Next-to-leading order multi-leg processes for the LHC

Thomas Binoth

1st October 2007

RADCOR

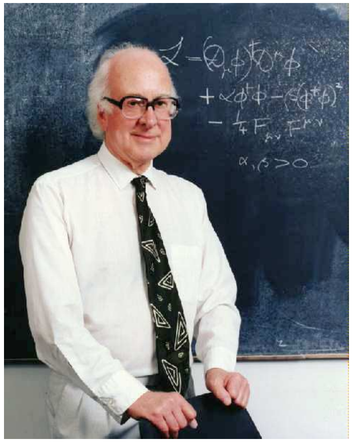
Galileo Galilei Institute, Florence, Italy

Content:

- Motivation: LHC @ NLO
- The GOLEM project
- Applications for LHC
- Summary

The advent of the LHC era

LHC: Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, start 2008

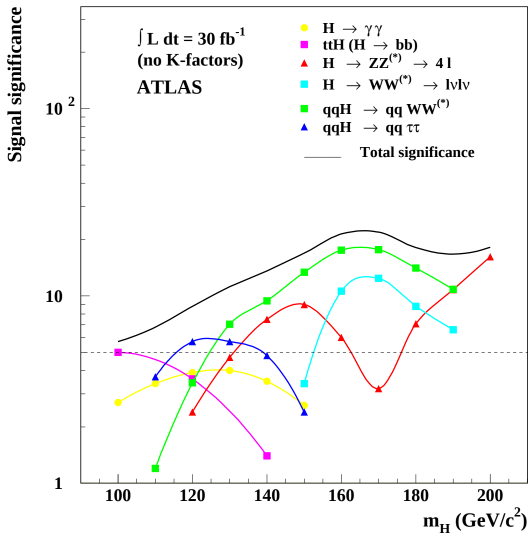


What do we expect?

- test Higgs mechanism
  - $\rightarrow$ SM Higgs boson: 114.4 GeV $< m_H < 200$ GeV (!)
  - $\rightarrow$ $V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3 \, H^3 + \lambda_4 \, H^4$
    SM: $\lambda_4 = \lambda_3 / v = 3 \, M_H^2 / v^2$
- explore physics beyond the Standard Model
  - $\rightarrow$ SM $\subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
    SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
  - $\rightarrow$ BSM something around 1 TeV (?)
- nothing ?!
  - $\rightarrow$ hint of a hidden sector (?)
  - $\rightarrow$ hint of strong interactions in the e.w. sector (?)

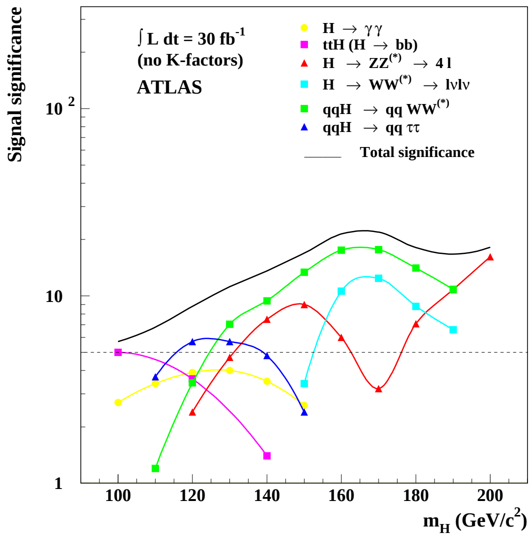
Discovery potential of the Higgs boson at the LHC

- most studies based on LO Monte Carlo tools
  - $\longrightarrow$ large uncertainties
  - $\longrightarrow$ some loop induced LO processes not included [e.g. $gg \to WW$]



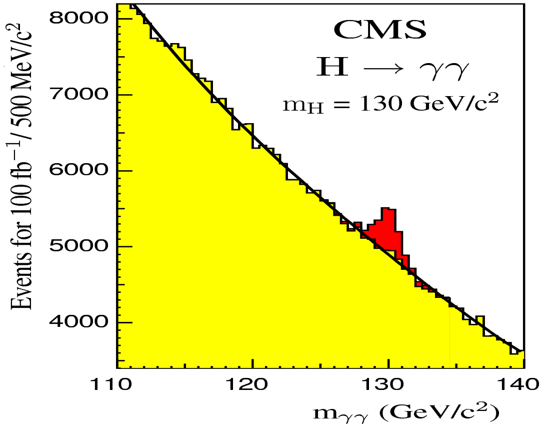
Discovery potential of the Higgs boson at the LHC

- most studies based on LO Monte Carlo tools
  - → large uncertainties
  - → some loop induced LO processes not included [e.g. $gg \to WW$]

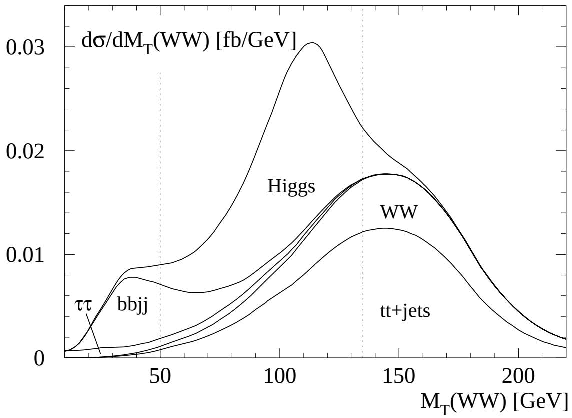


- Not all backgrounds can be measured
- Quantitative analysis of SM/BSM physics needs background control
- Nothing @ LHC = Bkgnd(experiment) - Bkgnd(theory) !

$PP \to H + X \to \gamma\gamma + X$

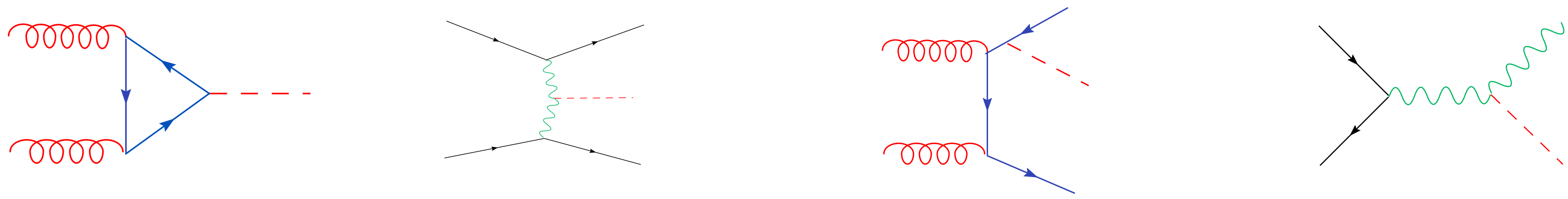


WBF: $H \to WW \to l^+l^- + \not{p}_T$



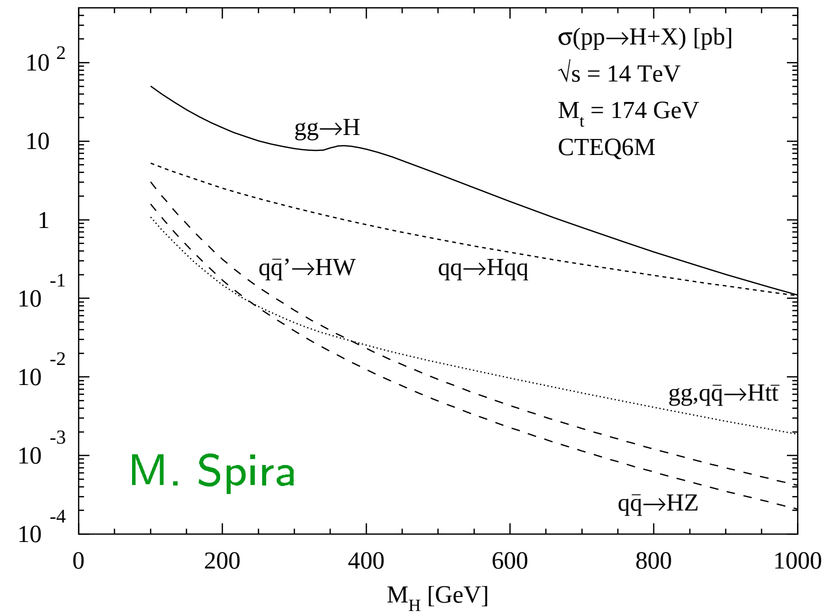
Kauer, Plehn, Rainwater, Zeppenfeld (2001)

