Two-loop EW Sudakov logarithms for massive fermion scattering

Stefano Pozzorini
MPI Munich

in collaboration with *A. Denner, B. Jantzen*

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Outline of the talk

(1) **EW logarithms at the TeV scale: strategies and results** [Fadin, Lipatov, Martin, Melles, Jantzen, Kühn, Moch, Penin, Smirnov, M.Ciafaloni, P.Ciafaloni, Comelli, Hori, Kawamura, Kodaira, Beenakker, Werthenbach, Denner, S. P.]

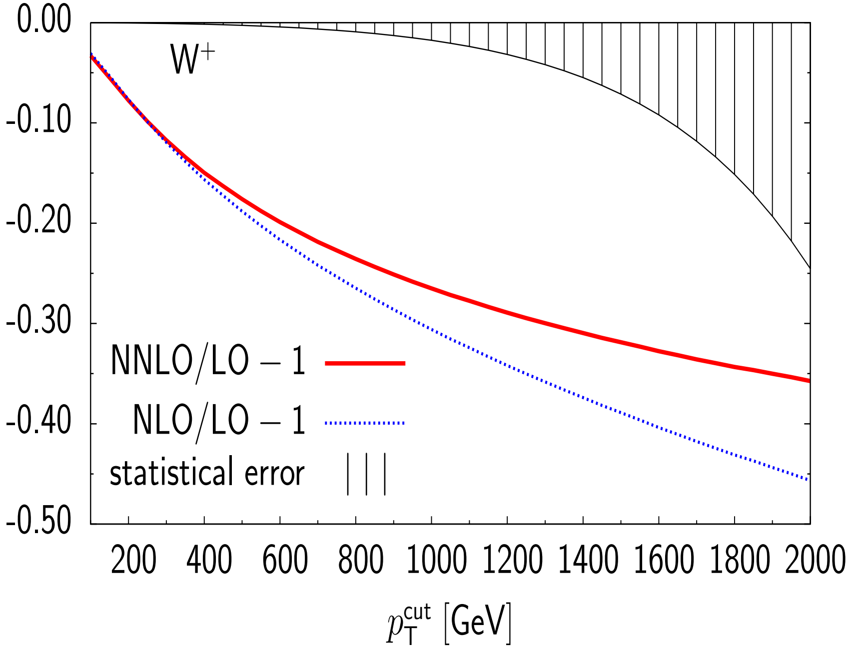
(2) **Two-loop logarithms for massive fermion scattering** [Denner, Jantzen, S.P.]

PART 1

Introduction: electroweak corrections at the TeV scale

Example: electroweak corrections to $pp \to W + \text{jet}$ at the LHC

$(\sigma^{NLO} - \sigma^{LO})/\sigma^{LO}$



Kühn, Kulesza, S.P., Schulze (2007)

At small p_T

- Corrections of $\mathcal{O}(\alpha) \sim 1\%$

At $p_T > 100$ GeV

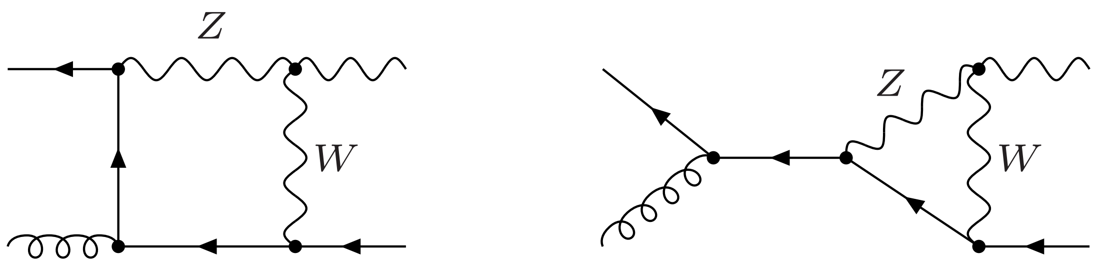
- **large negative corrections $\gg$ 1%**
- **increase with p_T**
- -30% **at $p_T \sim$ 1TeV !**

Origin: scattering energy $\gg$ characteristic scale of EW corrections

Large double logarithms

$$\frac{\delta\sigma}{\sigma} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2}\ln^2\left(\frac{s}{M_{W,Z}^2}\right) \simeq -26\% \qquad \text{at} \quad \sqrt{s} \sim 1\,\mathrm{TeV}$$

from vertex and box diagrams involving virtual W and Z bosons



Kuroda, Moultaka, Schildknecht (1991); Degrassi, Sirlin (1992); Beenakker, Denner, Dittmaier, Mertig, Sack (1993); Denner , Dittmaier, Schuster (1995); Denner, Dittmaier, Hahn (1997), Beccaria, Montagna, Piccinini, Renard, Verzegnassi (1998); Ciafaloni, Comelli (1999)

Affect all hard scattering processes at LHC, ILC, CLIC!

Asymptotic expansion of 1-loop EW corrections

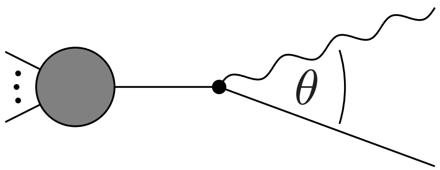
General form of M_W^2/s expansion

$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + C_1 \underbrace{\ln\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + \tilde{C}_1 \underbrace{\ln\left(\frac{s}{\mu_{\mathrm{R}}^2}\right)}_{\text{UV}} + C_0\right]$$

Terms of $\mathcal{O}(M_W^2/s)$ negligible for $s \sim 1\,\mathrm{TeV}^2$

$$C_k = \sum_{j=0}^{\infty} C_k^{(j)} \left(\frac{M_W^2}{s}\right)^j \quad \rightarrow \quad C_k^{(0)}$$

Mass singularities from **soft/collinear gauge bosons** coupling to external lines

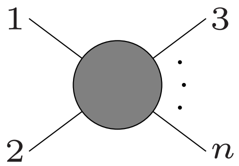


$$\Longrightarrow \int \frac{\mathrm{d}E}{E} \int \frac{\mathrm{d}\cos\theta}{(1-\cos\theta)}$$

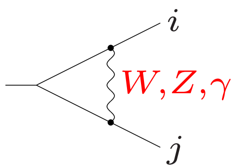
Analogies with QED and QCD? **Factorization and universality?**

Factorization and universality of one-loop EW logarithms [Denner, S.P. (2001)]

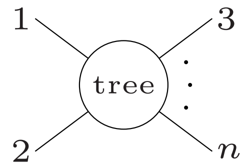
For **arbitrary processes** $(e, \nu, u, d, t, b, \gamma, Z, W^{\pm}, H, g)$



$$= \underbrace{\left(\frac{1}{2} \sum_{j \neq i} \right.}$$

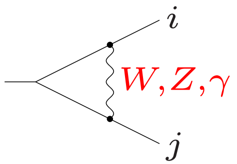


$$\left. \vphantom{\sum} \right) \times$$



universal

proven with **collinear Ward identities** for spontaneously broken YM theories



$$= \frac{\alpha}{4\pi} \left\{ \sum_{V=\gamma,Z,W} I_i^V I_j^V \ln^2 \frac{r_{ij}}{M_W^2} + 2 I_i^Z I_j^Z \ln \frac{r_{ij}}{M_W^2} \ln \frac{M_W^2}{M_Z^2} + \gamma_{ij}^{\mathrm{ew}} \ln \frac{s}{M_W^2} \right.$$

$$\left. + Q_i Q_j \sum_{k=i,j} \left[\ln \frac{r_{ij}}{m_k^2} \ln \frac{M_W^2}{\lambda^2} - \frac{1}{2} \ln^2 \frac{M_W^2}{m_k^2} - \ln \frac{M_W^2}{\lambda^2} - \frac{1}{2} \ln \frac{M_W^2}{m_k^2} \right] \right\}$$

Simple and general recipe for LL and NLL at one loop

1% precision at 1 TeV requires two-loop EW effects!

Leading two-loop logarithms

$$\frac{\delta\sigma}{\sigma} \sim \frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\% \qquad \text{at} \quad \sqrt{s} \sim 1\,\mathrm{TeV}$$

Asymptotic high-energy expansion for $M_W^2/s \ll 1$

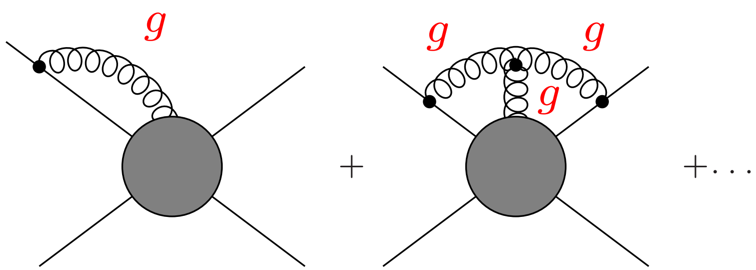
$$\alpha^2 \Bigg[C_4 \underbrace{\ln^4\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + C_3 \underbrace{\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + \tilde{C}_3 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)\ln\left(\frac{s}{\mu_{\mathrm{R}}^2}\right)}_{\text{soft}\times\text{UV}} + \ldots \Bigg]$$

Analogies with QCD? Exponentiation?

