

# **Electroweak corrections to hadronic production of gauge bosons at large transverse momentum**

Anna Kulesza



in collaboration with J. H. Kühn, S. Pozzorini and M. Schulze

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*RADCOR2007: Application of Quantum Field Theory to Phenomenology*

*01.10.2007, Florence*

# Introduction

LHC:

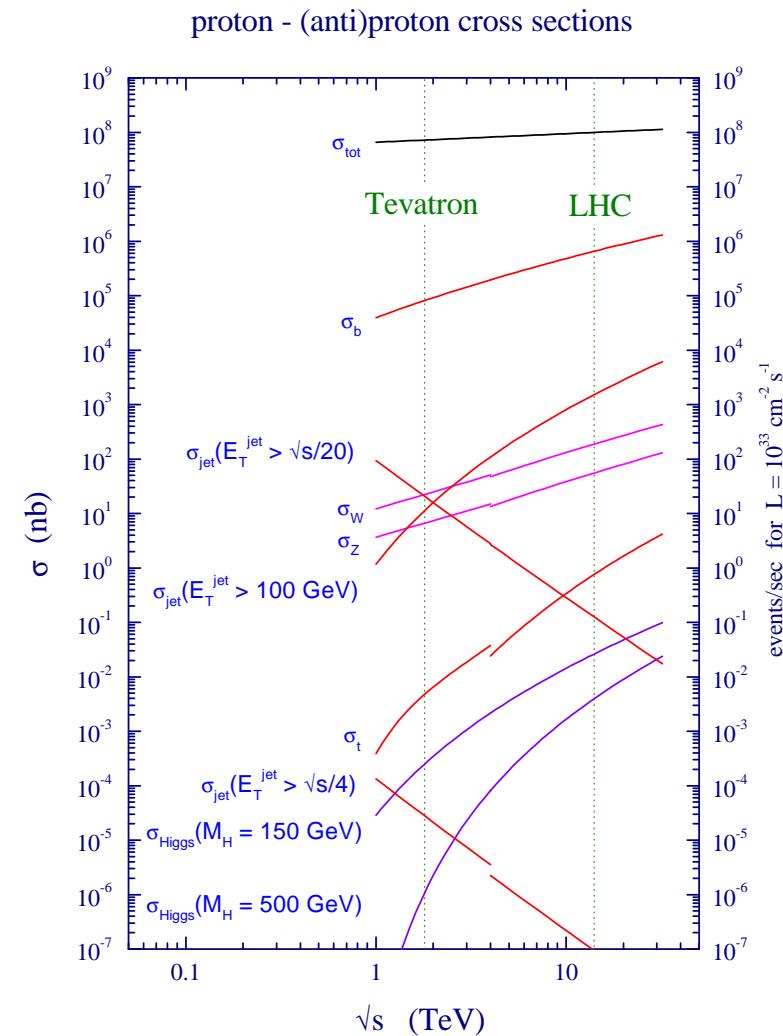
- $W/Z$  production: benchmark process
- Expected cross sections large

At low luminosity  $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$   
estimate

200  $W$  bosons  
50  $Z$  bosons per second!

⇒ LHC will be a  $W/Z$  factory  
(→ parton luminosity monitor)

[Dittmar, Pauss, Zürcher,'97]



# Introduction

## Associated gauge boson production $V + \text{jet}$

- Probe of the hard-scattering dynamics → important test of the SM physics
  - clean signatures: direct  $\gamma$ ,  $W/Z$ -boson ( $\rightarrow$  leptons)

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- Background to Higgs production

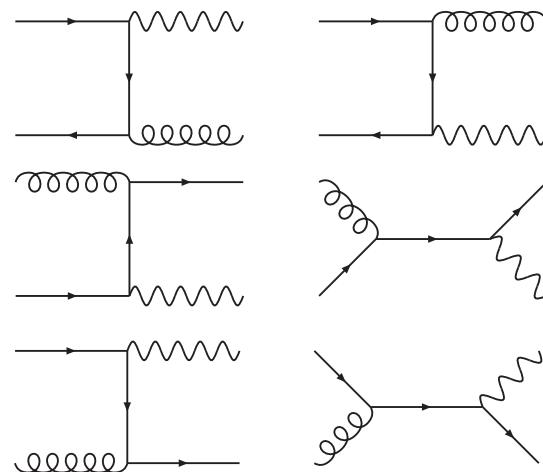
$(H \rightarrow \gamma\gamma, H \rightarrow WW^* \rightarrow l\nu l\nu, H \rightarrow ZZ \rightarrow ll\nu\nu, \dots)$

and SUSY searches at the LHC

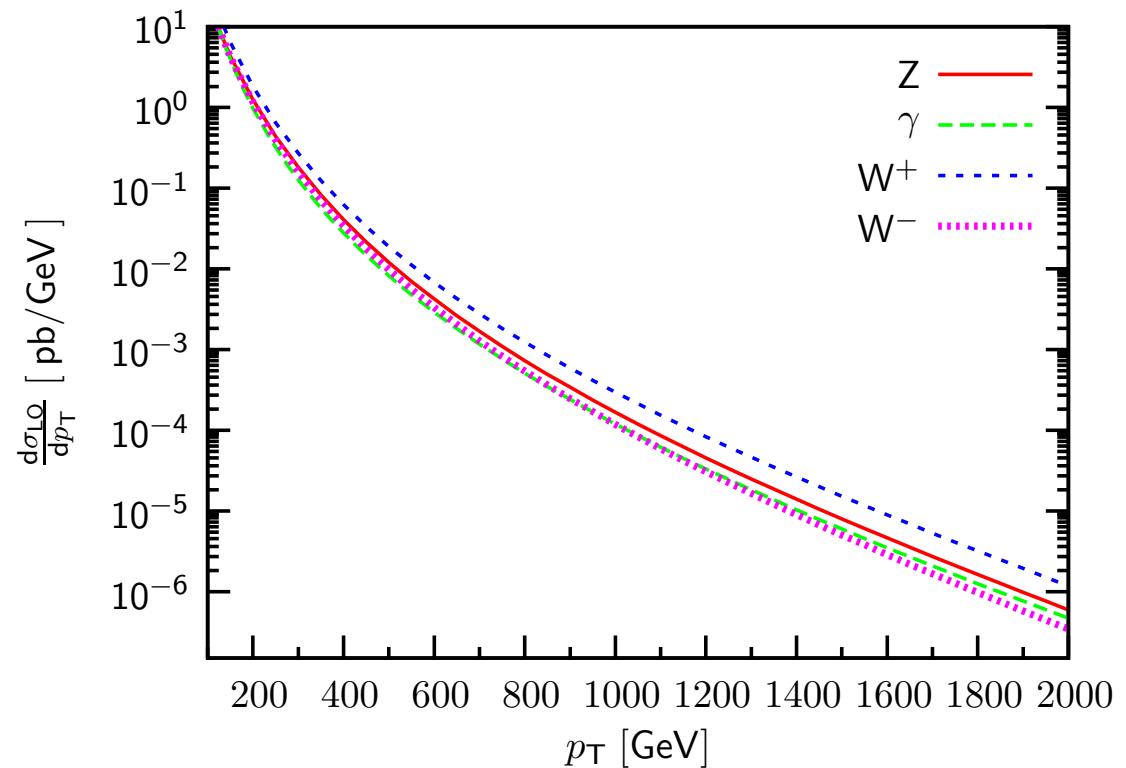
(typical signature  $E_T^{\text{miss}} + \text{jet} [+ l]$ )

# Introduction

## LO $p_T$ distribution at the LHC



$|\mathcal{M}|^2$  related by crossing symmetries



- Large cross sections at LO  $\Rightarrow$  good statistics;
- Reducing theoretical error requires calculation of radiative corrections:  
here EW

# Gauge boson production at large $p_T$

## Theoretical status of higher order corrections

- $\mathcal{O}(\alpha_S)$  QCD corrections [*Ellis, Martinelli, Petronzio'81*] [*Arnold, Reno'89*][*Arnold, Ellis, Reno'89*]  
[*Gonsalves, Pawłowski, Wai'89*] [*Giele, Glover, Kosower'93*] [*Melnikov, Petriello'06*]
- Implementations exist (*DYRAD* [*Giele, Glover, Kosower'93*], *MCFM* [*Campbell, Ellis'02*],  
*FEWZ* [*Melnikov, Petriello'06*], *JETPHOX* [*Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen*]...)

