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All-orders Symmetric Subtraction of Divergences for Massive YM Theory based on Nonlinearly Realized Gauge Group

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Based on

D.Bettinelli, A.Q., R.Ferrari, **arXiv:0705.2339 & arXiv:0709.0644**

Further references on the subtraction properties of nonlinearly realized theories:

hep-th/0701212, hep-th/0701197, hep-th/0611063, hep-th/0511032, hep-th/0506220, hep-th/0504023

Mass Generation in Non-Abelian Gauge Theories

Linear Representation of the Gauge Group
$\rightarrow$ Higgs Mechanism

- ✔ Physical Unitarity
- ✔ Power-counting Renormalizability
- ✔ (at least one) additional physical scalar particle

Mass Generation in Non-Abelian Gauge Theories

Non-Linear Representation of the Gauge Group → Stückelberg Mechanism

- ✔ Mass through the coupling with the flat connection

$$\frac{M^2}{2}(A_{a\mu} - F_{a\mu})^2$$

- ✔ Physical Unitarity [R.Ferrari, A.Q., JHEP 0411:019,2004]
- ✔ No additional physical scalar particle

Mass Generation in Non-Abelian Gauge Theories

Non-Linear Representation of the Gauge Group
→ Stückelberg Mechanism

Not power-counting renormalizable

How to subtract the divergences?

How many physical parameters are there?

Is the model unique?

How to subtract the divergences?

Lessons from the Nonlinear Sigma Model: The Local Functional Equation

Enforce the invariance of the path-integral SU(2) Haar measure under local left group transformations

Defining local functional equation

for the 1-PI vertex functional

$$\Big(- \partial_\mu \frac{\delta \Gamma}{\delta J_{a\mu}} + \epsilon_{abc} J_{c\mu} \frac{\delta \Gamma}{\delta J_{b\mu}} + \frac{1}{2} K_0 \phi_a + \frac{1}{2} \frac{\delta \Gamma}{\delta K_0} \frac{\delta \Gamma}{\delta \phi_a} + \frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta \Gamma}{\delta \phi_b} \Big)(x) = 0$$

How to subtract the divergences?

Lessons from the Nonlinear Sigma Model: The Hierarchy Principle

All the amplitudes involving at least one pion (descendant amplitudes) are fixed once those involving only insertions of the flat connection and the nonlinear sigma model constraint (ancestor amplitudes) are given.

Solution of the recursion generated by the local functional equation

[D.Bettinelli, A.Q, R.Ferrari, JHEP0703:065,2007]

How to subtract the divergences?

Lessons from the Nonlinear Sigma Model: The Weak Power-Counting Theorem

At every loop order there is only
a finite number of divergent ancestor amplitudes

$$\delta = (D-2)n + 2 - N_J - 2N_{K_0}$$

There is an infinite number of divergent amplitudes
involving pions already at one loop

Symmetries of nonlinearly realized Yang-Mills

Try with the standard framework of gauge theories

BRST symmetry $\rightarrow$ Slavnov-Taylor identity
(Physical Unitarity)

Stability equations (B-equation, ghost equation)

Is this enough to implement the hierarchy?

The answer is no.

Due to the antisymmetric character of the ghost fields the ST identity only fixes suitable antisymmetrized combinations of the pseudo-Goldstone amplitudes.

A counter-example

$$\mathcal{I} = \mathcal{S}_0\Big(\int d^D x\,(A^*_{a\mu} + \partial_\mu \bar{c}_a) A^\mu_a\Big)$$
$$= \int d^D x \left(A_{a\mu} \frac{\delta S}{\delta A_{a\mu}} - (A^*_{a\mu} + \partial_\mu \bar{c}_a)\partial^\mu c_a \right)$$

$$\mathcal{I}' = \int \Bigg(\frac{1}{g^2}\Big(-(D[F]_\mu I_\nu)_a (D[F]^\mu I^\nu)_a + (D[F]I)^2_a$$
$$- 3\epsilon_{abc}(D_\mu[F]I_\nu)_a I^\mu_b I^\nu_c - (I^2)^2 + I_{a\mu} I^\mu_b I_{a\nu} I^\nu_b \Big)$$
$$+ M^2 I^2 + \mathcal{S}_0((A^*_{a\mu} + \partial_\mu \bar{c}_a)\partial^\mu(\Omega^{-1}_{ap}\phi_p)) \Bigg) .$$