InfraRed Evolution Equation (IREE) for QCD matrix elements

Logarithmic dependence on soft-collinear cut-off $\frac{\partial \mathcal{M}}{\partial \ln(\mu_T)} = K(\mu_T)\mathcal{M}$

$\mathcal{M}(\mu_T) =$



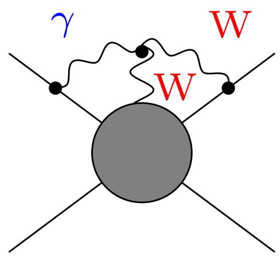
$$= \exp\left[-\int_{\mu_T}^{Q} \frac{d\mu}{\mu} K(\mu)\right] \mathcal{M}(Q)$$

$\mathbf{k}_T(\mathbf{gluon}) > \mu_T$

Gribov(1967); Kirschner, Lipatov (1982)

How to deal with mass gap in the electroweak gauge sector?

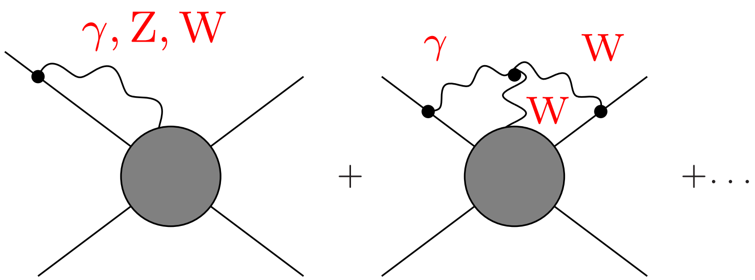
$M_\gamma = 0 \ll M_Z \sim M_W$:



$$\Rightarrow \quad \alpha^2 \frac{1}{\varepsilon} \ln^3\left(\frac{s}{M_W^2}\right)$$

Solution proposed by Fadin, Lipatov, Martin, Melles (2000)

SU(2)×U(1) regime: $\mu_\mathrm{T} > M_\mathrm{W,Z}$

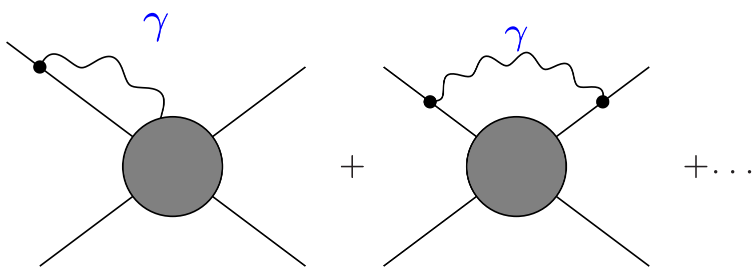


mass gap irrelevant ($M_\gamma = M_\mathrm{Z} = M_\mathrm{W}$)

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_\mathrm{T})} = K_\mathrm{EW}(\mu_\mathrm{T})\mathcal{M}$$

as in symmetric SU(2) × U(1) theory

$\mathrm{U(1)_{em}}$ regime: $\mu_\mathrm{T} < M_\mathrm{W,Z}$



weak boson frozen ($M_\mathrm{Z}, M_\mathrm{W} = \infty$)

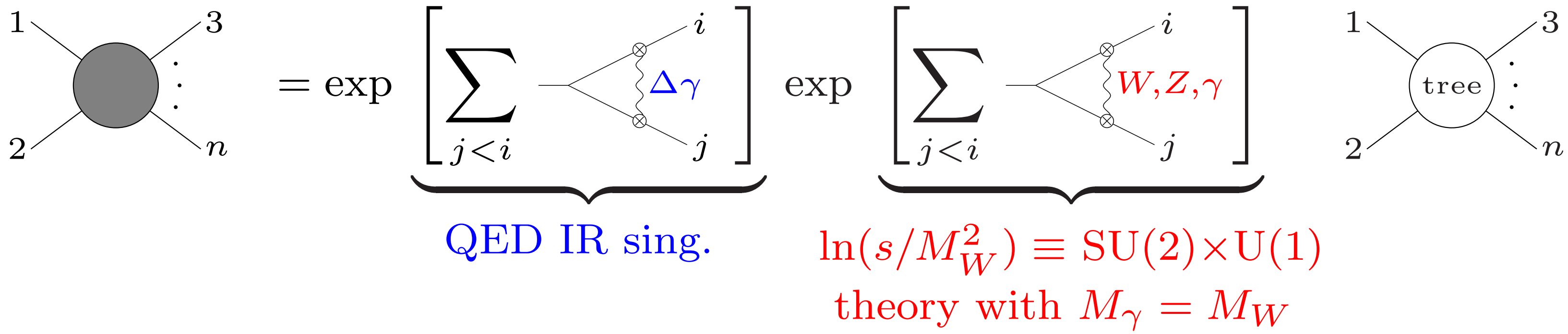
$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_\mathrm{T})} = K_\mathrm{QED}(\mu_\mathrm{T})\mathcal{M}$$

as in QED

Symmetry-breaking problem reduced to two problems with unbroken symmetry

Resummation formula based on IREE approach

Double exponentiation resulting from $M_\gamma \ll M_W \sim M_Z$



Existing applications

$$\alpha^2 \left[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right)}_{\text{Fadin, Lipatov, Martin, Melles (2000)}} + \underbrace{C_3 \ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Melles (2001–2004)}} + \underbrace{C_2 \ln^2\left(\frac{s}{M_W^2}\right)}_{\text{Kühn, Moch, Penin, Smirnov (2000–2003)}} + \underbrace{C_1 \ln^1\left(\frac{s}{M_W^2}\right)}_{\text{Jantzen, Kühn, Penin, Smirnov (2004,2005)}} \right]$$

First two terms: **arbitrary processes** (vev = 0)

Last two terms: **massless $f\bar{f} \to f'\bar{f}'$ processes** ($\sin^2\theta_{\mathrm{w}} = 0$)

Very recently: $e^+e^- \to W^+W^-$ [Kühn, Metzler, Penin], SCET approach [Manohar *et. al.*]

Two-loop calculations based on EW Feynman rules

The (few) existing results agree with the IREE

$$\alpha^2 \left[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right)}_{\text{Melles; Hori,Kawamura, Kodaira (2000)}} + \underbrace{C_3^{\text{ang}} \ln\left(\frac{t}{s}\right) \ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Denner, Melles,P. (2003)}} + \underbrace{C_3 \ln^3\left(\frac{s}{M_W^2}\right)}_{\text{S.P. (2004)}} \right]$$

Beenakker, Werthenbach (2000,2002) — Denner, Jantzen, S.P. (2006)

arbitrary processes involving $Z, W, H, b, t, \ldots$ — massless $f_1 f_2 \rightarrow f_3 \ldots f_n$

Technology for two-loop NLL for arbitrary processes now available

- electroweak collinear Ward identities for process-independent treatment
- automatic algorithm for 2-loop diagrams in NLL approximation

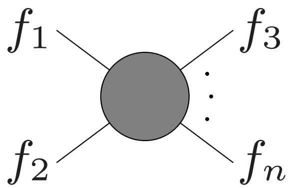
PART 2

Two-loop EW Sudakov logarithms for massive fermion scattering

[**Denner, Jantzen, S.P. (preliminary results)**]

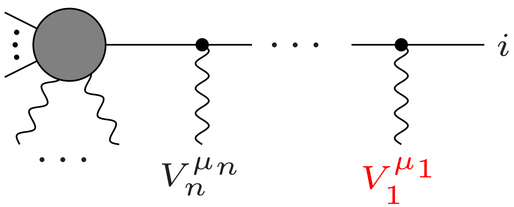
Feynman diagrams and soft-collinear approximation

Scattering of n fermions $(l, \nu_l, u, d, s, c, b, t)$

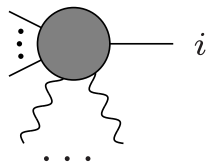


for $\quad Q^2 \gg M_W^2, M_Z^2, M_H^2, m_t^2 \quad$ and $\quad m_{f\neq t}^2 = 0$

