Perturbative approaches to the LSS in ΛCDM and beyond

Massimo Pietroni - INFN, Padova

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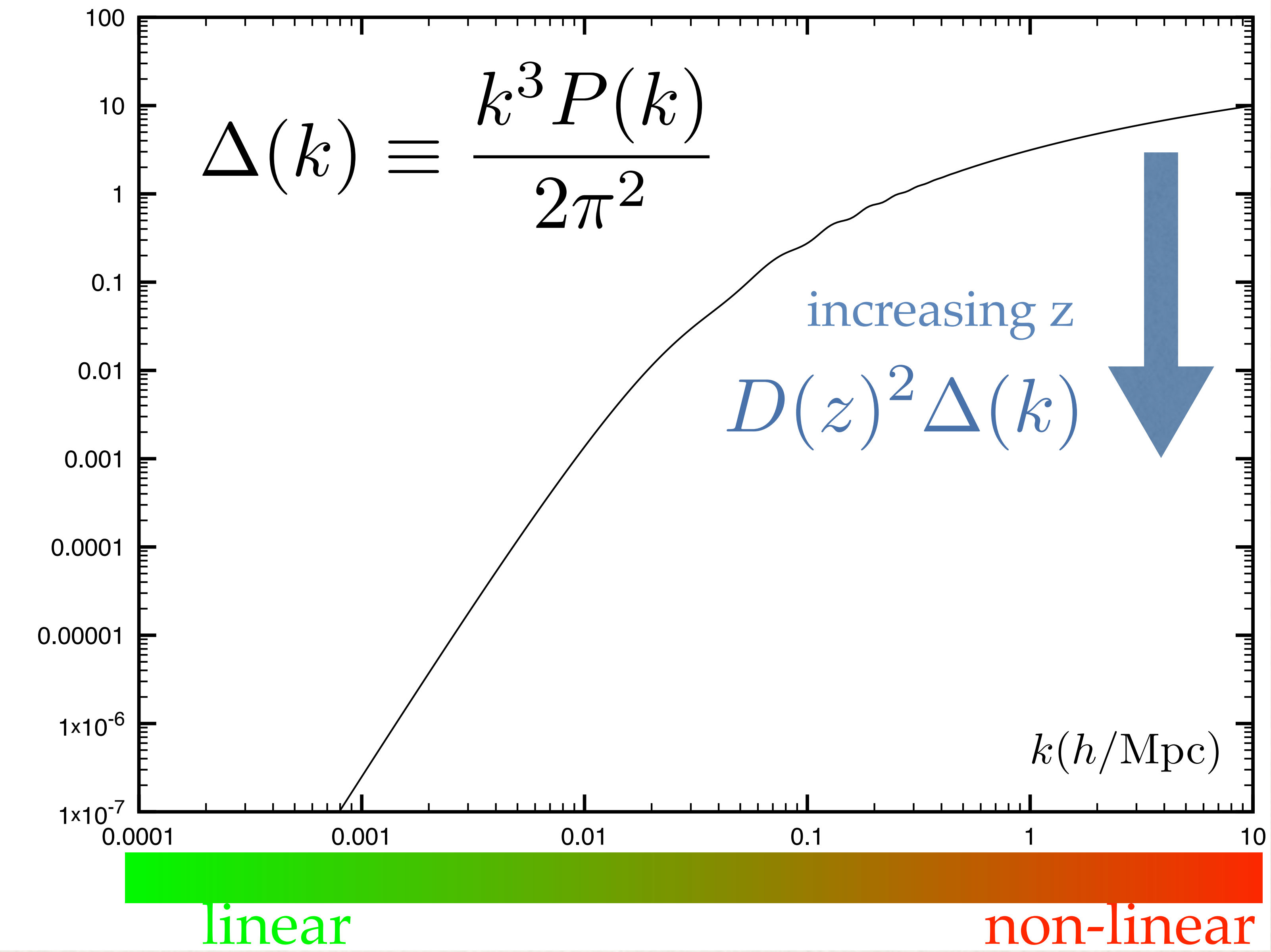
Outline

- IR effects on the nonlinear PS
- UV effects on the nonlinear PS
- Intermediate scales
- Putting all together: an improved TRG
- Scalar field (axion-like) DM

Linear and non-linear scales

linear Power Spectrum @z=0, ΛCDM



The nonlinear PS

$a, \cdots, d = 1$ density

$a, \cdots, d = 2$ velocity div.

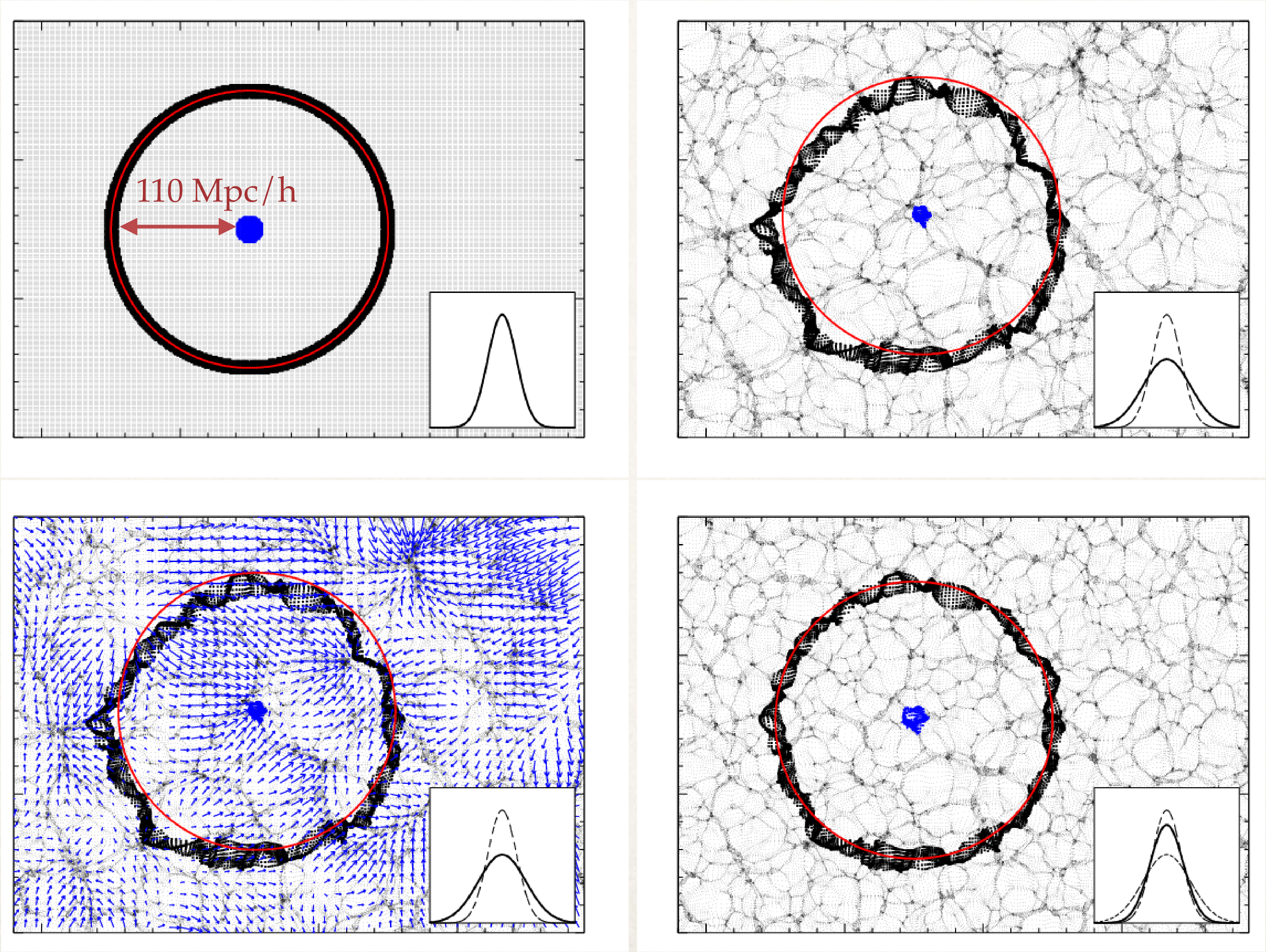
propagator $G_{ab}(k;z) = \left\langle \frac{\delta \varphi_a(\mathbf{k}, z)}{\delta \varphi_b(\mathbf{k}, z_{in})} \right\rangle' = \frac{\langle \varphi_a(\mathbf{k}, z) \varphi_b(-\mathbf{k}, z_{in}) \rangle'}{P^{lin}(k, z_{in})} + PNG$

$$P^{NL}_{ab}(k,z) = G_{ac}(k,z) G_{bd}(k,z) P^{lin}_{cd}(k,z) + P^{MC}_{ab}(k,z)$$

