NLO QCD CORRECTIONS TO VECTOR BOSON FUSION PROCESSES

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- VBF and Higgs Physics
- $Wjj$ and $Zjj$ production
- Vector boson pairs in VBF
- Virtual corrections
- Phenomenological implications
- Conclusions
Total Higgs production cross sections at the LHC

Vector boson fusion is an important ingredient in Higgs search at the LHC

\[ \sigma(pp \rightarrow H + X) \ [\text{pb}] \]
\[ \sqrt{s} = 14 \text{ TeV} \]
NLO / NNLO

[Krämer ('02)]
Higgs Search in Vector Boson Fusion

Most measurements can be performed at the LHC with statistical accuracies on the measured cross sections times decay branching ratios, $\sigma \times BR$, of order 10% (sometimes even better).

[Eboli, Hagiwara, Kauer, Plehn, Rainwater, D.Z. ...]
VBF signature

\[ \eta = \frac{1}{2} \log \frac{1 + \cos \theta}{1 - \cos \theta} \]

Characteristics:

- energetic jets in the forward and backward directions \((p_T > 20 \text{ GeV})\)
- Higgs decay products between tagging jets
- Little gluon radiation in the central-rapidity region, due to colorless \(W/Z\) exchange (central jet veto: no extra jets with \(p_T > 20 \text{ GeV}\) and \(|\eta| < 2.5\))
Statistical and systematic errors at LHC

Assumed errors in fits to couplings:

- QCD/PDF uncertainties
  - $\pm 5\%$ for VBF
  - $\pm 20\%$ for gluon fusion
- luminosity/acceptance uncertainties
  - $\pm 5\%$

Some of the lowest errors are achievable in VBF production of the Higgs boson
VBF processes at LHC

\( qq \rightarrow qqH \)  

- Higgs coupling measurements

\( qq \rightarrow qqZ \) and \( qq \rightarrow qqW \)  
Oleari, DZ: hep-ph/0310156

- \( Z \rightarrow \tau \tau \) as background for \( H \rightarrow \tau \tau \)

- measure central jet veto acceptance at LHC

\( qq \rightarrow qqWW, qq \rightarrow qqZZ, qq \rightarrow qqWZ \)  

- \( qqWW \) is background to \( H \rightarrow WW \) in VBF

- underlying process is weak boson scattering:  
\( WW \rightarrow WW, WW \rightarrow ZZ, WZ \rightarrow WZ \) etc.

\[ \Rightarrow \] measure weak boson scattering

Precise predictions require QCD corrections
Generic features of QCD corrections to VBF

\( t \)-channel color singlet exchange \( \implies \) QCD corrections to different quark lines are independent

Born and vertex corrections to upper line

No \( t \)-channel gluon exchange at NLO

Features are generic for all VBF processes
Real emission

Calculation is done using Catani-Seymour subtraction method

Consider $q(p_a)Q\rightarrow g(p_1)q(p_2)QH$. Subtracted real emission term

$$|M_{\text{emit}}|^2 - 8\pi\alpha_s \frac{C_F}{Q^2} \frac{x^2 + z^2}{(1-x)(1-z)} |M_{\text{Born}}|^2$$

with $1-x = \frac{p_1 \cdot p_2}{(p_1 + p_2) \cdot p_a}$, $1-z = \frac{p_1 \cdot p_a}{(p_1 + p_2) \cdot p_a}$

is integrable $\implies$ do by Monte Carlo

Integral of subtracted term over $d^3p_1$ can be done analytically and gives

$$\frac{\alpha_s}{2\pi} C_F \left( \frac{4\pi\mu_R^2}{Q^2} \right)^\epsilon \Gamma(1+\epsilon) |M_{\text{Born}}|^2 \left[ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 9 - \frac{4}{3} \pi^2 \right] \delta(1-x)$$

after factorization of splitting function terms (yielding additional “finite collinear terms”)

