SUPERGRAVITY at 4,0

Supersymmetric 6-D gravity with (4,0) Susy

GGI, October 2016

Theory X?

- Considerable evidence for mysterious interacting 6-D (2,0) non-lagrangian SCFT
- Key to understanding SYM in D<6, S-duality
- Similar story for gravity?
- IF there is an interacting (4,0) SCFT in 6-D, it would be exotic CONFORMAL theory giving SUGRA in D<6

(2,0) Theory

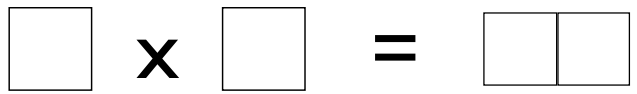
- Free (2,0) theory in 6-D: 2-form B, H=H*
- Reduces to 5-D N=4 Maxwell, F=dA
- Interacting (2,0) SCFT, non-lagrangian, reduces to 5-D SYM
- Strong coupling limit of 5-D SYM: (2,0) SCFT
- Stringy constructions: M5-brane, IIB on K3

(4,0) Theory

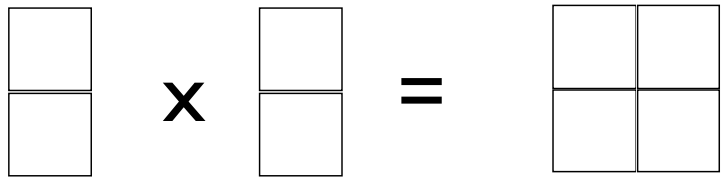
- Free (4,0) theory in 6-D: SCFT
- Reduces to 5-D linearised N=8 SUGRA
- Is there an interacting (4,0) SCFT? Non-lagrangian, reducing to 5-D SUGRA?
- Strong coupling limit of 5-D SUGRA?
- Exotic conformal theory of gravity?
- Highly symmetric (4,0) phase of M-theory?

Gravity = $(YM)^2$

- Free SUGRA ~ Free $(SYM)^2$



- Free (4,0) ~ Free ((2,0) theory)^2



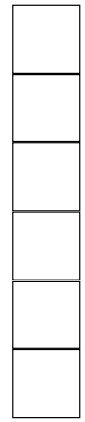
- Free (2,0) reduces to 5-D theory of photon + dual photon
- Free (4,0) reduces to 5-D theory of graviton + dual graviton + double dual graviton

5-D Superalgebra

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

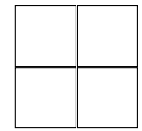
- Central charges Z,K
- Z Electric charges for Maxwell fields
- States with K ~ KK modes of 6-D (p,0) theory
- SYM: K carried by BPS solitons (from YM instantons)
- Does M-theory on T^6 have BPS states with K?
- Do they become massless at strong coupling?

Maxwell in D-dimensions

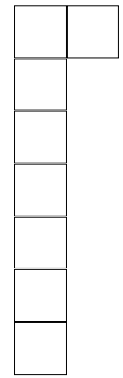
- Photon A_μ 
- Dual photon: n=D-3 form $\tilde{A}_{\mu_1 \ldots \mu_n}$ 
  $F = *\tilde{F}$
- Magnetic charges: D-4 branes.
  A has Dirac strings, or connection on non-trivial bundle, Ã well-defined
- Electric charges: 0-branes.
  Ã has Dirac string singularities, A OK
- YM? No non-abelian theory for Ã

Linearised Gravity

- **Graviton** $h_{\mu\nu}$  $(1,1)$

  Field strength $R_{\mu\nu\rho\sigma}$  $(2,2)$

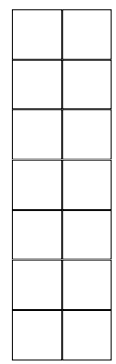
- **Dual Graviton**

  $\tilde{h}_{\mu_1 \ldots \mu_n \nu}$ $(n,1)$ 

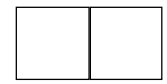
  n=D-3

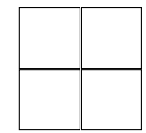
- **Double Dual Graviton**

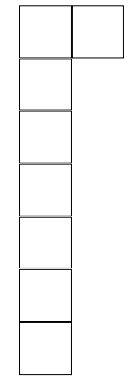
  $\tilde{\tilde{h}}_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}$

  (n,n) 