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*FEWZ* [*Melnikov, Petriello'06*], *JETPHOX* [*Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen*]...)
- $\mathcal{O}(\alpha)$  EW corrections to LO  $\mathcal{O}(\alpha\alpha_S)$  process (no QCD corrections)
  - $\gamma/Z$  production [*Maina, Moretti, Ross'04*]
  - $\gamma/Z/W^\pm$  production [*Kühn, A.K., Pozzorini, Schulze'05-07*] (analytic results and numerical predictions)
  - $W^\pm$  production [*Hollik, Kasprzik, Kniehl '07*]

# Introduction

→ see S. Pozzorini's talk

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- Systematic enhancements due to logarithmic (Sudakov) terms of the structure

$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha^2)$	
$\alpha \log^2 \left( \frac{\hat{s}}{M_W^2} \right)$	$\alpha^2 \log^4 \left( \frac{\hat{s}}{M_W^2} \right)$	leading log (LL)
$\alpha \log \left( \frac{\hat{s}}{M_W^2} \right)$	$\alpha \log^3 \left( \frac{\hat{s}}{M_W^2} \right)$	next – to – leading log (NLL)

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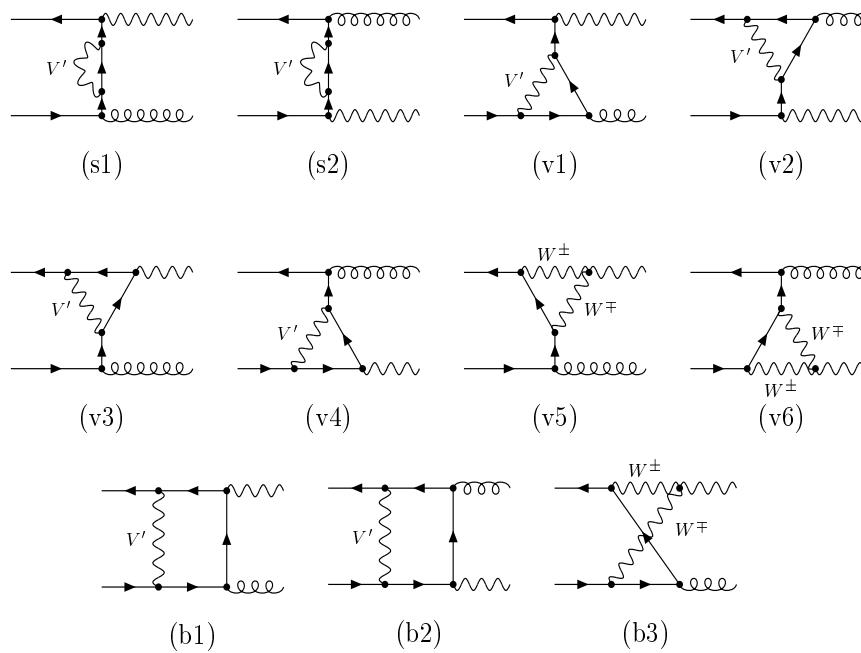
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- Origin: soft/collinear emission of **virtual massive** gauge bosons ( $W, Z$ )
  - real radiation possible to observe  $\implies$  no compensation of virtual emission by real radiation
  - finite logarithmic corrections  $\implies$  different from massless gauge theories such as QCD or QED

# $\mathcal{O}(\alpha)$ corrections to $q\bar{q} \rightarrow Vg$

$Z/\gamma$  production  
 $V' = W^\pm, Z$

[Kühn, A.K., Pozzorini, Schulze'05-07]



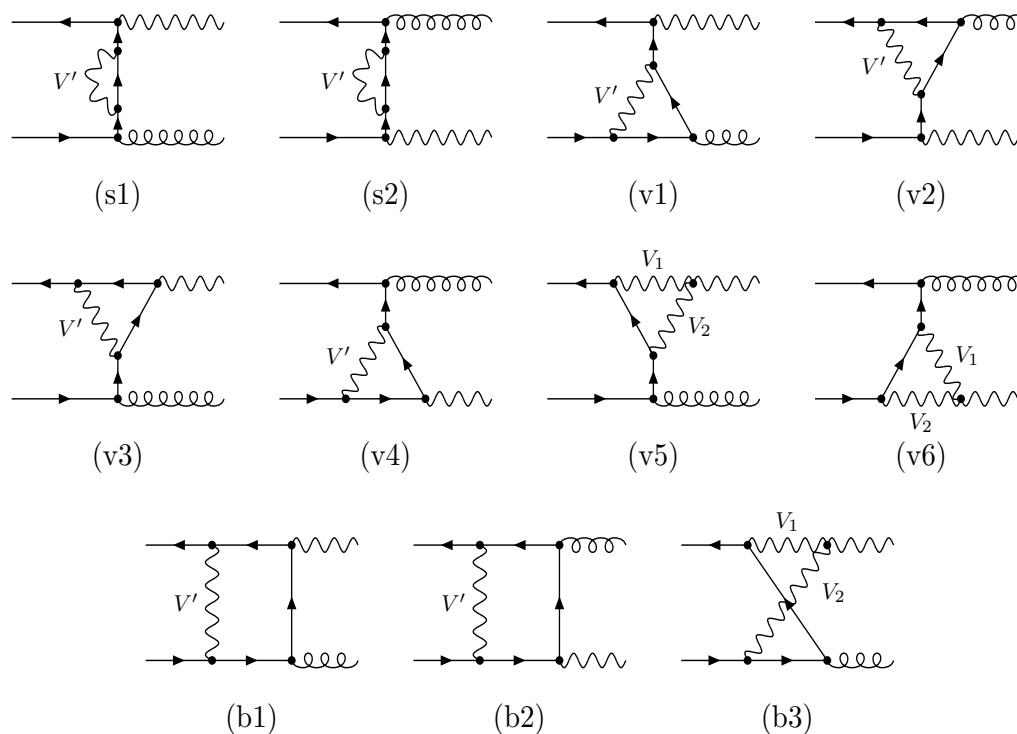
- Loop corrections: IR-finite
- Real corrections:  $W, Z$  emission assumed possible to be observed → not calculated

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$W^\pm$  production

$(V_1, V_2) = (V'', W^\pm)$  with  $V'' = (\gamma, Z)$ ,  $V' = (\gamma, Z, W^\pm)$  or  $V' = (\gamma, Z)$

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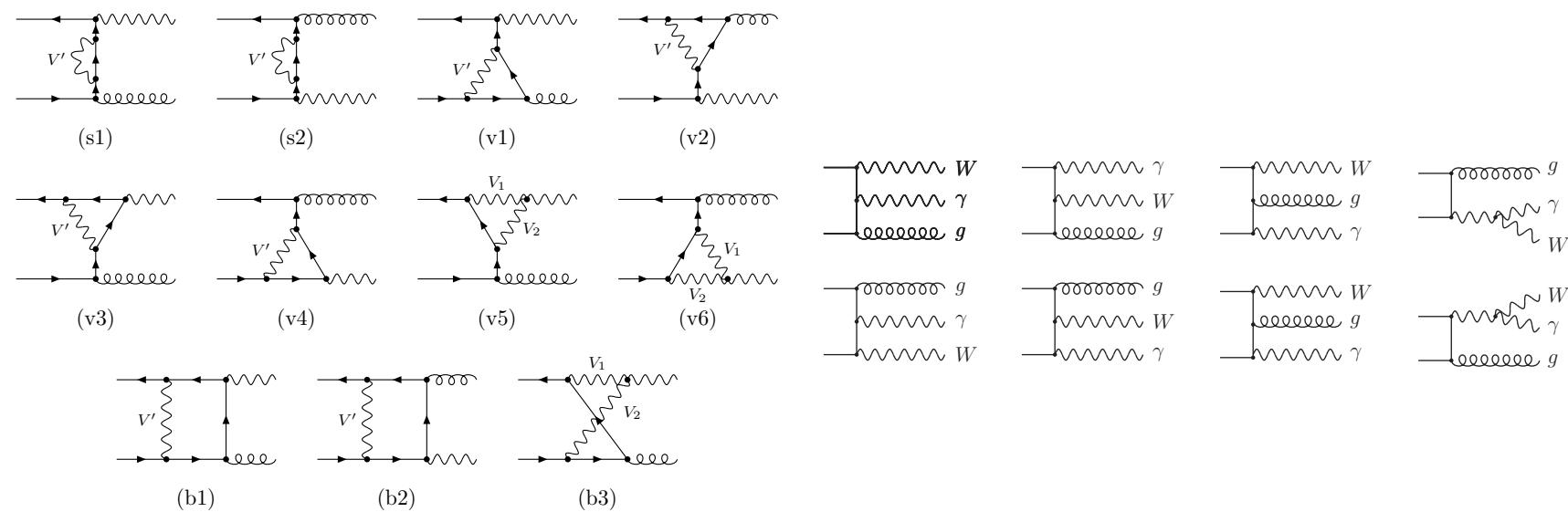


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photon emission IR-singular → needs to be calculated

# Results for $\mathcal{O}(\alpha)$ corrections to $q\bar{q} \rightarrow Vg$

[Kühn, A.K., Pozzorini, Schulze'05-07]

- Analytical one-loop result (Passarino - Veltman tensor reduction)

Schematically

$$\overline{\sum} |\mathcal{M}_{1,v}^{q_i q_j \rightarrow V g}|^2 \sim \sum_{i=1}^{N_V} \sum_{V'=A,Z,W^\pm} C_i^{V' V'} H_V^i(M_{V'})$$

coupling factor

$$N_V = 2 \text{ for } V = A, Z, \quad N_V = 4 \text{ for } V = W^\pm$$

$$H_V^i(M_{V'}^2) = \text{Re} \left[ \sum_{j=0} K_{V,j}^i(M_{V'}^2) J_j(M_{V'}^2) \right]$$

Basis of 14 scalar integrals

$$\begin{array}{lll} J_0(M_{V'}^2) = 1 & J_2(M_{V'}^2) \dots J_6(M_{V'}^2) = B_0(\dots) & J_{12}(M_{V'}^2) \dots J_{14}(M_{V'}^2) = \\ J_1(M_{V'}^2) = A_0(M_{V'}^2) & J_7(M_{V'}^2) \dots J_{11}(M_{V'}^2) = C_0(\dots) & \text{comb. of } C_0(\dots) \& D_0(\dots) \end{array}$$

- Compact expressions for coefficients  $K_{V,j}^i$  (rational functions of kin. variables)

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- Subtraction formalism

$$\sigma^{\text{NLO}} = \int_{m+1} \left( d\sigma^R - d\sigma^A \right) + \int_m \left( d\sigma^V + \int_1 d\sigma^A \right)$$

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- Dipole subtraction method
  - ⇒ photon radiation off massless or massive fermions: mass regularization [Dittmaier '00]
  - ⇒ QCD radiation off massless or massive partons: dimensional regularization, easily adopted to QED radiation [Catani, Seymour '97; Catani, Dittmaier, Seymour, Trócsanyi '02]

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- Results always obtained with two independent codes within the collaboration

