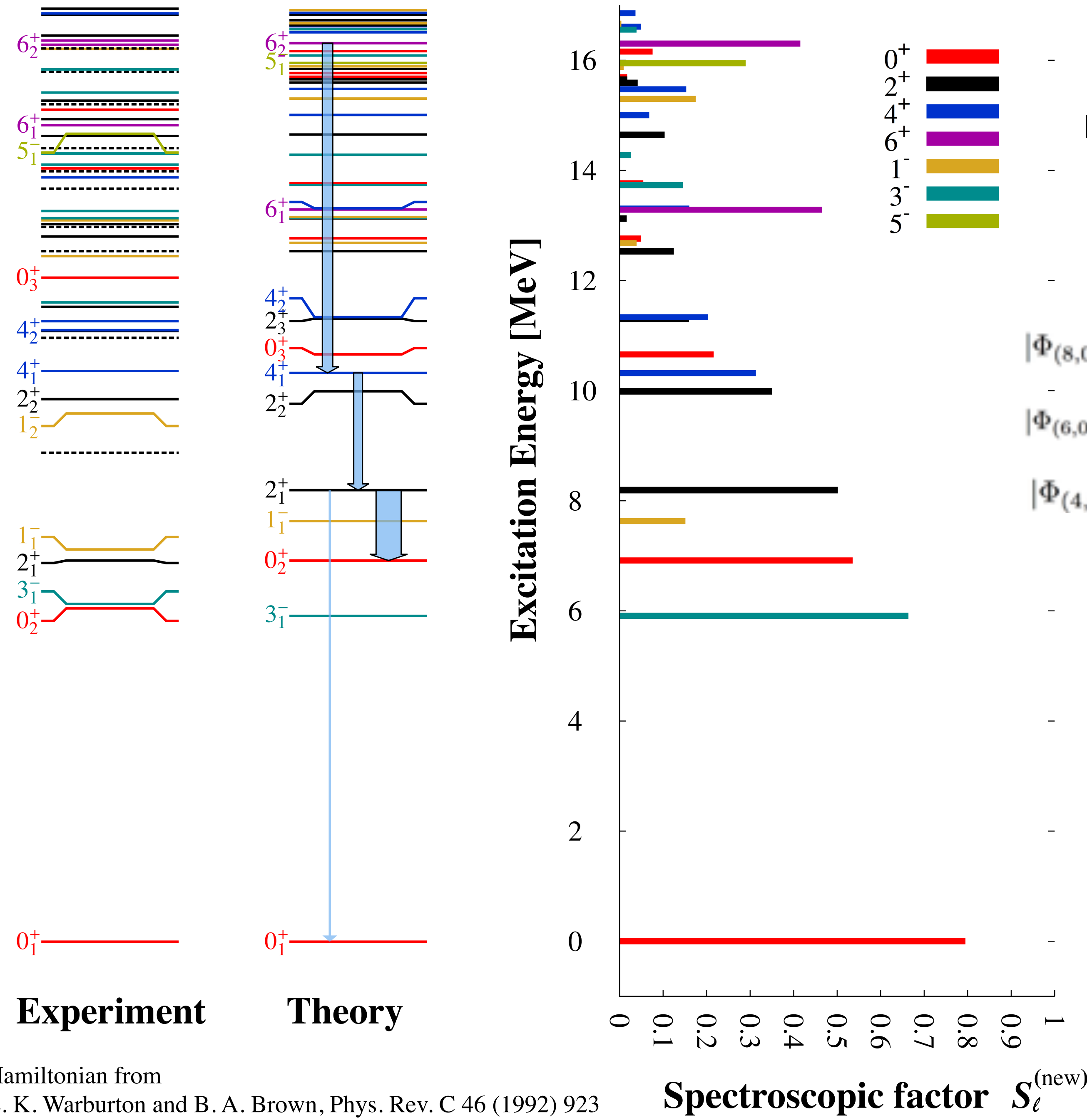


The sd valence space is considered with USDB interaction the operator is

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4]\,(8,0),:\,L\,S = T = 0\rangle$$

Clustering in light nuclei





p-sd shell model

SU(3) configurations

For positive parity

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4]\,(8,0),:LS=T=0\rangle$$

$$|\Phi_{(6,0):L}\rangle = |p^2(sd)^4[4]\,(6,0),:LS=T=0\rangle$$

$$|\Phi_{(4,0):L}\rangle = |p^4[4]\,(4,0),:LS=T=0\rangle$$

Hamiltonian from
E. K. Warburton and B. A. Brown, Phys. Rev. C 46 (1992) 923
Y. Utsuno and S. Chiba, Phys. Rev. C83 021301(R) (2011)

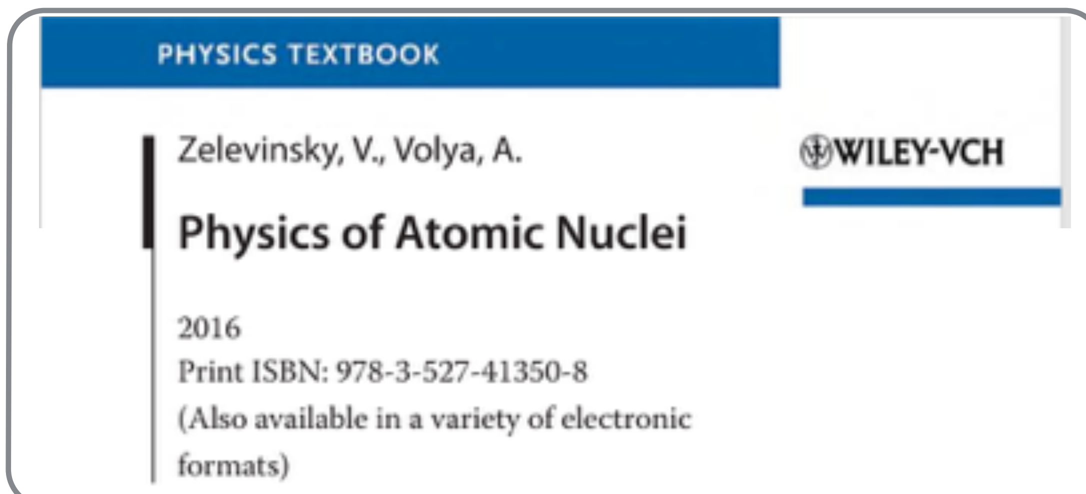
Atomic nucleus is an open quantum many-body system

- Nuclear physics as a cross-discipline science.
- From fundamental theory to applications.
- High performance computing

Support:

- GGI, school organizers
- U.S. Department of Energy DE-SC0009883
- Florida State University

Further reading:


PHYSICS TEXTBOOK

Zelevinsky, V., Volya, A.

WILEY-VCH

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