$P^{P}_{ab}(k;z)$ IR physics

intermediate and UV physics

Large scale flows and BAO's

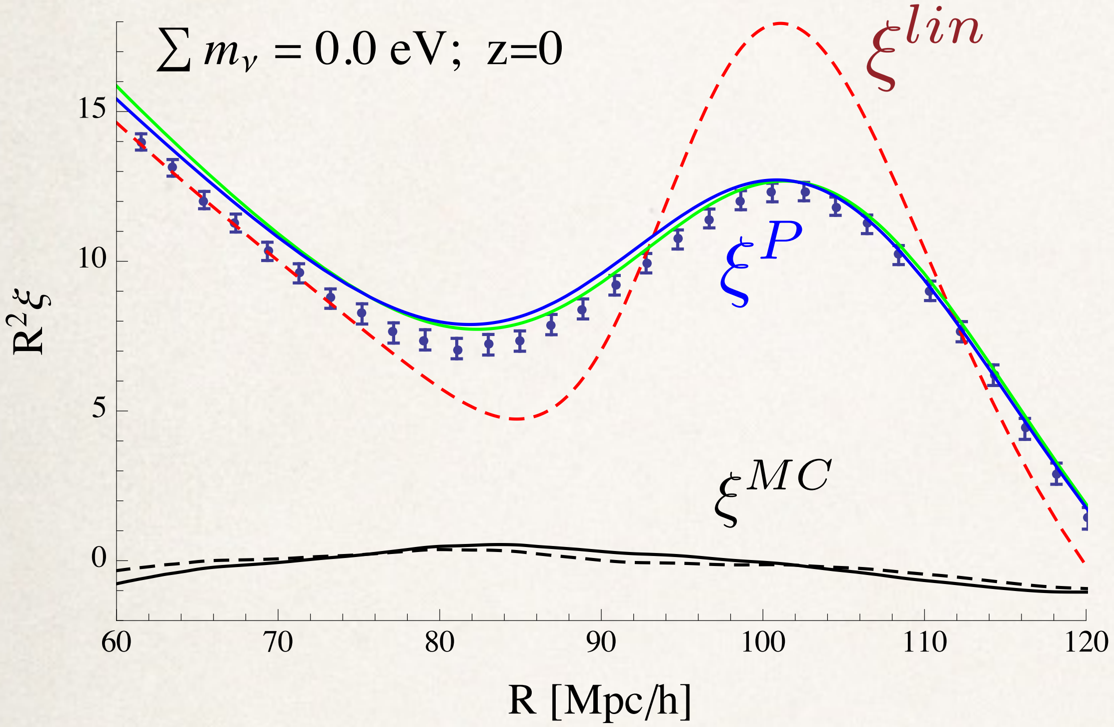


O(10 Mpc) displacements

reconstruction

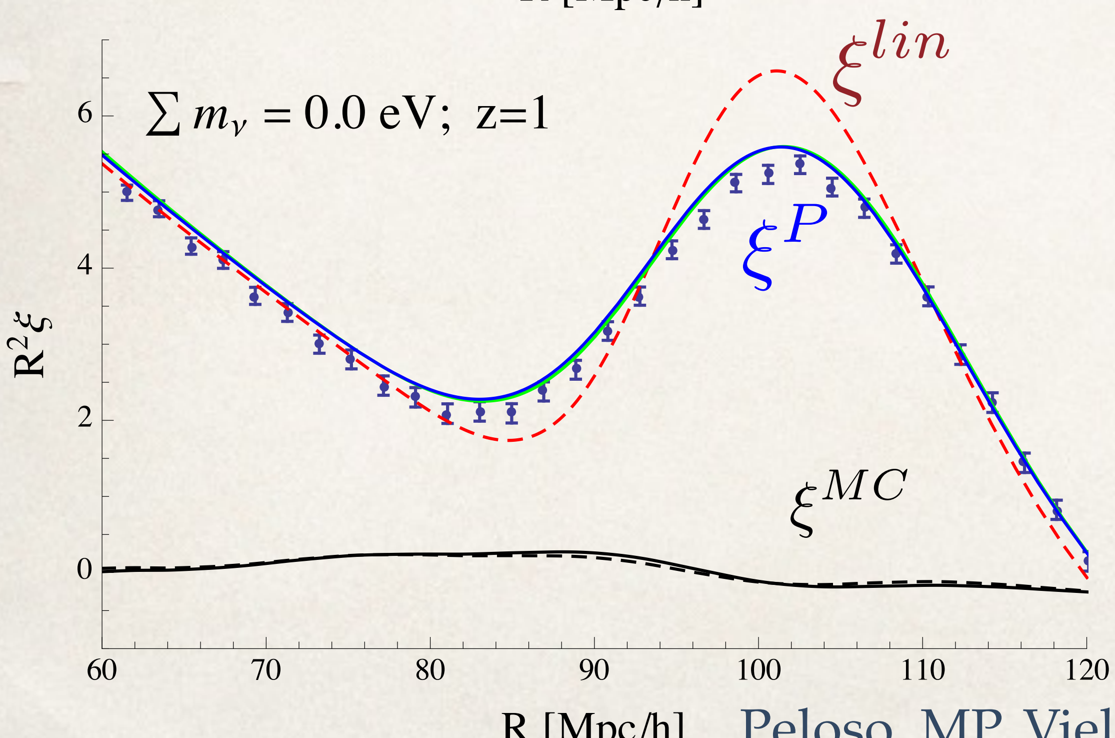
Padmanabhan et al 1202.0090

Effect on the Correlation Function



All the information on the BAO peak is contained in the propagator part

The widening of the peak can be reproduced by Zel'dovich approximation (and improvements of it)



The widening of the peak contains physical information (not a parameter to marginalize)

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

(simplified) Zel'dovich approximation

$$G^{Zeld}(k,z) = \mathrm{e}^{-\frac{k^2\sigma_v^2(z)}{2}}$$

$$P_{11}^{P}(k,z) = e^{-\frac{k^2\sigma_v^2(z)}{2}} P^{lin}(k;z)$$

$$\sigma_v^2(z) = \frac{1}{3}\int \frac{d^3q}{(2\pi)^3}\,\frac{P^{lin}(q,z)}{q^2}$$

linear velocity dispersion:
contains information on linear PS, growth factor,…

$$\delta\xi(R) = \frac{1}{2\pi^2}\int dq\, q^2\, \delta P^{lin}(q)\left(\frac{\sin(qR)}{qR}\, e^{-q^2\sigma_v^2} - \frac{1}{3}\frac{\xi_2(R)}{q^2R^2}\right)$$

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

How to include Bulk Motions

$$\bar{\delta}_\alpha(\mathbf{x},\tau) = \delta_\alpha(\mathbf{x} - \mathbf{D}_\alpha(\mathbf{x},\tau),\tau)$$

$$\mathbf{D}_\alpha(\mathbf{x},\tau) \equiv \int_{\tau_{in}}^{\tau} d\tau' \mathbf{v}_{\alpha,\mathrm{long}}(\mathbf{x},\tau') \simeq \mathbf{D}_\alpha(\tau)$$

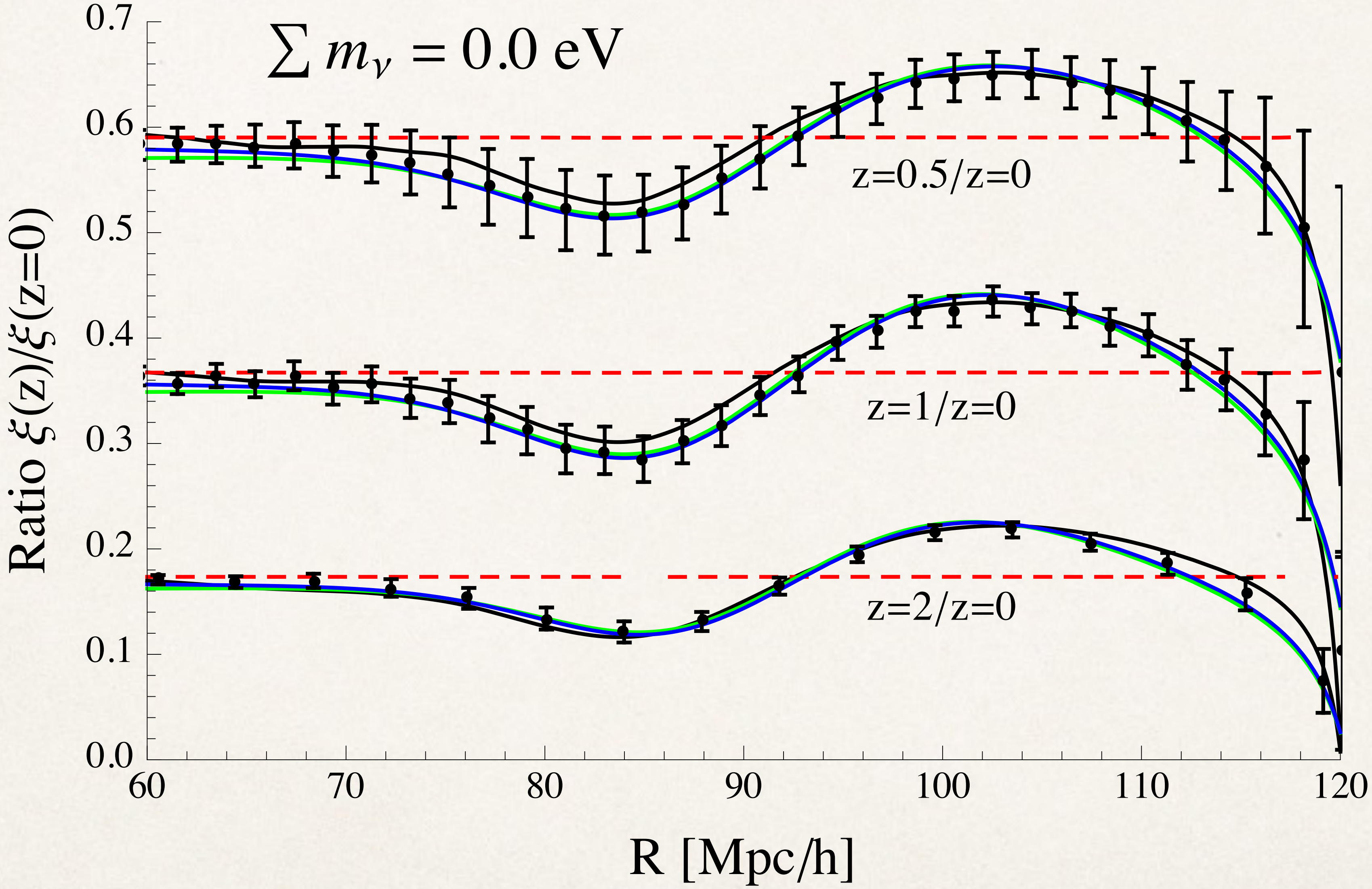
$$\langle \delta_\alpha(\mathbf{k},\tau)\delta_\alpha(\mathbf{k}',\tau')\rangle = \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle \langle e^{-i\mathbf{k}\cdot(\mathbf{D}_\alpha(\tau)-\mathbf{D}_\alpha(\tau'))}\rangle$$

$$= \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle e^{\frac{-k^2\sigma_v^2(D(\tau)-D(\tau'))^2}{2}}$$

$$\sigma_v^2 = -\frac{1}{3\mathcal{H}^2 f^2}\int^{\Lambda} d^3q \langle v^i_{long}(q) v^i_{long}(q)\rangle' = \frac{1}{3}\int^{\Lambda} d^3q \frac{P^0(q)}{q^2}$$

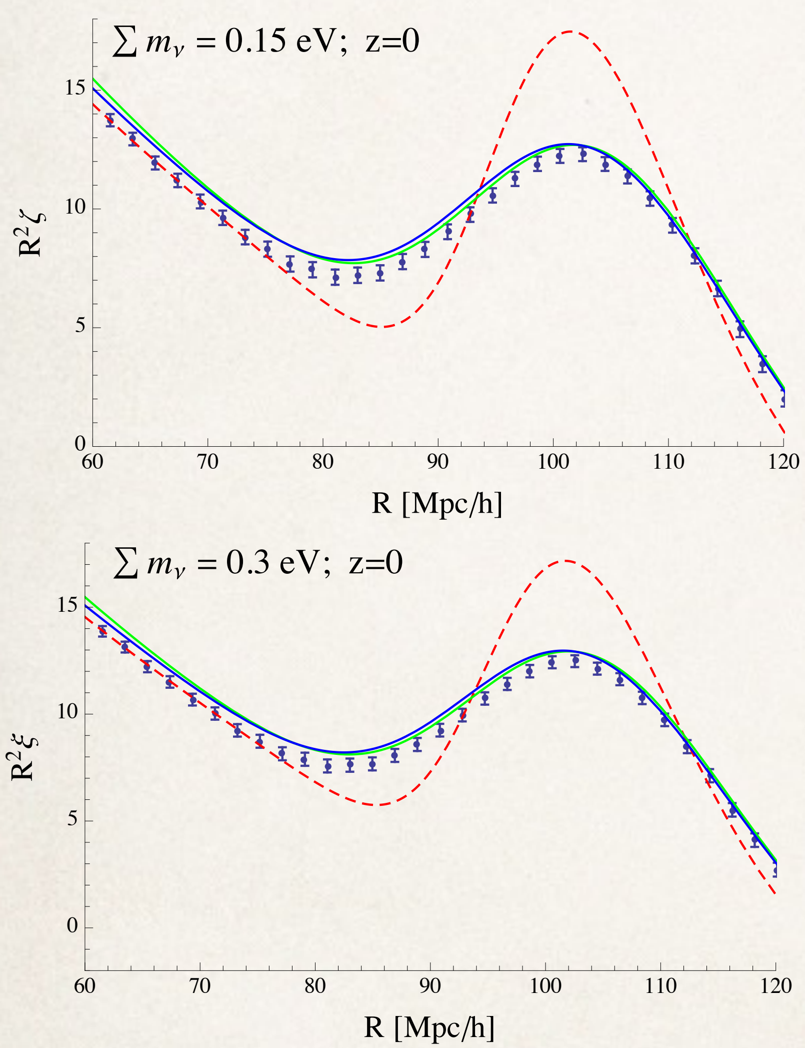
Resummations (~Zel'dovich)
take into account the large scale bulk motions