They coincide at $\vec{\phi} = 0$, but they have different projections on the monomial $\epsilon_{abc}\partial A^*_a c_b \phi_c$.

$$S = -\frac{1}{g^2}\int d^D x\, \frac{1}{4} G_{a\mu\nu} G^{\mu\nu}_a + \frac{M^2}{2}\int d^D x\, A^2_{a\mu} ,$$
$$I_{a\mu} = A_{a\mu} - F_{a\mu}$$

Symmetries of nonlinearly realized Yang-Mills

One also needs a local functional equation along the same lines of the nonlinear sigma model

Introduce a background connection and use a background (Landau) gauge-fixing

Symmetries of nonlinearly realized Yang-Mills

The local functional equation (bilinear!)

$$
\begin{aligned}
\mathcal{W}(\Gamma) \equiv \int d^D x \alpha_a^L(x) \Bigg(& -\partial_\mu \frac{\delta \Gamma}{\delta V_{a\mu}} + \epsilon_{abc} V_{c\mu} \frac{\delta \Gamma}{\delta V_{b\mu}} - \partial_\mu \frac{\delta \Gamma}{\delta A_{a\mu}} \\
& + \epsilon_{abc} A_{c\mu} \frac{\delta \Gamma}{\delta A_{b\mu}} + \epsilon_{abc} B_c \frac{\delta \Gamma}{\delta B_b} + \frac{1}{2} K_0 \phi_a + \underbrace{\frac{1}{2} \frac{\delta \Gamma}{\delta K_0} \frac{\delta \Gamma}{\delta \phi_a}} \\
& + \frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta \Gamma}{\delta \phi_b} + \epsilon_{abc} \bar{c}_c \frac{\delta \Gamma}{\delta \bar{c}_b} + \epsilon_{abc} c_c \frac{\delta \Gamma}{\delta c_b} \\
& + \epsilon_{abc} \Theta_{c\mu} \frac{\delta \Gamma}{\delta \Theta_{b\mu}} + \epsilon_{abc} A^*_{c\mu} \frac{\delta \Gamma}{\delta A^*_{b\mu}} + \epsilon_{abc} c^*_c \frac{\delta \Gamma}{\delta c^*_b} + \frac{1}{2} \phi^*_0 \frac{\delta \Gamma}{\delta \phi^*_a} \\
& + \frac{1}{2} \epsilon_{abc} \phi^*_c \frac{\delta \Gamma}{\delta \phi^*_b} - \frac{1}{2} \phi^*_a \frac{\delta \Gamma}{\delta \phi^*_0} \Bigg) = 0 \,.
\end{aligned}
$$

Bleaching

Introduce variables invariant

under the linearized local functional equation

(**bleached variables**)

$$\begin{aligned} a_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger(A_\mu - F_\mu)\Omega \\ &= \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega\,. \end{aligned}$$

Bleaching/2

By using bleached variables only

there are too many invariants

(like off-diagonal mass terms).