Linearised Gravity

- **Graviton** $h_{\mu\nu}$  $(1,1)$

  Field strength $R_{\mu\nu\rho\sigma}$  $(2,2)$

- **Dual Graviton** 

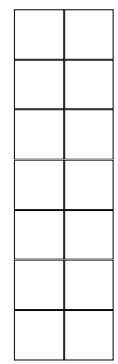
  $\tilde{h}_{\mu_1 \ldots \mu_n \nu}$ $(n,1)$

  $\tilde{R}_{\mu_1 \ldots \mu_{n+1}\, \rho\sigma}$

  $(n+1,2)$

  n=D-3

- **Double Dual Graviton**

  $\tilde{\tilde{h}}_{\mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}$ 

  (n,n)

  $\tilde{\tilde{R}}_{\mu_1 \ldots \mu_{n+1}\, \nu_1 \ldots \nu_{n+1}}$

  $(n+1,n+1)$

Field strengths are Dual:

$$R \qquad \tilde{R} = *R \qquad \tilde{\tilde{R}} = *R*$$

Duality Exchanges field equals and Bianchis

$$R_{\mu\rho\nu}{}^{\rho} = 0 \qquad \leftrightarrow \qquad \tilde{R}_{[\mu_1 \ldots \mu_n \mu_{n+1} \nu]\rho} = 0$$

$$R_{[\mu\nu\rho]\sigma} = 0 \qquad \leftrightarrow \qquad \tilde{R}_{\mu_1 \ldots \mu_n \rho}{}^{\nu}{}_{\nu} = 0$$

Electric and Magnetic Grav Sources $T, \tilde{T}$ for $h, \tilde{h}$

$\tilde{T}$: Dirac strings for h

T : Dirac strings for $\tilde{h}$

- Hull 2000: Dual graviton, double dual graviton in D dims, motivated by 6-D CFT
- West 2001: Dual graviton & E_{11}
- Bekaert, Boulager & Henneaux 2002: No interactions for dual graviton, no dual formulation of GR
- Non-linear action with both
  West 2001, Boulanger & Hohm 2008
  D=11 Sugra: Bergshoeff, de Roo & Hohm

D=6 (2,0) free theory

R-symmetry Sp(2)=USp(4)

Superconformal OSp(4/8*) ⊃ USp(4)xSO(6,2)

$$B_{MN} \qquad\qquad H = *H$$

5 scalars, 4 fermions

Reduce to D=5

$$B_{\mu\nu}, B_{\mu 5} = A_\mu \qquad\qquad H = *F$$

A,B dual, not independent

A, 5 scalars, 4 fermions: D=5 N=4 vector multiplet

Reduce to D=4

2 vector fields $B_{\mu i} = A_{\mu i} \qquad i = 1, 2$ $F_1 = *F_2$

SL(2,Z): diffeos on T^2 (A_1, A_2) doublet

Only one independent field, D=4 N=4 vector multiplet

SL(2,Z): $(A_1, \tilde{A}_1)$ doublet, E-M duality

D=6 Free (4,0) Theory

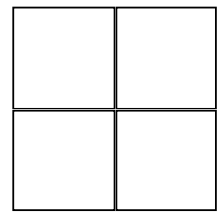
Hull

42 scalars

27 self-dual B_2: $H = *H$

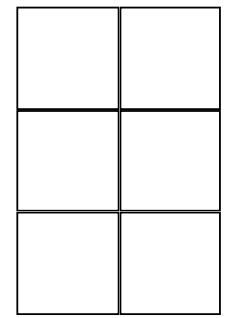
Gauge field

$$C_{MNPQ}$$



Curvature

$$G_{MNPQRS}$$



Self-dual: G=*G=G*

"Supergravity without a graviton"

Superconformal OSp(8/8*) ⊃ USp(8)xSO(6,2)

Reduce to D=5

27 $B_2 \rightarrow$ 27 vectors A_1, 42 scalars $\rightarrow$ 42 scalars

$$C_{\mu 5 \nu 5} = h_{\mu\nu}$$
$$C_{\mu\nu\rho 5} = \tilde{h}_{\mu\nu\,\rho}$$
$$C_{\mu\nu\rho\sigma} = \tilde{\tilde{h}}_{\mu\nu\rho\sigma}$$

Self-duality: Only one of these independent, dual gravitons

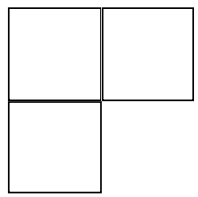
Spectrum of D=5 N=8 SUGRA!