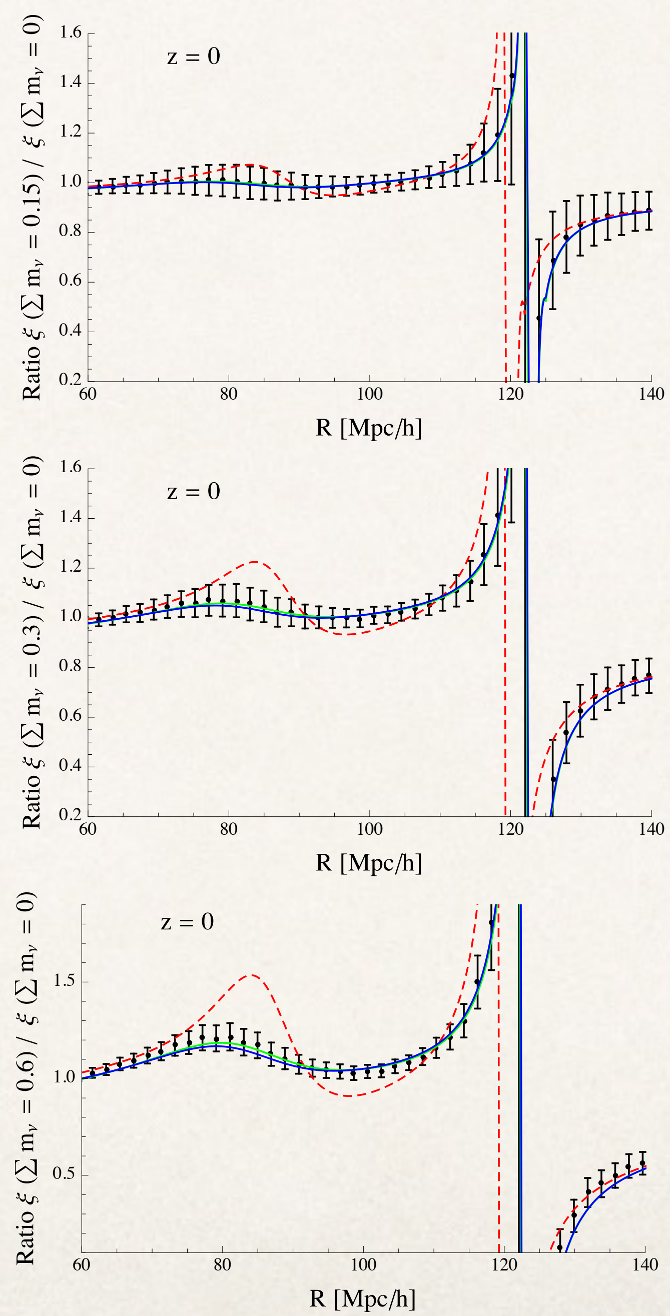
Redshift ratios



Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Massive neutrinos





$$P^P_{11}(k,z) = e^{-\frac{k^2\sigma_v^2(z)}{2}} P^{lin}(k;z)$$

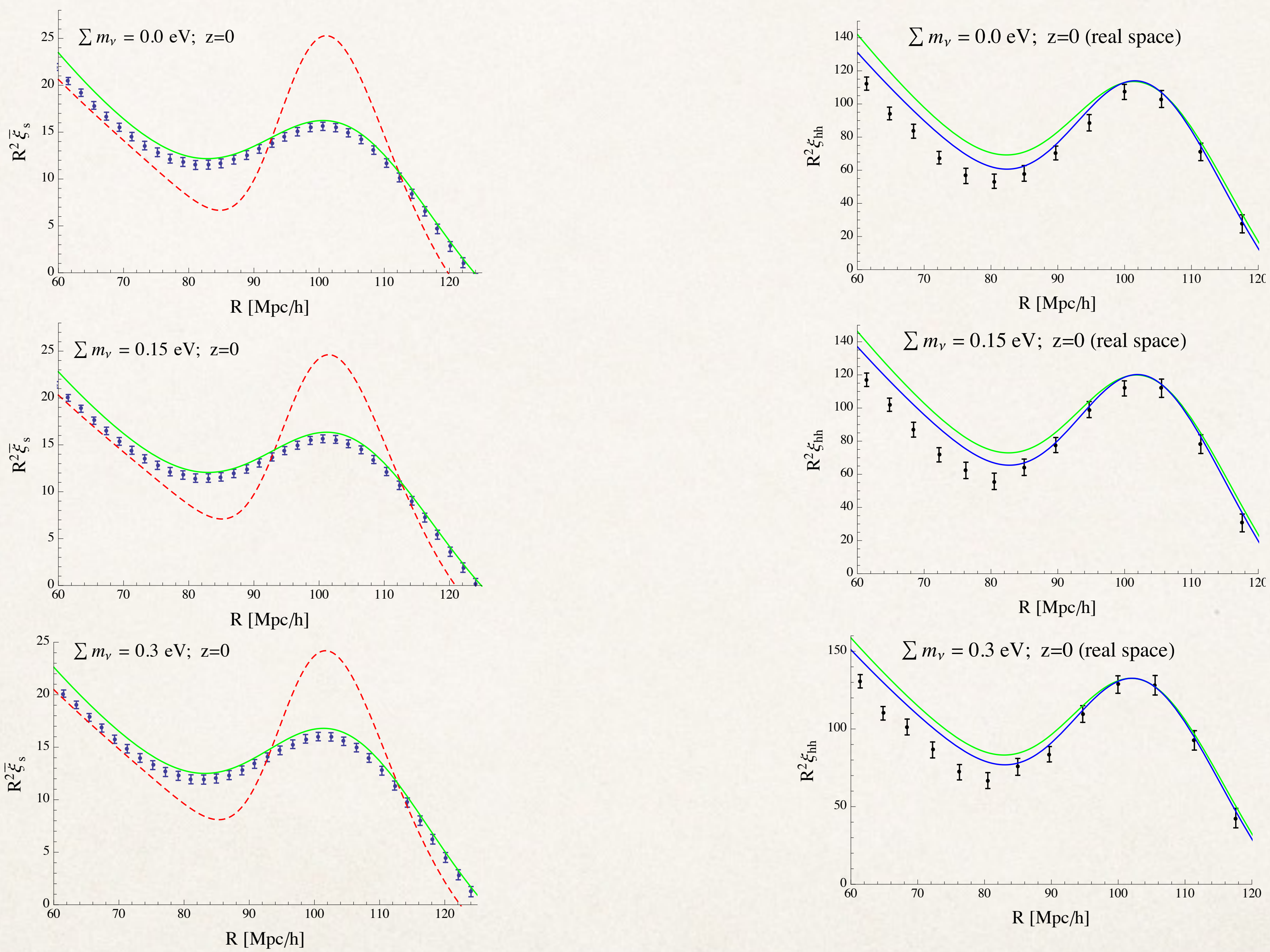
increasing neutrino masses, Plin decreases, but also damping decreases.

$$\sum m_\nu = 0.15 \quad \text{eV} \qquad \downarrow 0.6\%$$

$$\sum m_\nu = 0.3 \quad \text{eV} \qquad \uparrow 1.2\%$$

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

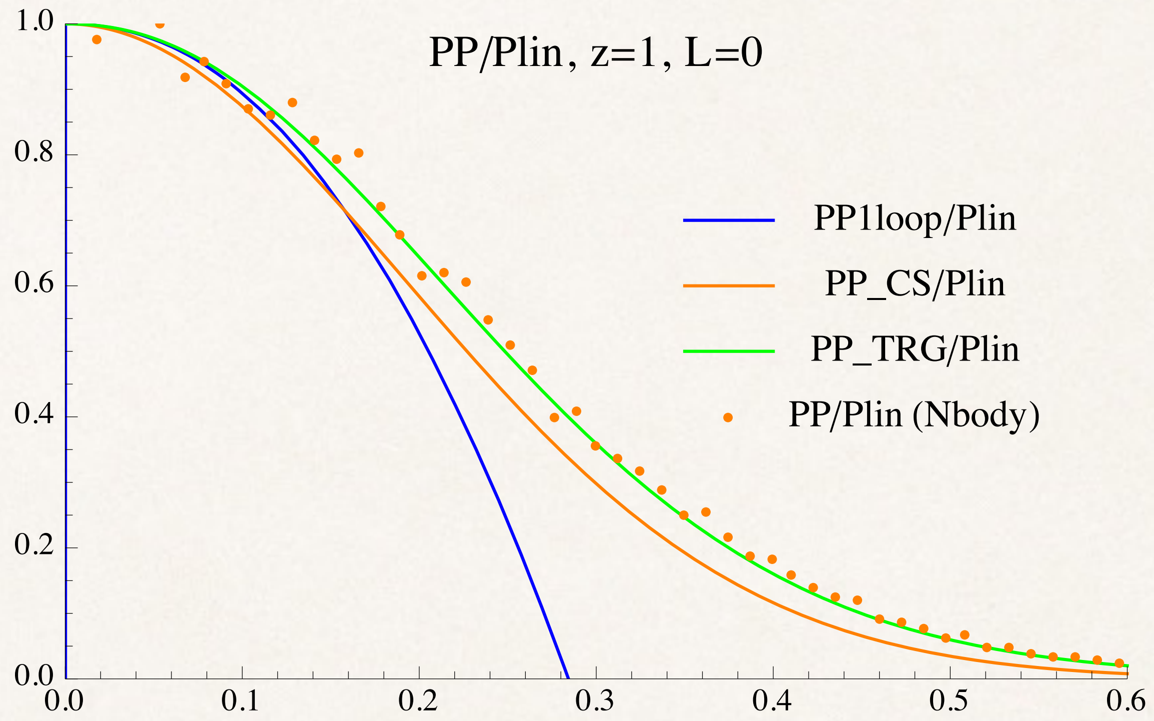
Massive neutrinos



Redshift space　　　　Halos

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Improving over Zel'dovich



$$\partial_\eta P^P_{ab}(k;\eta,\eta) = -\Omega_{ac} P^P_{cb}(k;\eta,\eta) - \Omega_{bc} P^P_{ac}(k;\eta,\eta)$$
$$+ \int_{\eta_{in}} ds \; \left[\Sigma_{ac}(k;\eta,s) P^P_{cb}(k;s,\eta) + \Sigma_{bc}(k;\eta,s) P^P_{ac}(k;\eta,s)\right]$$

Exact equation

$$\Sigma_{ab}(k;\eta,s) \to \Sigma^{1-loop}_{ab}(k;\eta,s) \qquad \text{for} \qquad k \to 0$$
$$\Sigma_{ab}(k;\eta,s) \to -k^2 \sigma_v^2(z) e^{\eta+s} g_{ab}(\eta;s) \qquad \text{for} \qquad k \to \infty$$

Anselmi, Matarrese, MP, 1011.4477

Peloso, MP, Viel, Villaescusa-Navarro, in preparation

Mode coupling-Response functions

The nonlinear PS is a functional of the initial one
(in a given cosmology and assuming no PNG):

SPT is an expansion around $P^0(q) = 0$

$$P_{ab}[P^0](\mathbf{k};\eta) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \; \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)} \right|_{P^0=0} P^0(\mathbf{q}_1)\cdots P^0(\mathbf{q}_n)$$

n=1 linear order (= "0-loop")
n=2 "1-loop"
…

Mode coupling-Response functions

Let's instead expand around a reference PS: $P^0(q) = \bar{P}^0(q)$

$$
\begin{aligned}
P_{ab}[P^0](\mathbf{k};\eta) &= P_{ab}[\bar{P}^0](\mathbf{k};\eta) \\
&\quad + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \; \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots \delta P^0(\mathbf{q}_n)} \right|_{P^0=\bar{P}^0} \delta P^0(\mathbf{q}_1)\cdots \delta P^0(\mathbf{q}_n)\,, \\
&= P_{ab}[\bar{P}^0](\mathbf{k};\eta) + \int \frac{dq}{q}\, K_{ab}(k,q;\eta)\, \delta P^0(q) + \cdots\,,
\end{aligned}
$$

$$\delta P^0(\mathbf{q}) \equiv P^0(\mathbf{q}) - \bar{P}^0(\mathbf{q})$$

Linear response function: $K_{ab}(k,q;\eta) \equiv q^3 \int d\Omega_{\mathbf{q}} \left. \frac{\delta P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q})} \right|_{P^0=\bar{P}^0}$

Non-perturbative (gets contributions from all SPT orders)

Key object for more efficient interpolators ?