S+B for the Higgs boson

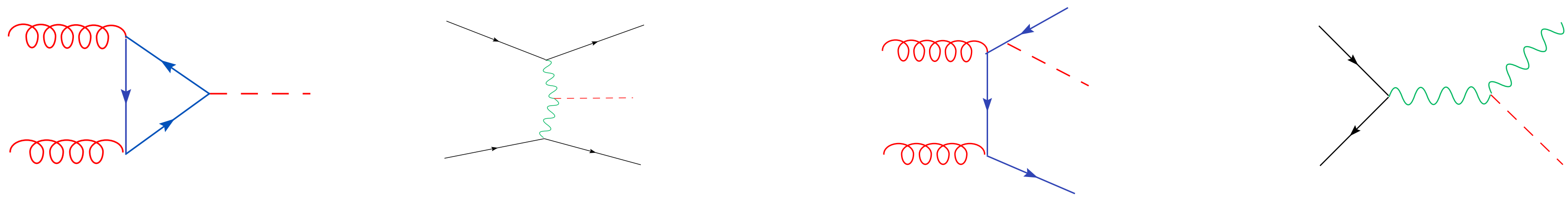


Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$

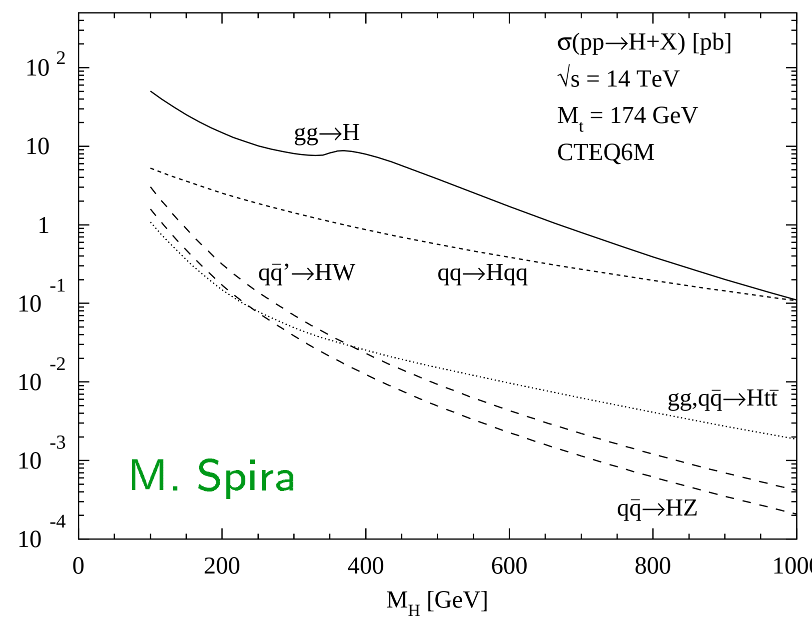


S+B for the Higgs boson



Signal:

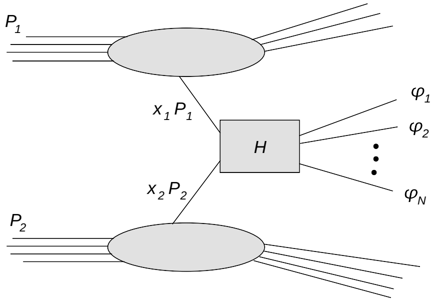
- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$



Backgrounds:

- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to V +$ up to 3 jets $\quad (V = \gamma, W, Z)$
- $PP \to VVV + 0, 1$ jet

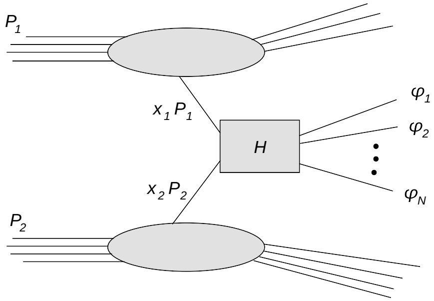
Parton model and scale uncertainties



$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\text{parton}_j(x_1 P_1) + \text{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

Parton model and scale uncertainties

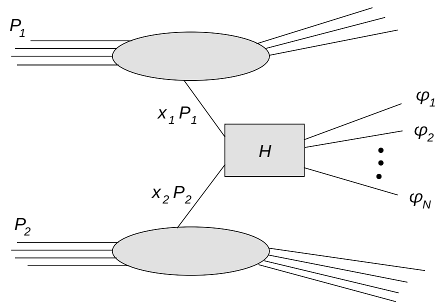


$$d\sigma(H_1 H_2 \to \phi_1 + \dots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\text{parton}_j(x_1 P_1) + \text{parton}_l(x_2 P_2) \to \varphi_1 + \dots \varphi_N, \alpha_s(\mu), \mu_F)$$

Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

Parton model and scale uncertainties



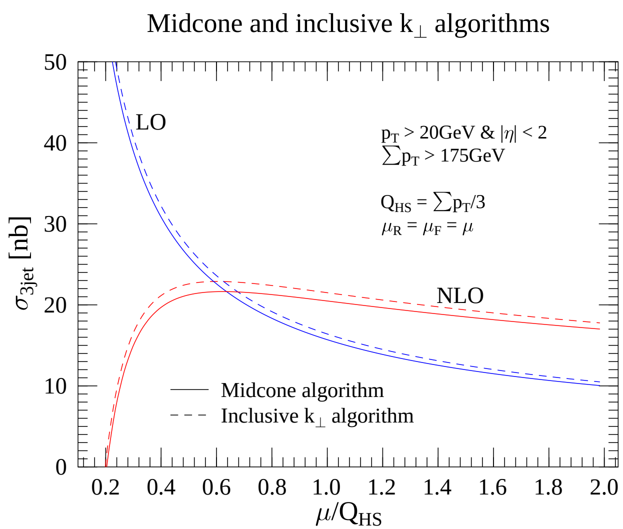
$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times\, d\hat{\sigma}(\mathrm{parton}_j(x_1 P_1) + \mathrm{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

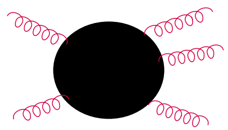
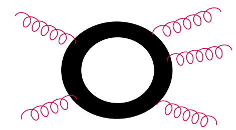
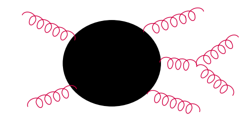
Example: 3 jet cross section at NLO

[Z. Nagy, Phys.Rev. D68 (2003)]



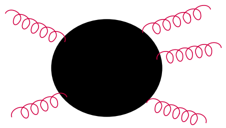
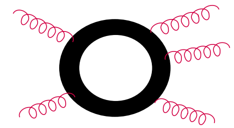
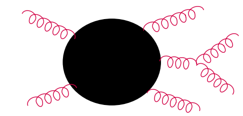
Higher order QCD calculations are mandatory to soften scale dependence !!!

Framework for NLO calculations

$\mathcal{M}_{\mathrm{LO}}$:  $\mathcal{M}_{\mathrm{NLO,virtual}}$:  $\mathcal{M}_{\mathrm{NLO,real}}$: 

$$\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \frac{1}{2s}\int dPS_N \;\; \mathcal{O}_N(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{LO}}|^2 \\
\sigma_{NLO} &= \frac{1}{2s}\int dPS_N \; \alpha_s \; \Big(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\mathrm{LO}}\mathcal{M}^*_{\mathrm{NLO,V}} + \mathcal{M}^*_{\mathrm{LO}}\mathcal{M}_{\mathrm{NLO,V}} \right] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \;\; |\mathcal{M}_{\mathrm{NLO,R}}|^2 \Big)
\end{aligned}$$

Framework for NLO calculations

$\mathcal{M}_{\text{LO}}$:  $\mathcal{M}_{\text{NLO,virtual}}$:  $\mathcal{M}_{\text{NLO,real}}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \frac{1}{2s}\int dPS_N \;\; \mathcal{O}_N(\{p_j\}) \;\; |\mathcal{M}_{\text{LO}}|^2 \\
\sigma_{NLO} &= \frac{1}{2s}\int dPS_N \; \alpha_s \; \Big(\mathcal{O}_N(\{p_j\}) \left[\mathcal{M}_{\text{LO}}\mathcal{M}^*_{\text{NLO,V}} + \mathcal{M}^*_{\text{LO}}\mathcal{M}_{\text{NLO,V}}\right] \\
&\qquad + \int dPS_1 \, \mathcal{O}_{N+1}(\{p_j\}) \;\; |\mathcal{M}_{\text{NLO,R}}|^2 \Big)
\end{aligned}
$$

- For IR-safe observables, $\mathcal{O}_{N+1} \underset{IR}{\rightarrow} \mathcal{O}_N$, IR divergences cancel
- treelevel LO, NLO contributions technically unproblematic
- IR subtraction: e.g. dipole method à la Catani, Seymour (massless); Dittmaier, Trocsanyi, Weinzierl, Phaf (massive).
- automated dipole subtraction (massless particles) Gleisberg, Krauss 9/2007.
- Bottleneck: virtual corrections

