P fluxes and exotic branes

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"Supergravity: what next?"

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Based on work with E. Bergshoeff, V. Penas and S. Risoli
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Introduction

Fluxes play a crucial role in string theory for moduli stabilisation, which is crucial for phenomenology

In this talk we will consider the low energy four dimensional theories that result when geometric and non-geometric fluxes are turned on

Here geometric means anything that has a ten-dimensional origin

Example: CY O3-orientifold of IIB with NS-NS and RR 3-form fluxes turned on compatibly with susy results in $\mathcal{N} = 1$ supergravity with GVW superpotential

$$W = \int (F_3 - iSH_3) \wedge \Omega$$

Introduction

Fluxes induce a gauging in the four dimensional low energy effective supergravity action

Gauging is described in terms of the embedding tensor

de Wit, Samtleben, Trigiante (2002)

Maximal theory in D=4: embedding tensor in the **912** of $E_{7(7)}$

Decomposing this representation under $SO(6,6)$ one finds

$$\mathbf{912} = \mathbf{32} \oplus \mathbf{220} \oplus \mathbf{352} \oplus \ldots.$$

The **32** repr corresponds to the RR fluxes

$$\theta_a \rightarrow \begin{cases} F \quad F_{mn} \quad F_{mnpq} \quad F_{mnpqrs} \quad (IIA) \\ \\ F_m \quad F_{mnp} \quad F_{mnpq} \qquad\qquad (IIB) \end{cases}$$

The **220** corresponds to the NS fluxes

$$\theta_{MNP} \rightarrow H_{mnp} \quad f_{mn}{}^{p} \quad Q_m{}^{np} \quad R^{mnp}$$

Introduction

The Q and R fluxes are non-geometric

Still, they make perfect sense in the four dimensional theory

T-duality rule:

$$H_{mnp} \xrightarrow{T^p} f_{mn}{}^p \xrightarrow{T^n} Q_m{}^{np} \xrightarrow{T^m} R^{mnp}$$

One derives the form of the superpotential simply applying T duality on known superpotential:

$$W = \int (H_3 + fJ_c + QJ_c^{(2)} + RJ_c^{(3)}) \wedge \Omega$$

Shelton, Taylor, Wecht (2005)

By T duality starting from H_{mnp} the components $f_{mn}{}^n$ and $Q_m{}^{mn}$ can not be turned on. We will always put them to zero

NS fluxes

We consider non-geometric fluxes in a specific model: IIA/IIB $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold

Aldazabal, Cámara, Font, Ibáñez (2006)

$T^6 = \bigotimes_{i=1}^{3} T^2_{(i)}$

Each torus has coordinates (x^i, y^i), basis of closed 2-forms is

$\omega_i = dx^i \wedge dy^i$

Kahler form:

$J = \sum_i A_i \omega_i$

Holomorphic 3-form:

$\Omega = (dx^1 + i\tau_1 dy^1) \wedge (dx^2 + i\tau_2 dy^2) \wedge (dx^3 + i\tau_3 dy^3)$

NS fluxes

O3 IIB orientifold: divide out by $\Omega_P(-1)^{F_L}\sigma_B$ where

$\sigma_B(x^i) = -x^i \qquad \sigma_B(y^i) = -y^i$

Complex moduli are

- complex structure moduli

  $U_i = \tau_i$

- complex Kahler moduli

  $J_c = C_4 + \frac{i}{2} e^{-\phi} J \wedge J = i \sum_i T_i \tilde{\omega}_i$

- axion-dilaton

  $S = e^{-\phi} + iC_0$

NS fluxes

O6 IIA orientifold: perform three T-dualities along $x^1 \quad x^2 \quad x^3$

Involution is now

$$\sigma_B(x^i) = x^i \qquad \sigma_B(y^i) = -y^i$$

τ_i are now real

Complexified holomorphic 3-form is

$$\Omega_c = C_3 + i\mathrm{Re}(C\Omega) = iS(dx^1 \wedge dx^2 \wedge dx^3) + iU_i(dx \wedge dy \wedge dy)^i$$