UV screening

Sensitivity of the nonlinear PS at scale k on a change of the initial PS at scale q:

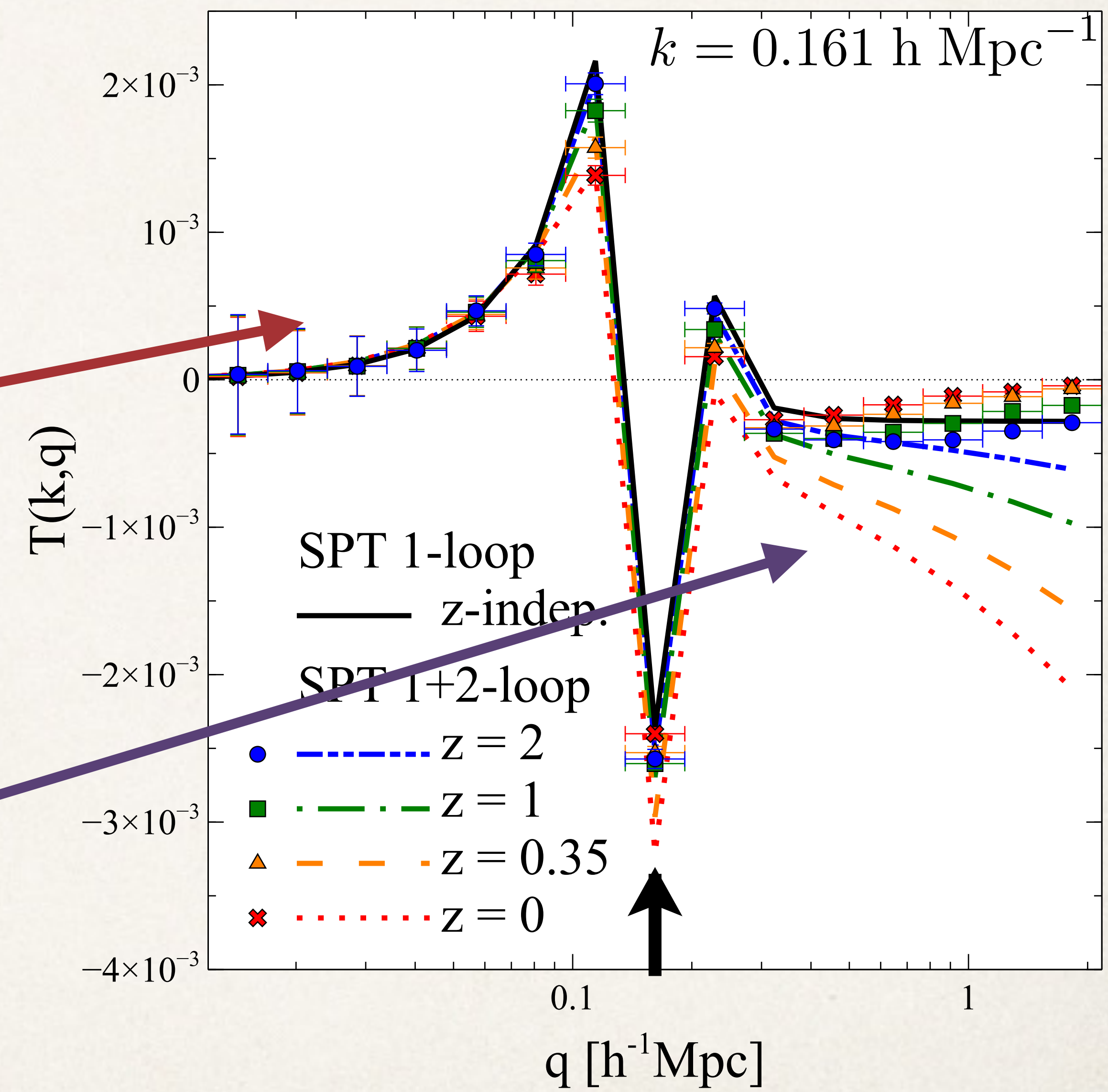
$$K(k, q; z) = q \frac{\delta P^{\rm nl}(k; z)}{\delta P^{\rm lin}(q; z)}$$

IR: "Galilean invariance"

$K(k, q; z) \sim q^3$

Peloso, MP 1302.0223

PT overpredicts the effect of UV scales on intermediate ones

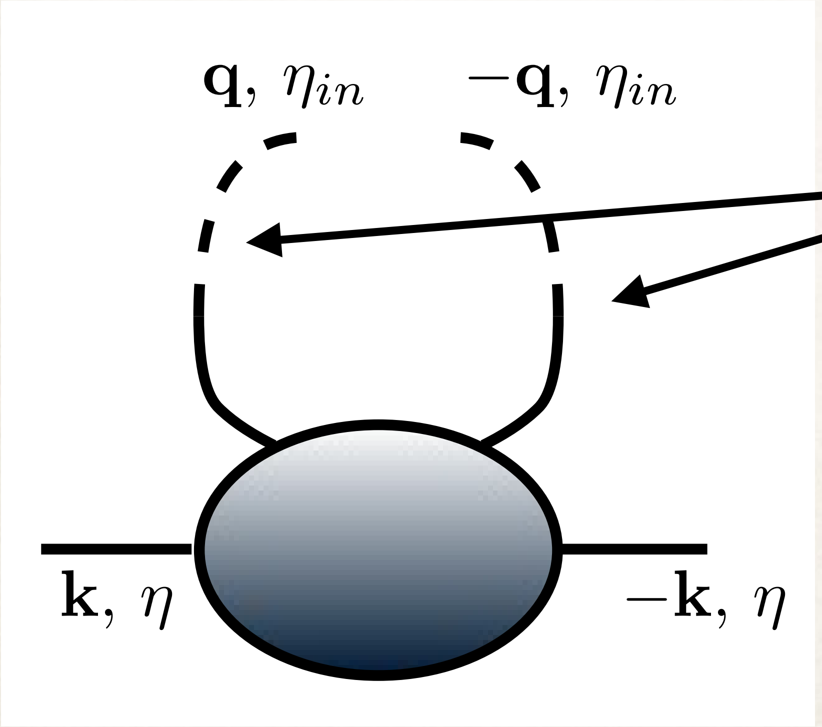


Nishimichi et al 1411.2970

UV screening

The effect of virialized structures on larger scales is screened
(Peebles ’80, Baumann et al 1004.2488, Blas et al 1408.2995).

However, the departure from the PT predictions starts at small k’s:
is it really a virialization effect?



$$e^{-\frac{q^2\sigma_v^2}{2}}$$

damped propagators!
(compare SPT: g=O(1))

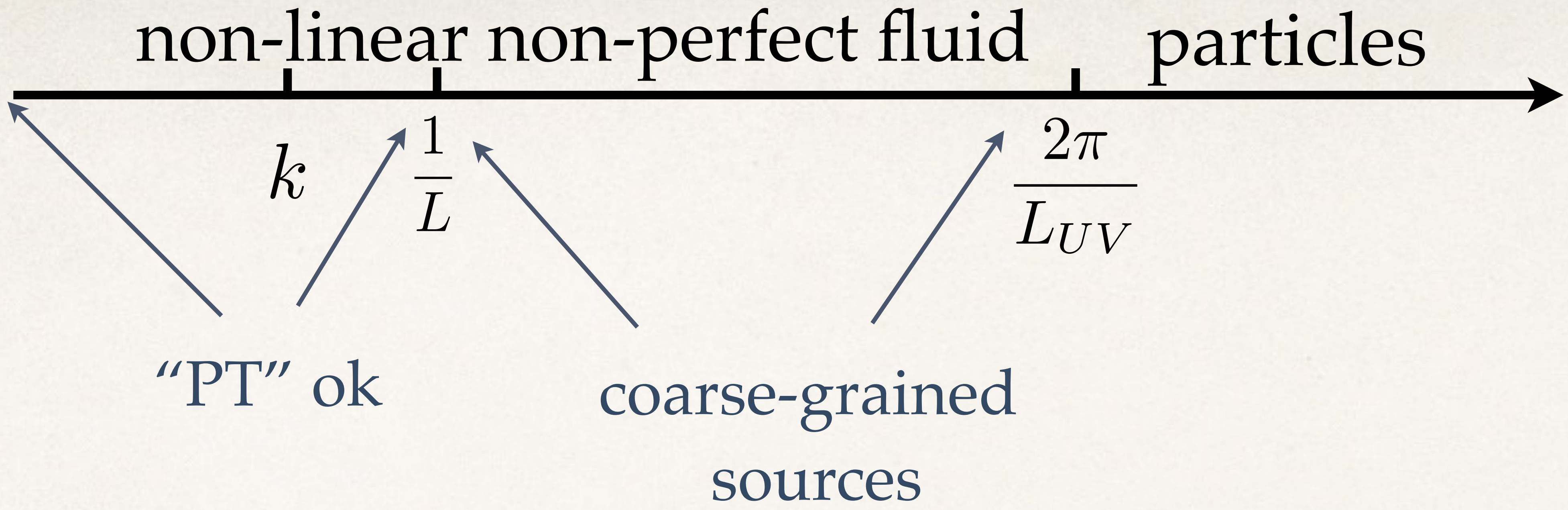
memory of initial substructures is largely lost

UV lessons

- SPT fails when loop momenta become too high ($q \gtrsim 0.4$ h/Mpc)
- The real response to modifications in the UV regime is mild
- Most of the cosmology dependence is on intermediate scales

Effective approaches to the UV

- General idea: take the UV physics from N-body simulations and use (resummed) PT only for the large and intermediate scales



Physics at k is independent on L, L_uv ("Wilsonian approach")

Expansion in sources:

$$\langle \delta\delta \rangle_J = \langle \delta\delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta J J \delta \rangle_{J=0} + \cdots$$

computed in PT with cutoff at 1/L

measured from simulations

Vlasov Equation

Liouville theorem+ neglect non-gravitational interactions:

$$\frac{d}{d\tau} f_{mic} = \left[\frac{\partial}{\partial \tau} + \frac{p^i}{am} \frac{\partial}{\partial x^i} - am \nabla_x^i \phi(\mathbf{x}, \tau) \right] f_{mic}(\mathbf{x}, \mathbf{p}, \tau) = 0$$

moments:

$$n_{mic}(\mathbf{x}, \tau) = \int d^3p f_{mic}(\mathbf{x}, \mathbf{p}, \tau)$$ density

$$\mathbf{v}_{mic}(\mathbf{x}, \tau) = \frac{1}{n_{mic}(\mathbf{x}, \tau)} \int d^3p \, \frac{\mathbf{p}}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau)$$ velocity

$$\sigma^{ij}_{mic}(\mathbf{x}, \tau) = \frac{1}{n_{mic}(\mathbf{x}, \tau)} \int d^3p \, \frac{p^i}{am} \frac{p^j}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau) - v^i_{mic}(\mathbf{x}, \tau) v^j_{mic}(\mathbf{x}, \tau)$$ velocity dispersion

