Multi-gluon amplitudes with heavy particles from SUSY, BCFW and CSW

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2.10.2007

Introduction

Since 2003: New methods (mostly) for massless QCD amplitudes

- CSW rules: MHV diagrams (Cachazo, Svrček, Witten 04)
- BCFW rules: on shell recursion (Britto, Cachazo, Feng/Witten, 04/05)

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QCD amplitudes with top quarks+ jets important at LHC:

- Top physics, Background to Higgs physics: $t\bar{t}H \to t\bar{t}b\bar{b}$
- Compact expression for tree amplitudes:
  input for unitarity method (Talks by D.Forde, P.Mastrolia, Z.Kunszt)

Overview

- SUSY identities for massive quarks and scalars
  (CS, S.Weinzierl, hep-th/0602012, JHEP 0603:030,2006)
- BCFW recursion (CS, S.Weinzierl, hep-ph/0703021, JHEP 0704:072,200)
- CSW diagrams for massive scalars (R.Boels, CS; in progress)

Color decomposition into color ordered partial amplitudes

$$\mathcal{A}_{n+2}(Q_i, g_1 \dots, g_n, \bar{Q}_i) = g^n \sum_{\sigma \in S_n} (T^{a_{\sigma(1)}} \dots T^{a_{\sigma(n)}})_{i,j} \, A_{n+2}\left(Q_i, g_{\sigma(1)}, \dots, g_{\sigma(n)}, \bar{Q}_j\right)$$

Spinor helicity methods

- Braket notation for Weyl spinors:

$$|k+\rangle = \lambda_{k,A} = \left(\tfrac{1+\gamma^5}{2}\right) u(k) \quad , \quad |k-\rangle = \bar{\lambda}_k^{\dot{A}} = \left(\tfrac{1-\gamma^5}{2}\right) u(k)$$

- antisymmetric spinor products

$$\langle pk \rangle = \langle p - |k+\rangle = \epsilon^{AB} \lambda_{p,B} \lambda_{q,A} \quad , \quad [pk] = \langle p + |k-\rangle = \epsilon_{\dot{B}\dot{A}} \bar{\lambda}^{p,\dot{B}} \bar{\lambda}^{q,\dot{A}} \quad ,$$

- Polarization vectors of the external gluons

$$\epsilon_\mu^\pm(k,q) = \pm \frac{\langle q \mp |\gamma_\mu| k \mp \rangle}{\sqrt{2}\,\langle q \mp | k \pm \rangle}$$

with q arbitrary light-like reference momentum

"Effective Supersymmetry" of QCD: (Parke, Taylor 1985; Kunszt 1986)

Tree level partial amplitudes for massless quarks are the same as for gluinos in a fictitious, unbroken, SUSY QCD.

SUSY transformations of helicity states of gluons and gluinos with Grassmann-valued spinor η:

$$\delta_\eta g^\pm(k) = \langle \eta \pm | k \mp \rangle \, \lambda^\pm(k) \qquad \delta_\eta \lambda^\pm(k) = - \langle \eta \mp | k \pm \rangle \, g^\pm(k)$$

SUSY Ward-Identities (Grisaru, Pendleton 1977)

$$0 = \langle 0 | [Q_{\text{SUSY}}, \psi_1 \dots \psi_n] | 0 \rangle = \sum_i A_n(\psi_1 \dots (\delta_\eta \psi_i) \dots \psi_n)$$

Fermionic MHV amplitudes (set $|\eta+\rangle \propto |j+\rangle$) (Parke, Taylor 1985; Kunszt 1986)

$$A_n(\bar{\lambda}_1^-, g_2^+, \dots g_j^-, \dots, \lambda_n^+) = \frac{\langle nj \rangle}{\langle 1j \rangle} A_n(g_1^-, g_2^+, \dots g_j^-, \dots, g_n^+)$$

$$= i 2^{n/2-1} \frac{\langle 1j \rangle^3 \langle nj \rangle}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

SUSY-relations for massive quarks

Spinors for massive quarks (Kleiss, Stirling 85,. . . , CS S.Weinzierl 05)

$$u(\pm) = \frac{1}{\langle p^\flat \pm | q \mp \rangle} (\not{p} + m) |q\mp\rangle \qquad \text{with} \quad p^\flat = p - \tfrac{p^2}{2p\cdot q} q.$$

Eigenstates of $(1 \pm \not{s}\gamma^5)$ with spin vector $s^\mu = \frac{p^\mu}{m} - \frac{m}{(p\cdot q)} q^\mu$.