# Results for $\mathcal{O}(\alpha)$ corrections to $q\bar{q}^{(-)} \rightarrow Vg$

- Final state  $W + \text{jet}$  with  $p_T^{\text{jet}} > p_T^{\text{jet,c}}$   
Collinear singularity ( $q\gamma W$  state)
  - Here: recombination of momenta for collinear  $q\gamma$  configurations  
if  $R(q, \gamma) = \sqrt{(n_q - n_\gamma)^2 + (\phi_q - \phi_\gamma)^2} < R_{\text{sep}}$  then  $\mathbf{p}_{\text{jet}} = \mathbf{p}_q + \mathbf{p}_\gamma$

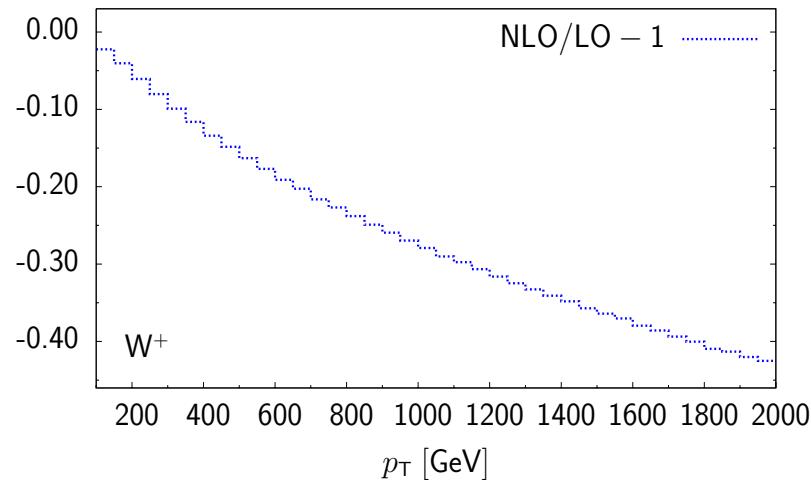
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- Initial state collinear singularities absorbed in the pdfs
  - $\mathcal{O}(\alpha)$  effects in MRST2004QED (NLO QCD evolution)
  - This is a LO calculation in QCD → MRST2001 LO pdfs
  - $\mathcal{O}(\alpha)$  effects known to be small [Roth, Weinzierl '04]
  - Photon-induced contributions suppressed by  $\alpha/\alpha_s$  wrt  $\mathcal{O}(\alpha^2\alpha_s) \Rightarrow$  not included here; found to be non-negligible numerically [Hollik, Kasprzik, Kniehl '07]

# $\mathcal{O}(\alpha)$ corrections to $pp \rightarrow W^+ + 1$ jet at the LHC

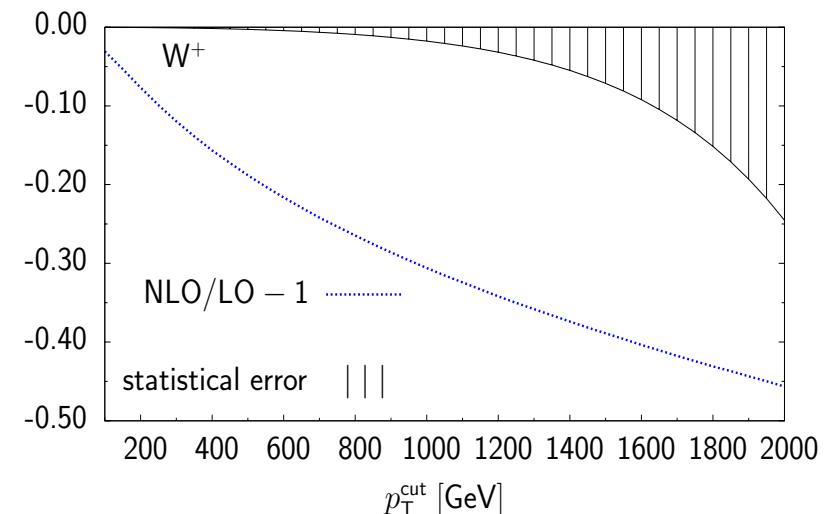
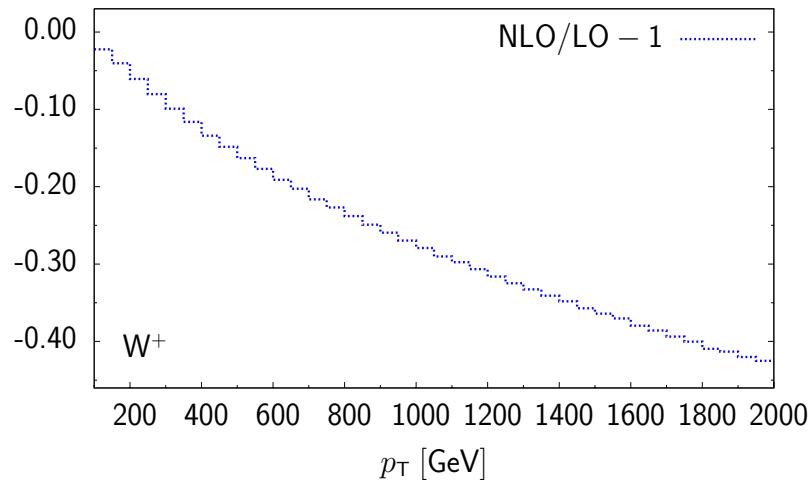


- Corrections negative; range from -15% at 500 GeV up to -42% at 2 TeV

LO MRST2001 pdf's,  $R_{\text{sep}} = 0.4$ ,  $p_T^{\text{jet},c} = 100 \text{ GeV}$ ,  $\alpha_s(M_Z) = 0.13$ ,  
 $\mu_F^{QCD} = \mu_R^{QCD} = p_T$ ,  $\mu_F^{QED} = M_W$ ,  $G_\mu$  scheme,  $\alpha(M_Z) = 1/127.9$ ,  $s_W^2 = 0.231$ ,  $M_Z = 91.19$   
 $\text{GeV}$ ,  $M_W = c_W M_Z$ ,  $m_t = 175 \text{ GeV}$ ,  $m_H = 130 \text{ GeV}$

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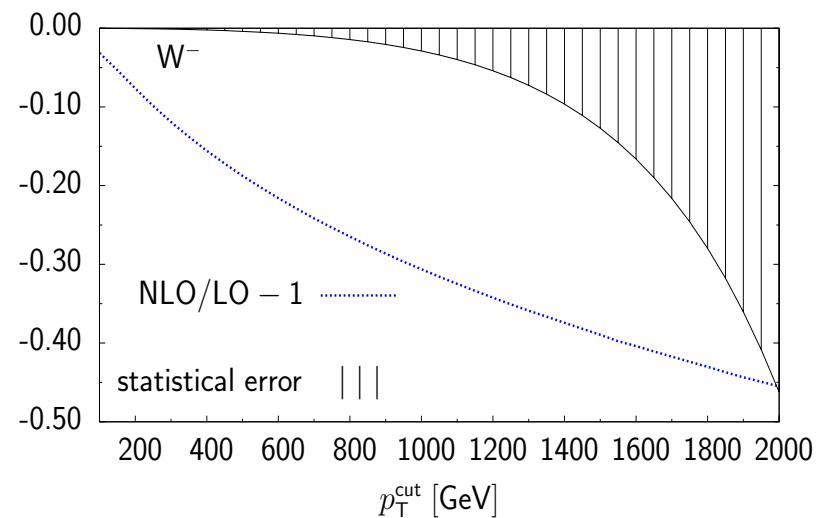
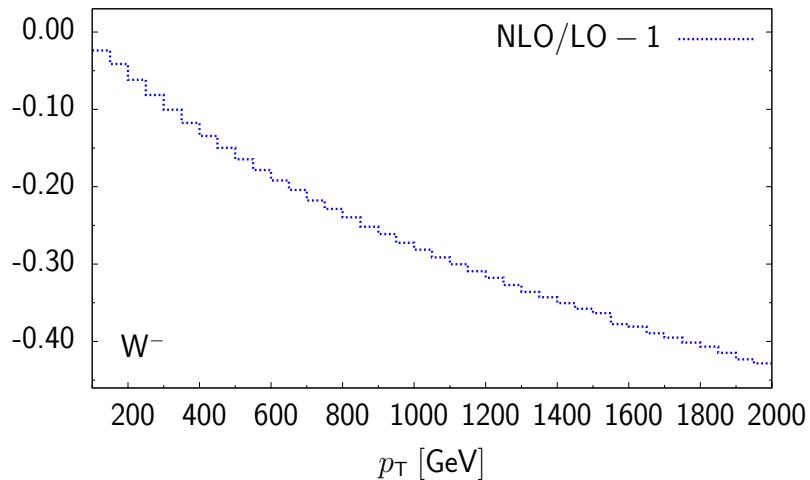
Integrated  $\Delta\sigma(p_T^{\text{cut}})$  vs.  $\Delta\sigma_{\text{stat}} = \frac{\sigma}{\sqrt{N}}$   
 $N = \mathcal{L} \times \text{BR}(Z \rightarrow l, \nu_l) \times \sigma_{\text{LO}}$   
 $\text{BR}(W^+ \rightarrow l, \nu_l) = 30.6\%, \mathcal{L} = 300 \text{ fb}^{-1}$



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- Size of the integrated corrections much bigger than the statistical error!