Status QCD@NLO for LHC:

$2 \to 2:$ everything you want (see e.g. MCFM by Campbell/Ellis)

Status QCD@NLO for LHC:

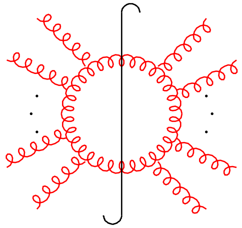
$2 \to 2:$ everything you want (see e.g. MCFM by Campbell/Ellis)

$2 \to 3:$

- $PP \to jjj$ Nagy (2002), Giele, Kilgore (1997); Bern, Dixon, Kosower (1993), Kunszt, Signer, Tróscsányi (1994).
- $PP \to Vjj$ MCFM; Campbell, Glover, Miller (1997).
- $PP \to Hjj$ [GF] LO Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld (2001); NLO $m_t \to \infty$ Campbell, Ellis, Zanderighi (2006).
- $PP \to Ht\bar{t}$ Reina, Dawson, Wackeroth, Orr (2001); Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas (2001).
- $PP \to \gamma\gamma\, j$ Del Duca, Maltoni, Nagy, Tróscsányi (2003); de Florian, Kunszt (1999); T.B., Guillet, Mahmoudi (2004).
- $PP \to Hjj$ [WBF] Figy, Oleari, Zeppenfeld (2003).
- $PP \to HHH$ Plehn, Rauch (2005); TB, Karg, Kauer, Ruckl (2006).
- $PP \to VVjj$ [WBF] Jager, Oleari, Zeppenfeld (2006).
- $PP \to ZZZ$ Lazopoulos, Melnikov, Petriello (2007) (numerical approach: sector decomposition and contour deformation; also applied to $gg \to t\bar{t}Z$).
- $PP \to t\bar{t}j$ S. Dittmaier, P. Uwer, S. Weinzierl (2007).

Status QCD@NLO for LHC:

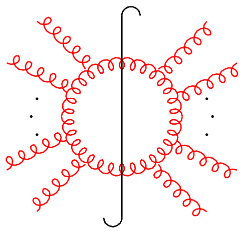
$2 \to 4:$ No LHC cross section done yet !!!

$$\mathcal{A}_{\text{1-loop}} \sim \sum_C \int d\text{PS}_C$$  $$+\mathcal{R}$$

1994-2006: 6 gluon one-loop amplitude

- cut-constructible part
  Bern, Dixon, Dunbar, Kosower (1994/95), Britto, Cachazo, Feng, Mastrolia (2005/2006), Bern, Bidder, Bjerrum-Bohr, Dixon, Dunbar, Ita, Perkins (2005/2006), Bedford, Brandhuber, Spence, Travaglini (2005)
- Numerical recursion based on Feynman diagrams
  Ellis, Giele, Zanderighi (2006)
- Numerical approach based on unitarity
  Ellis, Giele, Kunszt (2007)
  (inspired by Ossola, Papadopoulos, Pittau (2006))
- rational terms
  Berger, Bern, Dixon, Forde, Kosower (2005/06) "unitarity-bootstrap",
  Xiao, Yang, Zhu (2006) Feynman diagrams.

Status QCD@NLO for LHC:

$\mathcal{A}_{1\text{-loop}} \sim \sum_C \int d\mathrm{PS}_C$  $+\mathcal{R}$

$2 \to 4$: No LHC cross section done yet !!!

1994-2006: 6 gluon one-loop amplitude

- cut-constructible part
  Bern, Dixon, Dunbar, Kosower (1994/95), Britto, Cachazo, Feng, Mastrolia (2005/2006), Bern, Bidder, Bjerrum-Bohr, Dixon, Dunbar, Ita, Perkins (2005/2006), Bedford, Brandhuber, Spence, Travaglini (2005)
- Numerical recursion based on Feynman diagrams
  Ellis, Giele, Zanderighi (2006)
- Numerical approach based on unitarity
  Ellis, Giele, Kunszt (2007)
  (inspired by Ossola, Papadopoulos, Pittau (2006))
- rational terms
  Berger, Bern, Dixon, Forde, Kosower (2005/06) ”unitarity-bootstrap”,
  Xiao, Yang, Zhu (2006) Feynman diagrams.

Full $2 \to 4$ one-loop calculations for e^+e^-, $\gamma\gamma$ colliders:

- $\mathcal{O}(\alpha)$ $e^+e^- \to f\bar{f}f'\bar{f}'$ Denner, Dittmaier, Roth, Wieders (2005)
- $\mathcal{O}(\alpha)$ $e^+e^- \to HH\nu\nu$ GRACE collaboration (2005)
- $\mathcal{O}(\alpha_s)$ $\gamma\gamma \to b\bar{b}t\bar{t}$ Lei, Wen-Gan, Liang, Ren-You, Yi (2007)

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes
- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, T. Reiter, G. Sanguinetti

The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ Numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes
- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, T. Reiter, G. Sanguinetti

”GOLEM”...

- ... refers in the Bible to embryonic / incomplete substance.
- ... is created from mud $\rightarrow$ like Adam.
- ... maybe a creation of overambition $\rightarrow$ Mary Shelley’s Frankenstein.
- ... need to be instructed wisely $\rightarrow$ Göthes Zauberlehrling (The Sorcerer’s Apprentice).
- ... ”Wie er in die Welt kam”, film by Paul Wegener 1920.

Feynman diagrammatic approach:

$$\Gamma^{c,\lambda}(p_j,m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \ldots D_N} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\ldots,\mu_R} \, I_N^{\mu_1\cdots\mu_R}(p_j,m_j)$$

$$I_N^{\mu_1\cdots\mu_R}(p_j,m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1}\ldots k^{\mu_R}}{D_1\ldots D_N}\,,\; D_j = (k-r_j)^2 - m_j^2,\; r_j = p_1 \cdots + p_j$$

- Passarino-Veltman: $\rightarrow 1/\det(G)^R$, $G_{ij} = 2\,r_i \cdot r_j$ induce numerical problems
- projection on helicity amplitudes reduces $2\,k \cdot r_j = D_N - D_j + r_j \cdot r_j$
- Lorentz Tensor Integrals $\longrightarrow$ form factor representation à la Davydychev
- Reduction in Feynman parameter space

$$I_N^{\mu_1\cdots\mu_R} = \sum \tau^{\mu_1\cdots\mu_R}(r_{j_1},\ldots,r_{j_r},g^m)\, I_N^{n+2m}(j_1,\ldots,j_r)$$

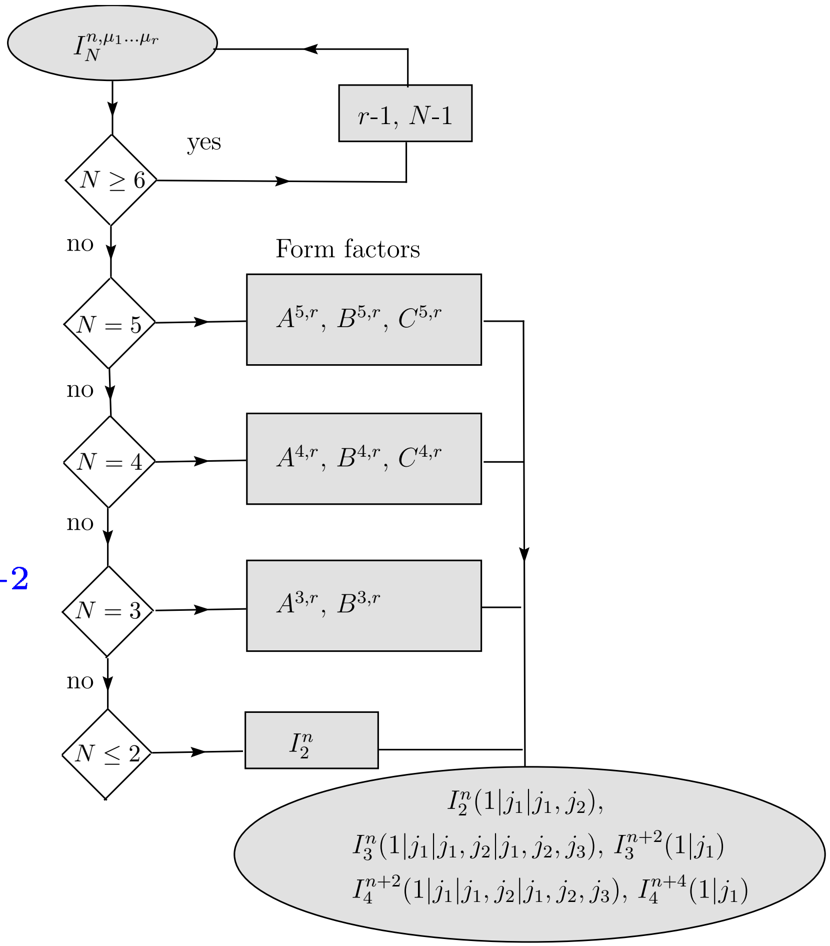
$$I_N^D(j_1,\ldots,j_r) = (-1)^N \Gamma(N-\frac{D}{2}) \int_0^\infty d^N z\, \delta(1-\sum_{l=1}^N z_l)\, \frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2} z\cdot \mathcal{S} \cdot z)^{N-D/2}}$$

$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

Schematic overview of N-point tensor integral reduction

T.B., J.P. Guillet, G. Heinrich (2000); T.B., Guillet, Heinrich, Pilon, Schubert (2005).