…

From particles to fluids

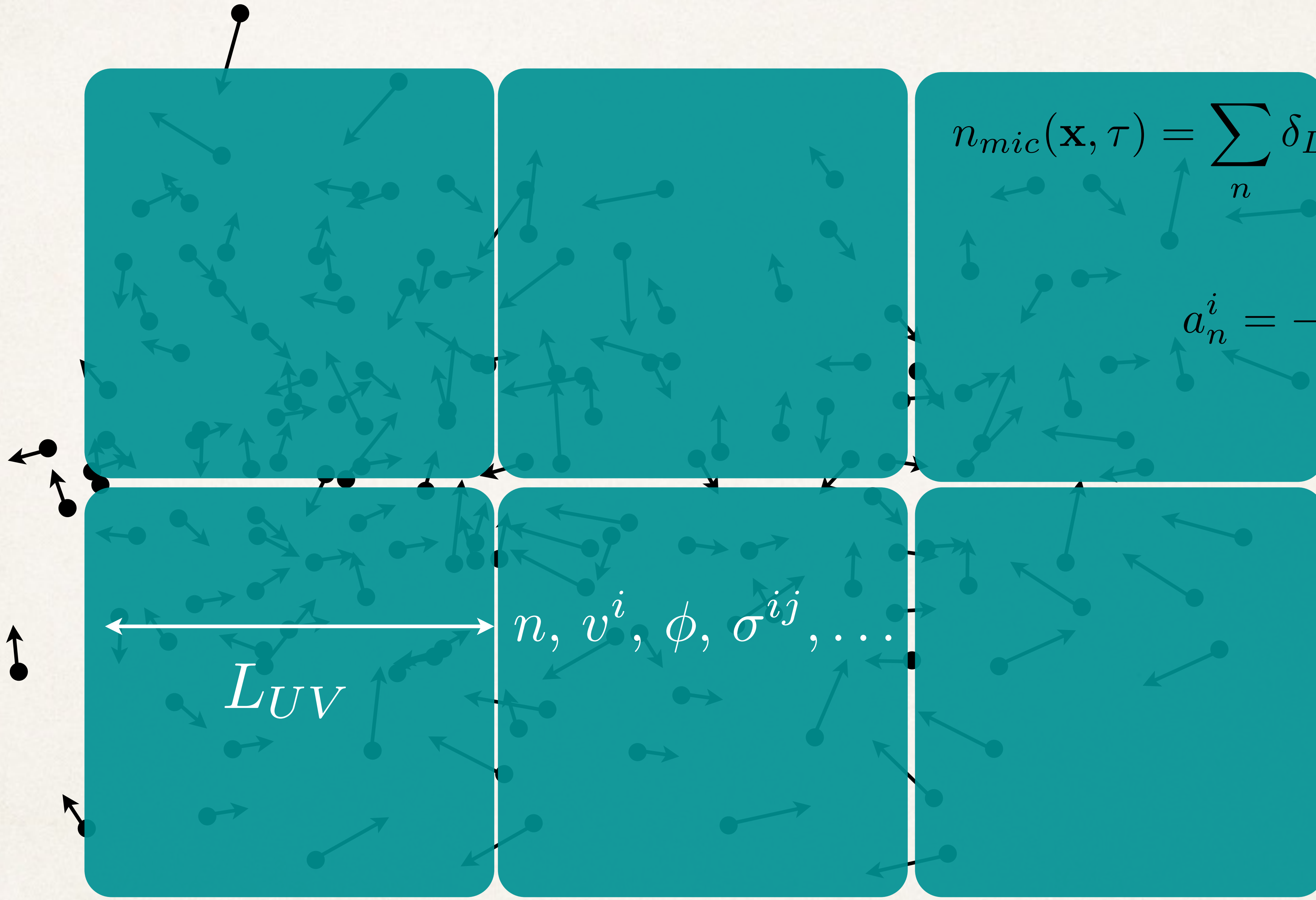
Buchert, Dominguez, ’05, Pueblas Scoccimarro, ’09, Baumann et al. ’10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 ....

$$n_{mic}(\mathbf{x},\tau) = \sum_n \delta_D(\mathbf{x}-\mathbf{x}_n(\tau)),$$

$$v_n^i = \dot{x}_n(\tau),$$

$$a_n^i = -\nabla_x^i \phi_{mic}(\mathbf{x},\tau)$$



$$f_{mic}(x,p,\tau) = \sum_n \delta_D(x - x_n(\tau))\,\delta_D(p - p_n(\tau))$$

$$f(x,p,\tau) = \frac{1}{V}\int d^3y\, W(y/L_{UV})\, f_{mic}(x+y,p,\tau)$$

Satisfies the “Vlasov eq.”

Coarse-grained Vlasov equation

large scales

$$\left[\frac{\partial}{\partial\tau} + \frac{p^i}{am}\frac{\partial}{\partial x^i} - am\nabla_x^i\phi(\mathbf{x},\tau)\frac{\partial}{\partial p^i}\right] f(\mathbf{x},\mathbf{p},\tau) =$$
$$am\left[\langle\frac{\partial}{\partial p^i} f_{mic}\nabla^i\phi_{mic}\rangle_{L_{UV}}(\mathbf{x},\mathbf{p},\tau) - \frac{\partial}{\partial p^i} f(\mathbf{x},\mathbf{p},\tau)\nabla_x^i\phi(\mathbf{x},\tau)\right]$$

short scales

$$\langle g\rangle_{L_{UV}}(\mathbf{x}) \equiv \frac{1}{V_{UV}}\int d^3y\,\mathcal{W}(y/L_{UV})g(\mathbf{x}+\mathbf{y})$$

$$\phi = \langle\phi_{mic}\rangle_{L_{UV}}$$

$$f = \langle f_{mic}\rangle_{L_{UV}}$$

Vlasov equation in the L_uv → 0 limit!

Taking moments…

Exact large scale dynamics for density and velocity fields

$$\frac{\partial}{\partial \tau}\delta(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta(\mathbf{x}))v^i(\mathbf{x})\right] = 0$$

$$\frac{\partial}{\partial \tau}v^i(\mathbf{x}) + \mathcal{H}v^i(\mathbf{x}) + v^k(\mathbf{x})\frac{\partial}{\partial x^k}v^i(\mathbf{x}) = -\nabla^i_x\phi(\mathbf{x}) - \underline{J^i_\sigma(\mathbf{x}) - J^i_1(\mathbf{x})}$$

$$\nabla^2\phi(\mathbf{x}) = \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{x})$$

$$n(\mathbf{x}) = n_0(1+\delta(\mathbf{x})) = n_0(1+\langle\delta_{mic}\rangle(\mathbf{x}))$$

$$v^i(\mathbf{x}) = \frac{\langle(1+\delta_{mic})v^i_{mic}\rangle(\mathbf{x})}{1+\delta(\mathbf{x})}$$

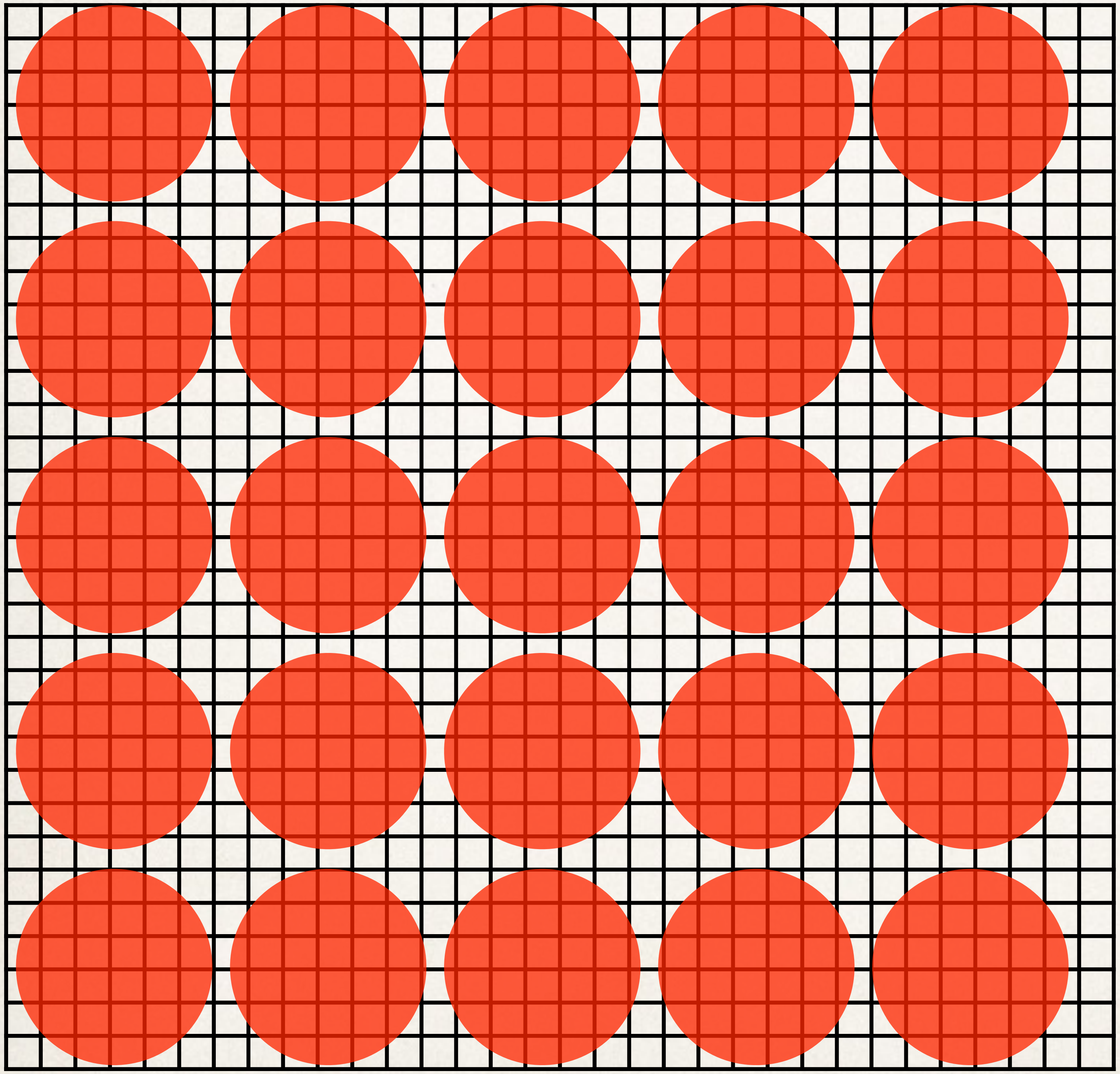
external input on UV-physics needed

$$J^i_\sigma(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\frac{\partial}{\partial x^k}(n(\mathbf{x})\sigma^{ki}(\mathbf{x}))$$

$$J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\left(\langle n_{mic}\nabla^i\phi_{mic}\rangle(\mathbf{x}) - n(\mathbf{x})\nabla^i\phi(\mathbf{x})\right)$$