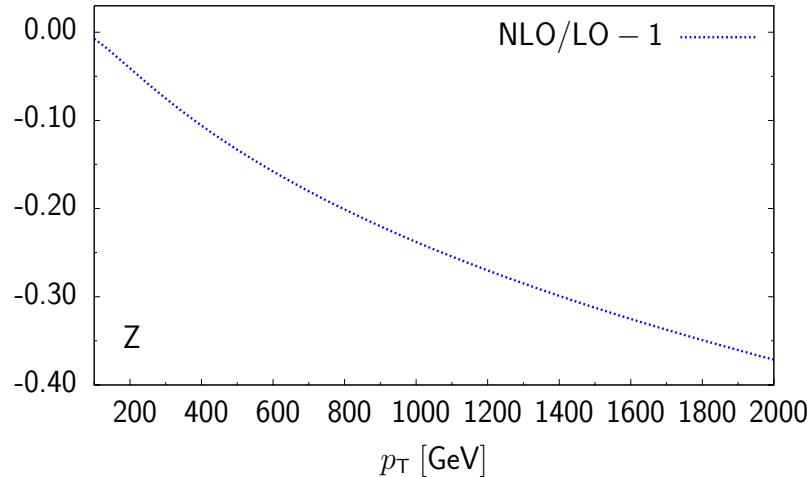
# $\mathcal{O}(\alpha)$ corrections to $pp \rightarrow W^- + 1$ jet at the LHC

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- Corrections negative; behaviour quantitatively very similar to  $W^+$
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# $\mathcal{O}(\alpha)$ corrections to $pp \rightarrow Z + 1$ jet at the LHC

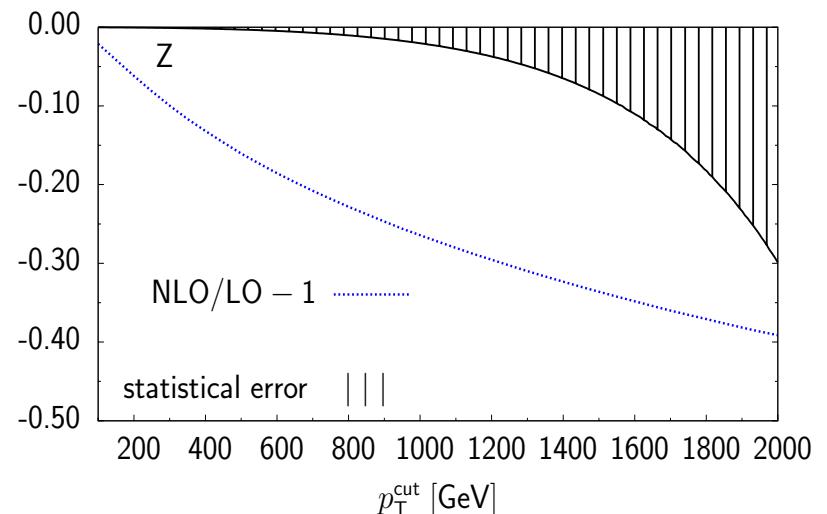
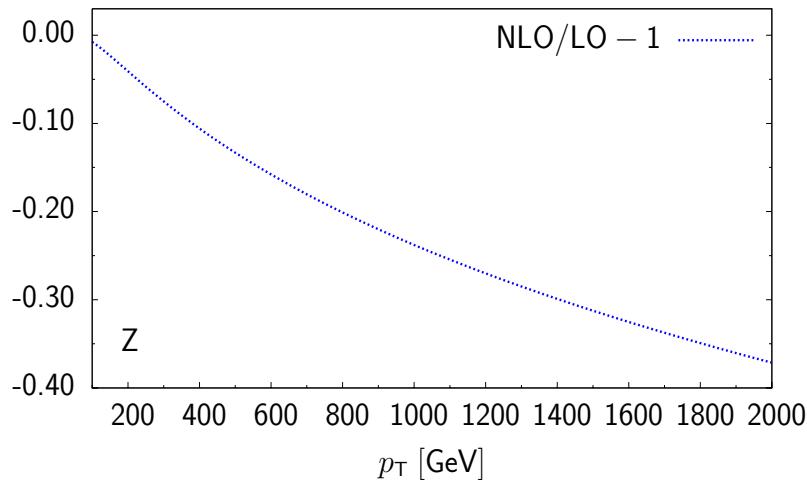


- Corrections negative; range from -13% at 500 GeV up to -37 % at 2 TeV

LO MRST2001 pdf's,  $\alpha_s(M_Z) = 0.13$ ,  $\mu_F = \mu_R = p_T$ ,  $\overline{MS}$  scheme,  $\alpha(M_Z) = 1/127.9$ ,  $s_W^2 = 0.231$ ,  
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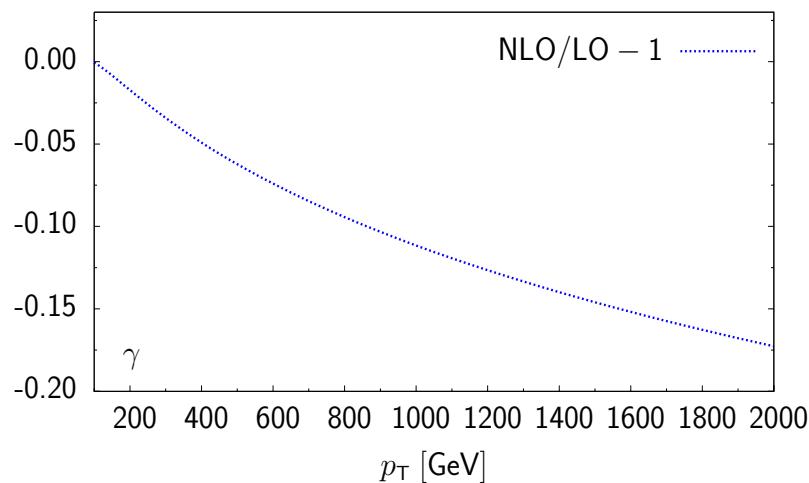
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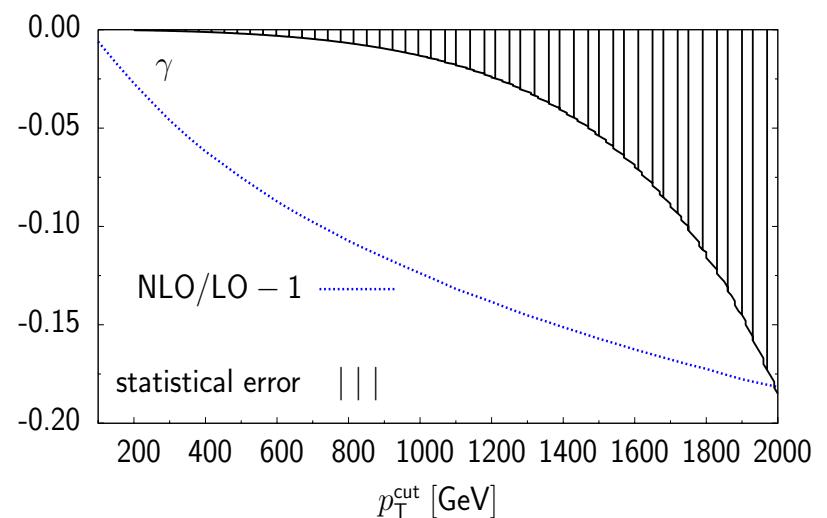
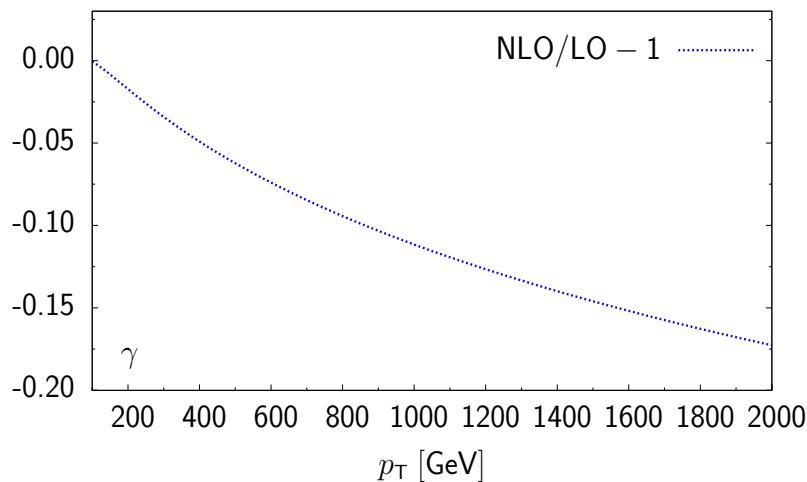


- Corrections negative; range from -6% at 500 GeV up to -17 % at 2 TeV

LO MRST2001 pdf's,  $\alpha_s(M_Z^2) = 0.13$ ,  $\mu_F = \mu_R = p_T$   
OS scheme,  $\alpha(0) = 1/137$ ,  $s_W^2 = 1 - M_W^2/M_Z^2$ ,  $M_Z = 91.19$  GeV,  $M_W = 80.39$  GeV

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# High-energy approximation of the one-loop result

NNLL approximation: high energy limit of the NLO result for  $\sum |\mathcal{M}^{q_i q_j}|^2$

$M_W^2/\hat{s} \rightarrow 0$  ( $\hat{t}/\hat{s}, \hat{u}/\hat{s}$  constant)

[Roth, Denner'96]

terms with  $\alpha \ln^2(\frac{\hat{s}}{M_W^2})$ ,  $\alpha \ln(\frac{\hat{s}}{M_W^2})$  and constants