Complex Kahler moduli are

$$J_c = B + iJ = i\sum_i T_i\omega_i$$

IIB and IIA moduli related by T-duality as $T_i \leftrightarrow U_i$

NS fluxes

We now turn on the geometric fluxes

In IIB, only F_3 and H_3 can be turned on, leading to the GVW superpotential which has the form

$$W_B = P_1(U) + SP_2(U)$$

In IIA, the superpotential is

$$W_A = \int [e^{J_c} \wedge F_{RR} + \Omega_c \wedge (H_3 + fJ_c)]$$

Grimm, Louis (2005)

Villadoro, Zwirner (2005)

This has the form

$$W_A = \tilde{P}_1(T) + U + S + UT + ST$$

Clearly the two do not match under T-duality

NS fluxes

IIB NS fluxes	IIA NS fluxes
$H_{x_1 x_2 x_3}$	?
$H_{y_i x_j x_k}$	?
$H_{y_i y_j x_k}$	${f_{y_i y_j}}^{x_k}$
$H_{y_1 y_2 y_3}$	$H_{y_1 y_2 y_3}$
?	${f_{x_i x_j}}^{x_k}$
?	${f_{x_i y_j}}^{y_k}$
?	$H_{x_i x_j y_k}$

IIB: only H fluxes

IIA: odd y's for H and even y's for f

NS fluxes

IIB NS fluxes	IIA NS fluxes
$H_{x_1x_2x_3}$ $H_{y_ix_jx_k}$ $H_{y_iy_jx_k}$ $H_{y_1y_2y_3}$	$R^{x_1x_2x_3}$ ${Q_{y_i}}^{x_jx_k}$ ${f_{y_iy_j}}^{x_k}$ $H_{y_1y_2y_3}$
${Q_{x_k}}^{x_ix_j}$ ${Q_{y_j}}^{x_iy_k}$ ${Q_{y_k}}^{x_ix_j}$ ${Q_{x_j}}^{x_iy_k}$ ${Q_{x_i}}^{y_jy_k}$ ${Q_{y_i}}^{y_jy_k}$	${f_{x_ix_j}}^{x_k}$ ${f_{x_iy_j}}^{y_k}$ $H_{x_ix_jy_k}$ ${Q_{x_i}}^{x_jy_k}$ $R^{x_iy_jy_k}$ ${Q_{y_i}}^{y_jy_k}$

NS fluxes

Due to the Q flux, the IIB superpotential has the form

$W_B = P_1(U) + SP_2(U) + TP_3(U)$

Due to the Q and R fluxes, the IIA superpotential has the form

$W_A = \tilde{P}_1(T) + S\tilde{P}_2(T) + U\tilde{P}_3(T)$

By identifying the fluxes, the two expressions match under T-duality

NS fluxes

The fluxes induce RR tadpoles

For instance, in IIA in the presence of F_0 and H_3 fluxes, the term

$\int C_7 \wedge H_3 F_0$

induces a D6-brane charge

These expressions are generalised by terms containing the non-geometric fluxes

All tadpole conditions have to be taken into account for consistency

P fluxes

In the IIB theory, by S duality the ${Q_m}^{np}$ flux is mapped to a new flux ${P_m}^{np}$

By requiring that the superpotential transforms properly under S duality, one obtains

$$W_B = \int [(F_3 - iSH_3) + (Q - iSP)J_c] \wedge \Omega$$

This implies that the superpotential has the form

$$W_B = P_1(U) + SP_2(U) + TP_3(U) + STP_4(U)$$

This flux also contributes to the tadpole conditions

In particular it induces a charge for the 7-brane which is S-dual of the D7-brane

P fluxes

The P flux $P_a{}^{bc}$ belongs to the representation of the embedding tensor which is the **352** representation of $SO(6,6)$