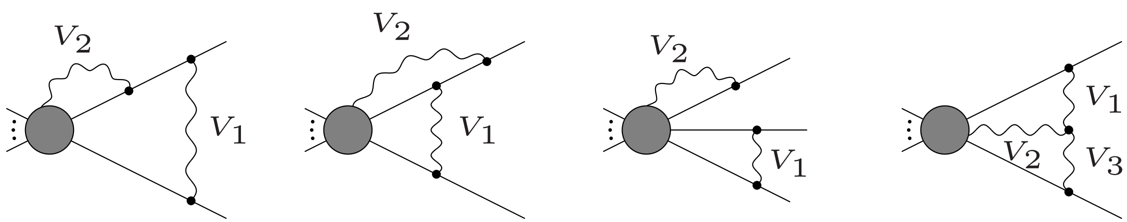
Soft-collinear fermion-boson vertices



$$= \frac{-2eI_i^{V_1}(p_i+q_1)^{\mu_1}}{(p_i+q_1)^2-m_1^2}\cdots\frac{-2eI_i^{V_n}(p_i+\tilde{q}_n)^{\mu_n}}{(p_i+\tilde{q}_n)^2-m_n^2} \times$$



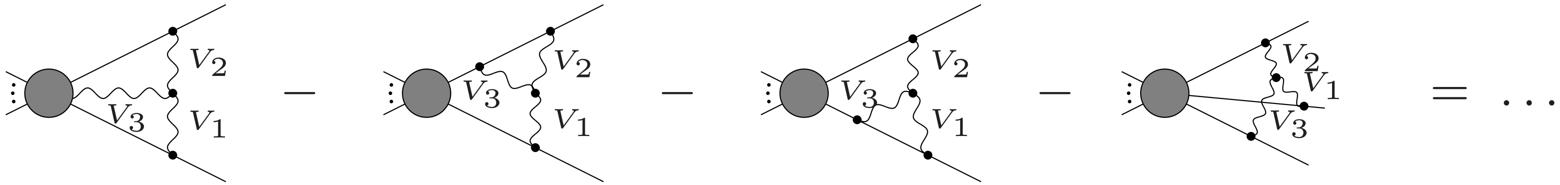
Two-loop diagrams that yield $\ln^3(Q^2/M^2)$ soft/collinear contributions



involve (at least) one soft/collinear gauge boson coupling to two external lines

(A) Non-factorizable two-loop diagrams

Collinear gauge bosons coupling to external and internal lines



Collinear Ward identities for SB non-abelian theories [Denner, Jantzen, S.P. (2001,2006)]

$$\ldots = \mu_0^{4\epsilon} \int \frac{\mathrm{d}^D q_1}{(2\pi)^D} \int \frac{\mathrm{d}^D q_2}{(2\pi)^D} \frac{4\mathrm{i}e^2 g_2 \varepsilon^{V_1 V_2 V_3}}{(q_1^2 - M_{V_1}^2)(q_2^2 - M_{V_2}^2)(q_3^2 - M_{V_3}^2)(p_i - q_2)^2 (p_j - q_1)^2}$$

$$\times \lim_{q_1^\mu \to 0} \lim_{q_2^\mu \to x p_i^\mu} (p_i - q_2)^{\mu_2} (p_j - q_1)^{\mu_1} \left[g_{\mu_1 \mu_2} (q_1 - q_2)^{\mu_3} + g_{\mu_2}^{\mu_3} (q_2 + q_3)_{\mu_1} - g_{\mu_1}^{\mu_3} (q_3 + q_1)_{\mu_2} \right]$$

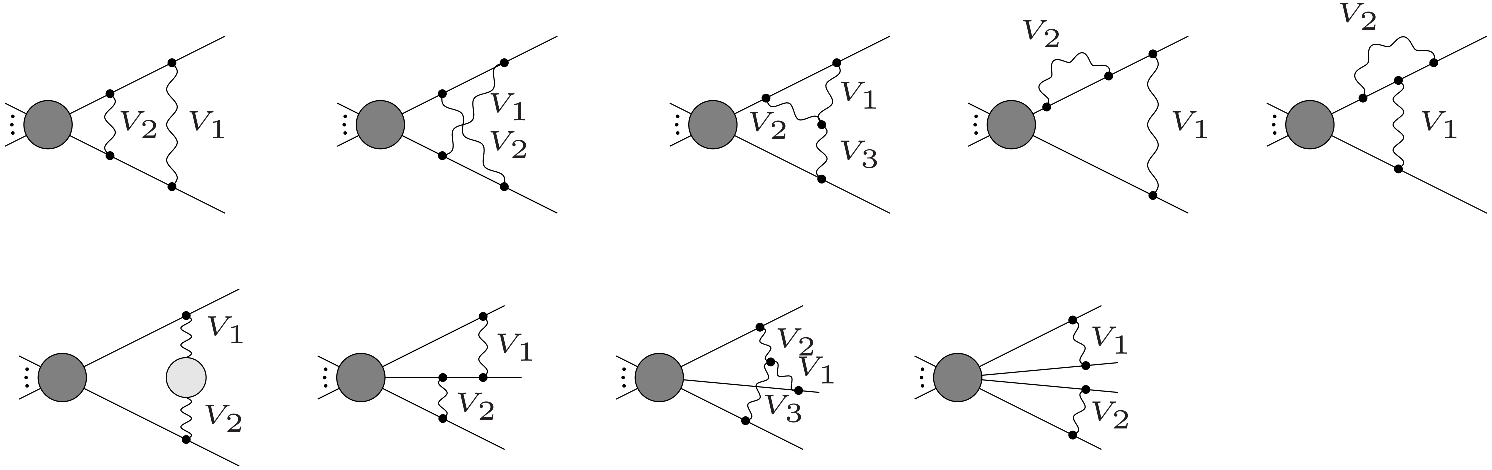
$$\times \sum_{\varphi_i', \varphi_j'} \Bigg\{ G_{\mu_3}^{[\underline{\bar{V}_3}\, \underline{\varphi_i'}]}(q_3, p_i - q_2) u(p_i, \kappa_i) + \frac{2(p_j + q_2)_{\mu_3}}{(p_j + q_2)^2} \sum_{\varphi_j''} e I^{\bar{V}_3}_{\varphi_j'' \varphi_j'} \mathcal{M}_0^{\varphi_1 \cdots \varphi_i' \cdots \varphi_j'' \cdots \varphi_n}$$

$$+ \sum_{\substack{k=1 \\ k \neq i,j}}^{n} \frac{2(p_k + q_3)_{\mu_3}}{(p_k + q_3)^2} \sum_{\varphi_k'} \mathcal{M}_0^{\varphi_1 \cdots \varphi_i' \cdots \varphi_j' \cdots \varphi_k' \cdots \varphi_n} e I^{\bar{V}_3}_{\varphi_k' \varphi_k} \Bigg\} I^{\bar{V}_1}_{\varphi_j' \varphi_j} I^{\bar{V}_2}_{\varphi_i' \varphi_i} = 0$$

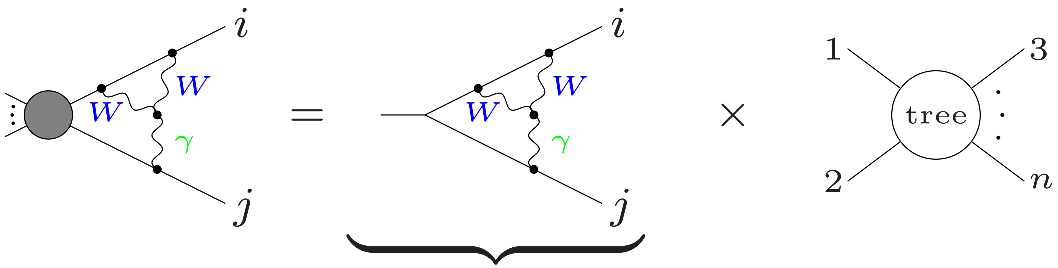
This cancellation mechanism permits process-independent treatment

(B) Factorizable two-loop diagrams

Soft/collinear gauge bosons coupling only to external lines



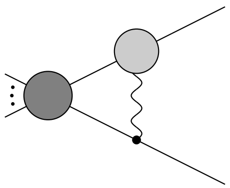
Factorization and explicit calculation using sector decomposition [Denner, S.P. (2004)] and expansion by regions [Jantzen, Smirnov (2006)]



$$\frac{\mathrm{i}e^4 \epsilon^{W\bar{W}\gamma} I_i^W I_i^{\bar{W}} I_j^{\gamma}}{s_\mathrm{W}} \left(\frac{s}{Q^2}\right)^{2\epsilon} \left[-\frac{1}{3}L^3\epsilon^{-1} - 5L^4 - 6\epsilon^{-3} - 6L\epsilon^{-2} - 2L^2\epsilon^{-1} + \frac{2}{3}L^3\right]$$