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP,
Villaescusa-Navarro, Viel, 1407.1342



$$L_{UV} = 1,\, 2,\, 4\ \mathrm{Mpc/h}$$

$$L_{UV}:\ \delta,\, v^i,\, J_1^i\,,\, J_\sigma^i$$

$$L:\ \bar{\delta},\, \bar{v}^i,\, \bar{J}_1^i,\, \bar{J}_\sigma^i$$

$$\mathcal{W}(R/L) = \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{L^3}\, e^{-\frac{R^2}{2L^2}}$$

$$L_{box} = 512\,\mathrm{Mpc/h} \qquad N_{particles} = (512)^3$$

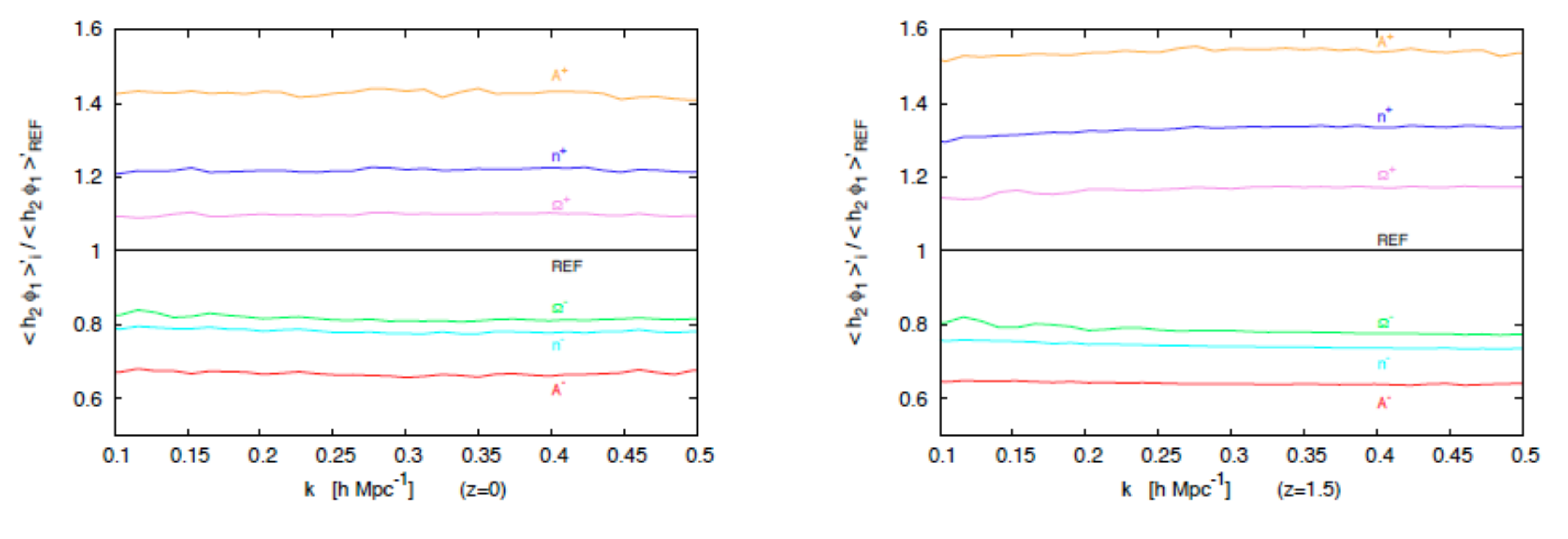
COSMOLOGY DEPENDENCE

Simulation Suite

Name	Ω_m	Ω_b	Ω_Λ	h	n_s	$A_s\,[10^{-9}]$
REF	0.271	0.045	0.729	0.703	0.966	2.42
A_s^-	0.271	0.045	0.729	0.703	0.966	1.95
A_s^+	0.271	0.045	0.729	0.703	0.966	3.0
n_s^-	0.271	0.045	0.729	0.703	0.932	2.42
n_s^+	0.271	0.045	0.729	0.703	1.000	2.42
Ω_m^-	0.247	0.045	0.753	0.703	0.966	2.42
Ω_m^+	0.289	0.045	0.711	0.703	0.966	2.42

$L_{box} = 512\,\mathrm{Mpc/h}$ $\qquad$ $N_{particles} = (512)^3$

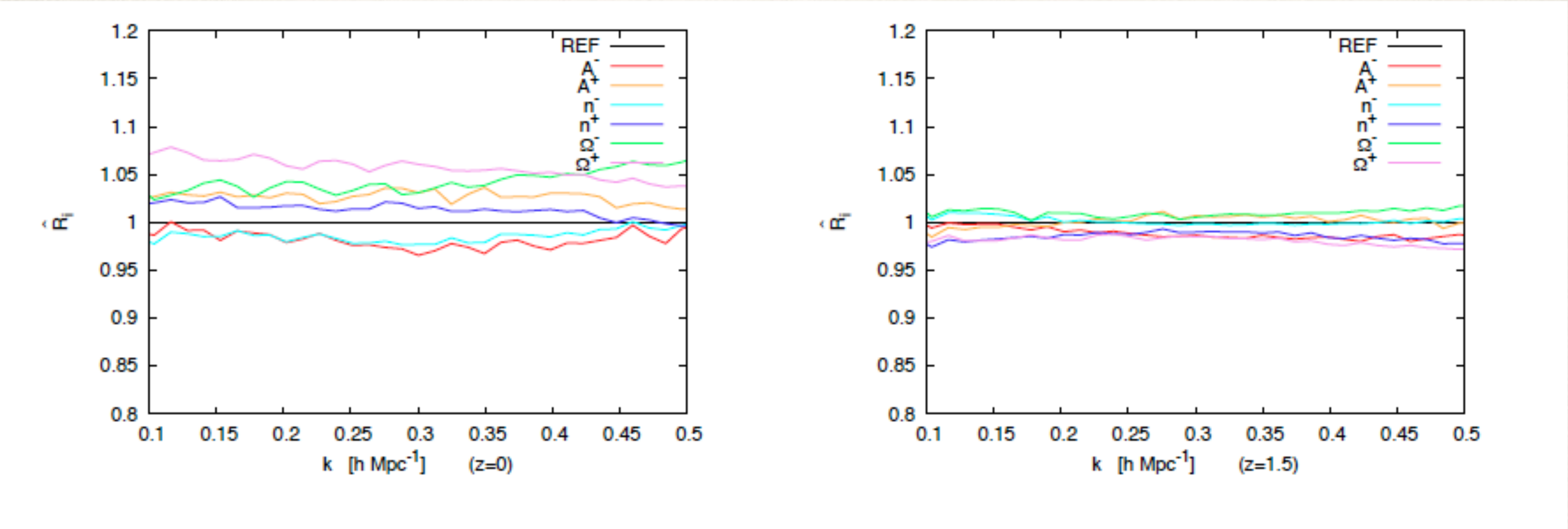
Ratios of UV source correlators



$$\frac{\langle J\delta\rangle_i}{\langle J\delta\rangle_{REF}}$$ From N-body

Scale-independent!!

Rescale using PT information



Amplitude rescaling captured by PT!!

Relation with EFToLSS

Baumann et al 1004.2488
Carrasco et al 1206.2926
...

$$\dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0 \ ,$$

$$\dot{v}_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a}\partial_i \phi_l = -\frac{1}{a\rho_l}\partial_j \left[\tau^{ij}\right]_\Lambda \ .$$

$J_1^i + J_\sigma^i$

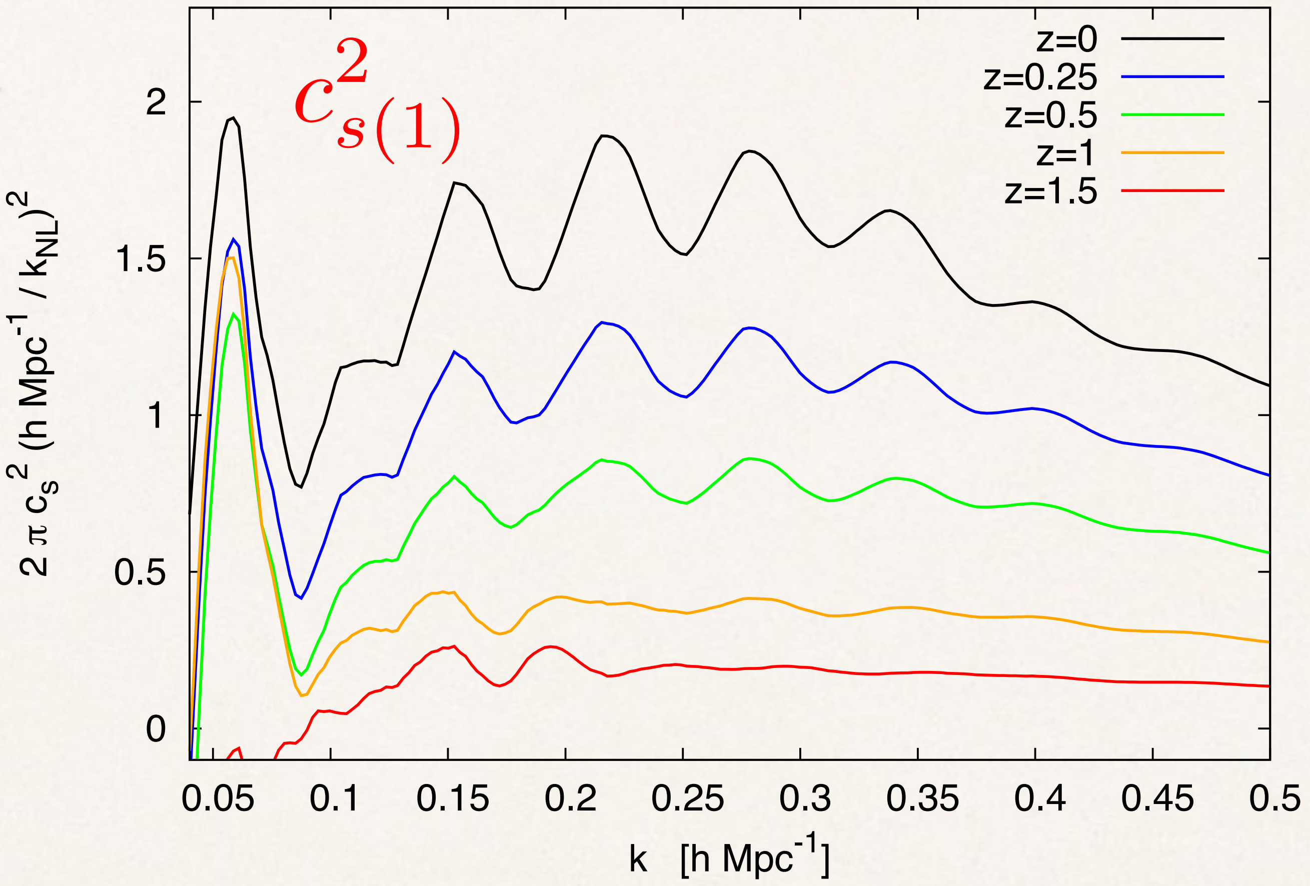
$$\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha}\delta^{ij}\partial_k v_l^k - \frac{3}{4}\frac{c_{sv}^2}{Ha}\left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3}\delta^{ij}\partial_k v_l^k\right)\right] + \Delta\tau^{ij} + \dots \ .$$

derivative expansion, or expansion in k/k_nl

coefficients should be scale independent, nice results for simple power law linear PS

The PS in 1-loop EFToLSS

$$P_{11}(k,\eta) \simeq P_{11}^{lin}(k,\eta) + P_{ss,11}^{1-\text{loop}}(k,\eta) - 2(2\pi)\, c^2_{s(1)} \frac{k^2}{k^2_{NL}} P^{lin}(k,\eta)\,,$$



higher orders+resummations needed to reduce the scale dependence

(see Senatore Zaldarriaga, 1404.5954)

Putting everything together

$$\partial_\eta P^{MC}_{ab}(k;\eta,\eta) = -\Omega_{ac} P^{MC}_{cb}(k;\eta)$$ linear growth

$$+ \int^\eta ds\, \Sigma_{ac}(k;\eta,s) P^{MC}_{cb}(k;s,\eta)$$ IR (propagator) effects

$$+ e^\eta \int d^3q \gamma_{acd}(k,q) B^{MC}_{cdb}(q,k;\eta)$$ Intermediate scales: (resummed) SPT