Way out: enforce also

global $SU_R(2)$ invariance

A summary

- ✓ Slavnov-Taylor identity
- ✓ Local functional equation
- ✓ B-equation (Landau gauge equation)

(the ghost equation follows
as a consequence of the above identities)

to be solved in the $\hbar$ expansion

Symmetries of nonlinearly realized Yang-Mills

- ST identity

$$\mathcal{S}(\Gamma) = \int d^D x \Big(\frac{\delta\Gamma}{\delta A^*_{a\mu}} \frac{\delta\Gamma}{\delta A^\mu_a} + \frac{\delta\Gamma}{\delta \phi^*_a} \frac{\delta\Gamma}{\delta \phi_a} + \frac{\delta\Gamma}{\delta c^*_a} \frac{\delta\Gamma}{\delta c_a}$$
$$+ B_a \frac{\delta\Gamma}{\delta \bar{c}_a} + \Theta_{a\mu} \frac{\delta\Gamma}{\delta V_{a\mu}} - K_0 \frac{\delta\Gamma}{\delta \phi^*_0} \Big) = 0$$

- Landau gauge equation

$$\frac{\delta\Gamma}{\delta B_a} = \frac{\Lambda^{D-4}}{g^2} D^\mu[V](A_\mu - V_\mu)_a$$

- Ghost equation

$$\frac{\delta\Gamma}{\delta \bar{c}_a} = \frac{\Lambda^{D-4}}{g^2} \Big(- D_\mu[V] \frac{\delta\Gamma}{\delta A^*_\mu} + D_\mu[A]\Theta^\mu \Big)_a$$

Feynman rules in the Landau gauge

The classical gauge-invariant action ...

$$
\begin{aligned}
S &= \frac{\Lambda^{(D-4)}}{g^2}\int d^D x \left(-\frac{1}{4} G_{a\mu\nu}[a] G_a^{\mu\nu}[a] + \frac{M^2}{2} a_{a\mu}^2 \right) \\
&= \frac{\Lambda^{(D-4)}}{g^2}\int d^D x \left(-\frac{1}{4} G_{a\mu\nu}[A] G_a^{\mu\nu}[A] + \frac{M^2}{2} (A_{a\mu} - F_{a\mu})^2 \right)
\end{aligned}
$$

... plus gauge-fixing terms plus couplings of antifields
with BRST transformations plus sources
for the local left transformations

Feynman rules in the Landau gauge

The tree-level vertex functional

$$
\begin{aligned}
\Gamma^{(0)} \quad = \quad & S + \frac{\Lambda^{D-4}}{g^2} \int d^D x \left(B_a (D^\mu[V](A_\mu - V_\mu))_a - \bar{c}_a (D^\mu[V] D_\mu[A] c)_a \right) \\
& + \frac{\Lambda^{D-4}}{g^2} \int d^D x \, \Theta^\mu_a \; (D_\mu[A]\bar{c})_a \\
& + \int d^D x \left(A^*_{a\mu} s A^\mu_a + \phi^*_0 s \phi_0 + \phi^*_a s \phi_a + c^*_a s c_a + K_0 \phi_0 \right) .
\end{aligned}
$$

Weak Power-Counting Formula

There is a week power-counting formula for the ancestor amplitudes

$$d(\mathcal{G}) \leq (D-2)n + 2 - N_A - N_c - N_V - N_{\phi_a^*} - 2(N_\Theta + N_{A^*} + N_{\phi_0^*} + N_{c^*} + N_{K_0}) \,.$$

Properties of the perturbative series

- In the Landau gauge the unphysical modes stay massless as a consequence of the Landau gauge equation

- One can drop all tadpole diagrams in DR (since in the Landau gauge all tadpole diagrams are massless)

One Loop

At one loop level
the relevant symmetries are

- the linearized ST identity
- the linearized local functional equation
- the Landau gauge equation

Compatibility condition $\left[\mathcal{S}_0, \mathcal{W}_0\right] = 0$

One Loop Solution

In the bleached variables the linearized local functional equation reads

$$\frac{\delta \Gamma^{(1)}}{\delta \phi_a(x)} = 0$$

Then one needs to solve a cohomological problem in the space of bleached variables

$$\mathcal{S}_0 \Gamma^{(1)} = 0$$

Bleached Variables/1

$$
\begin{aligned}
a_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger(A_\mu - F_\mu)\Omega \\
&= \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega \, .
\end{aligned}
$$

$$
\begin{aligned}
v_\mu &= a_{a\mu}\frac{\tau_a}{2} = \Omega^\dagger(V_\mu - F_\mu)\Omega \\
&= \Omega^\dagger V_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega \, .
\end{aligned}
$$