(C) UV singularities and renormalization

Soft-collinear gauge-boson exchange with one-loop UV insertions



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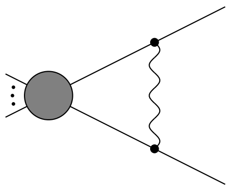
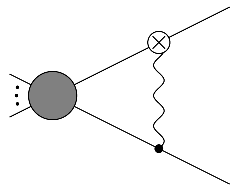
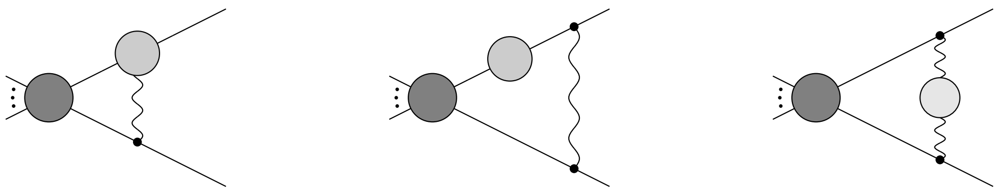


$$\times \underbrace{\left[\frac{1}{\epsilon}\left(\frac{Q}{\Lambda_{\text{loop}}}\right)^{2\epsilon} - \frac{1}{\epsilon}\right]}_{\overline{\text{MS}}\text{ subtraction at } \mu = Q} \quad +$$



finite renormalization

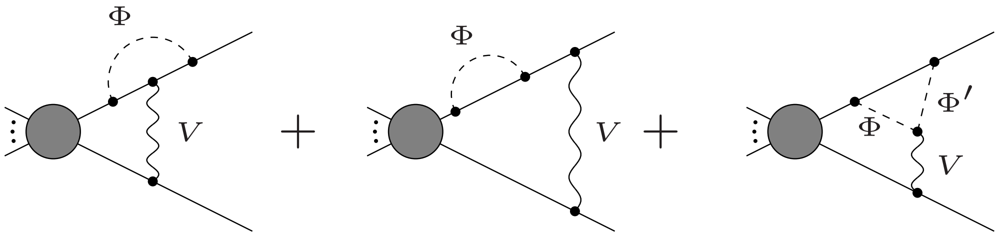
UV contributions only from subdiagrams with $\Lambda_{\text{loop}} \ll Q$



Involve virtual gauge bosons and scalar particles $(H, \chi, \phi^{\pm})$

(D) Yukawa contributions

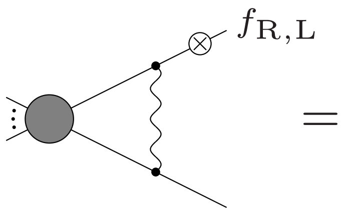
Cancellation due to global gauge invariance of $\mathcal{L}_{\mathrm{Yuk}} = -\bar{\Psi}\Phi\, G^{\Phi}_{\rho}\omega_{\rho}\Psi$



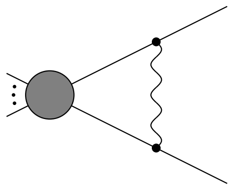
$$\Rightarrow \Big\{ \tfrac{1}{6}L^3 + \delta_{V\gamma}\left[L\epsilon^{-2} + L^2\epsilon^{-1} + \tfrac{1}{3}L^3\right.$$

$$\left. -\delta_{i,t}\left(\tfrac{1}{2}L\epsilon^{-2} + \tfrac{3}{4}L^2\epsilon^{-1} + \tfrac{7}{12}L^3\right) - \delta_{j,t}\left(\tfrac{1}{2}L\epsilon^{-2} + \tfrac{5}{4}L^2\epsilon^{-1} + \tfrac{7}{4}L^3\right)\right]\Big\}\, G^{\Phi}_{\mathrm{R}} \underbrace{\left[I^{V}_{\mathrm{R}} G^{\Phi}_{\mathrm{L}} - G^{\Phi}_{\mathrm{L}} I^{V}_{\mathrm{L}} - G^{\Phi'}_{\mathrm{L}} I^{V}_{\Phi'\Phi}\right]}_{=0}$$

Yukawa contr. only from WF renormalization $(C^{t,b}_{\mathrm{L}} = 1, C^{t}_{\mathrm{R}} = 2, C^{b}_{\mathrm{R}} = 0)$

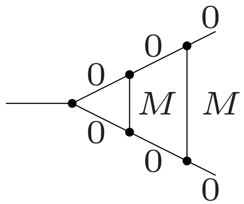


$$= -\frac{\alpha}{8\pi}\frac{m_{\mathrm{t}}^2}{4s_{\mathrm{W}}^2 M_{\mathrm{W}}^2} C^{f}_{\mathrm{R,L}} \underbrace{\frac{1}{\epsilon}\left[\left(\frac{Q}{m_{\mathrm{t}}}\right)^{2\epsilon} - 1\right]}_{\ln(Q^2/m_{\mathrm{t}}^2)} \times$$

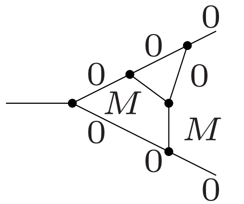


Algorithm based on sector decomposition [Denner, S.P. (2004)]

- arbitrary two-loop diagrams in the limit $L = \ln(Q^2/M^2) \gg 1$
- photons and light fermions massless in $D = 4 - 2\epsilon$

$(q_2 p_1)(q_2 p_2) \times$ 

$$= -\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[\frac{1}{16\epsilon} L^2 + \frac{1}{24} L^3\right]$$

$(q_2 p_1) \times$ 

$$= -\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \frac{1}{Q^2} \left[\frac{5}{48} L^4 + \frac{1}{12\epsilon} L^3 + \frac{1-2\gamma_{\mathrm{E}}}{12} L^3\right]$$

- completely automatized to **NLL accuracy**; computing time $= \mathcal{O}(10\mathrm{s})$

Multi-loop integrals with sector decomposition (one-slide summary)

Hepp(1966); Denner, Roth (1996); Binoth, Heinrich(2000); Denner, S.P. (2004)

(A) **L-loop integral with I propagators: Feynman parametrization**

$$G = \int_0^1 \prod_{i=1}^{I} \mathrm{d}\alpha_j \, \delta(1 - \sum_{s=1}^{I} \alpha_s) \frac{\Gamma(e)\mathcal{U}(\vec{\alpha})^{-e}}{\left[\mathcal{P}(\vec{\alpha}) + (M^2/Q^2)\mathcal{R}(\vec{\alpha})\right]^f} \quad \text{with} \quad M^2/Q^2 \ll 1$$

(B) **Isolate mass singularities in FP space: sector decomposition**

$$G' = \int_0^1 \prod_{i=1}^{m} \mathrm{d}\beta_i \int_0^1 \prod_{j=1}^{n} \mathrm{d}\alpha_j^f \frac{\mathcal{G}(\vec{\alpha};\vec{\beta})}{\left[\alpha_1\alpha_2\ldots\alpha_n + (M^2/Q^2)\mathcal{H}(\vec{\alpha};\vec{\beta})\right]^f} \Rightarrow \ln^n \text{ singularity!}$$

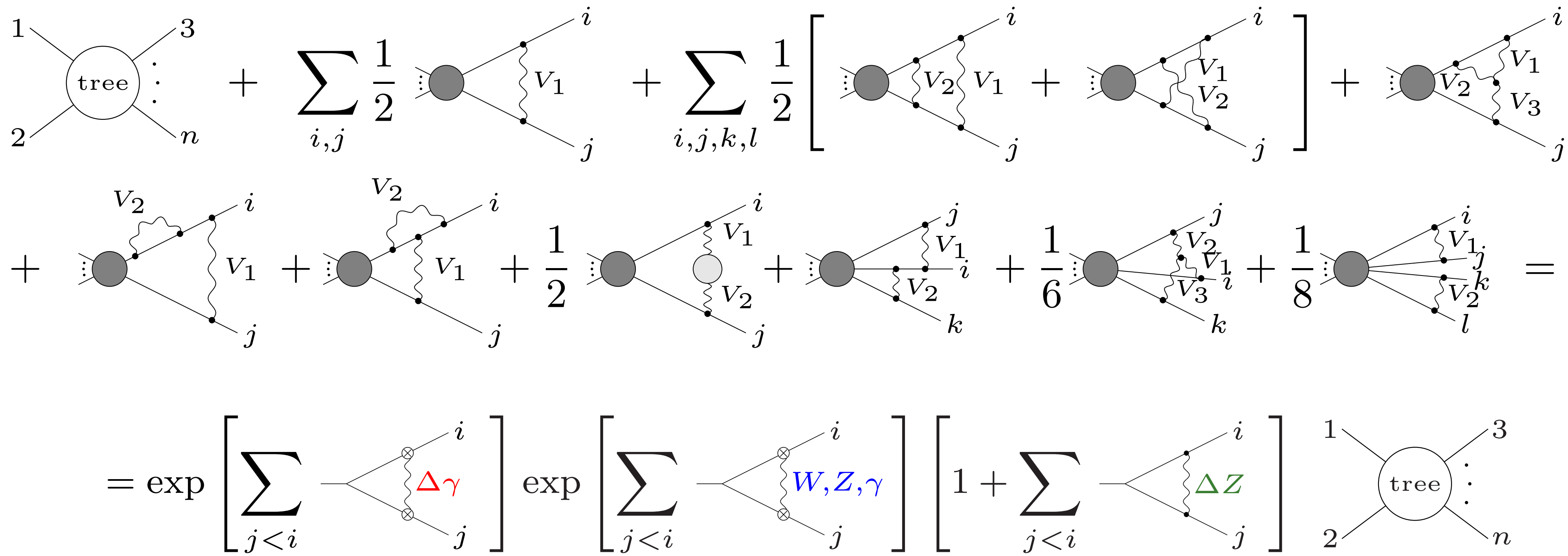
(C) **Extract logarithms: singular α_j-integrations**