"Helicity" amplitudes depend on q!

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SUSY toy model: Embed QCD with massive quark $Q = \binom{\psi_+}{\psi_-}$ in $N = 1$ SQCD $\Rightarrow$ two complex scalars $\phi_\pm$ as superpartners

Transformations of helicity states $((\bar{\phi}_\pm)^\dagger = \phi_\mp)$ (CS, S.Weinzierl, 06)

$$\delta_\eta \phi^- = [\eta k] Q^- + m \frac{[q\eta]}{[qk]} Q^+ \qquad \delta_\eta \phi^+ = \langle \eta k\rangle \, Q^+ + m \frac{\langle q\eta\rangle}{\langle qk\rangle} Q^-$$

$$\delta_\eta Q^+ = [k\eta] \phi^+ + m \frac{\langle q\eta\rangle}{\langle qk\rangle} \phi^- \qquad \delta_\eta Q^- = \langle k\eta\rangle \, \phi^- + m \frac{[q\eta]}{[qk]} \phi^+$$

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$$\delta_q \phi^- = [qk] Q^- \qquad \delta_q \phi^+ = \langle qk \rangle \, Q^+$$

$$\delta_q Q^+ = [k\eta] \phi^+ \qquad \delta_q Q^- = \langle kq \rangle \, \phi^-$$

Simplify for $|\eta\pm\rangle \propto |q\pm\rangle \Rightarrow$ similar to massless case!

SUSY-relations for massive quarks

Only positive helicity gluons:

Quark amplitude given by scalar amplitude

$$\langle 1q\rangle\, A_n(\bar{Q}_1^+,\ldots,g_{n-1}^+,Q_n^-) = \langle nq\rangle\, A_n(\bar{\phi}_1^+,\ldots,g_{n-1}^+,\phi_n^-)$$

(SYM Lagrangian $\Rightarrow$ no $\bar{\phi}^+\bar{\lambda}^+Q$ vertex)

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Compact expression for scalar amplitude known:

$$A(\bar{\phi}_1, g_2^+, \ldots, \phi_n) \;=\; \frac{i2^{n/2-1}m^2\,\langle 2+|\Pi_{j=3}^{n-2}\,(y_{1,j} - \not{k}_j\not{k}_{1,j-1})\,|(n-1)-\rangle}{y_{1,2}\ldots y_{1,n-2}\,\langle 23\rangle\,\langle 34\rangle\ldots\langle (n-2)(n-1)\rangle}$$

($k_{1,j} = \sum_1^j k_j$, $\quad y_{1,j} = k_{1,j}^2 - m^2$) (Ferrario, Rodrigo, Talavera 06)

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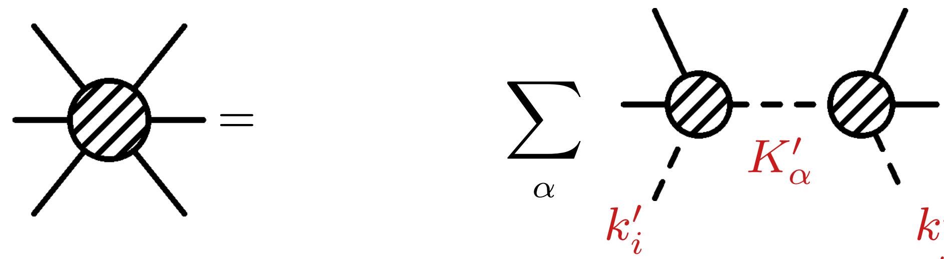
One negative helicity gluon:

Additional gluino contribution drops out for $|q+\rangle=|j+\rangle \Rightarrow$

$$A(\bar{Q}_1^+,\ldots,g_j^-,\ldots,Q_n^+)|_{|q+\rangle=|j+\rangle}=0$$

$$A(\bar{Q}_1^+,\ldots,g_j^-,\ldots,Q_n^-)|_{|q+\rangle=|j+\rangle}=\frac{\langle nj\rangle}{\langle 1j\rangle}A_n(\bar{\phi}_1^+,\ldots,g_j^-,\ldots\phi_n^-)$$