$(V = \gamma, Z)$

LL

NLL

NNLL

$$H_1^{V,A/N}(M_{V'}^2) \stackrel{\text{NNLL}}{=} \text{Re} \left[ g_0^{V,A/N}(M_{V'}^2) \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} + g_1^{V,A/N}(M_{V'}^2) \frac{\hat{t}^2 - \hat{u}^2}{\hat{t}\hat{u}} + g_2^{V,A/N}(M_{V'}^2) \right]$$

$$\begin{aligned} g_0^{V,A}(M_{V'}^2) &= -\log^2\left(\frac{-\hat{s}}{M_{V'}^2}\right) + 3\log\left(\frac{-\hat{s}}{M_{V'}^2}\right) + \frac{3}{2} \left[ \log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right) \right] + \frac{7\pi^2}{3} - \frac{5}{2} \\ g_0^{V,N}(M_W^2) &= 2 \left[ 2/(4-D) - \gamma_E + \log\left(\frac{4\pi\mu^2}{M_Z^2}\right) - \delta_{V\gamma} \log\left(\frac{M_W^2}{M_Z^2}\right) \right] + \log^2\left(\frac{-\hat{s}}{M_W^2}\right) - \log^2\left(\frac{-\hat{t}}{M_W^2}\right) - \log^2\left(\frac{-\hat{u}}{M_W^2}\right) \\ &\quad + \log^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2} \left[ \log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) \right] - 2\pi^2 + 2\delta_{VZ} \left( -\frac{\pi^2}{9} - \frac{\pi}{\sqrt{3}} + 2 \right) \end{aligned}$$

$$g_1^{V,N}(M_{V'}^2) = -g_1^{V,A}(M_{V'}^2) + \frac{3}{2} \left[ \log\left(\frac{\hat{u}}{\hat{s}}\right) - \log\left(\frac{\hat{t}}{\hat{s}}\right) \right] = \frac{1}{2} \left[ \log^2\left(\frac{\hat{u}}{\hat{s}}\right) - \log^2\left(\frac{\hat{t}}{\hat{s}}\right) \right]$$

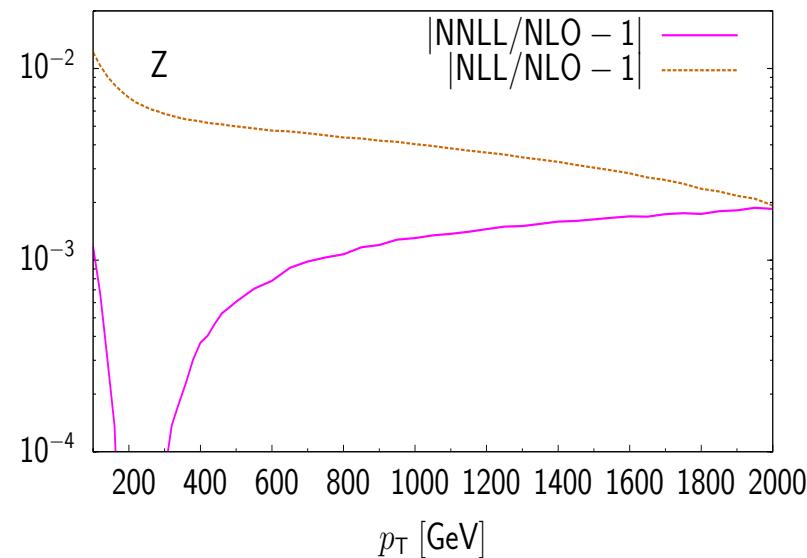
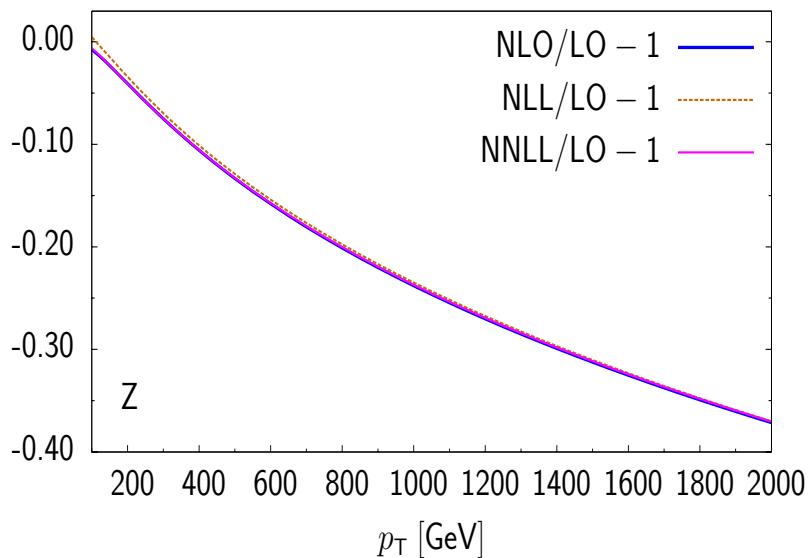
$$g_2^{V,N}(M_{V'}^2) = -g_2^{V,A}(M_{V'}^2) = -2 \left[ \log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right) \right] - 4\pi^2$$

$$g_i^{\gamma,A} = g_i^{Z,A} \text{ for } i = 0, 1, 2 \quad g_j^{\gamma,N} = g_j^{Z,N} \text{ for } j = 1, 2$$

→ extremely compact analytic formulae for NNLL approximation (also for  $W^\pm$ )

# High-energy approximation of the one-loop result

## $Z$ production at the LHC



- NLL approximation: percent (or better) level
  - ~ 1% deviation from NLO at low  $p_T$
  - ~ 0.2% deviation from NLO at  $p_T = 2$  TeV
- NNLL approximation: permille level
- Similar behaviour for  $W^\pm$  and  $\gamma$  production

# NLL approximation at two loops

Phys. Lett. B 609 (2005) 277, JHEP 0603 (2006) 059, Phys. Lett B 651 (2007) 160

- Electroweak corrections in the high energy region:  $|\hat{r}| \gg M_W^2 \sim M_Z^2$  for  $\hat{r} = \hat{s}, \hat{t}, \hat{u}$

# NLL approximation at two loops

Phys. Lett. B 609 (2005) 277, JHEP 0603 (2006) 059, Phys. Lett B 651 (2007) 160

- Electroweak corrections in the high energy region:  $|\hat{r}| \gg M_W^2 \sim M_Z^2$  for  $\hat{r} = \hat{s}, \hat{t}, \hat{u}$

→ Calculations based on results available in the literature:

## 1-loop

factorization and universality of 1-loop EW corrections at the LL and NLL level

$\alpha \log^2 \left( \frac{|\hat{r}|}{M_W^2} \right)$ ,  $\alpha \log \left( \frac{\hat{s}}{M_W^2} \right)$  for arbitrary processes [Denner, Pozzorini'01]

## 2-loop

LL  $\alpha^2 \log^4 \left( \frac{\hat{s}}{M_W^2} \right)$  and NLL “angular”  $\alpha^2 \log^3 \left( \frac{\hat{s}}{M_W^2} \right) \left( \frac{|\hat{r}|}{M_W^2} \right)$  terms for arbitrary processes [Denner, Melles, Pozzorini'03] + remaining NLL terms with  $\alpha^2 \log^3 \left( \frac{\hat{s}}{M_W^2} \right)$  from general resummation formula [Melles'02, '03]

[Denner, Janzen, Pozzorini'06]

# NLL approximation

## $W$ production

$$\overline{\sum} |\mathcal{M}_2^{\bar{q}q' \rightarrow W^\sigma g}|^2 \stackrel{NLL}{=} \overline{\sum} |\mathcal{M}_0^{\bar{q}q' \rightarrow W^\sigma g}|^2 \left[ 1 + \left( \frac{\alpha}{2\pi} \right) A^{(1)} + \left( \frac{\alpha}{2\pi} \right)^2 A^{(2)} \right]$$

$$A^{(1)} = - \left[ C_{qL}^{\text{ew}} (L_{\hat{s}}^2 - 3L_{\hat{s}}) + \frac{1}{s_W^2} (L_{\hat{t}}^2 + L_{\hat{u}}^2 - L_{\hat{s}}^2) \right]$$

$$L_{\hat{r}} = \log \left( \frac{|\hat{r}|}{M_W^2} \right), \quad \hat{r} = \hat{s}, \hat{t}, \hat{u}; \quad C_{qL}^{\text{ew}} = \frac{Y_{qL}^2}{4c_W^2} + \frac{3}{(4s_W^2)}$$

⇒ agreement with the NLL limit of the full one-loop result

# NLL approximation

## $W$ production

$$\overline{\sum} |\mathcal{M}_2^{\bar{q}q' \rightarrow W^\sigma g}|^2 \stackrel{NLL}{=} \overline{\sum} |\mathcal{M}_0^{\bar{q}q' \rightarrow W^\sigma g}|^2 \left[ 1 + \left( \frac{\alpha}{2\pi} \right) A^{(1)} + \left( \frac{\alpha}{2\pi} \right)^2 A^{(2)} \right]$$

$$A^{(1)} = - \left[ C_{qL}^{\text{ew}} (L_{\hat{s}}^2 - 3L_{\hat{s}}) + \frac{1}{s_W^2} (L_{\hat{t}}^2 + L_{\hat{u}}^2 - L_{\hat{s}}^2) \right]$$

