Non-Associative Flux Algebra in String and M-theory from Octonions

DIETER LÜST (LMU, MPI)

Supergravity @ 40, GGI Florence, October 27th, 2016

Donnerstag, 27. Oktober 16

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In collaboration with M. Günaydin & E. Malek,
arXiv:1607.06474

Supergravity @ 40, GGI Florence, October 27th, 2016

This talk is dedicated to my friend Ioannis Bakas

Outline:

I) Introduction

II) Non-associative R-flux algebra for closed strings

III) R-flux algebra from octonions

IV) M-theory up-lift of R-flux background

V) Non-associative R-flux algebra in M-theory

I) Introduction

Geometry in general depends on, with what kind of objects you test it.

Point particles in classical Einstein gravity „see“ continuous Riemannian manifolds.

- $[x^i, x^j] = 0$

Strings may see space-time in a different way.

We expect the emergence of a new kind of stringy geometry.

Closed strings in non-geometric R-flux backgrounds

$\Rightarrow$ non-associative phase space algebra:

$$[x^i, x^j] = i\frac{l_s^3}{\hbar} R^{ijk} p_k$$

D.L., arXiv:1010.1361;

R. Blumenhagen, E. Plauschinn, arXiv:1010.1263.

$$[x^i, p^j] = i\hbar\delta^{ij}\,, \qquad [p^i, p^j] = 0$$

$$\Longrightarrow \quad [x^i, x^j, x^k] \equiv \frac{1}{3}\left[[x^1, x^2], x^3\right] + \text{cycl. perm.} = l_s^3 R^{ijk}$$

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This algebra can be derived from closed string CFT.

R. Blumenhagen, A. Deser, D.L. , E. Plauschinn, F. Rennecke, arXiv:1106.0316

C. Condeescu, I. Florakis, D. L., arXiv:1202.6366

D. Andriot, M. Larfors, D.L. , P. Patalong:arXiv:1211.6437

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This algebra is also closely related to double field theory.

R. Blumenhagen, M. Fuchs, F. Hassler, D.L. , R. Sun, arXiv:1312.0719

Two questions:

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On the mathematical side:

How is the R-flux algebra related to other known non-associative algebras, in particular to the algebra of the octonions?

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Can one lift the R-flux algebra of closed strings to M-theory?

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On the mathematical side:

How is the R-flux algebra related to other known non-associative algebras, in particular to the algebra of the octonions?

Our conjecture:
the answers to these two questions are closely related

On the physics side:

Can one lift the R-flux algebra of closed strings to M-theory?

II) Non-geometric string flux backgrounds

Three-dimensional string flux backgrounds:

Chain of three T-duality transformations:

$$H_{ijk} \xrightarrow{T_i} f^i_{jk} \xrightarrow{T_j} Q^{ij}_k \xrightarrow{T_k} R^{ijk}\,, \qquad (i,j,k=1,\ldots,3)$$

(Hellerman, McGreevy, Williams (2002); C. Hull (2004); Shelton, Taylor, Wecht (2005); Dabholkar, Hull, 2005)

(i) T^3 with H-flux:

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2\,, \qquad B_{12} = Nx^3$$

H-flux: $\quad H_{123} = N$

(ii) Twisted torus tilde $\tilde{T}^3$: T-duality along x^1

$$ds^2 = \left(dx^1 - Nx^3 dx^2\right)^2 + (dx^2)^2 + (dx^3)^2 \,, \qquad B_2 = 0$$

$\tilde{T}^3$ is a U(1) bundle over T^2 :

Globally defined 1-forms:

$$\eta^1 = dx^1 - Nx^3 dx^2 \,, \qquad \eta^2 = dx^2 \,, \qquad \eta^3 = dx^3$$

$$d\eta^i = f^i_{jk} \eta^j \wedge \eta^k$$

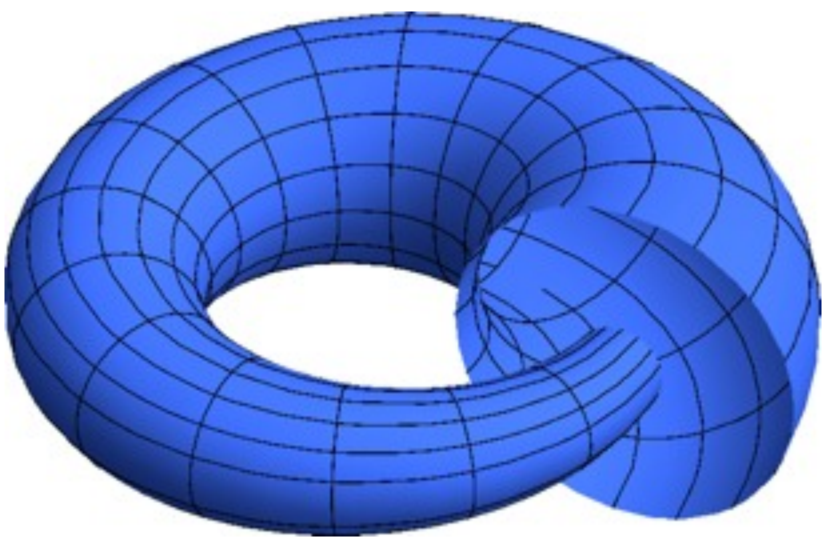
Geometric flux: $f^1_{23} = N$

(iii) Q-flux background: T-duality along x^2

$$ds^2 = \frac{(dx^1)^2 + (dx^2)^2}{1 + N^2\,(x^3)^2} + (dx^3)^2\,, \qquad B_{23} = \frac{N x^3}{1 + N^2\,(x^3)^2}$$

This background is globally not well defined, but it is patched together by a T-duality transformation.

$\Rightarrow$ T - fold C. Hull (2004)



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To make it well defined use double field theory:

W. Siegel (1993); C. Hull, B. Zwiebach (2009); C. Hull, O. Hohm, B. Zwiebach (2010,...)

SO(3,3) double field theory: Coordinates: $(x^1, x^2, x^3; \tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$

The dual background can then by described by „dual“ metric and a bi-vector:

M. Grana, R. Minasian, M. Petrini, D. Waldram (2008);
D. Andriot, O. Hohm, M. Larfors, D.L., P. Patalong (2011,2012);
R. Blumenhagen, A. Deser, E. Plauschinn, F. Rennecke, C. Schmid (2013);
D. Andriot, A. Betz (2013)

$$B_{ij}(x) \xleftrightarrow{T^{ij}} \beta^{ij}(x) = \frac{1}{2}\left((g-B)^{-1} - (g+B)^{-1}\right),$$

$$g(x) \xleftrightarrow{T^{ij}} \hat{g}(x) = \frac{1}{2}\left((g-B)^{-1} + (g+B)^{-1}\right)^{-1}.$$

$$Q_k^{ij} = \partial_k \beta^{ij}$$

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$$Q_k^{ij} = \partial_k \beta^{ij}$$

For the Q-flux background one obtains:

$$\hat{ds}^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 , \qquad \beta^{12} = N x^3$$

Q-flux: $Q_3^{12} = N$

Then one obtains from the CFT of the Q-flux background the following commutation relation among the coordinates:

I. Bakas, D.L., arXiv:1505.04004

Sigma-model for non-geometric backgrounds: A. Chatzistavrakidis, L. Jonke, O. Lechtenfeld, arXiv:1505.05457

$$[x^1, x^2] = N\tilde{p}^3$$

winding number = dual momentum

In general:

$$[x^i, x^j] = i\frac{l_s^2}{\hbar}\oint_{S_k^1} Q_k^{ij}(x)\; dx^k = i\frac{l_s^3}{\hbar}\; Q_k^{ij}\; \tilde{p}^k$$

(iv) R-flux background: T-duality along x^3

Buscher rule fails and one would get a background that is even locally not well defined.

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R-flux can be defined in double field theory:

$$x^k \stackrel{T^k}{\longleftrightarrow} \tilde{x}_k$$

$$\beta^{ij}(x^k) \stackrel{T^k}{\longleftrightarrow} \beta^{ij}(\tilde{x}_k)$$

$$R^{ijk} = 3\hat{\partial}^{[k}\beta^{ij]}$$

In our case we get:

$$\hat{ds}^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \,, \qquad \beta^{12} = N\tilde{x}_3$$

R-flux: $\quad R^{123} = N$

Strong constraint of DFT is violated by this background.

But it is still a consistent CFT background.

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momentum

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Two remarks:

- Mathematical framework to describe non-geometric string backgrounds:

  Group theory cohomology.

  $\Rightarrow$ 3-cycles, 2-cochains, $\star$ - products

  D. Mylonas, P. Schupp, R.Szabo, arXiv:1207.0926, arXiv:1312.162, arXiv:1402.7306.
  I. Bakas, D.Lüst, arXiv:1309.3172

- The same algebra appear in the context of the magnetic monopole.