- works for general N
- no inverse Gram determinants
- isolation of IR divergences simple
- tractable expressions
- form factors for $N \leq 6$ implemented in Fortran 90 code "golem90 v0.2" switch: fully / semi-numerical
- optional reduction to scalar integrals by FORM $\mathcal{M} = c_1 I_1^n + c_2 I_2^n + c_3 I_3^n + c_4 I_4^{n+2}$
- evaluation of rational terms



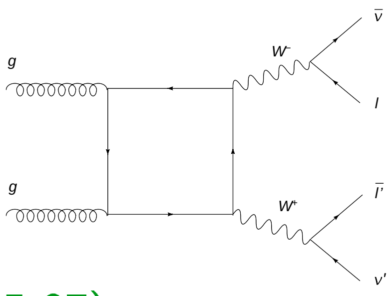
$$I_{N=3,4}^{n,n+2}(j_1,\ldots,j_r) \quad \sim \quad \int_0^1 \prod_{i=1}^{4} dz_i \, \delta(1-\sum_{l=1}^{4} z_l) \, \frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2}\, z\cdot\mathcal{S}\cdot z - i\delta)^{3-n/2}}$$

1

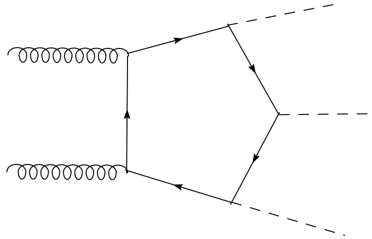
Computations with GOLEM:

Algorithm coded in FORM and FORTRAN 90

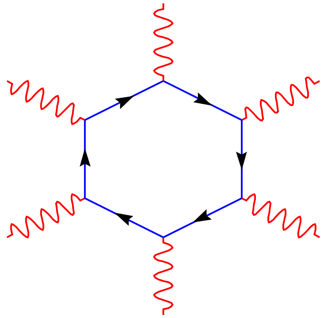
some recent applications ...



- $gg \to W^*W^* \to l\nu l'\nu'$, $gg \to Z^*Z^* \to l\bar{l}l'\bar{l}'$
  T.B., M. Ciccolini, M. Kramer, N. Kauer, P. Mertsch (2005-07)

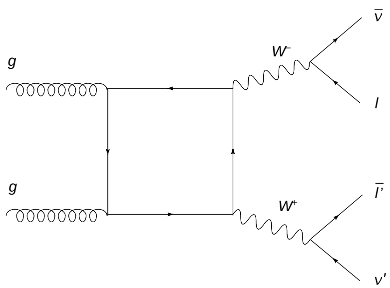


- $gg \to HH, HHH$
  T.B., S. Karg, N. Kauer, R. Rückl (2006)
- $PP \to Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
  J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
  T.B., G. Heinrich, T. Gehrmann, P. Mastrolia (2007)



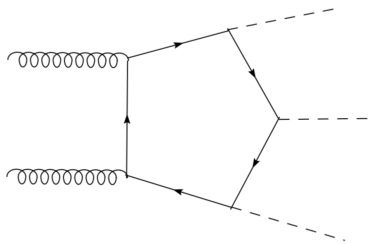
Computations with GOLEM:

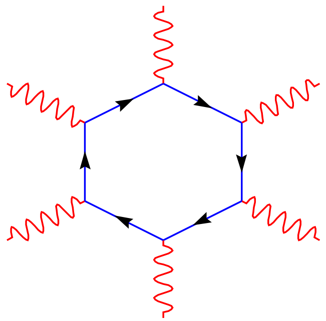
Algorithm coded in FORM and FORTRAN 90



some recent applications ...

- $gg \to W^*W^* \to l\nu l'\nu'$, $gg \to Z^*Z^* \to l\bar{l}l'\bar{l}'$
  T.B., M. Ciccolini, M. Kramer, N. Kauer, P. Mertsch (2005-07)
- $gg \to HH, HHH$
  T.B., S. Karg, N. Kauer, R. Rückl (2006)
- $PP \to Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
  J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
  T.B., G. Heinrich, T. Gehrmann, P. Mastrolia (2007)



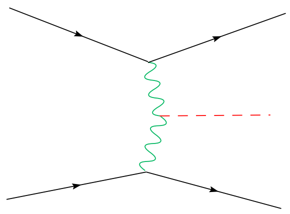


... and work in progress:

- $q\bar{q}VVg \to 0$, V=W,Z.
- $qq \to qqqq$, goal: $PP \to jjjj, jjbb, bbbb$.

The interference term for $PP \to Hjj$ to order $\mathcal{O}(\alpha^2\alpha_s^3)$

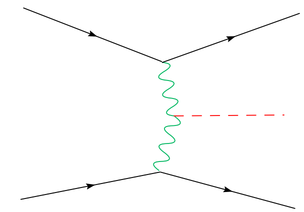
J. Andersen, T.B., G. Heinrich, J. Smillie 9/2007



- $PP \to Hjj$ in Weak Boson Fusion $\mathcal{O}(\alpha^4)$! promising process for Higgs search, properties and coupling measurements
- Gluon fusion contributions suppressible by "WBF" cuts: forward jet tagging, rapidity gaps, central jet veto, $\Delta\phi_{jj}$
- $\mathcal{O}(\alpha_s)$ corrections anomalously small $\sim +5\%$ (Han, Valencia, Willenbrock 1992, Figy, Oleari, Zeppenfeld 2003, Berger, Campbell 2004)
- e.w. $\mathcal{O}(\alpha)$ corrections $\sim -5\%$ (Ciccolini, Denner, Dittmaier 2007)

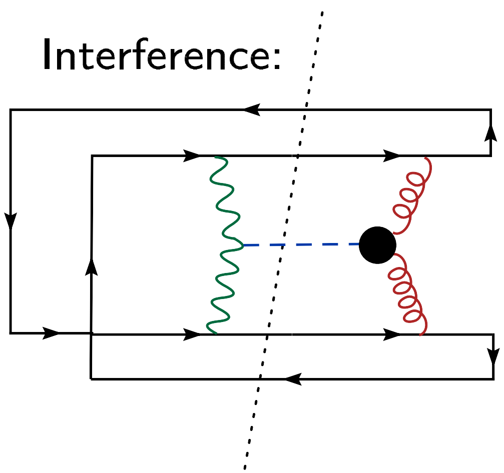
The interference term for $PP \to Hjj$ to order $\mathcal{O}(\alpha^2\alpha_s^3)$

J. Andersen, T.B., G. Heinrich, J. Smillie 9/2007



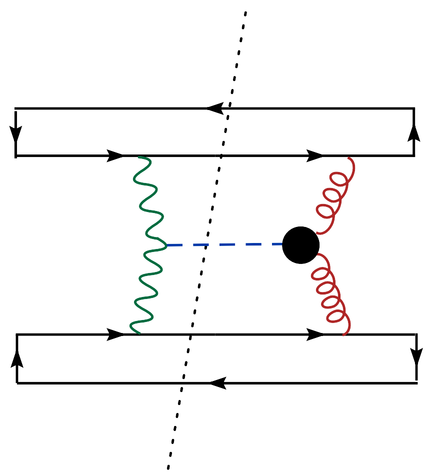
- $PP \to Hjj$ in Weak Boson Fusion $\mathcal{O}(\alpha^4)$! promising process for Higgs search, properties and coupling measurements
- Gluon fusion contributions suppressible by "WBF" cuts: forward jet tagging, rapidity gaps, central jet veto, $\Delta\phi_{jj}$
- $\mathcal{O}(\alpha_s)$ corrections anomalously small $\sim +5\%$ (Han, Valencia, Willenbrock 1992, Figy, Oleari, Zeppenfeld 2003, Berger, Campbell 2004)
- e.w. $\mathcal{O}(\alpha)$ corrections $\sim -5\%$ (Ciccolini, Denner, Dittmaier 2007)

Interference:



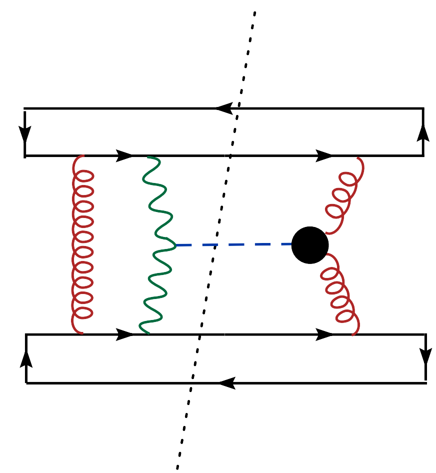
$\mathcal{O}(\alpha^2\alpha_s^2/N_F)$

$t \leftrightarrow u$ suppression



$\mathcal{O}(\alpha^2\alpha_s^2)$

forbidden by colour !