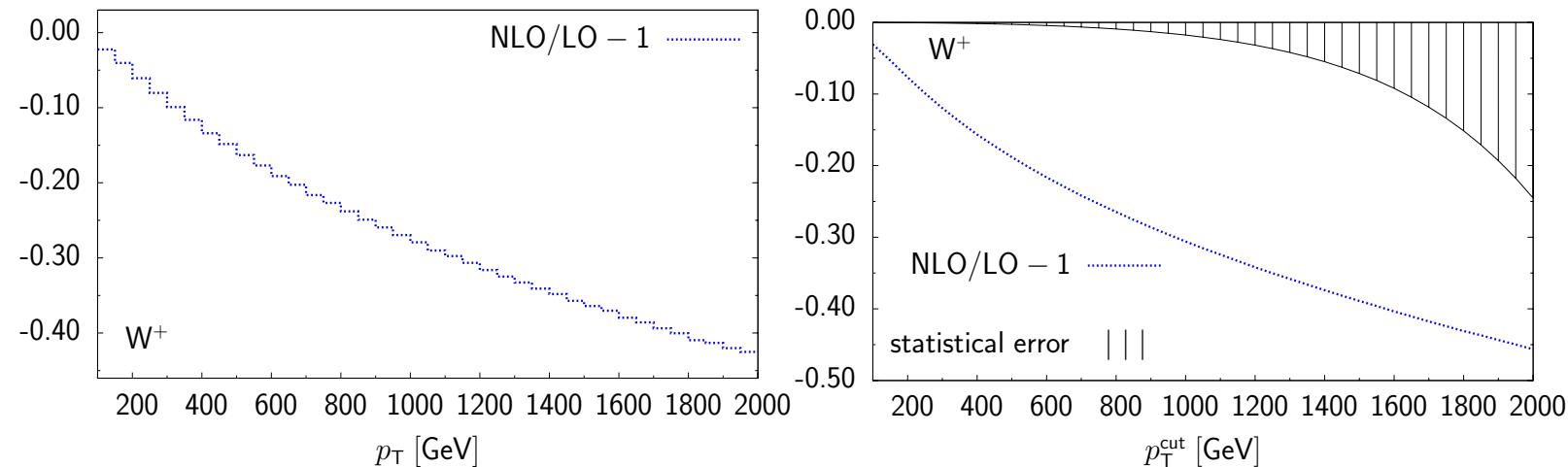
$$\begin{aligned} A^{(2)} &= \left\{ \frac{1}{2} \left( C_{qL}^{\text{ew}} + \frac{1}{s_W^2} \right) \left[ (L_{\hat{s}}^4 - 6L_{\hat{s}}^3) + \frac{1}{s_W^2} (L_{\hat{t}}^4 + L_{\hat{u}}^4 - L_{\hat{s}}^4) \right] \right. \\ &\quad \left. + \frac{1}{6} \left[ \frac{b_1}{c_W^2} \left( \frac{Y_{qL}}{2} \right)^2 + \frac{7}{8} \frac{b_2}{s_W^2} \right] L_{\hat{s}}^3 \right\} \end{aligned}$$

$$L_{\hat{r}} = \log \left( \frac{|\hat{r}|}{M_W^2} \right), \quad \hat{r} = \hat{s}, \hat{t}, \hat{u}; \quad C_{qL}^{\text{ew}} = \frac{Y_{qL}^2}{4c_W^2} + \frac{3}{4s_W^2}, \quad b_1 = -\frac{41}{6c_W^2}, \quad b_2 = \frac{19}{6s_W^2}$$

→ similar analytic results for neutral gauge boson production

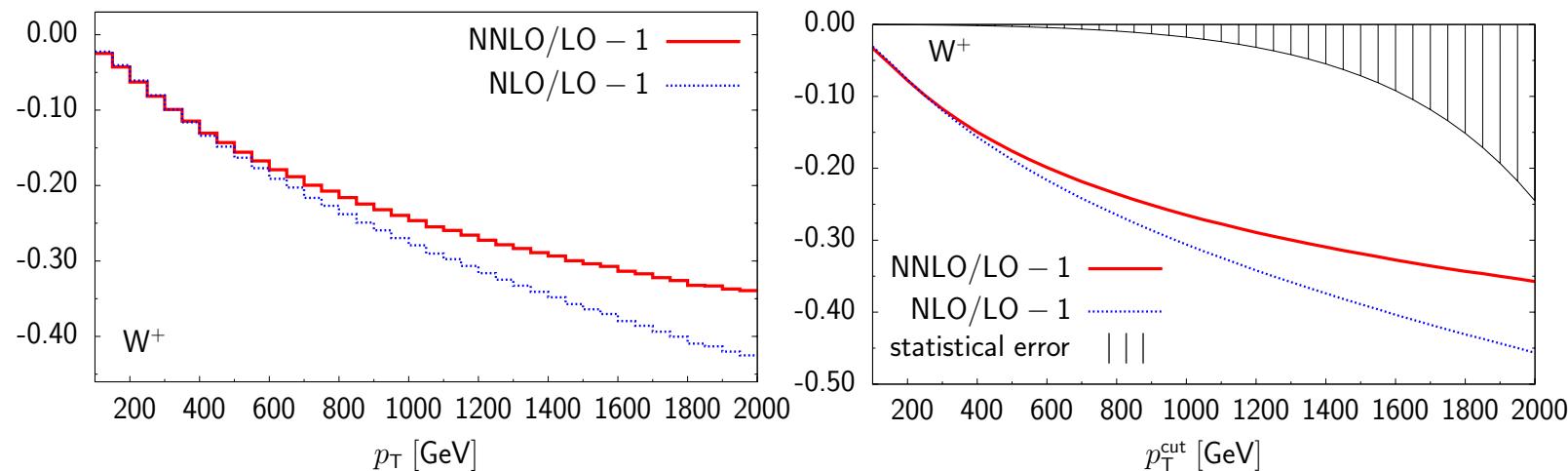
# NLL approximation: 2-loop results

Large  $p_T$   $W^+$ -boson production at the LHC



# NLL approximation: 2-loop results

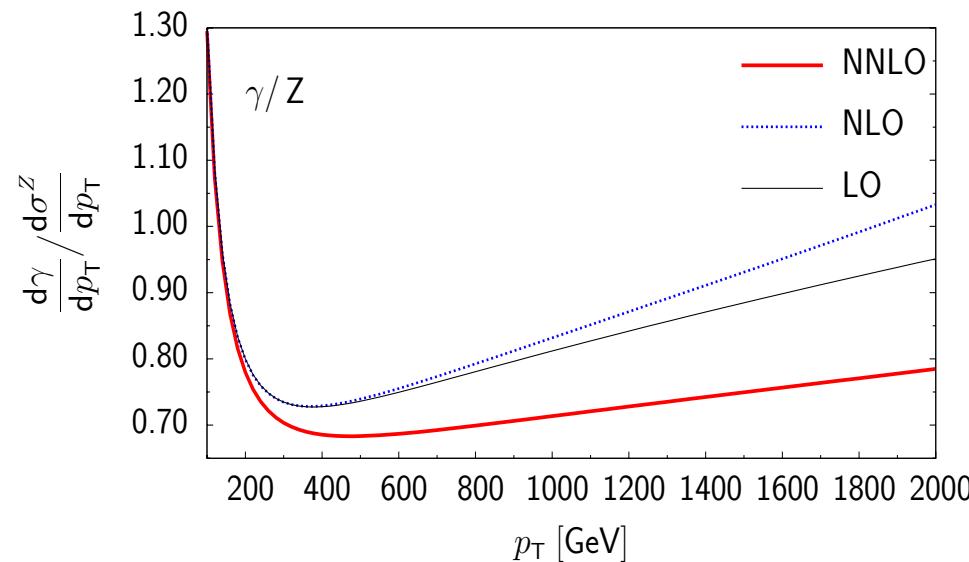
## Large $p_T$ $W^+$ -boson production at the LHC



- NLL 2-loop terms positive, up to 10% contribution (at 2 TeV)
- 1-loop + 2-loop NLL amount to up to -33% correction (at 2 TeV)
- For large range of  $p_T$  values 2-loop effects comparable with statistical error!
- very similar numbers for  $W^-$ , qualitatively similar behaviour for  $Z$  and  $\gamma$

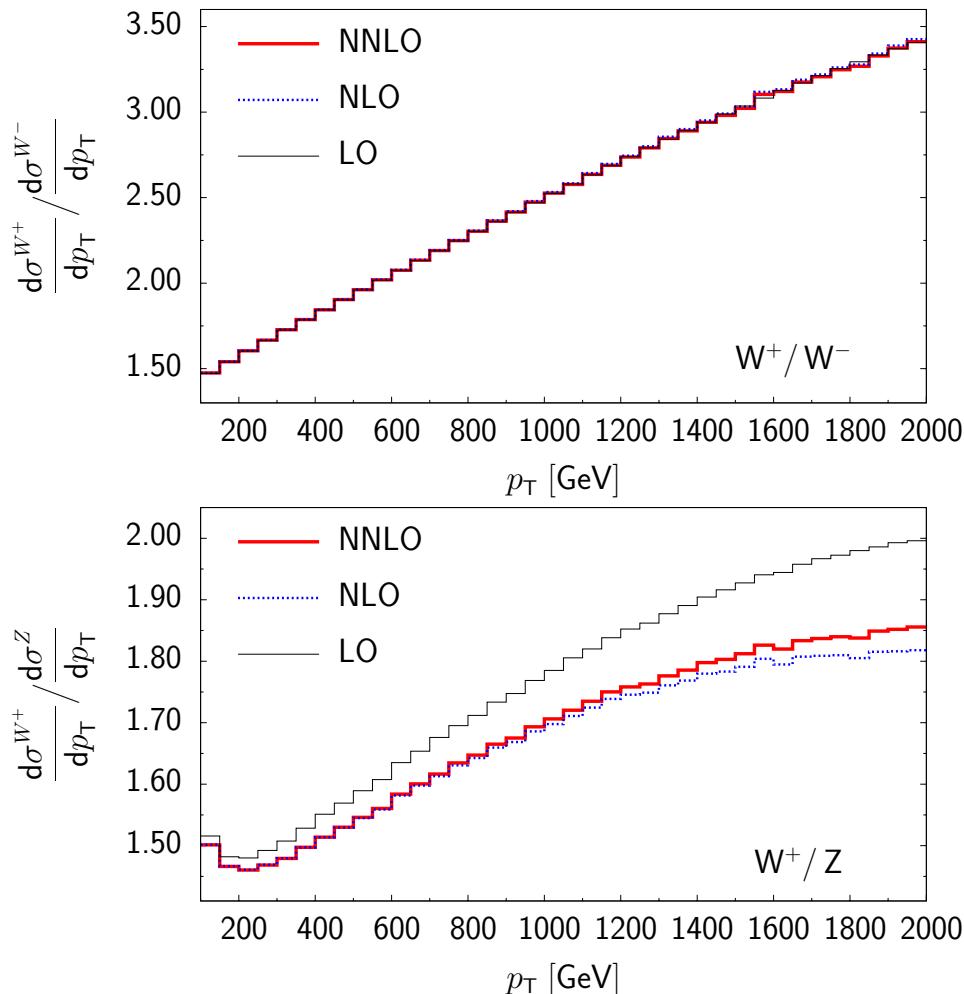
# Ratio of the $p_T$ distributions: $\gamma$ to $Z$