$$G' = \frac{1}{n!}\int_0^1 \prod_{i=1}^{m} \mathrm{d}\beta_i \left\{ \mathcal{G}(\vec{0},\vec{\beta})\ln^n\left(\frac{Q^2}{M^2}\right) + (n-1)\left\{ \mathcal{G}(\vec{0},\vec{\beta})\left[\ln[\mathcal{H}(\vec{0},\vec{\beta})] + \sum_{k=1}^{f-1}\frac{1}{k}\right] \right.\right.$$
$$\left.\left. + \sum_{j=1}^{n}\int_0^1 \frac{\mathrm{d}\alpha_j}{\alpha_j}\left[\mathcal{G}(0,\ldots,\alpha_j,\ldots,0,\vec{\beta}) - \mathcal{G}(\vec{0},\vec{\beta})\right] \right\} \ln^{n-1}\left(\frac{Q^2}{M^2}\right) + \mathcal{O}(\ln^{n-2}) \right\}$$

(D) **Compute LL and NLL coefficients: non-singular β_i-integrations**

(2-loop diagrams $\Rightarrow$ integrations 2-dimensional and simple)

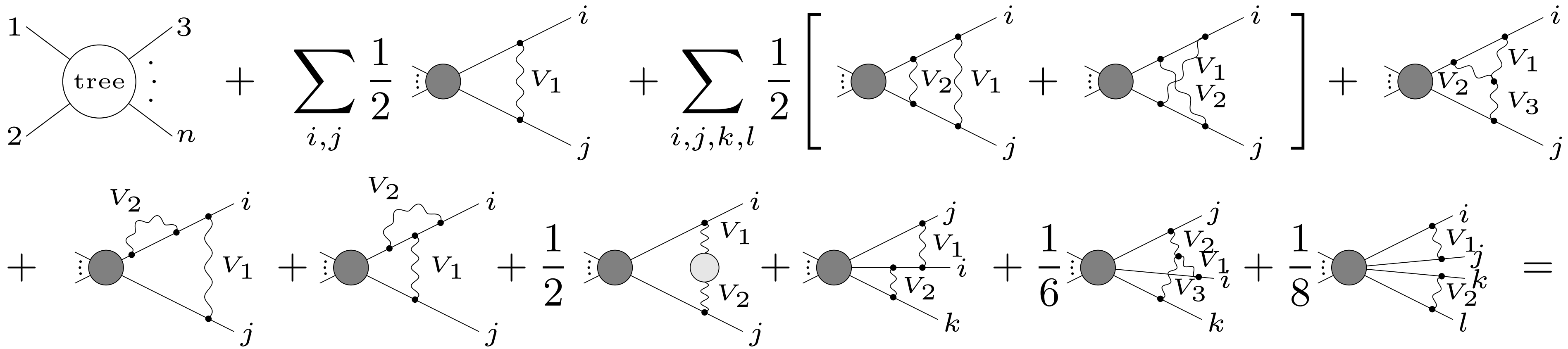
Two-loop NLL (preliminary) result for $f_1 f_2 \to f_3 \dots f_n$

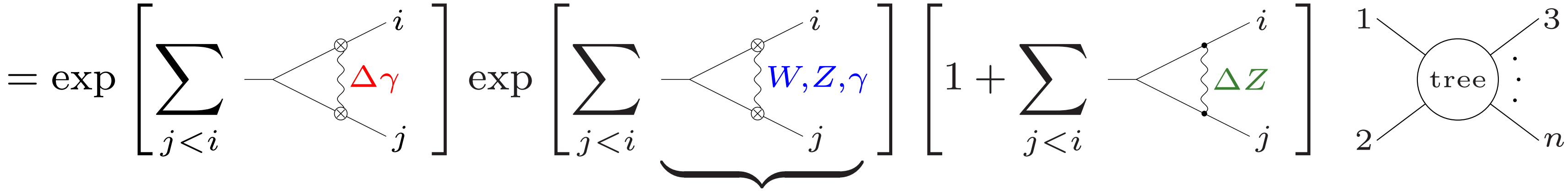


$\mathcal{O}(100)$ **inequivalent two-loop diagrams** $\Rightarrow$ **very simple result!**

- Two-loop $\equiv$ exp(1-loop)$\times$ Born
- Agreement with IREE [Kühn, Moch, Penin, Smirnov (2000); Melles (2003)]

Two-loop NLL (preliminary) result for $f_1 f_2 \to f_3 \dots f_n$

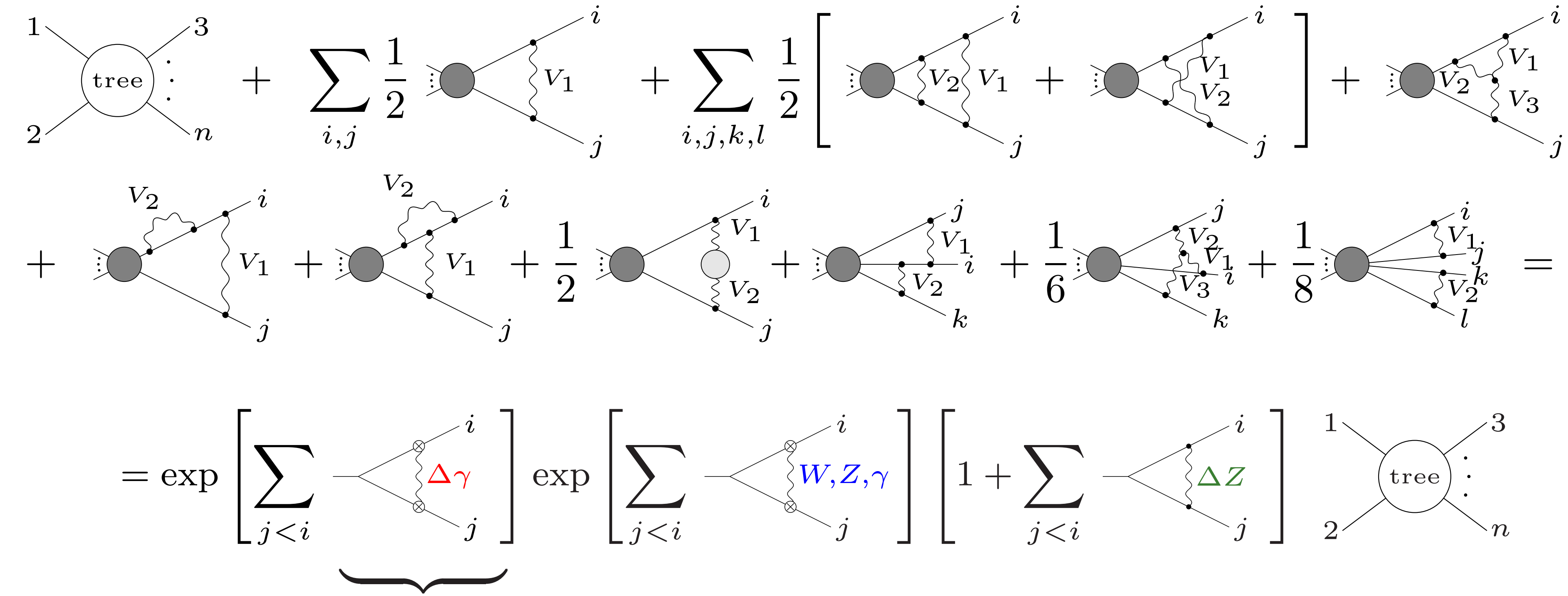




contains only $L = \ln(s/M_W^2)$ and behaves as in a symmetric SU(2)xU(1) theory with $M_W = M_Z = M_\gamma$

$$= \left(\frac{\alpha}{4\pi}\right) \sum_{V=B,W^a} I_i^{\bar{V}} I_j^V \Bigg[\underbrace{L^2 + \frac{2}{3}L^3\epsilon + \frac{1}{4}L^4\epsilon^2 - \left[\frac{3}{2} - \ln\left(\frac{r_{ij}}{s}\right)\right]\left(2L + L^2\epsilon + \frac{1}{3}L^3\epsilon^2\right)}_{K(\epsilon,\, M_\mathrm{W};\, r_{ij})} + \frac{C_\kappa^i}{C_i^\mathrm{ew}} \frac{m_\mathrm{t}^2}{4 s_\mathrm{W}^2 M_\mathrm{W}^2} \times \left(L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\Bigg] + \left(\frac{\alpha}{4\pi}\right)^2 \frac{1}{2\epsilon}\left[\left(\frac{-s}{\mu^2}\right)^\epsilon K(\epsilon, M_\mathrm{W}, s) - K(2\epsilon, M_\mathrm{W}, s)\right]\left(g_1^2 \frac{Y_i Y_j}{4} b_1^{(1)} + g_2^2 \frac{T_i^a T_j^a}{4} b_2^{(1)}\right)$$