Construct amplitudes from on-shell sub-amplitudes



Shifted on-shell momenta: (Britto, Cachazo, Feng/ Witten, 04/05)

$$k'_i = k_i - z_\alpha \eta \qquad k'_j = k_i + z_\alpha \eta$$

with $\eta^2 = 0 \qquad k_i \cdot \eta = k_j \cdot \eta = 0$

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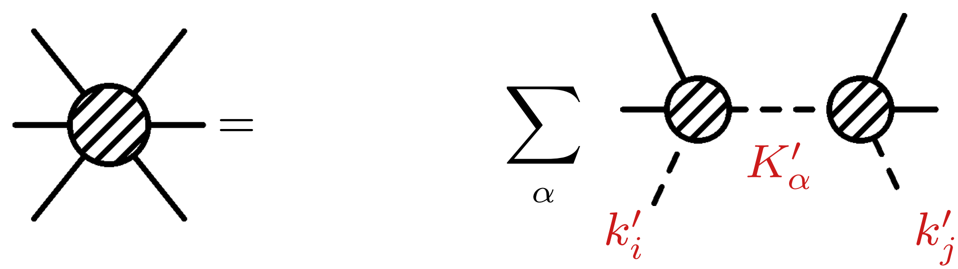

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$\Rightarrow \eta^\mu = \frac{1}{2} \langle i+|\gamma^\mu|j+\rangle$ for massless momenta

Corresponds to shifted spinors:

$$|i'+\rangle = |i+\rangle - z\,|j+\rangle \qquad |j'-\rangle = |j-\rangle + z\,|i-\rangle$$

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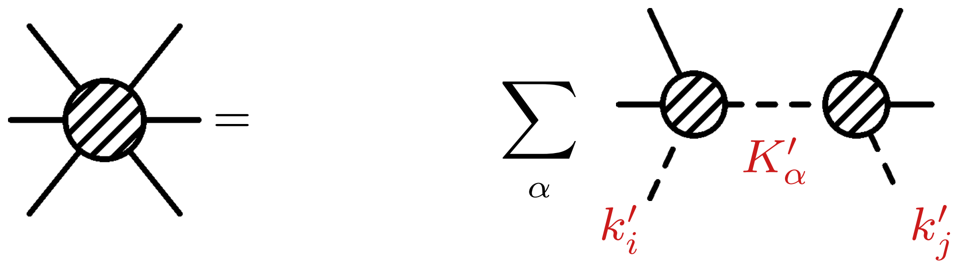

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Choose $z_\alpha = \frac{K_\alpha^2}{\langle i+|\not{K}_\alpha|j+\rangle} \quad \Rightarrow \quad K'^2_\alpha = 0$

Conditions for BCFW recursion:

$A(z)$ has simple poles, $A(z) \to 0$ for $z \to \infty$

Diagrammatical proof for (i^+, j^-) only (Draggiotis et.al.; Vaman, Yao; 05)

BCFW recursion relations

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Analysis of $z \to \infty$: Most dangerous diagrams: triple gluon vertices

$$A(z) \sim \underbrace{n \text{ propagators}}_{z^{-n}} \times \underbrace{(n+1) \text{ vertices}}_{z^{n+1}} \times \epsilon_i \times \epsilon_j \sim z \times \epsilon_i \times \epsilon_j$$

Scaling of gluon polarization vectors:

$$\epsilon^+(k_i') \sim \frac{1}{z}, \quad \epsilon^-(k_i') \sim z, \quad \epsilon^+(k_j') \sim z, \quad \epsilon^-(k_j') \sim \frac{1}{z}$$

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$\Rightarrow$ (i^+, j^-): $A(z) \sim \frac{1}{z}$ from powercounting (BCFW 05)

- (i^-, j^+): not allowed
- (i^+, j^+), (i^-, j^-): proof using three particle auxiliary shift

(Badger,Glover,Khoze,Svrček 05)

BCFW recursion for massive scalars (Badger,Glover,Khoze,Svrček 05)

- applied for shifted gluon lines
- shifted massive momenta defined ... not yet applied

Massive fermions (+gauge bosons)

- "stripped" amplitudes: (Badger, Glover, Khoze 05)

  remove spinors of internal quark lines:

$$\sum_{\sigma=\pm} A(\ldots, Q^{\sigma}_{K'}) \frac{\mathrm{i}}{K^2 - m^2} A(\bar{Q}^{-\sigma}_{K'}, \ldots) = A(\ldots, Q^{\bullet}_{K'}) \frac{\mathrm{i}(\not{K}' + m)}{K^2 - m^2} A(\bar{Q}^{\bullet}_{K'}, \ldots)$$

- 4-5 point amplitudes calculated (Ozeren,Stirling 06)

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BCFW relations for all born QCD amplitudes?