- Cancellation of theoretical uncertainties (PDFs,  $\alpha_S$ )
- Stability wrt. QCD corrections

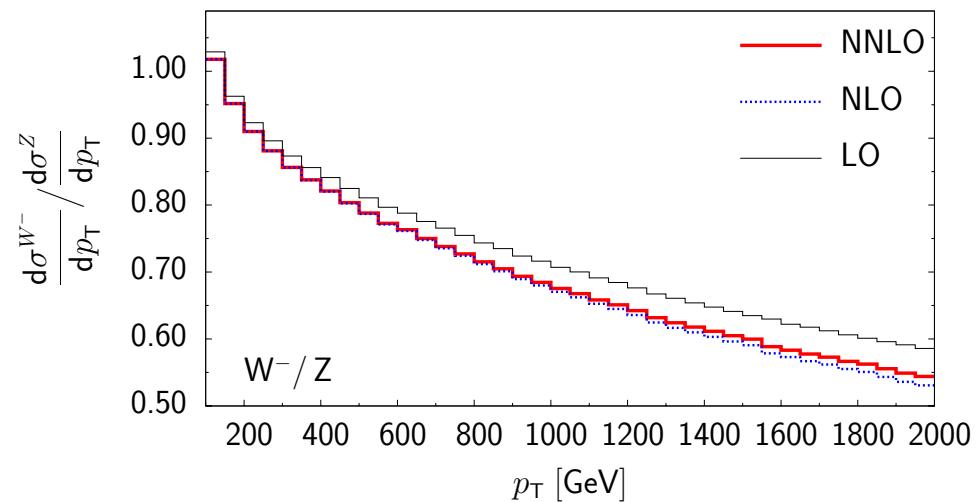


- Ratio of the LO distributions:  $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.7 - 0.8$
- EW corrections modify the ratio; strongest effect at large  $p_T$   
NLO:  $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.75 - 1$ , NNLO:  $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T} \sim 0.75 - 0.95$

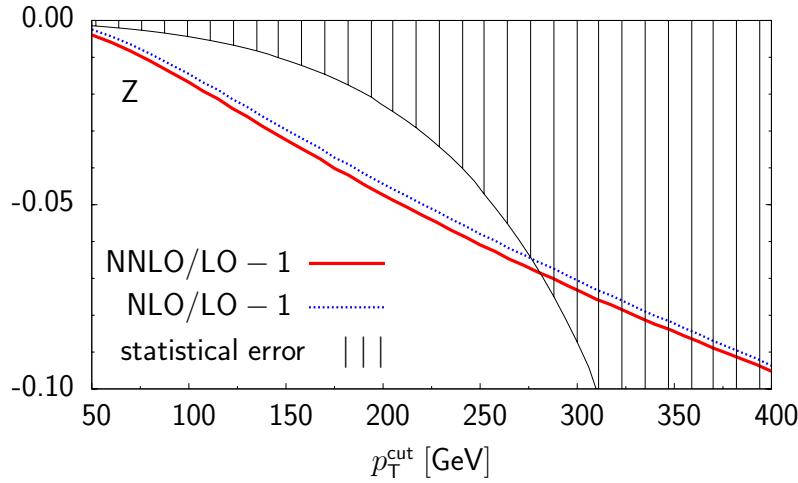
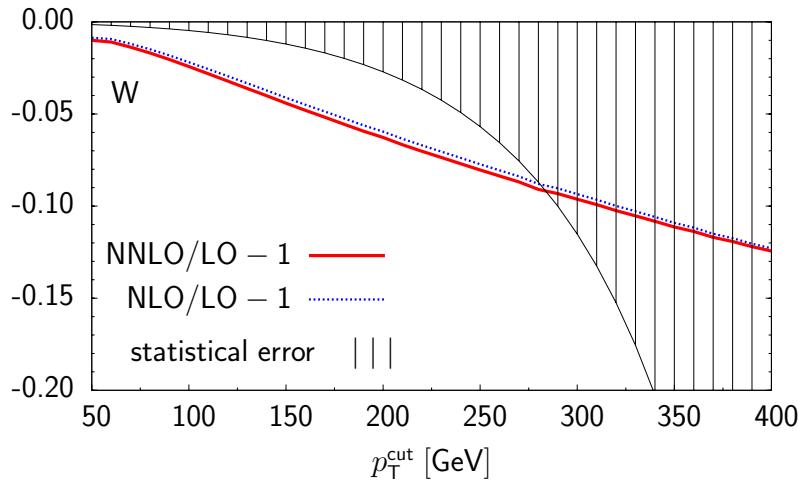
# Ratio of the $p_T$ distributions: $W^+$ to $W^-$ , $W^+$ to $Z$



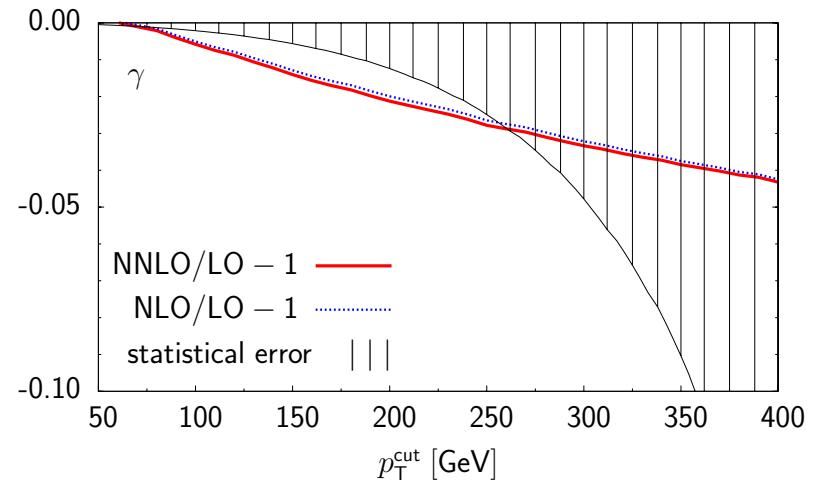
- $\frac{d\sigma^{W^+}}{dp_T} / \frac{d\sigma^{W^-}}{dp_T}$  not affected by EW corrections
- Above 1 TeV, EW corrections to  $\frac{d\sigma^{W^+} (W^-)}{dp_T} / \frac{d\sigma^Z}{dp_T}$  are of the 5 – 7% size



# Gauge boson production at the Tevatron



- $\mathcal{L} = 11\text{fb}^{-1}$
- NLO corrections bigger than stat. error
- NLL 2-loop corrections small



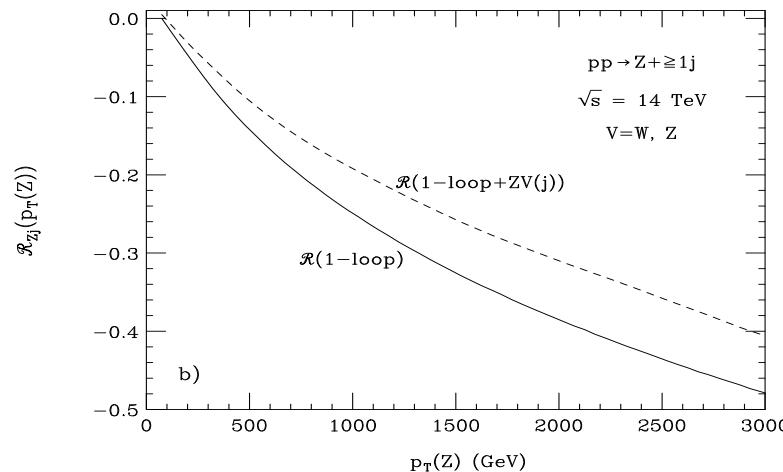
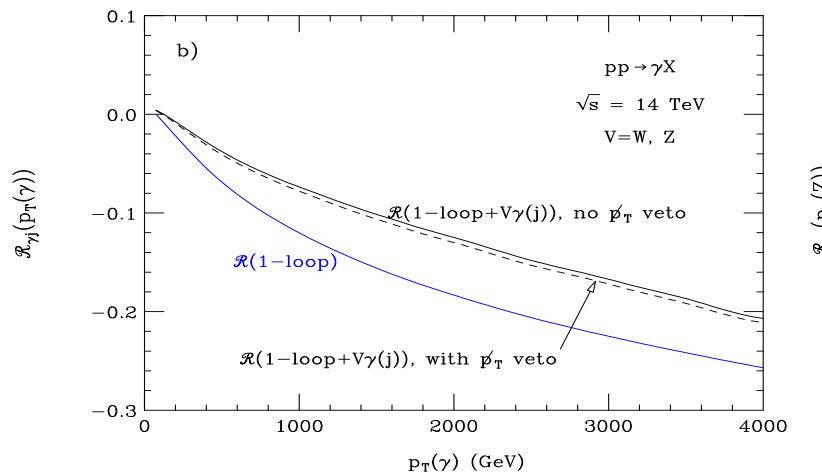
# Effects of real emission

[Baur'06]

- No real corrections for exclusive processes
- Effects from *real* weak boson emission for inclusive processes
- Violation of Bloch-Nordsieck theorem for non-abelian gauge theories  $\Rightarrow$  logarithmic terms survive