  R. Jackiw (1985); M. Günaydin, B. Zumino (1985)

  I. Bakas, D.L., arXiv:1309.3172

III) R-flux algebra from octonions

There exist four division algebras: over $\mathbb{R}\,,\mathbb{C}\,,\mathbb{Q}\,,\mathbb{O}$

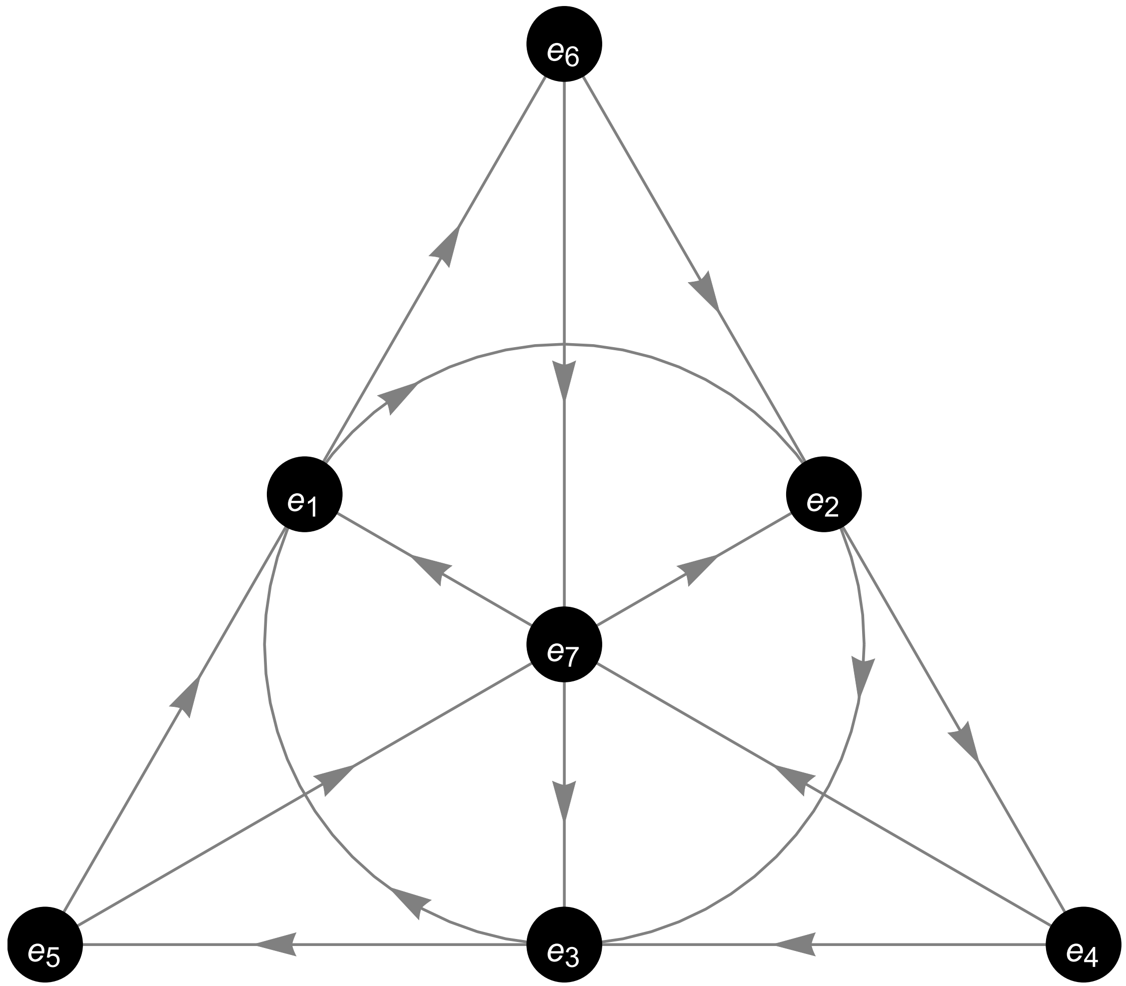
Division algebra of real octonions $\mathbb{O}$: non-commutative, non-associative

Besides the identity, there are seven imaginary units e_A

$$e_A e_B = -\delta_{AB} + \eta_{ABC}\, e_C \qquad (A = 1\ldots,7)$$

$$\eta_{ABC} = 1 \Longleftrightarrow (ABC) = (123),\ (516),\ (624),\ (435),\ (471),\ (572),\ (673)$$

Fano plane mnemonic:



Remark: Octonions generate a simple Malcev algebra

M. Günaydin, F. Gürsey (1973); M. Günaydin, D. Minic, arXiv:1304.0410.

Split indices: $e_i\,,\;\; e_{(i+3)} = f_i\,, \qquad\qquad \text{for } i = 1,2,3$