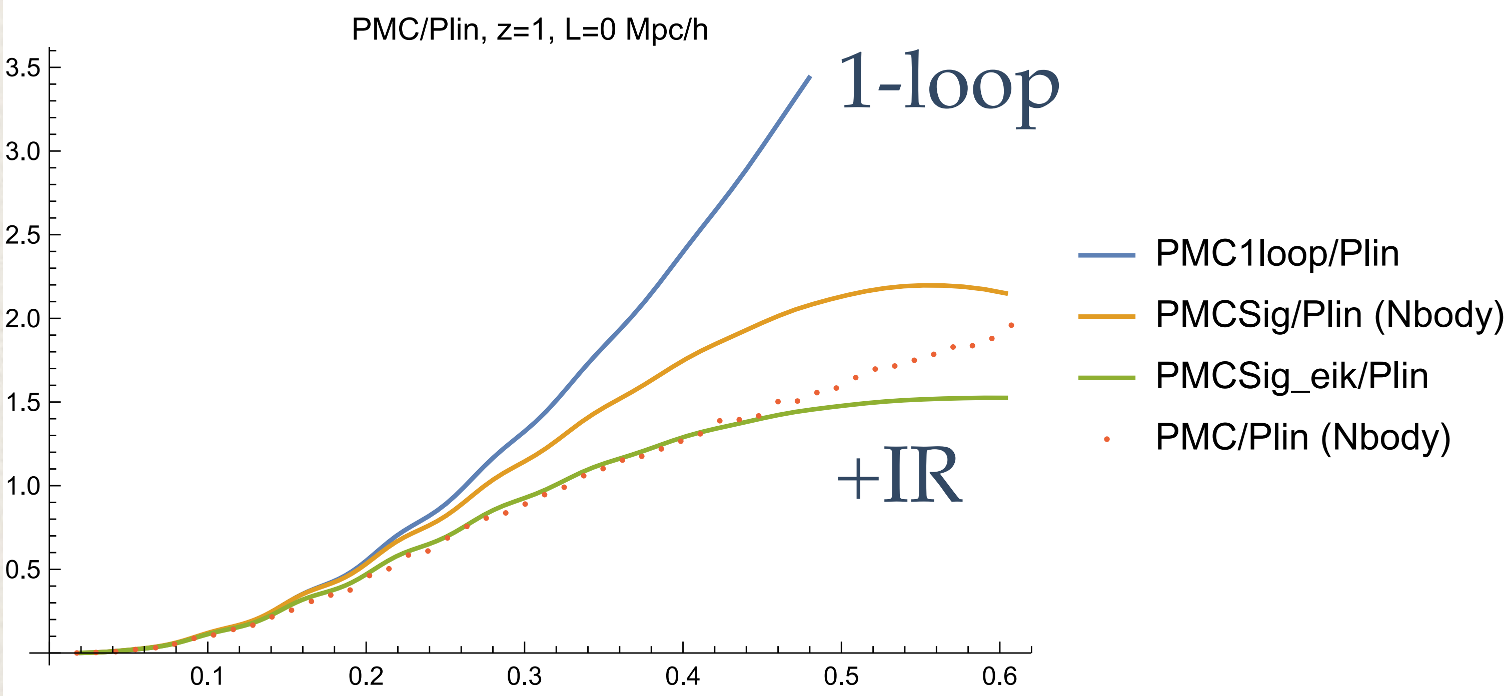
$$-\langle h_a(\mathbf{k},\eta)\varphi^{MC}_b(-\mathbf{k},\eta)\rangle$$ UV sources (from Nbody)

$$+(a \leftrightarrow b)$$

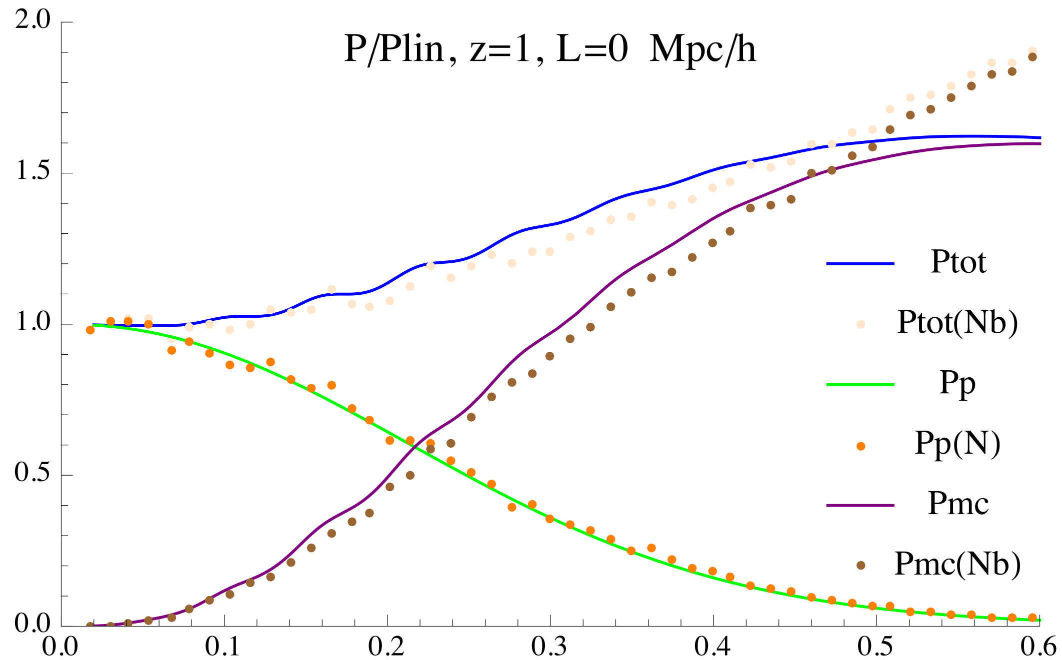
Improved TRG

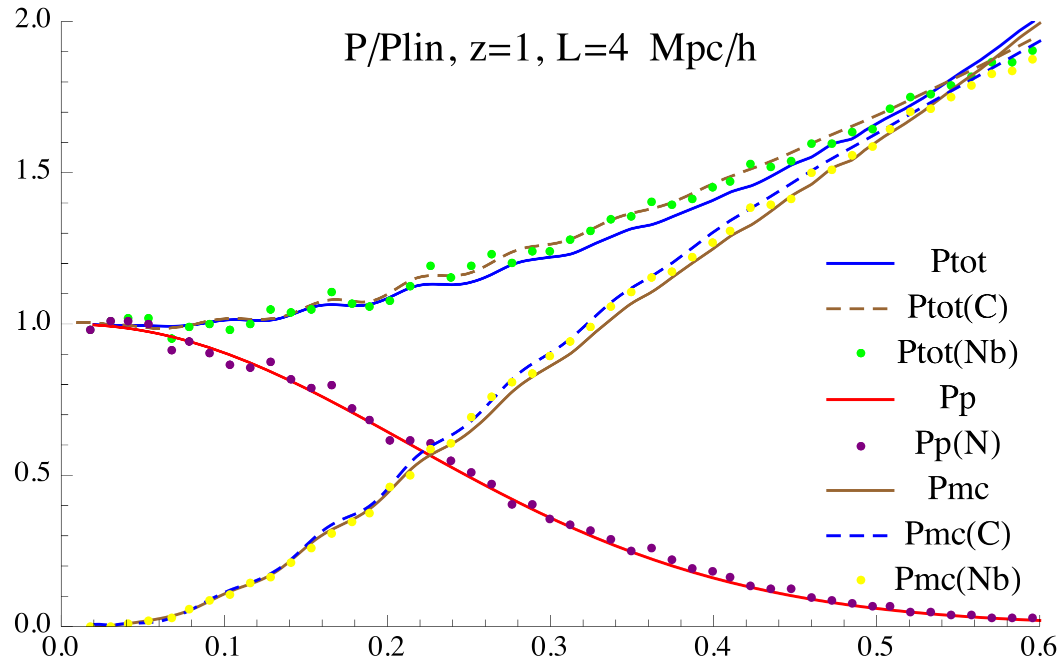
Peloso, MP, Viel, Villaescusa-Navarro, in preparation

Some results (preliminary)



+UV

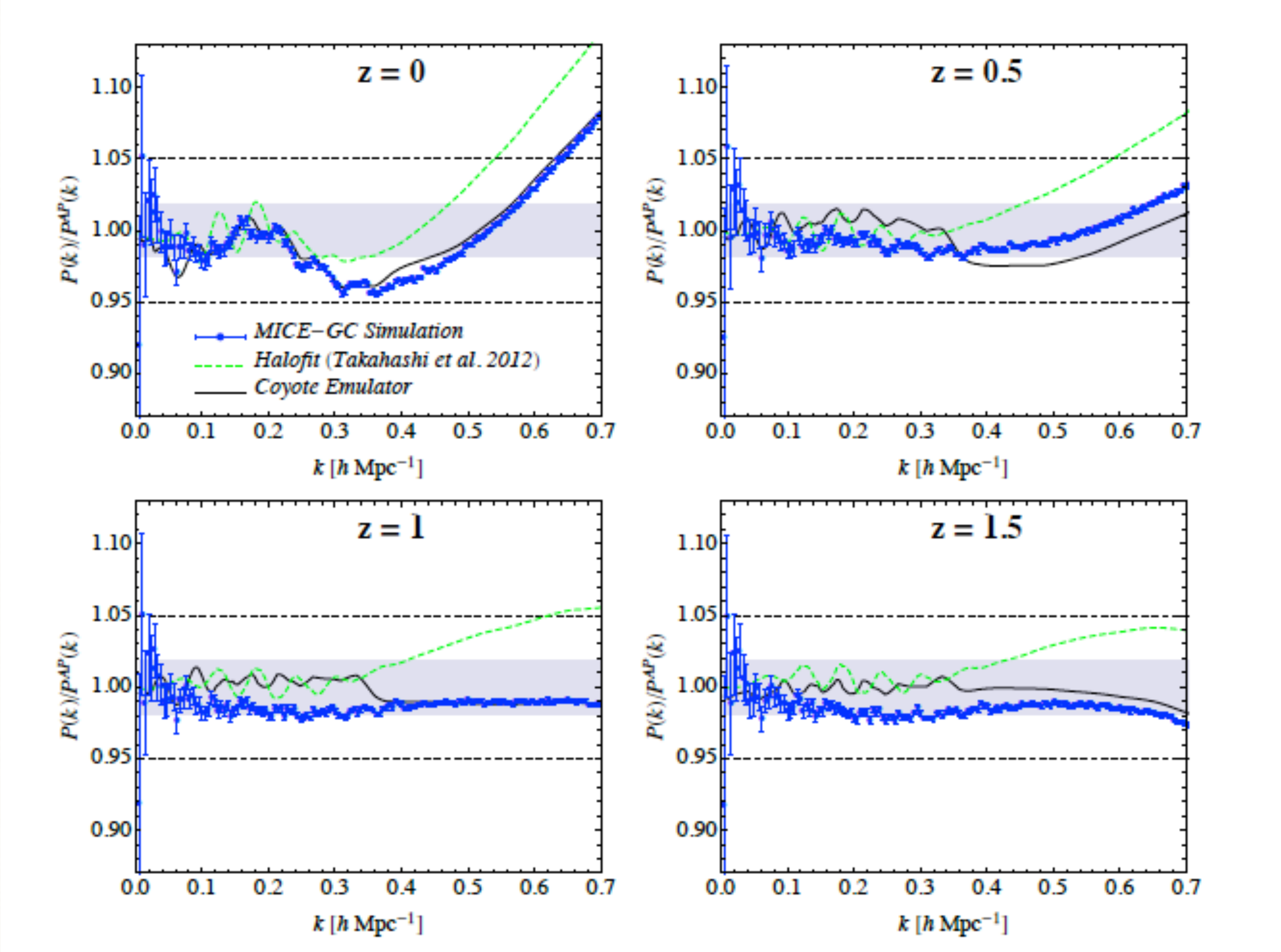




Intermediate scales at 1-loop SPT
no free parameter!