- allowed helicities?
- shift of massive quark lines?

BCFW recursion for massive quarks

Decompose general momenta into light-like $l_{i/j}$: (del Aguila, Pittau 04)

$$p_i = l_i + \alpha_j l_j \quad , \quad p_j = \alpha_i l_i + l_j$$

with $$\alpha_\ell = \frac{2p_i p_j \mp \sqrt{\Delta}}{2p_\ell^2}, \qquad \Delta = (2p_i p_j)^2 - 4p_i^2 p_j^2$$

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Define shifted spinors: (CS, S.Weinzierl 07)

$$u_i{}'(-) = u_i(-) - z\,|l_j+\rangle \quad , \quad \bar{u}_j'(+) = \bar{u}_j(+) + z\,\langle l_i+|$$

with reference spinors $|q_i\pm\rangle = |l_j\pm\rangle$, $|q_j\pm\rangle = |l_i\pm\rangle$

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Corresponds to shifted momenta

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Remark: Without fixing q one gets spurious poles in z:

$$u_i'(-) \stackrel{?}{=} \frac{(\not{p}_i' + m)\,|q-\rangle}{[p_i^\flat q]} \qquad \bar{u}_i'(+) \stackrel{?}{=} \frac{\langle q-|\,(\not{p}_i' + m)}{\langle q p_i^\flat\rangle - z\,\langle q l_j\rangle}$$

BCFW recursion for massive quarks

Recursion relation:

$$A_n(1,\dots,i,\dots,j,\dots,n) =$$

$$\sum_{\text{partitions},h=\pm} A_L(\dots,i',\dots K'^{h},\dots)\frac{i}{K^2-m_k^2}A_R(\dots,-K'^{-h},\dots j',\dots)$$

Intermediate massive quark: choose $|q_K+\rangle = |l_j+\rangle$ and $|q_K-\rangle = |l_i-\rangle$:

$$u'_K(-) = \frac{1}{\langle K^\flat + |l_i-\rangle}(\not{K}+m_k)\,|l_i-\rangle \qquad \bar{u}'_K(+) = \frac{1}{\langle l_j - |K^\flat+\rangle}\langle l_j-|\,(\not{K}+m_k)$$

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Conditions for $A(z)\to 0$

- (i^+,j^-) allowed if Q_i and Q_j are not joined by quark line

  (as for massless quarks: Luo, Wen; Badger et.al; Quigly, Rozali; 05)
- (g_i^+,g_j^+), (g_i^+,Q_j^+) , (g_i^-,g_j^-), (Q_i^-,g_j^-) allowed
- for (Q_i^+,Q_j^+), (Q_i^-,Q_j^-) three particle shift necessary

Proof for the case (i^+, j^+): auxilary (j^+, k^+, l^+) shift (Risager 05)

$$u_i'(-) = u_i(-) + y[p_k^\flat p_l^\flat]\,|l_j+\rangle \qquad u_k'(-) = u_k(-) + y[p_l^\flat l_i]\,|l_j+\rangle$$

$$u_l'(-) = u_l(-) + y[l_i p_k^\flat]\,|l_j+\rangle$$

where no two particles in $\{i, k, l\}$ belong to the same quark line

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$\Rightarrow$ recursion relation (following Badger, Glover, Khoze, Svrček 05)

$$A(y=0,z) = \sum_{\alpha,\lambda} A_L(y_\alpha, z, \lambda)\frac{i}{P_\alpha(z)^2 - m_\alpha^2} A_R(y_\alpha, z, -\lambda),$$

$i \in A_{L/R}$ and $j \in A_{R/L}$: $A(z) \xrightarrow[z\to\infty]{} 0$ using $y_\alpha \sim P_\alpha(z)^2 - m_\alpha^2 \sim z$