The divergence must be canceled by virtual corrections for all VBF processes

only variation: meaning of Born amplitude $M_{\text{Born}}$

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Most trivial case: Higgs production
Virtual correction is vertex correction only

\[ \mathcal{M}_V = \mathcal{M}_\text{Born} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left( \frac{4\pi \mu_R^2}{Q^2} \right)^\epsilon \Gamma(1 + \epsilon) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] + \mathcal{O}(\epsilon) \]

- Divergent piece canceled via Catani Seymour algorithm

Remaining virtual corrections are accounted for by trivial factor multiplying Born cross section

\[ |\mathcal{M}_\text{Born}|^2 \left( 1 + 2\alpha_s \frac{C_F}{2\pi} c_{\text{virt}} \right) \]

- Factor 2 for corrections to upper and lower quark line
- Same factor to Born cross section absorbs most of the virtual corrections for other VBF processes
Results for Higgs production

✓ Small QCD corrections of order 10%
✓ Tiny scale dependence of NLO result
  - ±5% for distributions
  - < 2% for $\sigma_{\text{total}}$
✓ K-factor is phase space dependent
✓ QCD corrections under excellent control
✗ Need electroweak corrections for 5% uncertainty
  Solved now $\Rightarrow$ talk by Denner

$m_H = 120$ GeV, typical VBF cuts
W and Z production

- 10 · · · 24 Feynman graphs
- ⇒ use amplitude techniques, i.e. numerical evaluation of helicity amplitudes
- However: numerical evaluation works in d=4 dimensions only
Virtual contributions

Vertex corrections: same as for Higgs case

For each individual pure vertex graph $\mathcal{M}^{(i)}$ the vertex correction is proportional to the corresponding Born graph

$$\mathcal{M}_V^{(i)} = \mathcal{M}_B^{(i)} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left( \frac{4\pi\mu_R^2}{Q^2} \right)^\epsilon \Gamma(1+\epsilon) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right]$$

New: Box type graphs (plus gauge related diagrams)

Vector boson propagators plus attached quark currents are effective polarization vectors

build a program to calculate the finite part of the sum of the graphs
Use tensor decomposition a la Passarino-Veltman
Split $B_0 \cdots D_{ij}$ functions into divergent and finite parts

With $s = (q_1 + q_2)^2, t = (k_2 + q_2)^2 = (k_1 - q_1)^2$ we get, for example,

$$
B_0(q^2) = \frac{\Gamma(1 + \epsilon)}{(-s)^\epsilon} \left[ \frac{1}{\epsilon} + 2 - \ln \frac{q^2 + i0^+}{s} + \mathcal{O}(\epsilon) \right]
$$

$$
= \frac{\Gamma(1 + \epsilon)}{(-s)^\epsilon} \left[ \frac{1}{\epsilon} + \tilde{B}_0(q^2) + \mathcal{O}(\epsilon) \right]
$$

$$
D_0(k_2, q_2, q_1) = \frac{\Gamma(1 + \epsilon)}{(-s)^\epsilon} \left[ \frac{1}{st} \left( \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{q_1^2 q_2^2}{t^2} \right) + \tilde{D}_0(k_2, q_2, q_1) + \mathcal{O}(\epsilon) \right]
$$

$$
D^{\mu\nu}(k_2, q_2, q_1) = \frac{\Gamma(1 + \epsilon)}{(-s)^\epsilon} \left( \frac{1}{\epsilon} \left( k_1^\mu k_1^\gamma d_2(q_1^2, t) + k_2^\mu k_2^\gamma d_2(q_2^2, t) \right) + \tilde{D}^{\mu\nu}(k_2, q_2, q_1) + \mathcal{O}(\epsilon) \right)
$$

with $d_2(q^2, t) = 1/(s(q^2 - t)^2) \left[ t \ln(q^2/t) - (q^2 - t) \right]$

Finite $\tilde{D}_{ij}$ have standard PV recursion relations $\implies$ determine them numerically

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Virtual corrections for quark line with 2 EW gauge bosons