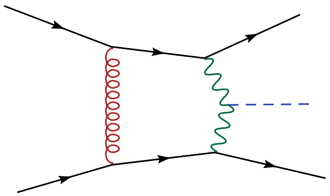


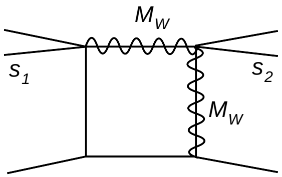
$\mathcal{O}(\alpha^2\alpha_s^3)$

colour channel opens for ZZ fusion!

The interference term for $PP \to Hjj$ to order $\mathcal{O}(\alpha^2\alpha_s^3)$

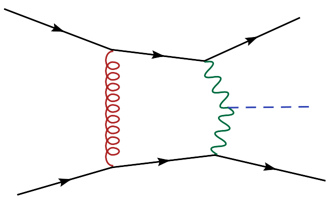
- 4 helicity amplitudes, analytical results $\mathcal{M} = c_2 I_2^n + c_3 I_3^n + c_4 I_4^{n+2}$
- 2 colour structurs, (octet, singlet)
- analytical representations for master integrals provided: 1 Bubble, 8 Triangles, 6 Box integrals
  $\longrightarrow$ to appear at http://www.ippp.dur.ac.uk/LoopForge
  see also: http://qcdloop.fnal.gov/ K.Ellis, L. Dixon

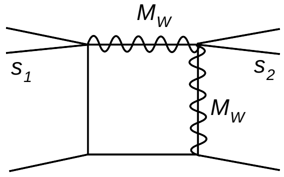




The interference term for $PP \to Hjj$ to order $\mathcal{O}(\alpha^2\alpha_s^3)$

- 4 helicity amplitudes, analytical results $\mathcal{M} = c_2 I_2^n + c_3 I_3^n + c_4 I_4^{n+2}$
- 2 colour structurs, (octet, singlet)
- analytical representations for master integrals provided:
  1 Bubble, 8 Triangles, 6 Box integrals
  $\to$ to appear at http://www.ippp.dur.ac.uk/LoopForge
  see also: http://qcdloop.fnal.gov/ K.Ellis, L. Dixon

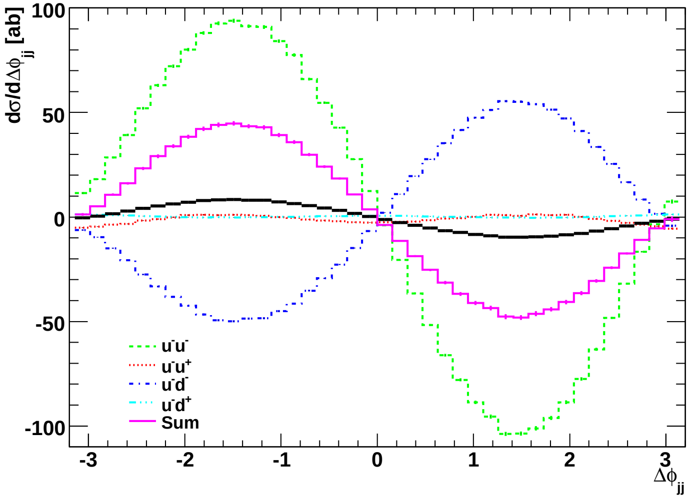


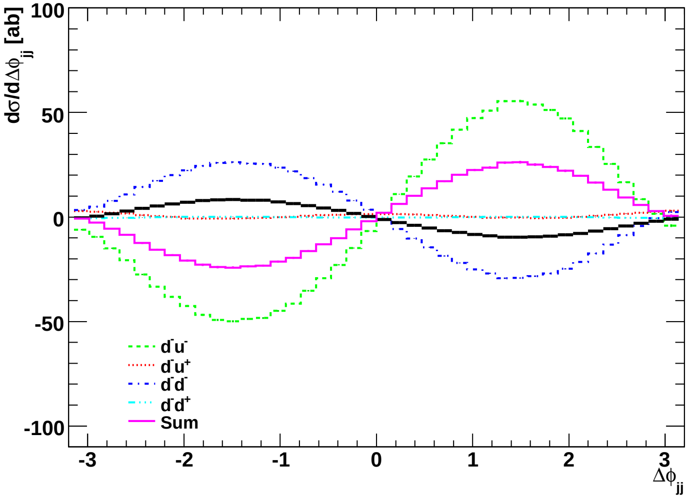


standard WBF enhancing cuts for tagged jets a, b:

- $p_{T\,a}, p_{T\,b} > 20$ GeV, $s_{ab} > (600\,\text{GeV})^2$
- $\eta_{a,b} < 5$, $\eta_a \cdot \eta_b < 0$, $|\eta_a - \eta_b| > 4.2$

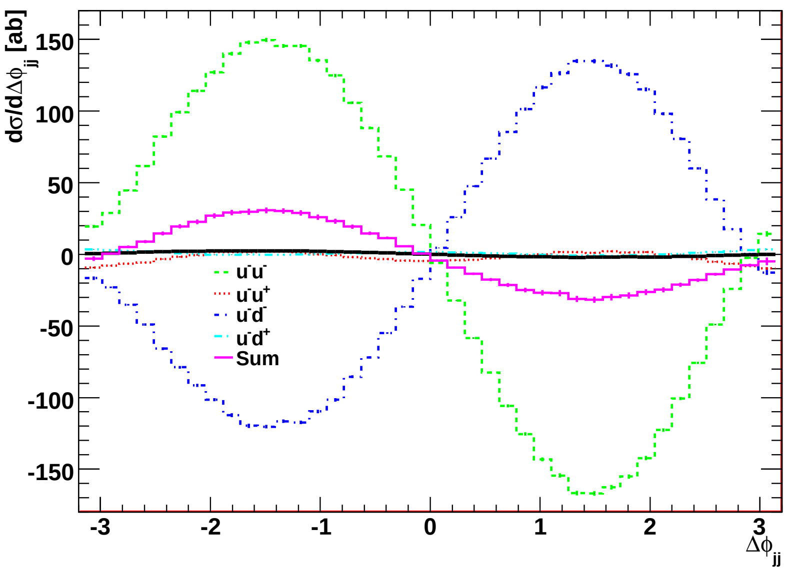
$\Delta\phi_{jj}$ distribution for valence quarks ($m_H = 115$ GeV):

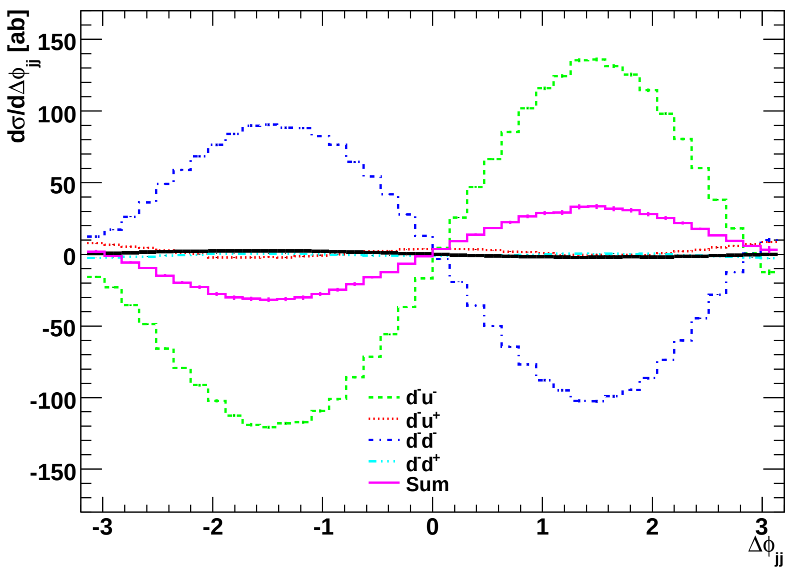




The interference term for $PP \to Hjj$ to order $\mathcal{O}(\alpha^2\alpha_s^3)$