Graviton, 27 vectors, 42 scalars Diffeos

Vectors from B_{MN}

Graviton from C_{MNPQ}

Diffeos from C gauge transformations. Parameter



Reduce to D=4

42 scalars → 42 scalars, Dual vector doublets $B_{\mu i} = A_{\mu i}$

Metrics $C_{\mu(ij)\nu} = -(h_{\mu\nu})_{ij}$

Curvatures: R_{21}=*R_{11}, R_{12} =R_{11}*, R_{22} =*R_{11}*

$$h_{21} = \tilde{h}_{11}, \qquad h_{22} = \tilde{\tilde{h}}_{11}$$

Just h_{11} independent

SL(2,Z) on torus:
(A_1,A_2) doublets, E-M duality
Triplet h_{ij}: gravitational triality

5-D SYM at Strong Coupling

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

Z electric charges: carried by W-bosons etc
YM instanton in R^4 lifts to BPS soliton in 5-D
K proportional to instanton number n, (2,0) short mult.

$$M \propto \frac{n}{g^2_{YM}}$$

Light at strong coupling: KK tower for 6'th dimension
Decompactifies to (2,0) theory in 6D as $g^2_{YM} \to \infty$

Witten, Rozali

(2,0) Interacting CFT

D=5 non-renormalizable, defined within string theory
e.g. D4 brane theory
Strong coupling limit defined within string theory
e.g. multiple D4 branes → multiple M5 branes
No direct construction of interacting (2,0) theory.
Reduce on T^2 gives interacting N=4 SYM
and SL(2,Z) S-duality from torus diffeos

g_{YM} dimensionful. Limit is one to high energies

$$E >> (g_{YM})^{-2} \qquad\qquad E(g_{YM})^2 \to \infty$$

SUGRA at Strong Coupling

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

If there are BPS states carrying K, with spectrum

$$M \propto \frac{n}{l_{Plank}}$$

Become light in strong coupling (high energy) limit

$$E \times l_{Plank} \to \infty$$

Decompactification limit with K-states as a KK tower?
If so, must decompactify to a (4,0) theory in 6D as (4,0) short multiplet

D=5 N=8 Superalgebra

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab}\left(\Gamma^\mu C\right)_{\alpha\beta} P_\mu + C_{\alpha\beta}(Z^{ab} + \Omega^{ab} K)$$

K carried by KK monopoles Gibbons & Perry

Z^{ab} carried by charged 0-branes (from wrapped M-branes)

BPS bound

$$M \geq |K|$$

Full D=5 M-theory on S^1:

No killing vectors, full KK tower etc

Has $E_7(Z)$ symmetry

Includes duality $P^5 \leftrightarrow K$

D>5: D-5 form charge K carried by KK monopoles CMH

Gravitational Instantons Carry K

- Nx(time), N gravitational instanton

  N Gibbons-Hawking multi-instanton space with general sources.

- Metric has Dirac string singularities in general, but connection well-defined

- If all charges are equal, singularities can be removed by identifying under discrete group: ALE or ALF instanton. But if not equal, singular.

- Should string singularities be allowed in quantum gravity? In M-theory?

Symmetry of (4,0)

Free theory:

Conventional field theory in flat background

Background diffeomorphisms + gauge trans

$$\delta C_{MN\,PQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

Reduce to D=5 or D=4:

Combine $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

2 Symmetries are the same for $g_{\mu\nu}$

On T^2, background diffeos give SL(2,Z) S-duality of both spin-1 and spin-2 fields in D=4

Interacting D=6 theory:

Can't combine background η_{MN} & field C_{MNPQ}

Don't expect D=6 diffeos, but exotic symmetries that give D=5 diffeomorphisms

Without D=6 diffeomorphisms, no reason to expect SL(2,Z) and hence no "derivation" of gravitational S-duality (unlike free case)

Without D=6 diffeomorphisms, should spacetime be replaced by something more exotic?
This should be consistent with free limit being a conventional field theory

(2,0) & (4,0) 6-D CFTs

- No local covariant interacting field theory
- D=5 BPS electric 0-branes and magnetic strings lift to self-dual strings in D=6. Tension to zero in conformal limit
- Large superconformal symmetry: (4,0) has 32+32 susys
- YM and graviton in D=5 lift to self-dual tensor gauge fields
- D=5 g_{YM} & l_{planck} from R_6 as no scale in 6-D

M-Theory

- M-theory on T^6 has D=5 N=8 SUGRA as low energy limit
- D=5 branes lift to self-dual strings in D=6. Tension to zero in strong coupling limit
- Is strong coupling limit a 6D theory with (4,0) SUSY, with exotic conformal gravity?
- Highly symmetric phase of M-theory?