$i, j \in A_{L/R}$: use induction hypothesis

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Special cases $\Rightarrow$ (Q_i^+, g_j^+), $(Q_i^+, \bar{Q}_j^+)$ not allowed

$$A_4({Q'}_i^+, g^+, Q, {g'}_j^+) \underset{z\to\infty}{\neq} 0 \qquad A_4(Q_i^+, Q, {Q'}_j^+, Q') \underset{z\to\infty}{\neq} 0, \qquad (m_i \neq 0)$$

BCFW recursion for massive quarks

Application: Amplitudes with g_2^- from shift $(i,j) = (Q_1^\pm, g_2^-)$:

$$\bar{u}_1'(-) = \bar{u}_1(-) - z\,\langle 2-| \qquad , |2'-\rangle = |2-\rangle + z\,|l_1-\rangle$$

Amplitude expressed in terms of known quantities:

$$A_n(\bar{Q}_1^{\lambda_1}, g_2^-, g_3^+ \ldots, Q_n^{\lambda_n}) =$$

$$\sum_{j=3}^{n} A(\bar{Q}'^{\lambda_1}_1, g^+_{k'_{2,j}}, g^+_{j+1}, \ldots, Q_n^{\lambda_n}) \frac{i}{k_{2,j}^2} A_{\mathsf{MHV}}(g^-_{-k'_{2,j}}, g'^-_2, \ldots, g_j^+)$$

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Example: with $|\Phi_{k,n}-\rangle = \prod_{j=k}^{n-2}\left(1 - \frac{\not{p}_j \not{p}_{1,j}}{y_{1,j}}\right)|(n-1)-\rangle$.

$$A_n(\bar{Q}_1^+, g_2^-, \ldots, Q_n^-) = \frac{i 2^{n/2-1}\langle n^\flat 2\rangle}{\langle 1^\flat 2\rangle\langle 23\rangle \ldots \langle (n-2)(n-1)\rangle}$$

$$\sum_{j=3}^{n-1} \frac{\langle 2 - |\not{k}_1 \not{k}_{2,j}|2+\rangle^2}{k_{2,j}^2 \langle 2 - |\not{k}_1 \not{k}_{2,j}|j+\rangle} \left(\delta_{j,n-1} + \delta_{j\neq n-1} \frac{m^2 \langle 2 - |\not{k}_{2,j}|\Phi_{j+1,n}-\rangle \langle j(j+1)\rangle}{y_{1,j}\langle 2 - |\not{k}_1 \not{k}_{2,j}|(j+1)+\rangle}\right)$$

Simpler calculation than from shift of gluons (Forde, Kosower 05)

MHV diagrams: (Cachazo, Svrček, Witten 04)

All massless born QCD amplitudes from MHV vertices (Parke,Taylor 86)

$$A_{\text{MHV}}(g_1^+, \dots, g_i^-, \dots, g_j^-, \dots g_n^+) = \mathrm{i}2^{n/2-1} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

with off-shell continuation $|k+\rangle \to \not{k}\,|\eta-\rangle$

- External Higgs or gauge bosons

  (Dixon, Glover ,Khoze 04; Bern, Forde, Kosower, Mastrolia 04)

- Loop diagrams in SUSY theories (Brandhuber, Spence, Travaglini 04)

  $+$ Proposals for non-SUSY theories

  (Ettle,Fu, Fudger, Mansfield, Morris; Brandhuber, Spence, Travaglini, Zoubos, 07)

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  + Proposals for non-SUSY theories

  (Ettle,Fu, Fudger, Mansfield, Morris; Brandhuber, Spence, Travaglini, Zoubos, 07)

- Several derivations:
  - Generalized BCFW recursion (Risager 05)
  - Field-redefinition in light-cone QCD (Mansfield 05)
  - Yang-Mills theory on twistor space (Boels, Mason, Skinner 06)

- Light-cone decomposition

$$A_\pm = \frac{1}{\sqrt{2}}(A_0 \mp A_3), \qquad A_{z/\bar{z}} = \frac{1}{\sqrt{2}}(-A_1 \pm \mathrm{i}A_2)$$

  impose light-cone gauge $A_+ = 0$,

- eliminate A_- by e.o.m $\Rightarrow$ Lagrangian for physical fields A_z, $A_{\bar{z}}$:

$$\mathcal{L}^{(2)} + \mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--}$$

Light-cone derivation of CSW rules

- Light-cone decomposition

$$A_\pm = \frac{1}{\sqrt{2}}(A_0 \mp A_3), \qquad A_{z/\bar z} = \frac{1}{\sqrt{2}}(-A_1 \pm \mathrm{i}A_2)$$