Valence and sea quarks:





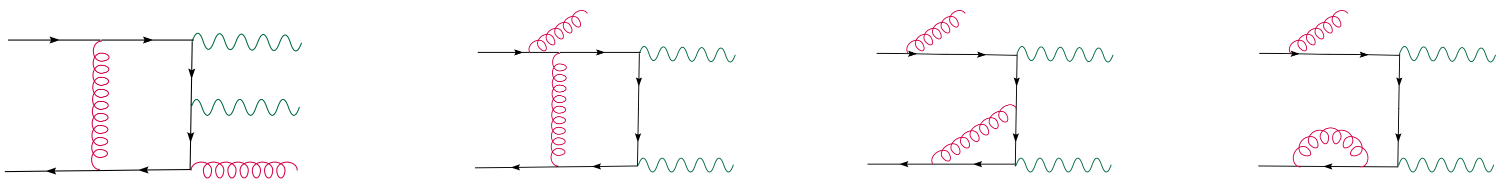
- sum over weak isospin/hypercharge $\longrightarrow$ cancellations
- amplitude phases $\longrightarrow$ destructive interference
- sum of sea and valence quarks $\longrightarrow$ accidental cancellation
- $\int d\Delta\phi_{ll}|d\sigma/d\Delta\phi_{ll}| \sim 0.01$ fb
- $\int dPS|d\sigma/dPS| \sim 0.03$ fb
- WBF $d\sigma/d\Delta\phi_{ll} \sim 240$ fb/rad (flat) $\Rightarrow$ interference effect negligible

The $q\bar{q}VVg \to 0$ amplitude

- Goal: $PP \to VV$ jet at NLO
  T.B., S. Karg, N. Kauer, J.Ph.-Guillet, G. Sanguinetti

The $q\bar{q}VVg \to 0$ amplitude

- Goal: $PP \to VV$ jet at NLO
  T.B., S. Karg, N. Kauer, J.Ph.-Guillet, G. Sanguinetti



The $q\bar{q}VVg \to 0$ amplitude

- Goal: $PP \to VV$ jet at NLO
  T.B., S. Karg, N. Kauer, J.Ph.-Guillet, G. Sanguinetti

$$q(p_1,\lambda_1,c_1) + \bar{q}(p_2,\lambda_2,c_2) + V(p_3,\lambda_3) + \bar{V}(p_4,\lambda_4) + g(p_5,\lambda_5,a_5) \to 0$$

$$\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} = \varepsilon^{\lambda_3}_{3,\mu_3}\,\varepsilon^{\lambda_4}_{4,\mu_4}\,\varepsilon^{\lambda_5}_{5,\mu_5}\,\langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle$$

- t'Hooft-Veltman scheme, γ_5 rules:
  $k_j = \hat{k}_j$, $k = \hat{k} + \tilde{k}$, $\gamma = \hat{\gamma} + \tilde{\gamma}$, $\{\gamma_5, \hat{\gamma}\} = 0$, $[\gamma_5, \tilde{\gamma}] = 0$
- 36 helicity amplitudes
- 3 colour structures

Helicity projection for $q\bar{q}VVg \to 0$

$$k_3 = \frac{1}{2\beta}[(1+\beta)\,p_3 - (1-\beta)\,p_4], \quad k_4 = \frac{1}{2\beta}[(1+\beta)\,p_4 - (1-\beta)\,p_3],$$

$$k_3^2 = k_4^2 = 0, \; \beta = \sqrt{1 - 4\,M_V^2/s_{34}}$$

$$\varepsilon^{+}_{3\,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 4^-|\mu|3^-\rangle}{\langle 43\rangle}, \quad \varepsilon^{-}_{3\,\mu} = \frac{1}{\sqrt{2}}\frac{\langle 3^-|\mu|4^-\rangle}{[34]}, \quad \varepsilon^{0}_{3\,\mu} = \frac{(1+\beta)k_{3\,\mu} - (1-\beta)k_{4\,\mu}}{2\,M_V}$$

Helicity projection for $q\bar{q}VVg \to 0$

$$k_3 = \frac{1}{2\beta}[(1+\beta)\,p_3 - (1-\beta)\,p_4], \quad k_4 = \frac{1}{2\beta}[(1+\beta)\,p_4 - (1-\beta)\,p_3],$$

$$k_3^2 = k_4^2 = 0,\ \beta = \sqrt{1 - 4\,M_V^2/s_{34}}$$

$$\varepsilon_{3\,\mu}^{+} = \frac{1}{\sqrt{2}}\frac{\langle 4^-|\mu|3^-\rangle}{\langle 43\rangle}, \quad \varepsilon_{3\,\mu}^{-} = \frac{1}{\sqrt{2}}\frac{\langle 3^-|\mu|4^-\rangle}{[34]}, \quad \varepsilon_{3\,\mu}^{0} = \frac{(1+\beta)k_{3\,\mu} - (1-\beta)k_{4\,\mu}}{2\,M_V}$$

Use to define projectors on helicity amplitudes, schematically:

$$\begin{aligned}\mathcal{M}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} &= \mathcal{P}^{\lambda_3\lambda_4\lambda_5}_{\mu_3\mu_4\mu_5}\,\langle 2^{\lambda_2}|\Gamma^{\mu_3\mu_4\mu_5}|1^{\lambda_1}\rangle \\ &= (\text{global spinorial factor}) \times (\text{contracted tensor integrals})\end{aligned}$$

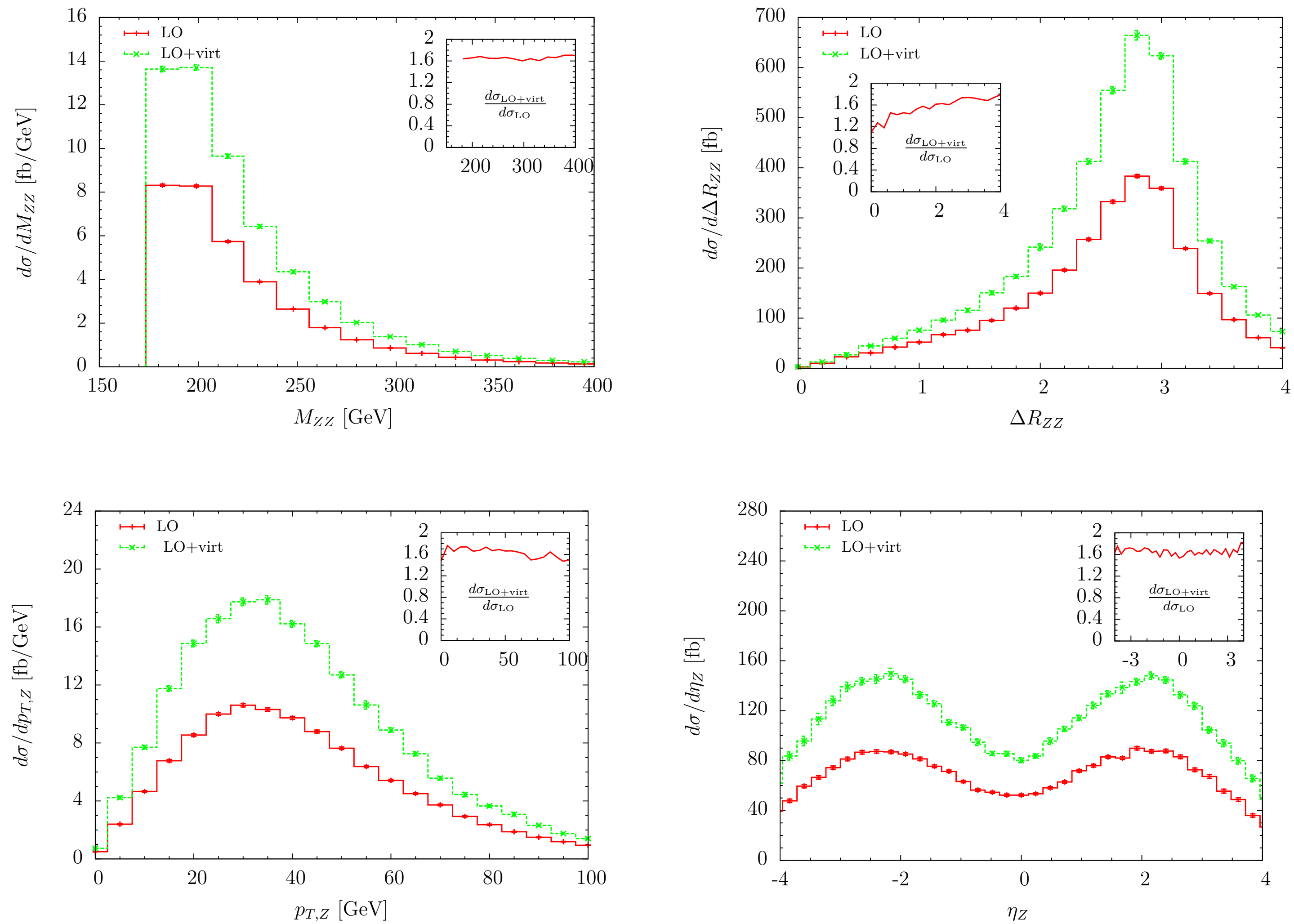
- Lorentz indices saturated, at most rank 1 5-point functions
- spinor products can be treated as global factors
- analytical expressions allow for further simplifications (Maple/Mathematica)
- evaluation time of a helicity amplitude about 0.5 seconds