Conclusions

- Dual gravitons and gravitational S-duality work well for free theory
- For $D \geq 5$, charge K carried by KK monopoles, and branes from D=4 instantons. Related to NUT charge and magnetic charge of KK monopoles
- For D=4 SYM or linearised SUGRA, S-duality from (2,0) or (4,0) theory on T^2

(4,0): All Four Nothing?

- Key question: are there BPS states with K?
- Extra dimension from strong coupling?
- (4,0) theory as a limit of M-theory?
  Vast symmetry and unusual features
- Not usual spacetime, no metric or diffeos
- Is (4,0) CFT a decoupling limit of (4,0) sector of M-theory?

Mass and Dual Mass

$$R_{\mu\nu} = t_{\mu\nu} \qquad t_{\mu\nu} = T_{\mu\nu} + \frac{1}{D-2}\eta_{\mu\nu}T$$

$$\tilde{R}_{\mu_1\ldots\mu_n\rho\,\nu}{}^{\rho} = \tilde{t}_{\mu_1\ldots\mu_n\,\nu}$$

$$\tilde{t}_{\mu_1\ldots\mu_n\,\nu} = \tilde{T}_{\mu_1\ldots\mu_n\,\nu} + \frac{n}{2}\eta_{\nu[\mu_1}\tilde{T}_{\mu_2\ldots\mu_n]\rho}{}^{\rho}$$

$$R_{[\mu\nu\,\sigma]\tau} = \frac{1}{n!}\epsilon_{\mu\nu\,\sigma}{}^{\mu_1\mu_2\ldots\mu_n}\tilde{t}_{\mu_1\mu_2\ldots\mu_n}$$

Just 2 kinds

Electric and Magnetic Grav Sources $T, \tilde{T}$

$\tilde{T}$: Dirac strings for h

T : Dirac strings for $\tilde{h}$

Non-Linear Gravity with Killing Vector $\frac{\partial}{\partial y}$

$g_{\mu\nu} \rightarrow (g_{mn}, g_{my}, g_{yy})$

Graviphoton in D-1 dimensions $A_m \sim g_{my}$

Dualise in D-1 dimensions: D-4 form $\tilde{A}_{m_1 \ldots m_{D-4}}$

D=4: Scalar NUT potential a.

SL(2,R) Ehlers symmetry. 2 scalars (a, g_{yy}) in $\frac{SL(2,R)}{U(1)}$

D=5: E-M duality for A,Ã

Electric charge: P^y

Magnetic charge: KK monopole

This E-M duality part of U-duality in M-theory

M-Theory Compactified on a Torus

D=4:

28 vector fields

28 electric + 28 magnetic charges

$E_7(Z)$ symmetry

D=5:

27 vector fields

27 electric charges Z^{ab} + 27 magnetic strings

$E_6(Z)$ symmetry

"Topological" charge K, carried by KK monopoles

Reduce 5→4:

Graviphoton $g_{\mu 5}$ Electric charge: P^5 Magnetic charge: K

K-Charge in D=5

Spacetime M asymptotic to $\bar{M}$
k asymptotic to Killing vector on $\bar{M}$

$$\Delta\omega = \omega - \bar{\omega}$$

Difference in spin connections: Asymptotic tensor
ADM Momentum for k: Integral at spatial infinity Σ^3

$$P[k] = \frac{1}{16\pi^2}\int_{\Sigma^3} *(e^A_\wedge e^B_\wedge k)_\wedge \Delta\omega_{AB}$$

Nestor

K-charge

$$K = \frac{1}{16\pi^2}\int_{\Sigma^3} e^A_\wedge e^B{}_\wedge \Delta\omega_{AB}$$

Hull

K and NUT Charge

NUT Charge: Reduce on Killing vector
N is magnetic charge for graviphoton in D=4

KK Monopole spacetime: (Taub-NUT)x(time)
NUT charge N
S^1 fibre, asymptotically radius R=|N|

$$K=RN=N|N|$$