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$$\mathcal{L}^{(2)} + \mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--}$$

- Canonical transformation $A_z \to B[A_z]$ (Mansfield 05)

  eliminates $\mathcal{L}^{(3)}_{++-}$ and generates MHV-type vertices:

$$\mathcal{L}^{(3)}_{++-} + \mathcal{L}^{(3)}_{+--} + \mathcal{L}^{(4)}_{++--} \Rightarrow \sum_n \mathcal{L}^{(n)}_{+\cdots+--}$$

- Explicit solution (Ettle,Morris 06)

$$A_z(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{\infty} \widetilde{dk_i} \, \frac{(g\sqrt{2})^{n-1} \langle \nu p \rangle^2}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1)n \rangle \langle \nu n \rangle} B(k_1) \ldots B(k_n)$$

  Similar solution for $A_{\bar z} \sim \sum_n B_1 \ldots \bar{B} \ldots B_n$

Application to massive scalars (R. Boels, CS, in progress)

- Lagrangian in light-cone gauge

$$\mathcal{L}^{(2)}(\bar\phi\phi) + \mathcal{L}^{(3)}(\bar\phi A_z \phi) + \mathcal{L}^{(3)}(\bar\phi A_{\bar z} \phi) + \mathcal{L}^{(4)}(\bar\phi A_z A_{\bar z} \phi) + \mathcal{L}^{(4)}(\bar\phi\phi\bar\phi\phi)$$

- eliminate $\mathcal{L}^{(3)}(\bar\phi A_z \phi)$ by transformation for massless scalars

$$\phi(p) = \sum_{n=1}^{\infty} \int \prod_{i=1}^{n} \widetilde{dk_i}\, \frac{(g\sqrt{2})^{n-1} \langle \nu n \rangle}{\langle \nu 1 \rangle \langle 12 \rangle \ldots \langle (n-1) n \rangle} B(k_1) \ldots B(k_{n-1}) \xi(k_n)$$

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$$\phi(p) = \sum_{n=1}^{\infty}\int\prod_{i=1}^{n}\widetilde{dk_i}\,\frac{(g\sqrt{2})^{n-1}\langle\nu n\rangle}{\langle\nu 1\rangle\langle 12\rangle\ldots\langle(n-1)n\rangle}B(k_1)\ldots B(k_{n-1})\xi(k_n)$$

- but mass term not invariant:

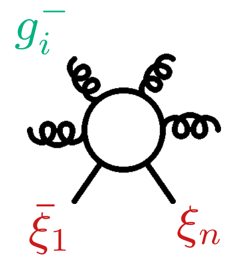
$$-m^2\bar\phi(p)\phi(-p) = \sum_{n=2}^{\infty}\int\prod_{i=1}^{n}\widetilde{dp_i}\,\mathcal{V}_{1,\ldots,n}\bar\xi(k_1)B(k_2)\ldots B(k_{n-1})\xi(k_n)$$

$\Rightarrow$ new CSW-vertex $\quad \mathcal{V}_{1,\ldots,n} = (g\sqrt{2})^{n-2}\dfrac{-m^2\langle 1n\rangle}{\langle 12\rangle\ldots\langle(n-1)n\rangle}$

Same result using Twistor Yang-Mills approach

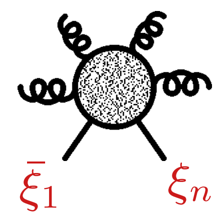
Summary of vertices: (four-scalar g^+ vertex not shown)

massless MHV vertices



$$= i2^{n/2-1} \frac{\langle in \rangle^2 \langle 1i \rangle^2}{\langle 12 \rangle \ldots \langle (n-1)n \rangle \langle n1 \rangle}$$

holomorphic vertex $\sim m^2$



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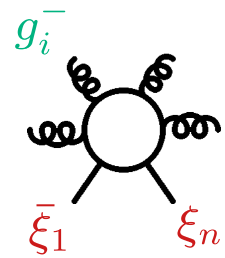