Results for $\mathcal{M}^{--+++}(q\bar{q} \to ZZg)$ (preliminary)

- Phase space integration for virtual corrections only, divergent parts set to 0
- Real emission part under construction
- $p_{T\,\mathrm{gluon\ jet}} > 20$ GeV; Z, g, beampipe separation: $\theta_{ij} > 1.5^0$

Results for $\mathcal{M}^{--+++}(q\bar{q} \to ZZg)$ (preliminary)

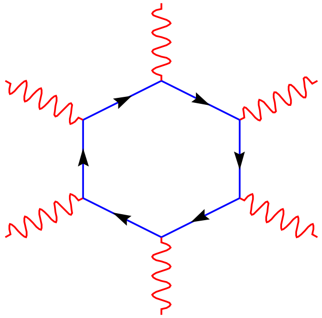
- Phase space integration for virtual corrections only, divergent parts set to 0
- Real emission part under construction
- $p_{T\,\text{gluon jet}} > 20$ GeV; Z, g, beampipe separation: $\theta_{ij} > 1.5^0$



Analytical evaluation of $\gamma\gamma \to \gamma\gamma\gamma\gamma$

- Four independent helicity amplitudes: $++++++\pm$, $++++--$, $+++---$
- Mahlon: $\mathcal{M}^{+++++\pm} = 0$, compact analytical result for $\mathcal{M}^{++++--}$
- Feynman diagrammatic approach for rational parts:
  T.B., G. Heinrich, J.Ph. Guillet (2006)

$$\mathcal{R}[\mathcal{M}^{+++++\pm}] = \mathcal{R}[\mathcal{M}^{++++--}] = \mathcal{R}[\mathcal{M}^{+++---}] = 0$$

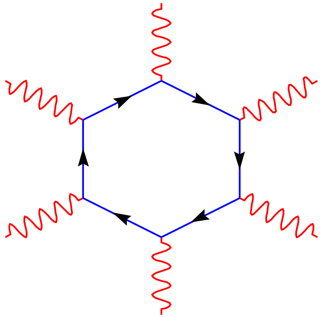


Analytical evaluation of $\gamma\gamma \to \gamma\gamma\gamma\gamma$

- Four independent helicity amplitudes: $++++++\pm$, $++++--$, $+++---$
- Mahlon: $\mathcal{M}^{+++++\pm} = 0$, compact analytical result for $\mathcal{M}^{++++--}$
- Feynman diagrammatic approach for rational parts:
  T.B., G. Heinrich, J.Ph. Guillet (2006)

$$\mathcal{R}[\mathcal{M}^{+++++\pm}] = \mathcal{R}[\mathcal{M}^{++++--}] = \mathcal{R}[\mathcal{M}^{+++---}] = 0$$

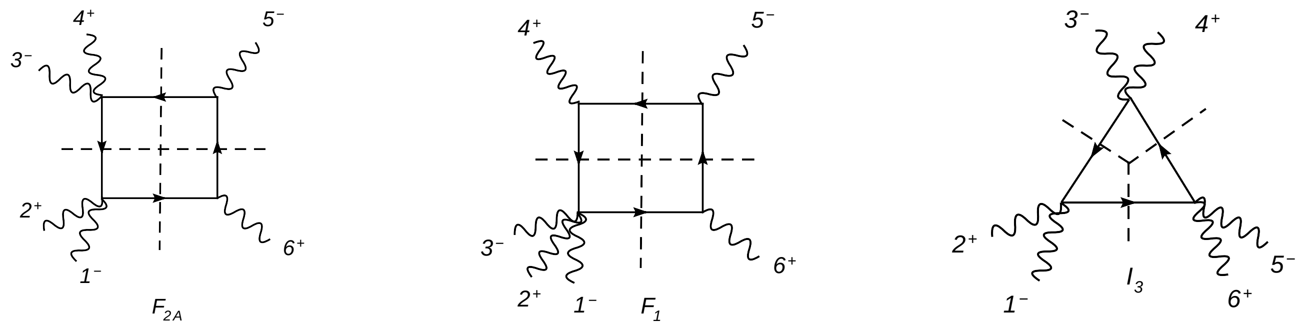
- Rational parts evaluation by-product of GOLEM project
- 6-photon amplitude cut-constructible



Analytical evaluation of $\gamma\gamma \to \gamma\gamma\gamma\gamma$

T.B., T. Gehrmann, G. Heinrich, P. Mastrolia 03/07
numerical methods: Nagy, Soper 09/06
numerical with unitarity cuts: Ossola, Pittau, Papadopoulos 04/07

- $\mathcal{M}^{-+-+-+}$: sum over permutations of 2 types of boxes and a triangle:



coefficient of "two adjacent off-shell legs box" $F_{2A}(s_{56}, s_{345}, s_{12}, s_{34})$:

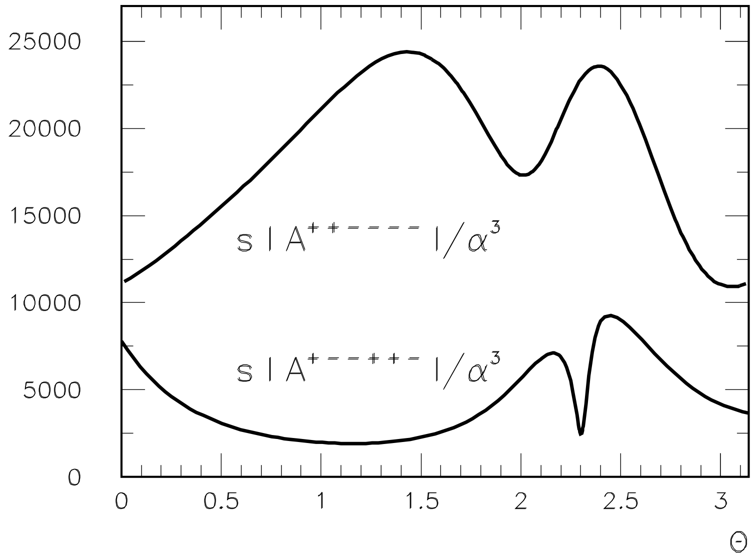
$$d_{2A}(s_{56}, s_{345}, s_{12}, s_{34}) = \frac{r_{2A}}{s_{345}s_{56}}\Big(s_{345}^2\langle 6\,1\rangle^2[4\,5]^2 + \langle 1|P_{34}|5]^2\langle 6|P_{12}|4]^2\Big)$$

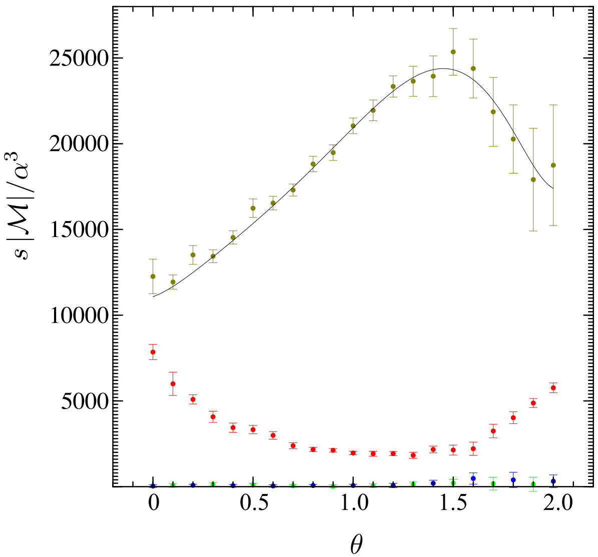
$$r_{2A} = -16\frac{s_{345}s_{56}^2}{\langle 2\,6\rangle[3\,5]\langle 6|P_{12}|3]\langle 2|P_{34}|5]\langle 6|P_{34}P_{12}|6\rangle[5|P_{34}P_{12}|5]}\,, \quad P_{ij} = P_i + P_j$$