[Catani, M.Ciafaloni, P. Ciafaloni, Comelli]

- Moderate effects at the LHC

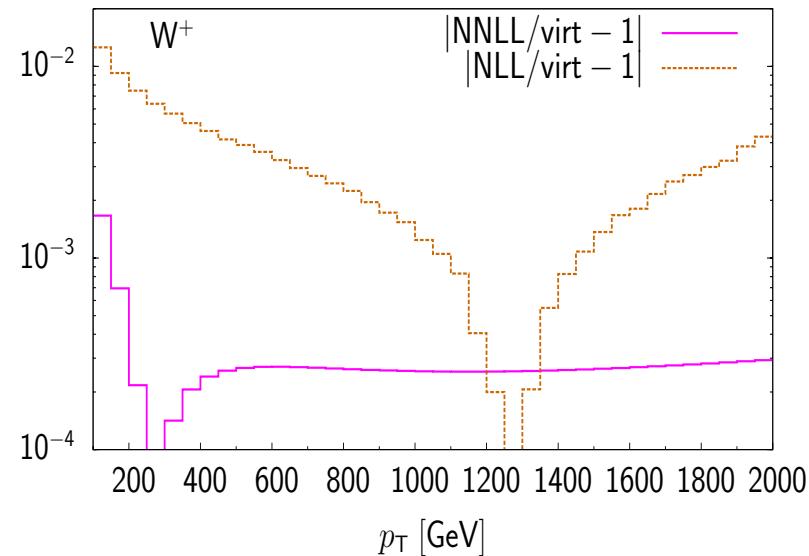
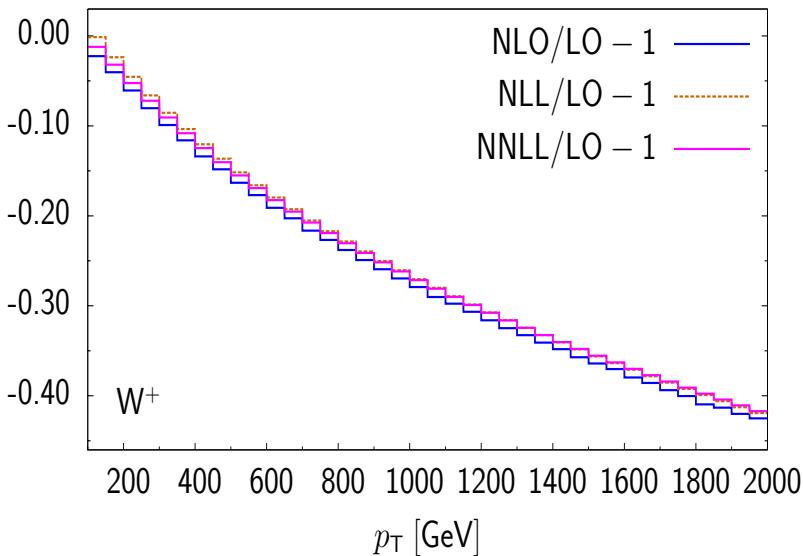


# Summary

- Analytic results for the full  $\mathcal{O}(\alpha)$  correction to the  $p_T$  distribution of  $W$ -bosons,  $Z$ -bosons and direct photons
- NNLL approximation of the NLO: compact expression, excellent approximation
- NLL approximation: 1-loop and 2-loop corrections
- Conclusion: EW corrections important for the precise knowledge of the production cross sections at large  $p_T$  (large logs at TeV scales!)
  - Negative 1-loop corrections of the order of tens of percent at high  $p_T$  at the LHC
  - Positive 2-loop NLL corrections of the order of several percent at high  $p_T$  at the LHC  $\Rightarrow$  relevant for the analysis!
  - Ratio  $\frac{d\sigma^\gamma}{dp_T} / \frac{d\sigma^Z}{dp_T}$  and  $\frac{d\sigma^{W^\pm}}{dp_T} / \frac{d\sigma^Z}{dp_T}$ : significant effects due to EW corrections at large  $p_T$
  - Same study for the Tevatron: corrections less significant numerically

# Backup: High-energy approximation of the one-loop

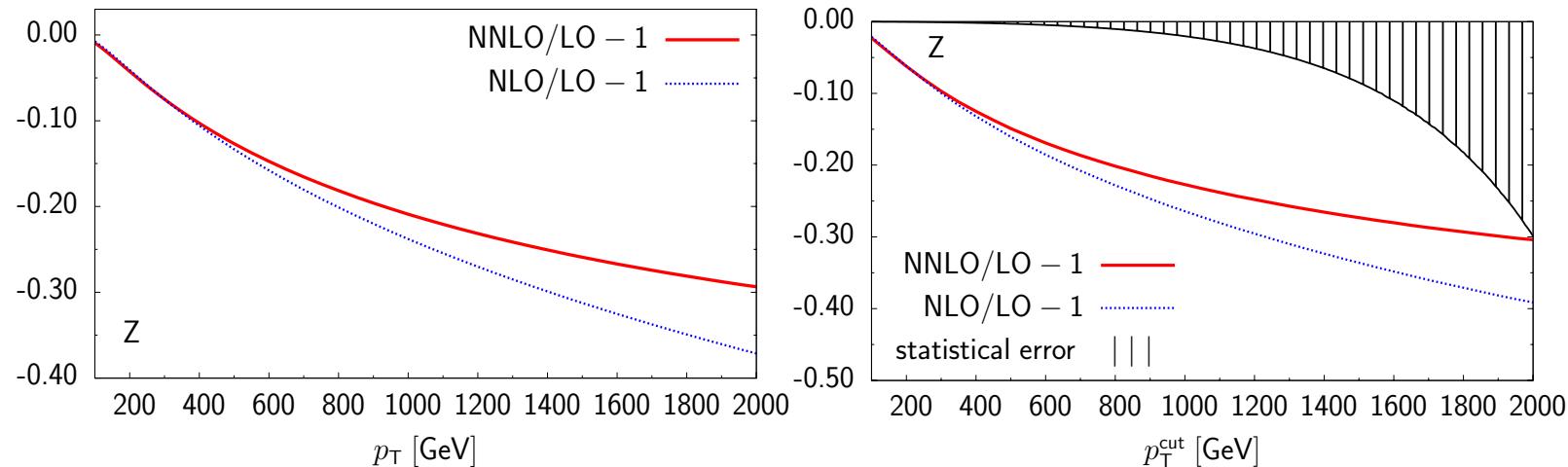
## $W^+$ production at the LHC



- NLL approximation: percent (or better) level
  - ~ 1% deviation from NLO at low  $p_T$
  - ~ 0.2% deviation from NLO at  $p_T = 2$  TeV
- NNLL approximation: permille level
- Similar behaviour for  $W^-$  production

# Backup: NLL approximation: 2-loop results

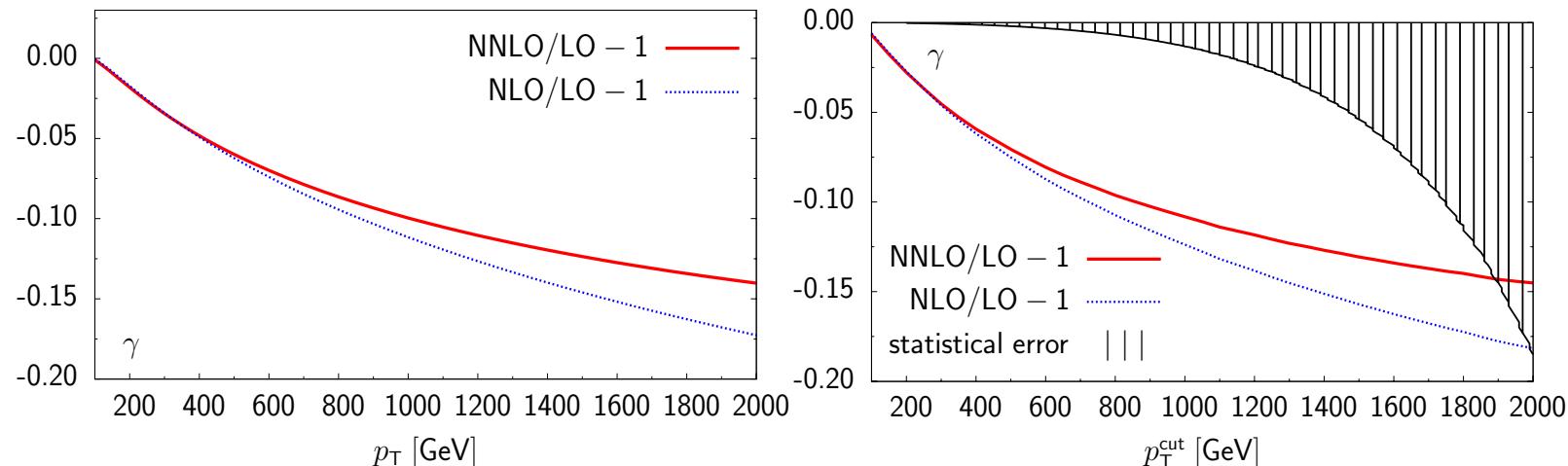
## Large $p_T$ $Z$ -boson production at the LHC



- NLL 2-loop terms positive, up to 8% contribution (at 2 TeV)
- 1-loop + 2-loop NLL amount to up to -30% correction (at 2 TeV)
- For large range of  $p_T$  values 2-loop effects comparable with statistical error!

# Backup: NLL approximation: 2-loop results

Large  $p_T$  photon production at the LHC



- NLL 2-loop terms positive, up to 3% contribution (at 2 TeV)
- 1-loop + 2-loop NLL amount to up to -14% correction (at 2 TeV)
- For large range of  $p_T$  values 2-loop effects comparable with statistical error!