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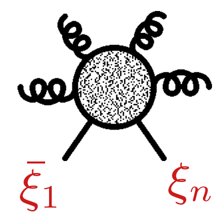
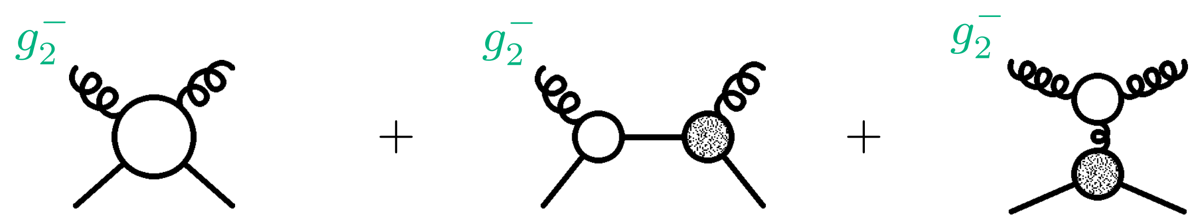


$$= \mathrm{i}2^{n/2-1}\frac{-m^2\langle 1n\rangle}{\langle 12\rangle\ldots\langle (n-1)n\rangle}$$

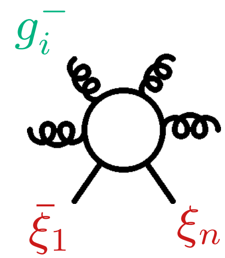
Example: $A_4(\bar{\xi}_1, g_2^-, g_3^+, \xi_4)$ (setting $|\eta+\rangle = |3+\rangle$)



$$= 2\mathrm{i}\frac{\langle 12\rangle^2\langle 24\rangle^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} + \frac{\sqrt{2}\mathrm{i}\langle 12\rangle\langle 2k_{1,2}\rangle}{\langle 1k_{1,2}\rangle}\frac{\mathrm{i}}{k_{1,2}^2 - m^2}\frac{-\sqrt{2}\mathrm{i}m^2\langle k_{1,2}4\rangle}{\langle k_{1,2}3\rangle\langle 34\rangle}$$

Summary of vertices: (four-scalar g^+ vertex not shown)

massless MHV vertices



$$= \mathrm{i}2^{n/2-1}\frac{\langle in\rangle^2\langle 1i\rangle^2}{\langle 12\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

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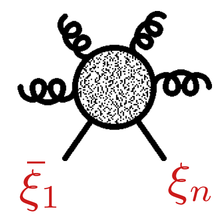

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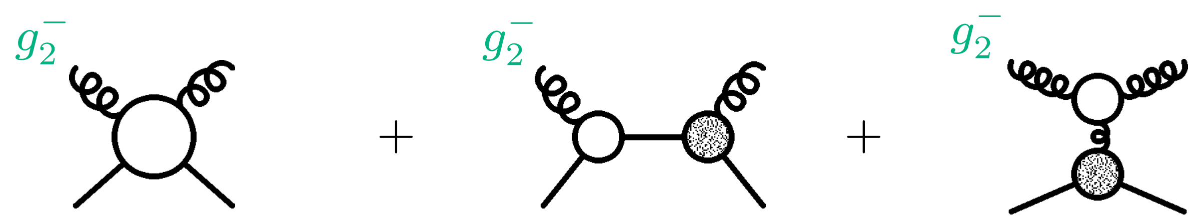

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$$= 2\mathrm{i}\frac{\langle 3+|\not{k}_1|2+\rangle}{\langle 3-|\not{k}_4|3-\rangle}\frac{\langle 2-|\not{k}_4|3-\rangle^2}{\langle 23\rangle\langle 3+|\not{k}_4\not{k}_1|3-\rangle} = 2\mathrm{i}\frac{\langle 3+|\not{k}_1|2+\rangle^2}{2(k_3\cdot k_4)\langle 23\rangle[23]}$$

Summary and outlook

SUSY relations of massive quarks to massive scalars

$\Rightarrow$ can use compact amplitudes with

massive scalars$+N$ gluons (Ferrario, Rodrigo, Talavera 06)

On-shell recursion with massive quarks

- Shift of massive quark lines
- Clarified allowed helicities
- Closed expressions for amplitudes $A_n(\bar{Q}_1^{\lambda_1}, g_2^-, \ldots, Q_n^{\lambda_n})$

CSW rules for massive scalars

- Massless MHV vertices + extra holomorphic vertex
- not an "on-shell" formalism
- number of "mass" vertices not fixed
  $\Rightarrow$ Berends-Giele might help
- Derivations should extend to massive particles with spin