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$$[e_i, e_j] = 2\epsilon_{ijk} e_k\,, \qquad [e_7, e_i] = 2 f_i\,,$$
$$[f_i, f_j] = -2\epsilon_{ijk} e_k\,, \qquad [e_7, f_i] = -2 e_i\,,$$
$$[e_i, f_j] = 2\delta_{ij} e_7 - 2\epsilon_{ijk} f_k$$

$$\begin{aligned}
[e_i, e_j, f_k] &= 4\epsilon_{ijk} e_7 - 8\delta_{k[i} f_{j]}\,,\\
[e_i, f_j, f_k] &= -8\delta_{i[j} e_{k]}\,,\\
[f_i, f_j, f_k] &= -4\epsilon_{ijk} e_7\,,\\
[e_i, e_j, e_7] &= -4\epsilon_{ijk} f_k\,,\\
[e_i, f_j, e_7] &= -4\epsilon_{ijk} e_k\,,\\
[f_i, f_j, e_7] &= 4\epsilon_{ijk} f_k
\end{aligned}$$

Associator $\quad [X, Y, Z] \equiv (XY)Z - X(YZ)$

Contraction of octonionic Malcev algebra:

$$p_i = -i\lambda \frac{1}{2} e_i \,, \qquad x^i = i\lambda^{1/2} \frac{\sqrt{N}}{2} f_i \ , \ I = i\lambda^{3/2} \frac{\sqrt{N}}{2} e_7$$

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$$\lambda \to 0$$

$$[f_i, f_j] = -2\epsilon_{ijk} e_k \quad \Longrightarrow \quad [x^i, x^j] = iN\epsilon^{ijk} p_k$$

$$[e_i, e_j] = 2\epsilon_{ijk} e_k \quad \Longrightarrow \quad [p_i, p_j] = 0$$

$$[f_i, e_j] = -\delta^i_j e_7 + \epsilon^i{}_{jk} f_k \quad \Longrightarrow \quad [x^i, p_j] = i\delta^i_j I$$

$$[x_i, I] = 0 = [p_i, I]$$

$$[f_i, f_j, f_k] = -4\epsilon_{ijk} e_7 \quad \Longrightarrow \quad [x^i, x^j, x^k] = N\epsilon^{ijk} I$$

Agrees with non-associative R-flux algebra !

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- e^7 additional M-theory coordinate

$\Rightarrow$ Four coordinates: f_1, f_2, f_3, e_7

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Will be closely related to SL(4) /SO(4) exceptional field theory.

IV) M-theory up-lift of R-flux background

Consider IIA string: the duality chain splits into two possible T-dualities:

$$H_{ijk} \xrightarrow{T_{ij}} Q_k^{ij}\,, \qquad f^i_{jk} \xrightarrow{T_{jk}} R^{ijk}$$

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Uplift to M-theory: add additional circle $S^1_{x^4}$

- 3-dim IIA flux background $\Leftrightarrow$ 4-dim M-theory flux background
- two T-dualities $\Leftrightarrow$ 3 U-dualities

(Need third duality along the M-theory circle to ensure right dilaton shift.)

(i) twisted torus $\tilde{T}^3 \times S^1_{x^4}$

$$ds_4^2 = \left(dx^1 - N x^3 dx^2\right)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2\,, \qquad C_3 = 0$$

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(ii) R-flux background: dualise along x^2, x^3 and x^4 .

This leads to a locally not well defined space.

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Use SL(5) exceptional field theory:

D. Berman, M. Perry, arXiv:1008.1763

10 generalized coordinates:

$$x^A \quad \leftrightarrow \quad x^{[ab]} \qquad (A = 1, \ldots, 10\,; \; a, b = 1 \ldots, 5)$$

- 4 coordinates of T^4 : $x^\alpha = x^{5\alpha}\,, \qquad (\alpha = 1, \ldots, 4)$
- 6 dual coordinates: $(\tilde{x}^{41}, \tilde{x}^{42}, \tilde{x}^{43}; \tilde{x}^{21}, \tilde{x}^{31}, \tilde{x}^{32})$

wrapped F1 wrapped D2

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6 of SO(4)

wrapped F1 wrapped D2

SL(5) Flux-background:

C. Blair, E. Malek, arXiv:1412.0635.

dual metric: $$\hat{g}_{\alpha\beta} = \left(1+V^2\right)^{-1/3}\left[\left(1+V^2\right)g_{\alpha\beta} - V_\alpha V_\beta\right],$$

tri-vector: $$\Omega^{\alpha\beta\gamma} = \left(1+V^2\right)^{-1} g^{\alpha\rho}g^{\beta\sigma}g^{\gamma\delta}C_{\rho\sigma\delta}\,,$$

$$\hat{ds}_7^2 = \left(1+V^2\right)^{-1/3} ds_7^2\,.$$

$$V^\alpha = \frac{1}{3!|e|}\epsilon^{\alpha\beta\gamma\delta}C_{\beta\gamma\delta}$$

R-flux: $$R^{\alpha,\beta\gamma\delta\rho} = 4\hat{\partial}^{\alpha[\beta}\Omega^{\gamma\delta\rho]}$$

$$\hat{\partial}^{\alpha\beta} = \partial^{\alpha\beta} + \Omega^{\alpha\beta\gamma}\partial_\gamma\,, \qquad \partial^{\alpha\beta} = \frac{\partial}{\partial x^{\alpha\beta}}$$

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A particular choice of R-flux breaks SL(5) to SO(4).

In this way we obtain a well defined R-flux background in M-theory, which is dual to twisted torus:

$$\hat{ds}_7^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2\,, \qquad \Omega^{134} = N\tilde{x}^{24}$$

$$R^{4,1234} = N$$

The R-flux breaks the section condition of exceptional field theory.

But it should be still a consistent M-theory background.

Four coordinates: x^1, x^2, x^3, x^4

What are the possible conjugate momenta (or windings)?

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- Alternatively consider Freed-Witten anomaly:

R-Flux with momentum p_4 along x^4

$$\Updownarrow$$

R-Flux with D0 branes.

Dualize to IIB: H-flux with D3-branes

This is forbidden by the Freed-Witten anomaly.

$\Rightarrow$ No momentum modes along the x^4 direction !

So we see that the phase space space of R-flux background in M-theory is seven-dimensional:

$$x^1, x^2, x^3, x^4\,;\, p_1, p_2, p_3$$

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Missing momentum condition in covariant terms:

$$p_\alpha R^{\alpha,\beta\gamma\delta\rho} = 0$$

This condition is not the same as section condition.

V) Non-associative R-flux algebra in M-theory

Identify

$$X^i = \frac{1}{2} i \sqrt{N} l_s^{3/2} \lambda^{1/2} f_i \,, \qquad X^4 = \frac{1}{2} i \sqrt{N} l_s^{3/2} \lambda^{3/2} e_7 \,, \qquad P^i = -\frac{1}{2} i \hbar \lambda e_i$$

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Octonionic algebra = conjectured M-theory algebra

$$\boxed{\begin{aligned}
[P_i, P_j] &= -i\lambda\hbar\epsilon_{ijk} P^k\,, \qquad \left[X^4, P_i\right] = i\lambda^2 \hbar X_i\,,\\
\left[X^i, X^j\right] &= \frac{i l_s^3}{\hbar} R^{4,ijk4} P_k\,, \qquad \left[X^4, X^i\right] = \frac{i\lambda l_s^3}{\hbar} R^{4,1234} P^i\,,\\
\left[X^i, P_j\right] &= i\hbar\delta^i_j X^4 + i\lambda\hbar\epsilon^i{}_{jk} X^k\,,\\
\left[X^\alpha, X^\beta, X^\gamma\right] &= l_s^3 R^{4,\alpha\beta\gamma\delta} X_\delta\,,\\
\left[P_i, X^j, X^k\right] &= 2\lambda l_s^3 R^{4,1234} \delta_i^{[j} P^{k]}\,,\\
\left[P^i, X^j, X^4\right] &= \lambda^2 l_s^3 R^{4,ijk4} P_k\,,\\
[P_i, P_j, X_k] &= -\lambda^2\hbar^2\epsilon_{ijk} X^4 + 2\lambda\hbar^2 \delta_{k[i} X_{j]}\,,\\
[P_i, P_j, X_4] &= \lambda^3\hbar^2\epsilon_{ijk} X_k\,,\\
[P_i, P_j, P_k] &= 0\,.
\end{aligned}}$$

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However the algebra is SO(4) invariant:

$$X^\alpha \ : \quad (2,2) \text{ of } SU(2,2)$$

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Further Modification compared to the string case:

$$[X^i, P_j] = i\hbar\delta^i_j X^4 + i\lambda\hbar{\epsilon^i}_{jk}X^k$$

V) Outlook & open questions

Non-associative algebras occur in M-theory at many places:

- Multiple M2-brane theories and 3-algebras

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- $Spin(7), G_2$ backgrounds

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Generalization to higher dimensional exceptional field theory?