Divergent terms in 4 Feynman graphs combine to multiple of corresponding Born graph

\[ M_{\text{boxline}}^{(i)} = M_B^{(i)} F(Q) \]
\[ \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] \]
\[ + \frac{\alpha_s(\mu_R)}{4\pi} C_F \tilde{M}_\tau(q_1, q_2)(-e^2)g_\Gamma V_{1f_1} V_{2f_2} \]
\[ + \mathcal{O}(\epsilon) \]

with \( F(Q) = \frac{\alpha_s(\mu_R)}{4\pi} C_F \left( \frac{4\pi\mu_R^2}{Q^2} \right) \epsilon \Gamma(1 + \epsilon) \)

\( \tilde{M}_\tau(q_1, q_2) = \tilde{D}_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu \) is universal virtual \( qqVV \) amplitude: use like HELAS calls in MadGraph

The external vector bosons correspond to \( V \to l_1 \bar{l}_2 \) decay currents or quark currents
Born sub-amplitude is multiplied by same factor as found for pure vertex corrections
⇒ when summing all Feynman graphs the divergent terms multiply the complete $M_B$

Complete virtual corrections

$$M_V = M_B \cdot F(Q) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] + \tilde{M}_V$$

where $\tilde{M}_V$ is finite, and is calculated with amplitude techniques.

The interference contribution in the cross-section calculation is then given by

$$2 \text{Re} \left[ M_V M_B^* \right] = |M_B|^2 F(Q) \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] + 2 \text{Re} \left[ \tilde{M}_V M_B^* \right]$$

The divergent term, proportional to $|M_B|^2$, cancels against the subtraction terms just like in the Higgs case.
Most recent: $qq \to qqWW, qqZZ, qqWZ$ at NLO

- example: $WW$ production via VBF with leptonic decays: $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu + 2j$
- Spin correlations of the final state leptons
- All resonant and non-resonant Feynman diagrams included
- $NC \implies 181$ Feynman diagrams at LO
- $CC \implies 92$ Feynman diagrams at LO

Use modular structure, e.g. leptonic tensor

Calculate once, reuse in different processes

Speedup factor $\approx 70$ compared to MadGraph for real emission corrections
Virtual corrections involve up to pentagons

The sum of all QCD corrections to a single quark line is simple

\[
\mathcal{M}_V^{(i)} = \mathcal{M}_B^{(i)} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left( \frac{4\pi\mu_R^2}{Q^2} \right)^\varepsilon \Gamma(1+\varepsilon)
\]

\[
\left[ -\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{\text{virt}} \right] + \tilde{\mathcal{M}}_{V_1V_2V_3,\tau}^{(i)}(q_1, q_2, q_3) + O(\varepsilon)
\]

- Divergent pieces sum to Born amplitude: canceled via Catani Seymour algorithm
- Use amplitude techniques to calculate finite remainder of virtual amplitudes

The external vector bosons correspond to \( V \to l_1 \bar{l}_2 \) decay currents or quark currents

Pentagon tensor reduction with Denner-Dittmaier is stable at 0.1% level
Gauge invariance tests

Numerical problems flagged by gauge invariance test: use Ward identities for pentline and boxline contributions

\[ q_2^{\mu_2} \tilde{E}_{\mu_1 \mu_2 \mu_3} (k_1, q_1, q_2, q_3) = \tilde{D}_{\mu_1 \mu_3} (k_1, q_1, q_2 + q_3) - \tilde{D}_{\mu_1 \mu_3} (k_1, q_1 + q_2, q_3) \]

With Denner-Dittmaier recursion relations for \( E_{ij} \) functions the ratios of the two expressions agree with unity (to 10% or better) at more than 99.8% of all phase space points.

