Nuclear Reaction Physics

Lectures at GGI FNHP 2016

Alexander Volya
Florida State University

Supported by:
GGI, FSU
US.Department of Energy

Resonant pole of the S-matrix

$k_p = \kappa - i\varkappa$ $\qquad S(k) \propto \frac{1}{k - k_p}$

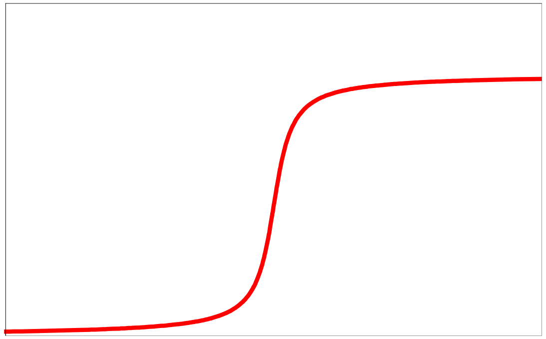
$-k_p^*$ is also a pole and $-k_p$ and k_p^* are both zeros

$$S(k) = e^{2i\eta} \frac{(k + k_p)(k - k_p^*)}{(k - k_p)(k + k_p^*)}$$

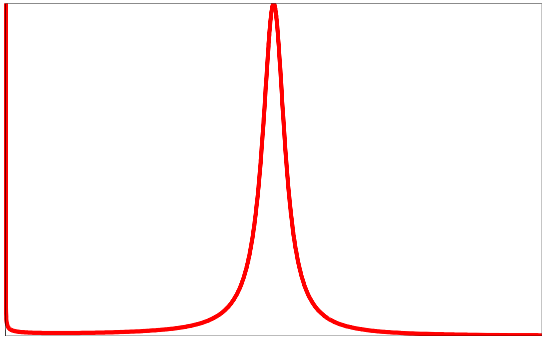
$$S(k) = e^{2i\eta} \frac{k^2 - 2ik\varkappa - |k_p|^2}{k^2 + 2ik\varkappa - |k_p|^2} = e^{2i\eta} \frac{E - E_p - \frac{i}{2}\Gamma_p}{E - E_p + \frac{i}{2}\Gamma_p}.$$

$\varkappa \ll \kappa$ $\qquad \mathcal{E}_p = E_p - \frac{i}{2}\Gamma_p = \frac{\hbar^2}{2\mu} k_p^2.$

Phase shift and Cross section



$$\delta(E) = \eta - \arctan\left[\frac{\Gamma_p/2}{E - E_p}\right]$$



$$\sigma_{BW} = \frac{\pi}{k^2}|S-1|^2 = \frac{\pi}{k^2}\frac{\Gamma^2}{(E-E_0)^2 + \frac{\Gamma^2}{4}}.$$

Cross section is maximum

$$S - 1 = (e^{2i\eta} - 1) + e^{2i\eta}\frac{-i\Gamma}{E - E_p + \frac{i}{2}\Gamma}$$

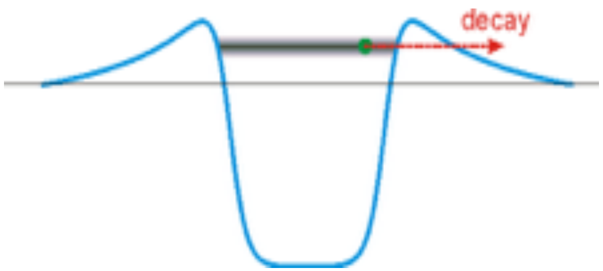
$$\sigma = \frac{4\pi}{k^2}\sin^2\eta + \sigma_{BW} - 4\Re\left(\frac{\Gamma e^{i\eta}\sin\eta}{E - E_0 + \frac{i}{2}\Gamma}\right)$$

Quantum mechanics of decay

Why exponential decay?

$$\frac{dN(t)}{dt} = -\Gamma N(t) \quad N(t) = N(0)\, e^{-\Gamma t}$$

Time evolution and decay in quantum mechanics

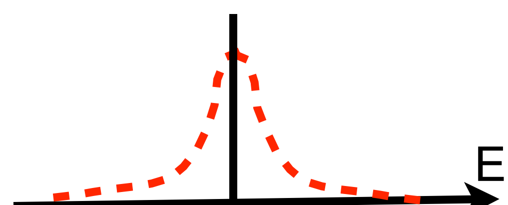


$$\psi(t) = e^{-iHt/\hbar}\psi(0)$$

Survival amplitude and probability

$$A(t) = \langle\psi(0)|\psi(t)\rangle = \langle\psi(0)|e^{-iHt/\hbar}|\psi(0)\rangle \qquad P(t) = |A(t)|^2$$

Resonance wave function



$$\psi_R(t) = \exp\left[-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t\right]\psi_R(0)$$

$$|\psi_R(t)|^2 = |\psi_R(0)|^2 \; e^{-\Gamma t}$$

$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}$$

Why and when decay cannot be exponential

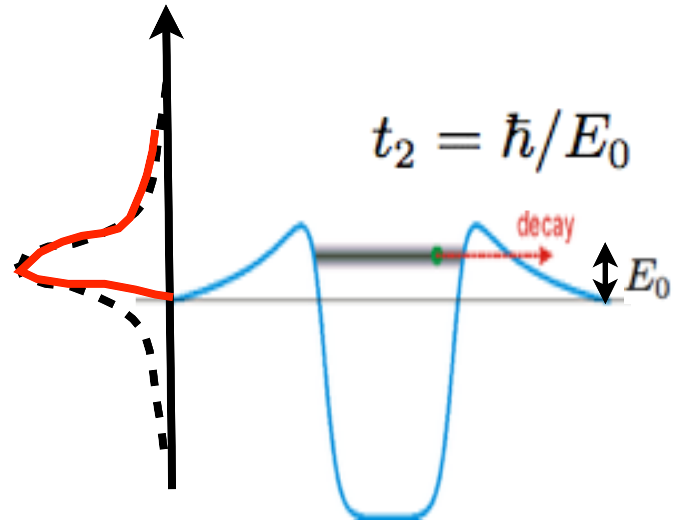
Initial state "memory" time $e^{-iHt/\hbar} \approx 1 - iHt/\hbar \ldots \quad t_1 = \hbar/(\Delta E) \quad t < t_1$

Internal motion in quasi-bound state

$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}$$

$$t < t_2$$

$$t_2 = \hbar/E_0$$



Remote power-law $t > t_3$

There are "free" slow-moving non-resonant particles, they escape slowly

$$N(t) \propto \frac{\Delta x}{vt} = \frac{\hbar}{mv^2 t} = \frac{2\hbar}{E_0 t} \propto |\psi_N(t)|^2 \qquad \Delta x = \frac{\hbar}{mv} \quad t_3 = \frac{\hbar}{\Gamma}\ln\left(\frac{E_0}{\Gamma}\right)$$

Example ^{14}C decay: E_0=0.157 MeV t_2=10^{-21} s $\quad \ln\left(\frac{E_0}{\Gamma}\right)$=73

Pole and non-resonant contribution

Decay amplitude

$$A(t) = \sum_{k_p} R_p e^{-ik_p^2 t} + A^{(NR)}(t)$$

$$\gamma_p = -2\Im(k_p^2)$$

First pole

$$k_1 = 2.75794 - i\,0.140433$$

Second pole

$$k_2 = 5.71348 - i\,0.370148$$

Non-resonant contribution

$$A_{nn'}^{(NR)}(t) = \frac{1+i}{\pi^{5/2}\sqrt{2}(1+G)^2 nn'} \frac{1}{t^{3/2}}$$