- coefficient of triangle ($I_{3,3\text{mass}}$) and one-mass box, F_1, a few lines
- analytical result also obtained with GOLEM, full agreement

Analytical evaluation of $\gamma\gamma \to \gamma\gamma\gamma\gamma$

- Evaluation of $s|\mathcal{M}^{++----}|/\alpha_s^3$, $s|\mathcal{M}^{+--++-}|/\alpha_s^3$ for rotated kinematics
- nontrivial phase structure





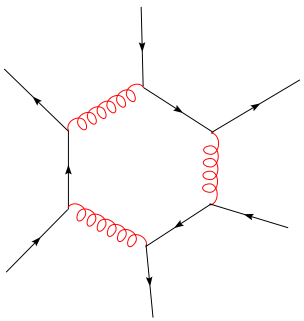
Soper, Nagy (2006)

- Dip at $\theta = 2.3$: $p_{T\,34} = p_{T\,56} << \sqrt{s}$, $p_T^2 = \det G_{1,2,34,56}/2/s^2$
- Vanishing determinants, $\det(G)$, $\det(S)$ related to integrable pinch singularities in Feynman integrands → Numerical problems inside phase space!

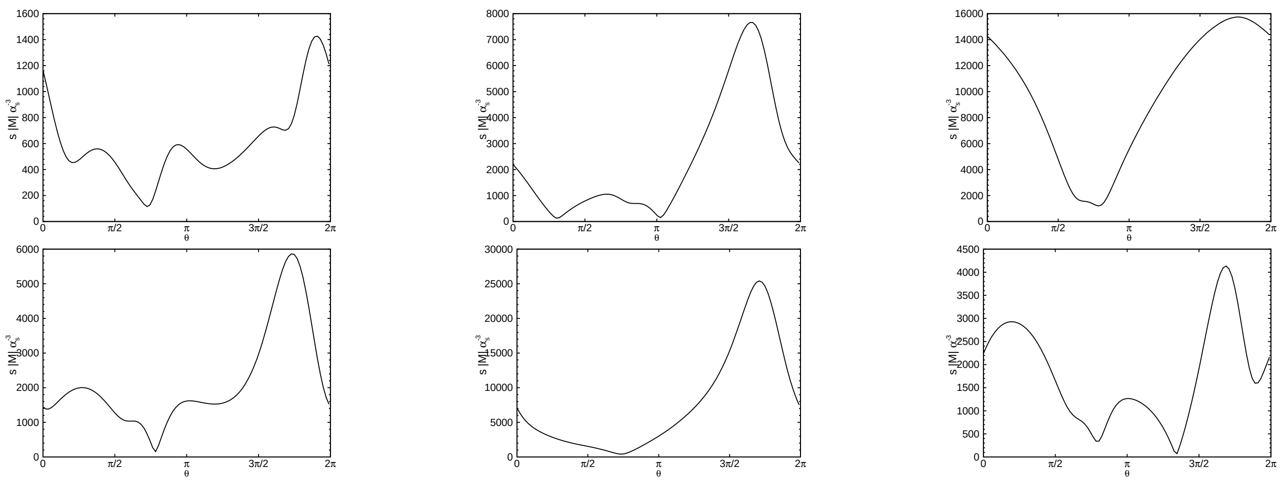
The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude

- Part of $PP \to 4$ jets, $bbbb$ at NLO.
  [T.B., J.Ph.-Guillet, A. Guffanti, G. Heinrich, T. Reiter]

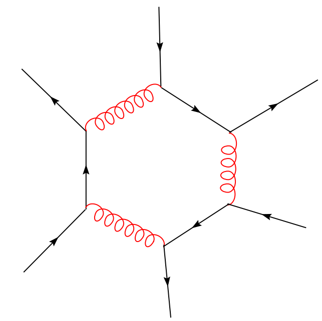
The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude



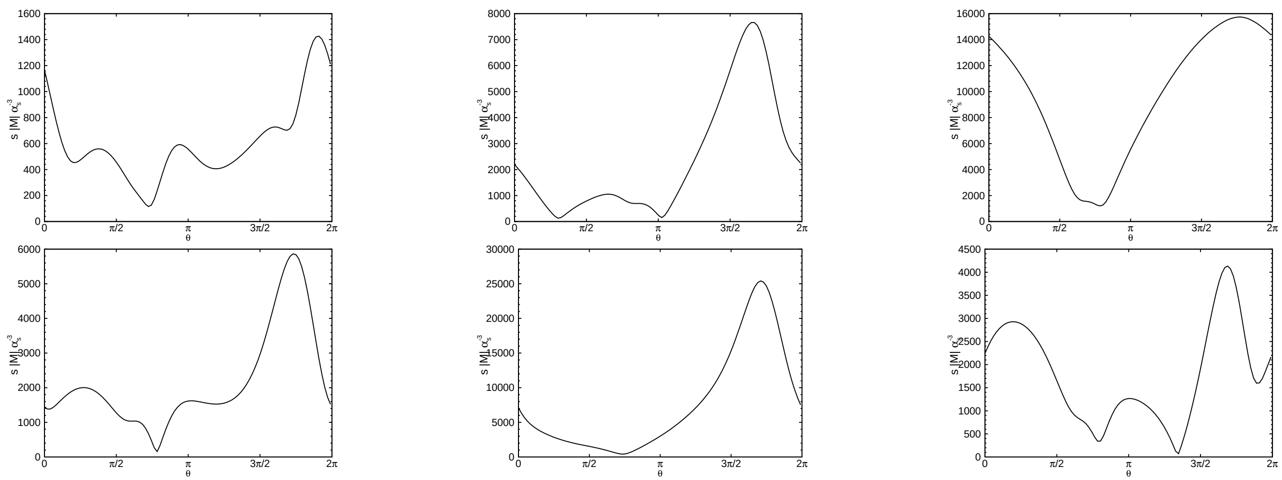
- Part of $PP \to 4$ jets, $bbbb$ at NLO.
  [T.B., J.Ph.-Guillet, A. Guffanti, G. Heinrich, T. Reiter]
- two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- six different colour structures, slice of phase-space of finite part:



The $q\bar{q} \to q\bar{q}q\bar{q}$ amplitude



- Part of $PP \to 4$ jets, $bbbb$ at NLO.
  [T.B., J.Ph.-Guillet, A. Guffanti, G. Heinrich, T. Reiter]
- two helicity amplitudes needed: $\mathcal{A}^{++++++}$, $\mathcal{A}^{++++--}$
- six different colour structures, slice of phase-space of finite part:



- algebraic reduction $\longrightarrow$ Master integrals
- semi-numerical reduction with Fortran 90 code "golem90 v0.2"
- Amplitude evaluation $\mathcal{O}(s)$, rank 3 6-point form factor ~ 30 ms
- phase space integration in progress

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

NLO multileg processes still challenging

- Lots of activity: algebraic, numeric, string inspired $\longrightarrow$ 6-gluon amplitude done...
- ... but still no complete genuine $2 \rightarrow 4$ process

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

NLO multileg processes still challenging

- Lots of activity: algebraic, numeric, string inspired $\longrightarrow$ 6-gluon amplitude done...
- ... but still no complete genuine $2 \rightarrow 4$ process

GOLEM approach for 1-loop multi-leg processes

- several $2 \rightarrow 2$, $2 \rightarrow 3$ processes evaluated
- can be used for rational terms only
- $\mathcal{O}(\alpha^2 \alpha_s^3)$ interference effect in $PP \rightarrow Hjj$ small
- six-photon amplitudes evaluated
- $q\bar{q} \rightarrow VVg$, $q\bar{q} \rightarrow q\bar{q}q\bar{q}$ at NLO in progress

Summary

LHC phenomenology needs and deserves (!) at least NLO precision

NLO multileg processes still challenging

- Lots of activity: algebraic, numeric, string inspired $\longrightarrow$ 6-gluon amplitude done...
- ... but still no complete genuine $2 \rightarrow 4$ process

GOLEM approach for 1-loop multi-leg processes

- several $2 \rightarrow 2$, $2 \rightarrow 3$ processes evaluated
- can be used for rational terms only
- $\mathcal{O}(\alpha^2 \alpha_s^3)$ interference effect in $PP \rightarrow Hjj$ small
- six-photon amplitudes evaluated
- $q\bar{q} \rightarrow VVg$, $q\bar{q} \rightarrow q\bar{q}q\bar{q}$ at NLO in progress

LHC = Long and Hard Calculations ...

- ...but results from many groups are on the horizon !