This is the 'gravitino' representation θ_{Ma}

By decomposing the whole representation under $GL(6,\mathbb{R})$ one gets

$$\theta_{Ma} \to \begin{cases} P_m{}^n \quad P_m{}^{n_1 n_2 n_3} \quad P_m{}^{n_1 \dots n_5} \quad P^{m,n} \quad P^{m,n_1 n_2 n_3} \quad P^{m,n_1 \dots n_5} \\ P_m \quad P_m{}^{n_1 n_2} \quad P_m{}^{n_1 \dots n_4} \quad P^{m,n_1 n_2} \quad P^{m,n_1 \dots n_4} \quad P^{m,n_1 \dots n_6} \end{cases}$$

where the first line is IIA and the second line is IIB

Bergshoeff, Penas, FR, Risoli (2015)

The fluxes $P^{m,n_1 \dots n_p}$ belong to mixed symmetry representations (completely antisymmetric part vanishes)

P fluxes

What happens to a given flux under T-duality?

We should remember that the fluxes belong to a vector-spinor representation

We should treat the m upstairs and downstairs indices as forming the vector index M, while the n indices form the spinor representation

As a consequence, one derives the following T-duality rules

$$P_m{}^{n_1 \dots n_p} \xrightarrow{T^m} P^{m, n_1 \dots n_p m}$$

$$P_m{}^{n_1 \dots n_p} \xrightarrow{T^{n_p}} P_m{}^{n_1 \dots n_{p-1}}$$

$$P^{m, n_1 \dots n_p} \xrightarrow{T^{n_p}} P^{m, n_1 \dots n_{p-1}}$$

We are only interested in the following components: if the m index is down, it is different from any of the n indices, while if it is up it has to be parallel to the n indices

P fluxes

Performing 3 T-dualities from IIB to IIA along the three x directions one therefore obtains

IIB P fluxes	IIA P fluxes
$P_{x_i}{}^{x_j x_k}$	P^{x_i, x_i}
$P_{y_i}{}^{x_j x_k}$ $P_{x_i}{}^{y_j x_k}$	$P_{y_i}{}^{x_i}$ $P^{x_i, x_i x_j y_j}$
$P_{x_i}{}^{y_j y_k}$ $P_{y_i}{}^{y_j x_k}$	$P^{x_i, x_i x_j x_k y_j y_k}$ $P_{y_i}{}^{x_i x_j y_j}$
$P_{y_i}{}^{y_j y_k}$	$P_{y_i}{}^{x_i x_j x_k y_j y_k}$

But in the IIA theory more fluxes are allowed...

IIB P fluxes	IIA P fluxes
$P_{x_i}{}^{x_j x_k}$	P^{x_i,x_i}
$P_{y_i}{}^{x_j x_k}$ $P_{x_i}{}^{y_j x_k}$	$P_{y_i}{}^{x_i}$ $P^{x_i,x_i x_j y_j}$
$P_{x_i}{}^{y_j y_k}$ $P_{y_i}{}^{y_j x_k}$	$P^{x_i,x_i x_j x_k y_j y_k}$ $P_{y_i}{}^{x_i x_j y_j}$
$P_{y_i}{}^{y_j y_k}$	$P_{y_i}{}^{x_i x_j x_k y_j y_k}$
$P^{x_i,x_1 x_2 x_3 y_i}$	$P_{x_i}{}^{y_i}$
$P^{y_i,x_1 x_2 x_3 y_i}$ $P^{x_i,x_i x_j y_i y_k}$	P^{y_i,y_i} $P_{x_i}{}^{y_i x_k y_k}$
$P^{x_i,x_i y_1 y_2 y_3}$ $P^{y_i,y_i x_i y_j x_k}$	$P_{x_i}{}^{x_j x_k y_1 y_2 y_3}$ $P^{y_i,y_i x_j y_j}$
$P^{y_i,y_1 y_2 y_3 x_i}$	$P^{y_i,y_1 y_2 y_3 x_j x_k}$

P fluxes

In the IIB theory, only the fluxes $P_m{}^{np}$ and $P^{m,n_1\dots n_4}$ are allowed