Two-loop NLL (preliminary) result for $f_1 f_2 \to f_3 \dots f_n$

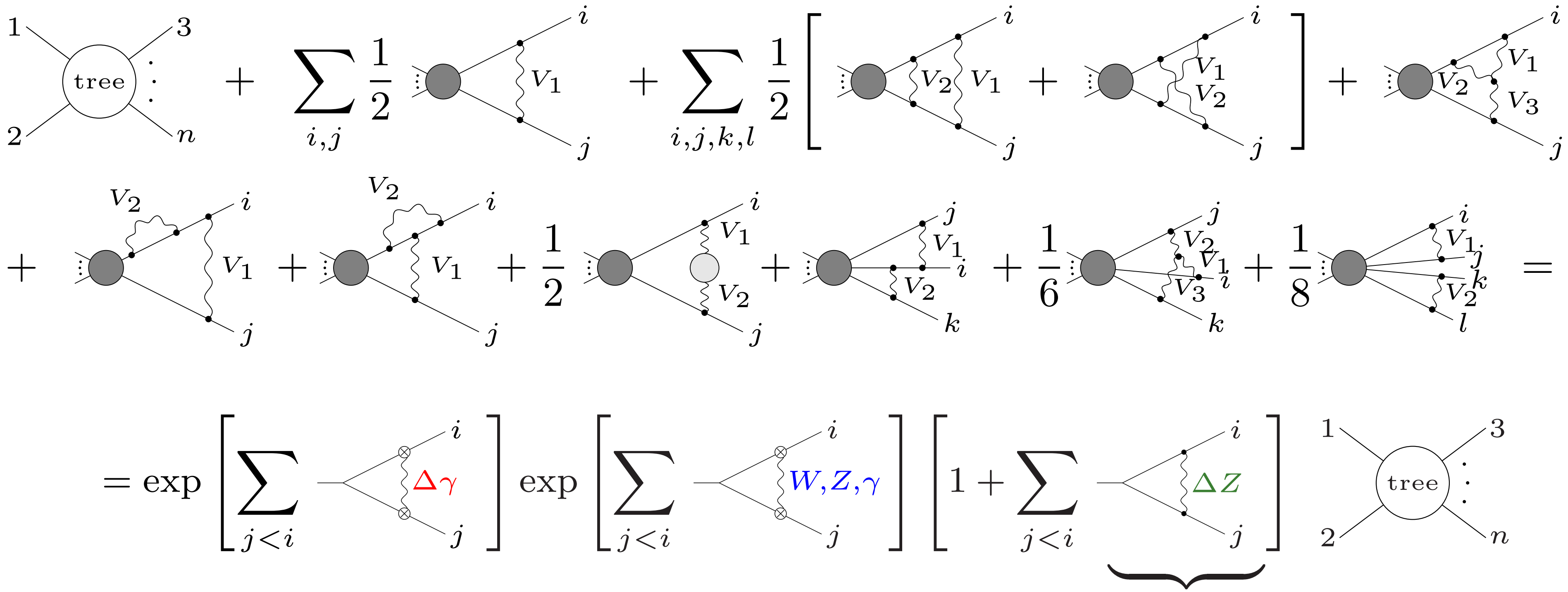


photonic singularities factorize and behave as in QED

$$= \left(\frac{\alpha}{4\pi}\right) Q_i Q_j \underbrace{\left\{\epsilon^{-2}\left[2\left(\frac{r_{ij}}{s}\right)^{-\epsilon} - \left(\frac{m_i^2}{s}\right)^{-\epsilon} - \left(\frac{m_j^2}{s}\right)^{-\epsilon}\right] + \frac{1}{2}\epsilon^{-1}\left[6 - \left(\frac{m_i^2}{s}\right)^{-\epsilon} - \left(\frac{m_j^2}{s}\right)^{-\epsilon}\right] - K(\epsilon, M_W; r_{ij})\right\}}_{\Delta K(\epsilon, 0, r_{ij})}$$

$$+ \left(\frac{\alpha}{4\pi}\right)^2 \frac{1}{2\epsilon}\left[\left(\frac{-s}{\mu^2}\right)^{\epsilon} \Delta K(\epsilon, 0, s) - \Delta K(2\epsilon, 0, s)\right] e^2 Q_i Q_j b^{(1)}_{\mathrm{QED}}$$

Two-loop NLL (preliminary) result for $f_1 f_2 \to f_3 \ldots f_n$

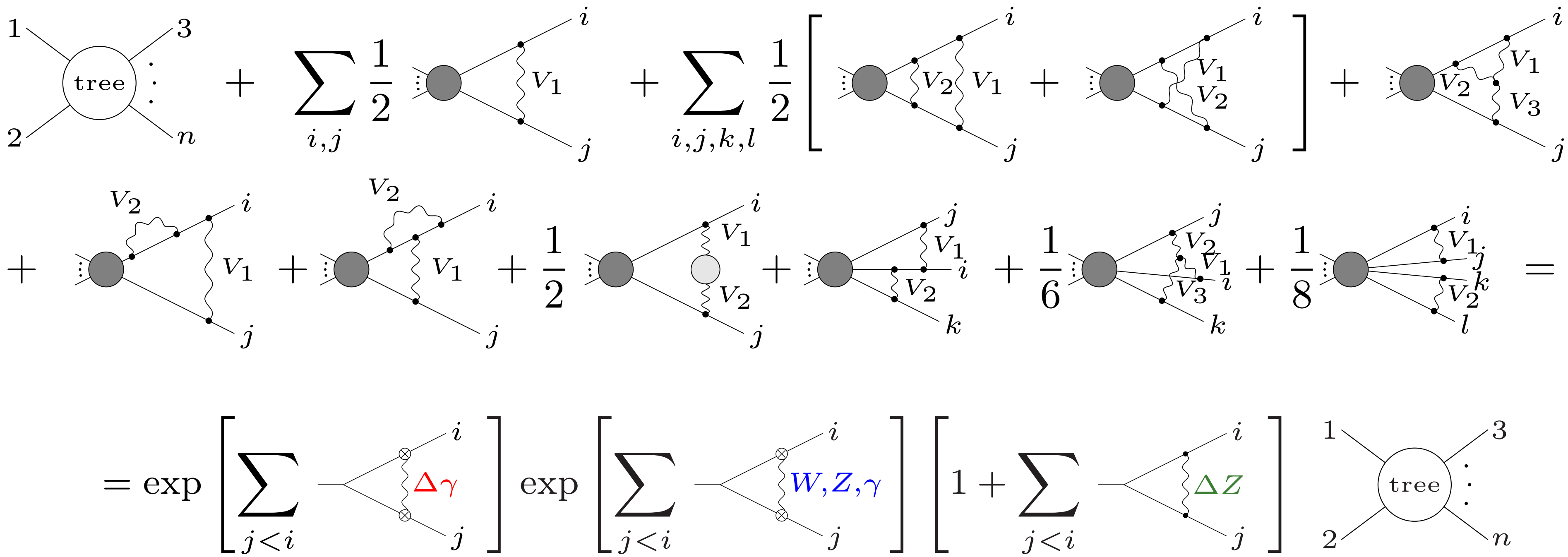


Mixing correction depending on Z-W mass difference

$$= -\left(\frac{\alpha}{4\pi}\right) I_i^Z I_j^Z \ln\left(\frac{M_Z^2}{M_W^2}\right) \left[2L + 2L^2\varepsilon + L^3\varepsilon^2\right]$$

$\Rightarrow \mathcal{O}(10^{-3})$ **effect at two loops**

Two-loop NLL (preliminary) result for $f_1 f_2 \to f_3 \dots f_n$



- these results applicable to $e^+e^- \to b\bar{b}$, $q\bar{q} \to \mu^+\mu^-$, $u\bar{d} \to t\bar{b}$, $gg \to b\bar{b}$, ...
- similar analysis can be performed for processes with γ, W, Z, H