Ward identities reduce importance of computationally slow pentagon contributions when contracting with \( W^\pm \) polarization vectors

\[ J_\pm^\mu = x_\pm q_\pm^\mu + r_\pm^\mu \]

choose \( x_\pm \) such as to minimize pentagon contribution from remainders \( r_\pm \) in all terms like

\[ J_+^{\mu_1} J_-^{\mu_2} \tilde{E}_{\mu_1 \mu_2 \mu_3} (k_1, q_+, q_-, q_0) = r_+^{\mu_1} r_-^{\mu_2} \tilde{E}_{\mu_1 \mu_2 \mu_3} (k_1, q_+, q_-, q_0) + \text{box contributions} \]

Resulting true pentagon piece contributes to the cross section at permille level \( \Rightarrow \) totally negligible for phenomenology
Study LHC cross sections within typical VBF cuts

- Identify two or more jets with $k_T$-algorithm ($D = 0.8$)
  \[ p_{Tj} \geq 20 \text{ GeV}, \quad |y_j| \leq 4.5 \]

- Identify two highest $p_T$ jets as tagging jets with wide rapidity separation and large dijet invariant mass
  \[ \Delta y_{jj} = |y_{j1} - y_{j2}| > 4, \quad M_{jj} > 600 \text{ GeV} \]

- Charged decay leptons ($\ell = e, \mu$) of $W$ and/or $Z$ must satisfy
  \[ p_{T\ell} \geq 20 \text{ GeV}, \quad |\eta_\ell| \leq 2.5, \quad \Delta R_{j\ell} \geq 0.4, \quad m_{\ell\ell} \geq 15 \text{ GeV}, \quad \Delta R_{\ell\ell} \geq 0.2 \]
  and leptons must lie between the tagging jets
  \[ y_{j,min} < \eta_\ell < y_{j,max} \]

For scale dependence studies we have considered

- $\mu = \xi m_V$ fixed scale
- $\mu = \xi Q_i$ weak boson virtuality: $Q_i^2 = 2k_{q_1} \cdot k_{q_2}$
Stabilization of scale dependence at NLO

Jäger, Oleari, DZ hep-ph/0603177

\[ m_{H} = 120 \text{ GeV} \]
\[ m_{WW} > 130 \text{ GeV} \]

- solid: NLO $\mu_{F}=\mu_{R}=\xi m_{W}$
- dotdash: NLO $\mu_{F}=\xi m_{W}$
- dashes: NLO $\mu_{R}=\xi m_{W}$
- dots: LO $\mu_{F}=\xi m_{W}$
Transverse momentum distribution of the softer tagging jet

- Shape comparison LO vs. NLO depends on scale
- Scale choice $\mu = Q$ produces approximately constant $K$-factor
- Ratio of NLO curves for different scales is unity to better than 2%: scale choice matters very little at NLO

Use $\mu_F = Q$ at LO to best approximate the NLO results
ZZ production in VBF, $ZZ \rightarrow e^+ e^- \mu^+ \mu^-$

4-lepton invariant mass distribution without/with Higgs resonance

Good agreement of LO and NLO due to low scale choice $\mu = m_Z$. Alternative choice $\mu = m_H$ or $\mu = m_{4\ell}$ leads to smaller LO cross section at high $m_{4\ell}$
Conclusions

- LHC will observe a SM-like Higgs boson in multiple channels, with 5...20% statistical errors
  \[ \Rightarrow \] great source of information on Higgs couplings

- Whether or not a light Higgs is observed, weak boson scattering, i.e. $VVjj$ production by VBF, is an important testing ground for the physics underlying $SU(2) \times U(1)$ breaking

- NLO QCD corrections and improved simulation tools are crucial for precise measurements with full LHC data.

**NLO QCD correction for VBF now available in VBFNLO:**
parton level Monte Carlo for $Hjj, Wjj, Zjj, W^+W^-jj, ZZjj$ production
by Bozzi, Figy, Hankele, Jäger, Klämke, Oleari, Worek, DZ, ...

[http://www-itp.physik.uni-karlsruhe.de/~vbfnloweb/](http://www-itp.physik.uni-karlsruhe.de/~vbfnloweb/)