We have a rule for the fluxes that are present in the IIA theory

We get the superpotentials

$$W_B = \int[(F_3 - iSH_3) + (Q - iSP_1^2)J_c + P^{1,5}J_c^2] \wedge \Omega$$

$$W_A = \int[e^{J_c}F_{RR} + \Omega_c(H_3 + fJ_c + QJ_c^2 + RJ_c^3 + P_1^1\Omega_c$$
$$+(P^{1,1} + P_1^3)\Omega_c J_c + (P^{1,3} + P_1^5)\Omega_c J_c^2 + (P^{1,5}\Omega_c J_c^3))]$$

The IIB superpotential has the form

$$W_B = P_1(U) + SP_2(U) + TP_3(U) + STP_4(U) + T^2P_5(U)$$

In the IIA case one gets exactly the same form, with $U \leftrightarrow T$

The two expression match under T-duality

The IIB superpotential was originally obtained in

Aldazabal, Andés, Cámara, Graña (2010)

P fluxes

Precisely like the NS fluxes, also the P fluxes induce tadpoles that must be cancelled by introducing branes

The NS fluxes induce charges for the RR potentials

We have mentioned already that the ${P_m}^{np}$ flux induces also a charge for the S-dual of the D7-brane

What happens to this brane under T-duality?

The full web of branes of the maximal theories in any dimensions has been derived in a series of papers

Bergshoeff, FR (2011)
Bergshoeff, Marrani, FR (2012)

Exotic branes

We classify the branes according to how their tension scales with the dilaton in the string frame, $T \sim g_S^\alpha$

- $\alpha = 0$: fundamental branes
- $\alpha = -1$: D-branes
- $\alpha = -2$: NS 5-branes
- $\alpha = -3$: S-dual of D7-brane

In $D = 4$ we are interested in spacefilling branes. These branes are charged under 4-form potentials, and we know the $SO(6,6)$ representations of all these potentials

- $\alpha = -1 : C_{4,a}$ $(\mathbf{32})$
- $\alpha = -2 : D_{4,MNPQ}$ $(\mathbf{495})$
- $\alpha = -3$: $E_{4,MNa}$ $(\mathbf{1728})$

Where do the branes in these representations come from?

Exotic branes

For D-branes ($\alpha = -1$) you just need the potentials of the 10-dim theory

For NS branes ($\alpha = -2$) you need the mixed-symmetry potentials

D_6 $D_{7,1}$ $D_{8,2}$ $D_{9,3}$ $D_{10,4}$

The extra indices correspond to the fact that the corresponding brane solution must have isometries

Lozano-Tellechea, Ortín (2001)

Bergshoeff, Ortín, FR (2011)

$D_{7,1} \rightarrow D_{6x,x}$ KK monopole

$D_{8,2} \rightarrow D_{6xy,xy}$ T-fold

de Boer, Shigemori (2010)

These branes are exotic

Exotic branes

For the $\alpha = -3$ brane one needs the potentials

$$E_{4,MNa} \rightarrow \begin{cases} E_8 \; E_{8,2} \; E_{8,4} \; E_{9,2,1} \; E_{8,6} \; E_{9,4,1} \; E_{10,2,2} \; E_{10,4,2} \; E_{10,6,2} \\ E_{8,1} \; E_{8,3} \; E_{9,1,1} \; E_{8,5} \; E_{9,3,1} \; E_{9,5,1} \; E_{10,3,2} \; E_{10,5,2} \end{cases}$$

where the first line is IIB and the second is IIA

We find the following T-duality rule:

- $\alpha = -1:$ $\quad 0 \longleftrightarrow 1 \quad C_{\dots} \xrightarrow{T^x} C_{\dots x}$
- $\alpha = -2:$ $\quad 0 \longleftrightarrow 1,1 \quad D_{\dots} \xrightarrow{T^x} D_{\dots x,x}$
  $1 \longleftrightarrow 1 \quad D_{\dots x} \xrightarrow{T^x} D_{\dots x}$
- $\alpha = -3:$ $\quad 0 \longleftrightarrow 1,1,1 \quad E_{\dots} \xrightarrow{T^x} E_{\dots x,x,x}$
  $1 \longleftrightarrow 1,1 \quad E_{\dots x} \xrightarrow{T^x} E_{\dots x,x}$