with resummations of the MC part

Anselmi, MP, 1205.2235



Anselmi, Lopez-Nacir, Sefusatti, 2014

Scalar field (axion-like) DM

$$(\Box - m_a^2)\phi = 0$$

$$\Box = -(1-2V)(\partial_t^2 + 3H\partial_t) + a^{-2}(1+2V)\nabla^2 - 4\dot{V}\partial_t$$

$$m_a \gg H$$

$$\phi = (m_a\sqrt{2})^{-1}(\psi e^{-im_a t} + \psi^* e^{im_a t})$$

$$i\dot{\psi} - 3iH\psi/2 + (2m_a a^2)^{-1}\nabla^2\psi - m_a V\psi = 0.$$

Shrödinger-Poisson

Perturbations

$$\psi = Re^{iS} \qquad \rho_a = R^2$$

$$\vec{v_a} = (m_a a)^{-1}\nabla S$$

Madelung

$$\dot{\bar{\rho}}_a + 3H\bar{\rho}_a = 0$$

$$\dot{\delta}_a + a^{-1}\vec{v_a}\cdot\nabla\delta_a + a^{-1}(1+\delta_a)\nabla\cdot\vec{v_a} = 0,$$

$$\dot{\vec{v_a}} + H\vec{v_a} + a^{-1}\left(\vec{v_a}\cdot\nabla\right)\vec{v_a} = -a^{-1}\nabla\left(V+Q\right)$$

$$Q = -\frac{1}{2m_a^2a^2}\frac{\nabla^2\sqrt{1+\delta_a}}{\sqrt{1+\delta_a}}.$$

"Quantum" term, deviations from CDM

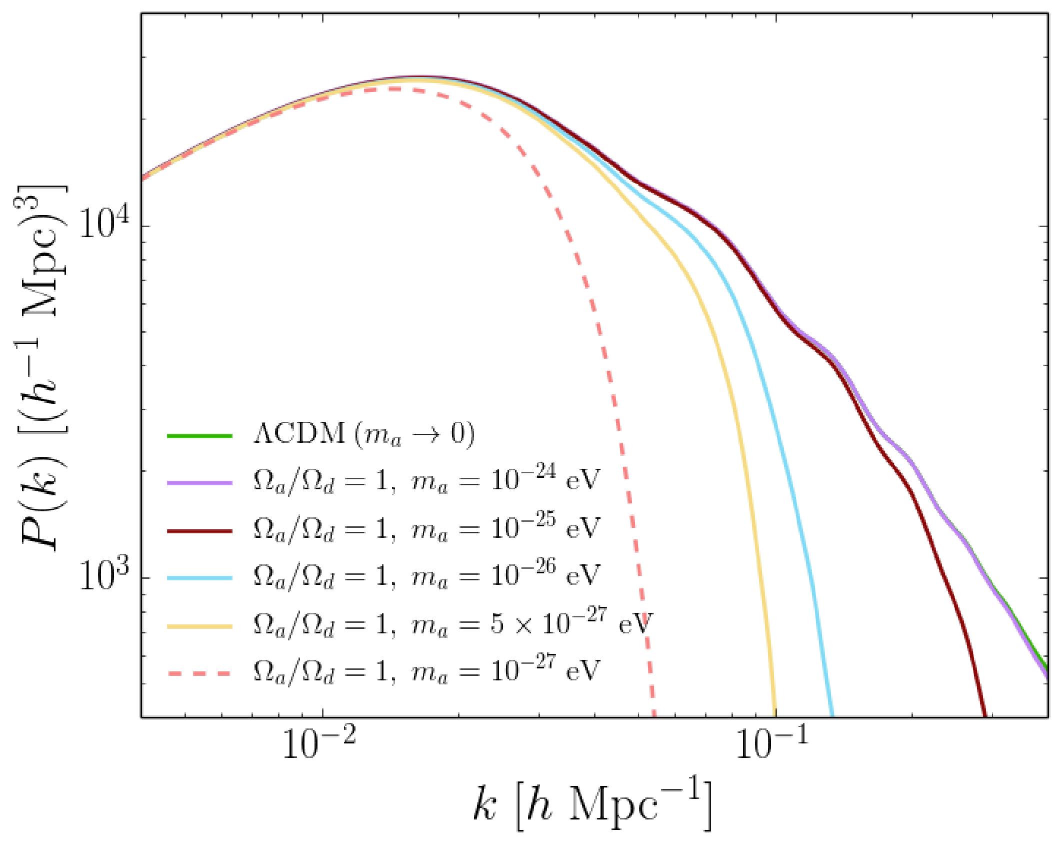
Linear Theory

$$\frac{\partial \delta_a(\mathbf{k},\tau)}{\partial \tau} + \theta(\mathbf{k},\tau) = 0$$

$$\frac{\partial \theta(\mathbf{k},\tau)}{\partial \tau} + \mathcal{H}(\tau)\theta(\mathbf{k},\tau) + \frac{3}{2}\mathcal{H}^2(\tau)\delta_a(\mathbf{k},\tau) - \frac{k^4}{4m_a^2 a^2} = 0$$

$$k_J = \sqrt[4]{6}\sqrt{m_a a \mathcal{H}} \approx 1.6 a \sqrt{m_a H}$$

Axion Jeans scale



Hlozek, Grin, Marsch, Ferreira 1410.2896

Nonlinear perturbations

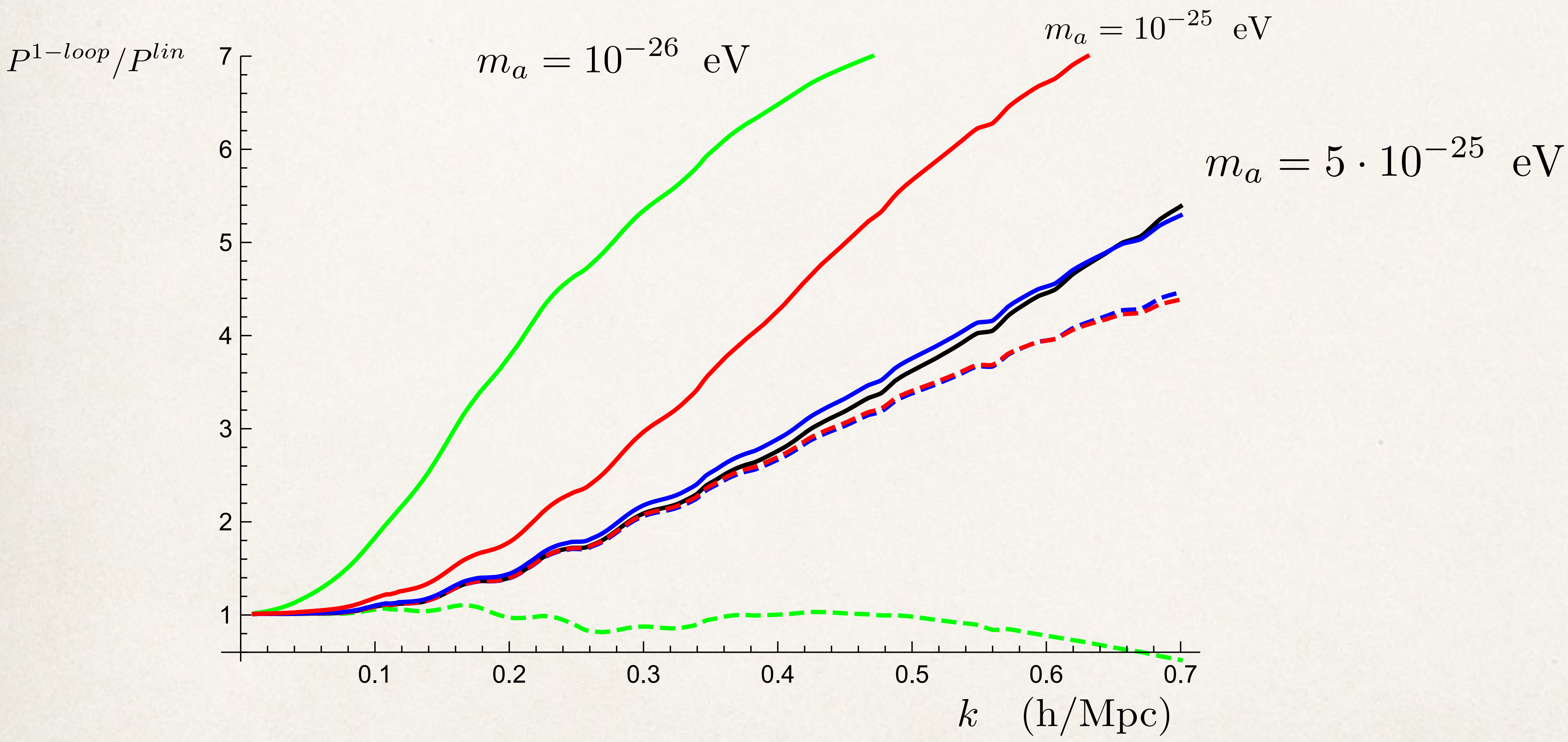
$$\frac{\partial \delta_a(\mathbf{k},\tau)}{\partial \tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{p}\, d^3\mathbf{q}\, \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\alpha(\mathbf{q},\mathbf{p})\theta(\mathbf{q},\tau)\delta_a(\mathbf{p},\tau)$$

$$\frac{\partial \theta(\mathbf{k},\tau)}{\partial \tau} + \mathcal{H}(\tau)\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta_a(\mathbf{k},\tau) - \frac{\mathbf{k}^4}{4m_a^2a^2}\delta_a(\mathbf{k},\tau)$$
$$+ \int d^3\mathbf{p}\, d^3\mathbf{q}\, \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\beta(\mathbf{q},\mathbf{p})\theta(\mathbf{p},\tau)\theta(\mathbf{q},\tau)$$
$$+ \int d^3\mathbf{p}\, d^3\mathbf{q}\, \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\frac{\mathbf{k}^2(\mathbf{k}^2+\mathbf{q}^2+\mathbf{p}^2)}{16m_a^2a^2}\delta_a(\mathbf{q},\tau)\delta_a(\mathbf{p},\tau)$$

From expanding Q to 2nd order
~ k^4: UV catastrophe!

$$\alpha(\mathbf{q},\mathbf{p}) = \frac{(\mathbf{p}+\mathbf{q})\cdot\mathbf{q}}{\mathbf{q}^2} \qquad \beta(\mathbf{q},\mathbf{p}) = \frac{(\mathbf{q}+\mathbf{p})^2\mathbf{q}\cdot\mathbf{p}}{\mathbf{q}^2\mathbf{p}^2}$$

SPT fails at all scales



TRG provides the proper UV cutoff (E. Noda, MP,in progress)

Summary

- The IR effects are well understood and implemented in most of the approaches on the market
- Widening of the BAO peak well understood, analytically
- SPT fails at high loop momenta: UV screening completely missed
- Resummations and effective UV approaches must and can be combined (interpolators from linear response function?)