Conclusions

Hard reactions at $Q^2 \sim 1\,\mathrm{TeV}^2$ receive large two-loop EW corrections

$$\frac{\delta\sigma}{\sigma} \sim \frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{Q^2}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%$$

important for precision at LHC,ILC,CLIC

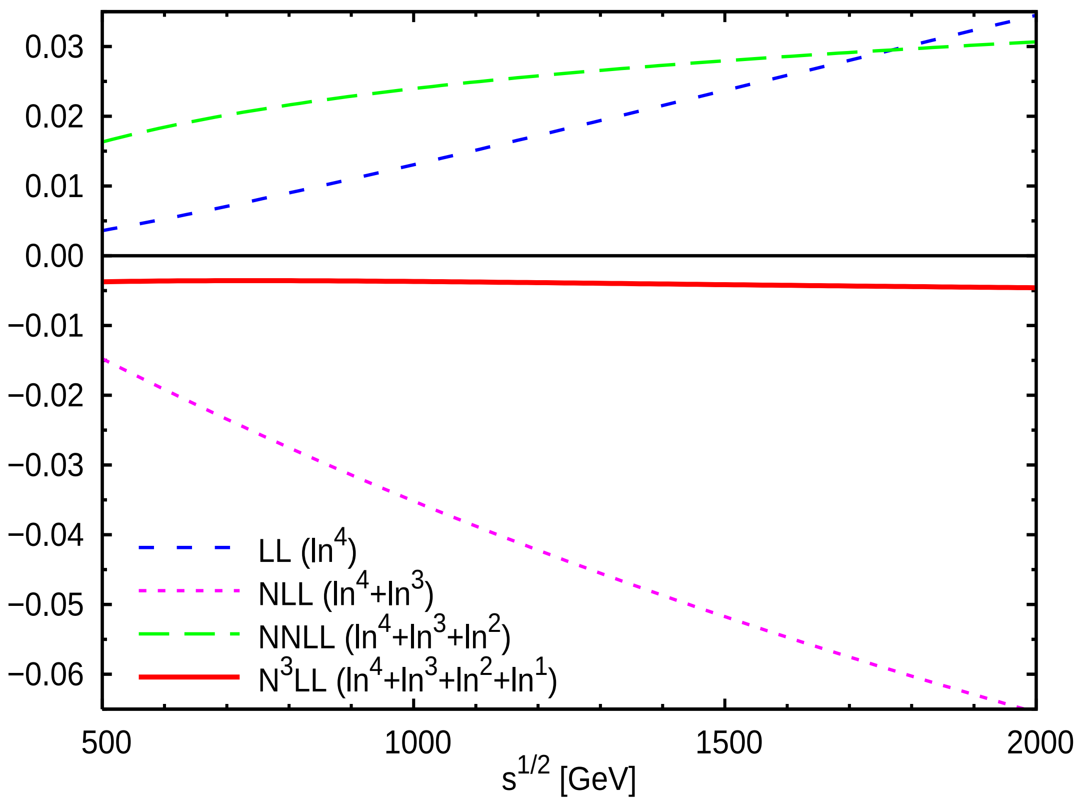
- LL well known for arbitrary SM processes
- NLL predictions based on IREE approach + few explicit calculations

Method to derive subleading two-loop logarithms diagrammatically

- Collinear Ward identities + algorithmic treatment of loop integrals
- Explicit NLL results for $f_1 f_2 \to f_3 \dots f_n$ (massless and massive)
- Highly automatized at NLL level and applicable to arbitrary processes

Relative 2-loop logarithmic corrections to $\sigma(\mathrm{e}^+\mathrm{e}^- \to d\bar{d})$ [Jantzen, Kühn, Penin, Smirnov (2005)]

$$\left(\frac{\alpha}{4\pi\sin^2\theta_w}\right)^2\left[2.79\,\ln^4\left(\frac{s}{M_\mathrm{W}^2}\right) - 51.98\,\ln^3\left(\frac{s}{M_\mathrm{W}^2}\right) + 321.34\,\ln^2\left(\frac{s}{M_\mathrm{W}^2}\right) - 757.35\,\ln^1\left(\frac{s}{M_\mathrm{W}^2}\right)\right]$$



Subleading logarithms

- increasingly large coefficients
- alternating signs

Importance of logarithmic effects

− total 2-loop correction small

+ residual theoretical error $\mathcal{O}(10^{-3})$

Behaviour of log expansion

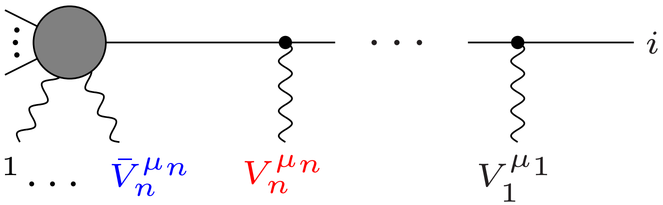
− oscillating, bad convergence

+ better convergence expected for gauge-boson production

Cancellation mechanism for non-factorizable contributions

Soft-collinear fermion-boson vertices (Dirac structure disappears)

$\lim_{q_k^\mu \to x_k p_i^\mu}$

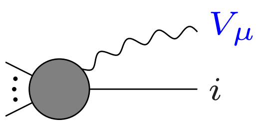


$$= G^{\bar{V}_1 \cdots \bar{V}_n \, i}_{\mu_1 \ldots \mu_n}(-q_1, \ldots, -q_n, p_i + \tilde{q}_n)\, u(p_i, \kappa_i)$$

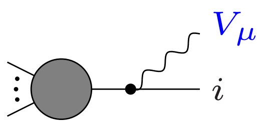
$$\times \frac{-2eI_i^{V_n}(p_i + \tilde{q}_n)^{\mu_n}}{(p_i + \tilde{q}_n)^2} \cdots \frac{-2eI_i^{V_1}(p_i + q_1)^{\mu_1}}{(p_i + q_1)^2}$$

Collinear Ward identities for spontaneously broken non-abelian theories

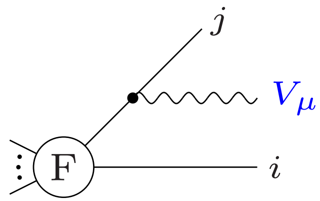
$\lim_{q^\mu \to x p_i^\mu} q^\mu \times \Bigg[$



$-$



$- \sum_{\substack{j=1 \\ j \neq i}}^{n}$



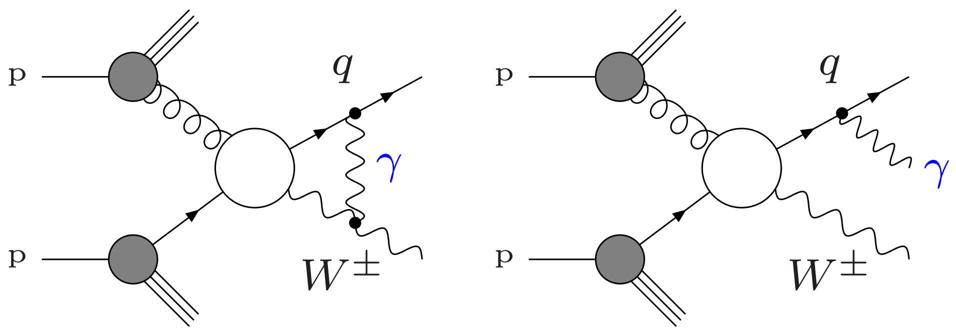
$\Bigg] = 0$

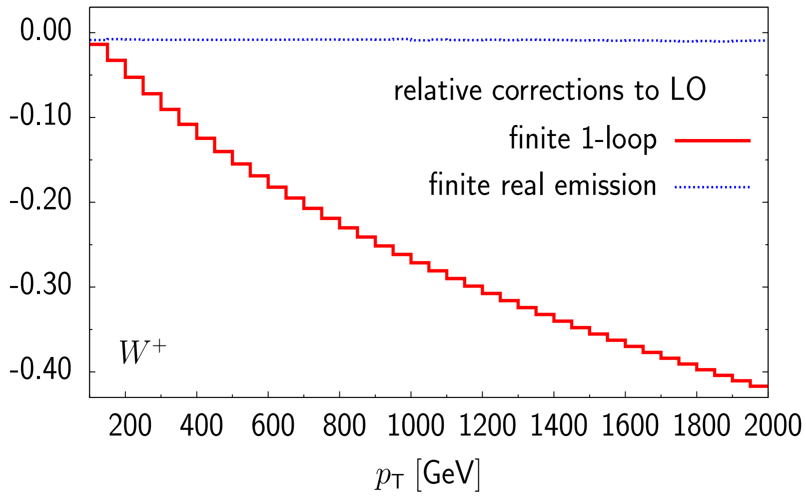
Denner, S.P. (2001)

derived from BRS symmetry and valid for arbitrary processes

Separation of photonic singularities for pp → Wj [Kühn, Kulesza, S.P., Schulze (2007)]

Cancellation of virtual-photon divergencies requires real bremstrahlung. Needed techniques (dipole subtraction) not available beyond one loop.