Exotic branes

Back to our $\mathcal{N} = 1$ model

Using all our T-duality rules we can now figure out what are all the tadpole conditions induced by all our fluxes and which branes can be included to cancel these tadpoles

The NS fluxes have $\alpha = 0$, while the P fluxes have $\alpha = 1$

$$\int(\text{potential})_{\alpha_1}(\text{flux})_{\alpha_2}(\text{flux})_{\alpha_3} \qquad \alpha_1 + \alpha_2 + \alpha_3 = -2$$

We find a fully consistent picture, in which the terms of the fluxes associated to the components that are not branes can be consistently put to zero, while the brane components can be cancelled by the branes

Exotic branes

IIB		IIA	
potential	internal component	internal component	potential
$D_{7,1}$	$D_{x_i y_i x_j , x_j}$	$D_{x_i y_i x_k , x_k}$	$D_{7,1}$
	$D_{x_i y_i y_j , y_j}$	$D_{x_i y_i x_j y_j x_k , x_j y_j x_k}$	$D_{9,3}$
$D_{9,3}$	$D_{x_i y_i x_j y_j x_k , x_j y_j x_k}$	$D_{x_i y_i y_j , y_j}$	$D_{7,1}$
	$D_{x_i y_i x_j y_j y_k , x_j y_j y_k}$	$D_{x_i y_i y_j x_k y_k , y_j x_k y_k}$	$D_{9,3}$

IIB		IIA	
potential	internal component	internal component	potential
E_8	$E_{x_i y_i x_j y_j}$	$E_{x_i y_i x_j y_j x_k, x_i x_j x_k, x_k}$	$E_{9,3,1}$
$E_{8,4}$	$E_{x_i y_i x_j x_k, x_i y_i x_j x_k}$	$E_{x_i y_i x_j x_k, y_i}$	$E_{8,1}$
	$E_{x_i y_i x_j y_j, x_i y_i x_j y_j}$	$E_{x_i y_i x_j y_j x_k, y_i y_j x_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_k, x_i y_i x_j y_k}$	$E_{x_i y_i x_j y_j x_k, y_i x_k y_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i y_j y_k, x_i y_i y_j y_k}$	$E_{x_i y_i x_j y_j x_k y_k, y_i x_j y_j x_k y_k, x_j x_k}$	$E_{10,5,2}$
$E_{9,2,1}$	$E_{x_i y_i x_j y_j x_k, x_i x_k, x_i}$	$E_{x_i x_j y_j x_k, x_j}$	$E_{8,1}$
	$E_{x_i y_i x_j y_j y_k, x_i y_k, x_i}$	$E_{y_i x_j y_j x_k y_k, x_j x_k y_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_j x_k, y_i x_k, y_i}$	$E_{x_i y_i x_j y_j x_k, x_i y_i x_j, y_i}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_j y_k, y_i y_k, y_i}$	$E_{x_i y_i x_j y_j x_k y_k, x_i y_i x_j x_k y_k, y_i x_k}$	$E_{10,5,2}$
$E_{10,4,2}$	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i x_j y_j, x_i y_i}$	$E_{y_i x_j y_j x_k y_k, y_i y_j x_k, y_i}$	$E_{9,3,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i x_j x_k, x_j x_k}$	$E_{x_i y_i y_j y_k, y_i}$	$E_{8,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_j x_k y_k, x_i y_j}$	$E_{y_i x_j y_j x_k y_k, x_j y_j y_k, y_k}$	$E_{9,3,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i y_j y_k, y_j y_k}$	$E_{x_1 y_1 x_2 y_2 x_3 y_3, y_i x_j y_j x_k y_k, y_j y_k}$	$E_{10,5,2}$

Conclusions

- We have an explicit T-dual expression for the superpotential with P fluxes included
- We have a new T-duality rule for the P fluxes
- We have a universal T-duality rule for all the branes in string theory
- We have a complete consistent expression for tadpole conditions including exotic branes

Conclusions

To be done in the next ten years:

- Extend to the remaining fluxes and branes
- Study moduli stabilisation
- Embed in DFT
- Dynamics of exotic branes