Strategy: gauge-invariant splitting

- $\sigma^{\text{fin}}_{\text{virt}} = \sigma_{\text{virt}}(M_\gamma = M_W)$
- $\sigma^{\text{fin}}_{\gamma}$ = virtual-photon singularities +photon bremsstrahlung

One-loop calculation for pp → Wj

- $\sigma^{\text{fin}}_{\text{virt}}$ = large negative corrections
- $\sigma^{\text{fin}}_{\gamma} \leq 1\%$ for fully inclusive γ

Origin of $1/\varepsilon$ and $\ln(Q^2/M^2)$ singularities

$$G \propto \int_0^1 \mathrm{d}^I\vec{\alpha}\,\delta(1-\sum_{r=1}^{I}\alpha_r)\frac{\Gamma(e)}{[\mathcal{U}(\vec{\alpha})]^e\,[\mathcal{F}(\vec{\alpha})]^f}$$

Polynomials ($\mathcal{T}$ = trees, $\mathcal{C}$ =cuts)

$$\mathcal{U}(\vec{\alpha}) = \sum_{\mathcal{T}} \alpha_{\mathcal{T}_1}\ldots\alpha_{\mathcal{T}_L}$$

$$-\mathcal{F}(\vec{\alpha}) = \sum_{\mathcal{C}} s_{\mathcal{C}}\,\alpha_{\mathcal{C}_1}\ldots\alpha_{\mathcal{C}_{L+1}} - \mathcal{U}(\vec{\alpha})\sum_{r=1}^{I}\alpha_r M_r^2 + \mathrm{i}\varepsilon$$

UV and mass singularities ($s_{\mathcal{C}} = s,t,u < 0$)

$\mathcal{U}(\vec{\alpha}) = 0 \;\Rightarrow\;$ UV sing. $\qquad\qquad \mathcal{F}(\vec{\alpha}) = 0 \;\Rightarrow\;$ mass sing.

Singular regions ($\mathcal{U} = 0, \mathcal{F} = 0$)

$$\{\vec{\alpha}|\alpha_{i_1} = \ldots = \alpha_{i_n} = 0\}$$

Crucial for factorization of singularities in FP space!

Step 2: **Sector decomposition**

Goal: **factorization of mass singularities** from $\mathcal{F}(\vec{\alpha})$

$$\int_0^1 \frac{\mathrm{d}^{I-1}\vec{\alpha}}{\underbrace{\left[Q^2\mathcal{P}(\vec{\alpha}) + M^2\mathcal{R}(\vec{\alpha})\right]}_{\mathcal{F}(\vec{\alpha})}^f \ldots} \Rightarrow \int_0^1 \frac{\mathrm{d}^{I-1}\vec{\alpha}}{\left[Q^2\,\underbrace{\hat{\mathcal{P}}(\vec{\alpha})}_{\neq 0}\,\alpha_1\ldots\alpha_k + M^2\hat{\mathcal{R}}(\vec{\alpha})\right]^f \ldots}$$

Sector decomposition for overlapping singularities $\mathcal{P}(\vec{\alpha})|_{\alpha_1=\ldots=\alpha_k=0} = 0$

(A) **partition of** $[0,1]^{I-1}$ **into sectors** $\Omega_1,\ldots,\Omega_k$

$$\Omega_j = \{\vec{\alpha}|\alpha_1,\ldots,\alpha_k \leq \alpha_j\}$$

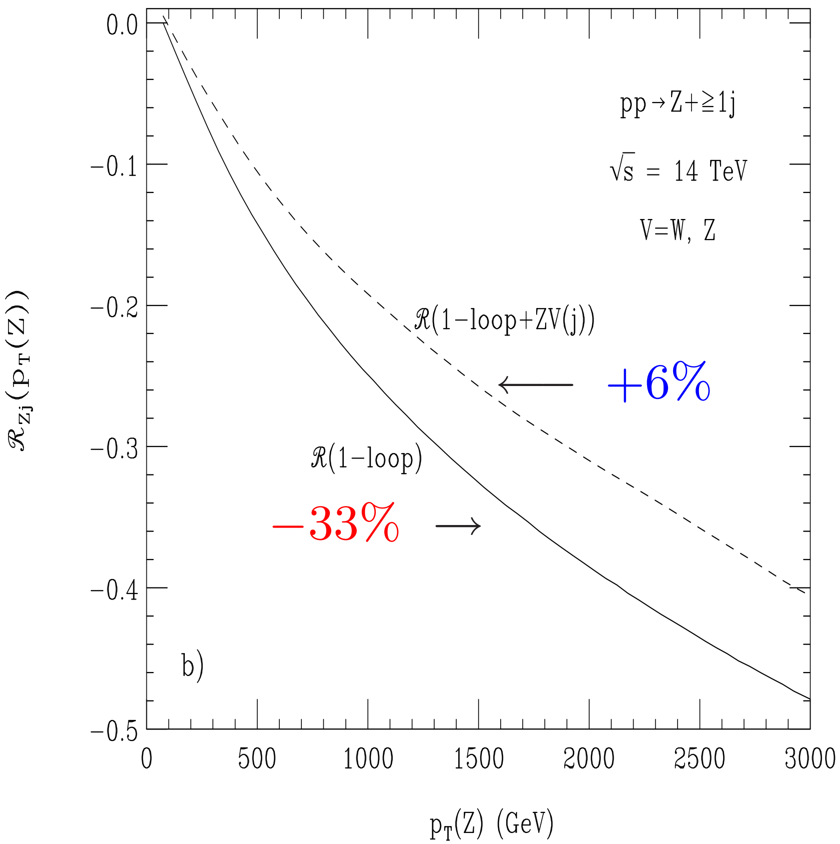
(B) **remapping** $\Omega_j \longrightarrow [0,1]^{I-1}$ yields **factorization** in Ω_j-sector

$$\alpha_k \longrightarrow \alpha_k\alpha_j \quad \text{for} \quad k \neq j \qquad \Rightarrow \qquad \mathcal{P}(\vec{\alpha}) \longrightarrow \alpha_j\,\hat{\mathcal{P}}_j(\vec{\alpha})$$

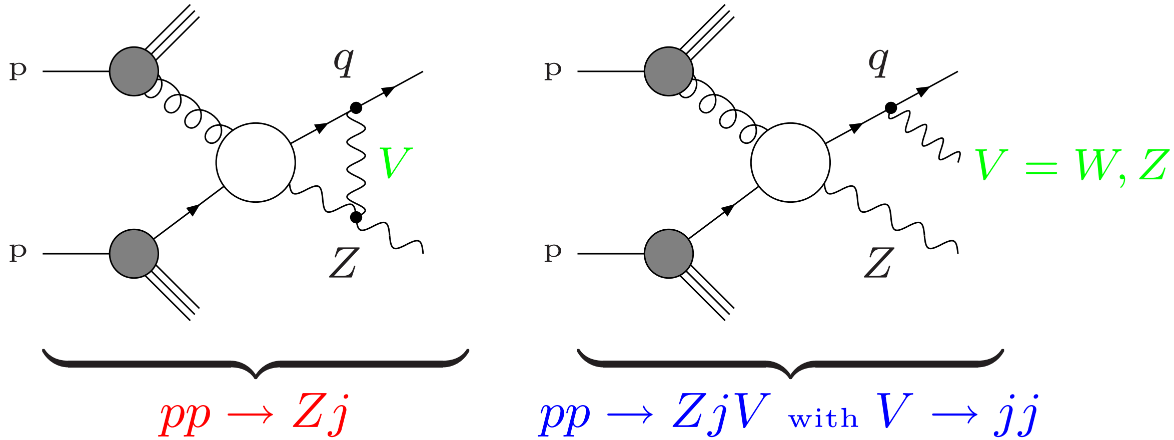
(C) **iterate** until $\hat{\mathcal{P}}_j(\vec{\alpha}) \neq 0$

Real W and Z emission for $pp \to Zj$ [Baur (2006)]

“ Since the number of jets is not fixed in a measurement of the Z boson p_T distribution, $\mathcal{O}(\alpha_s \alpha^2)$ ZVj production with $V \to jj$ has to be included when calculating weak radiative corrections ”



Virtual and real $\mathcal{O}(\alpha)$ corr. to $pp \to Zj$



- W, Z emission can be non-negligible and partially cancel EW virtual corrections
- depends on observable definition and can be reduced by jet veto