Gauge variables

$$
\widetilde{I} = \Omega^\dagger I \Omega \, ,
$$

$$
\widetilde{B}_a, \widetilde{\bar{c}}_a, \widetilde{c}_a, \widetilde{\Theta}_{a\mu}, \widetilde{A^*}_{a\mu}, \widetilde{c}^*_a \, .
$$

Variables
in the adj. representation
under the local left
transformations

Bleached Variables/2

$$\left.\begin{aligned}
K &= K_0 - i\frac{\delta\Gamma^{(0)}}{\delta\phi_a}\tau_a = K_0 + iK_a\tau_a \,, \\
\tilde{K} &= \Omega^\dagger K \,, \\
\Omega^* &= \phi_0^* + i\phi_a^*\tau_a \,, \\
\tilde{\Omega}^* &= \Omega^\dagger\Omega^*
\end{aligned}\right\} \text{ SU(2) doublets}$$

$$\widetilde{\phi_0^*} = \frac{1}{v_D}(\phi_0\phi_0^* + \phi_a\phi_a^*) \,,$$

$$\widetilde{\phi_a^*} = \frac{1}{v_D}(\phi_0\phi_a^* - \phi_a\phi_0^* - \epsilon_{abc}\phi_b^*\phi_c) \,.$$

$$\widetilde{K_0} = \frac{1}{v_D}\Big(\frac{v_D^2 K_0}{\phi_0} - \phi_a\frac{\delta}{\delta\phi_a}\Big(\Gamma^{(0)}\Big|_{K_0=0}\Big)\Big)$$

Linearized ST Transforms of Bleached Variables/1

$$\mathcal{S}_0 \Omega = i c \Omega \,,$$

$$\mathcal{S}_0 a_\mu = 0 \,,$$

$$\mathcal{S}_0 \widetilde{c} = -\frac{i}{2}\{\widetilde{c}, \widetilde{c}\} \,,$$

$$\mathcal{S}_0 v_\mu = \widetilde{\Theta}_\mu - D_\mu[v]\tilde{c} \,,$$

$$\mathcal{S}_0 \widetilde{\Theta}_\mu = -i\{\widetilde{c}, \widetilde{\Theta}_\mu\} \,,$$

Linearized ST Transforms of Bleached Variables/2

$$\mathcal{S}_0 \widehat{A}^*_\mu = \frac{\Lambda^{D-4}}{g^2}\Big[D^\rho G_{\rho\mu}[a] + M^2 a_\mu\Big] \,,$$

$$\mathcal{S}_0 \widetilde{\Omega^*} - i\widetilde{c}\, \widetilde{\Omega}^* - \widetilde{K} \,,$$

$$\mathcal{S}_0 \widetilde{K} = -i\widetilde{c}\, \widetilde{K} \,,$$

$$\mathcal{S}_0 \widetilde{c}^* = (D^\mu[a]\widetilde{\widehat{A}^*}_\mu) - \frac{i}{4}(\widetilde{\Omega}^*)^\dagger + \frac{i}{8} Tr[(\widetilde{\Omega}^*)^\dagger]\mathbf{1} \,.$$

The linearized ST transforms of bleached variables are bleached.

One Loop Invariants

Cohomologically non-trivial

$$\mathcal{I}_1 = \int d^D x\, Tr\ \partial_\mu a_\nu \partial^\mu a^\nu\ ,$$

$$\mathcal{I}_2 = \int d^D x\, Tr\ (\partial a)^2\, ,$$

$$\mathcal{I}_3 = i \int d^D x\, Tr(\partial_\mu a_\nu [a^\mu, a^\nu])\, ,$$

$$\mathcal{I}_4 = \int d^D x\, Tr(a^2)\ Tr(a^2)\, ,$$

$$\mathcal{I}_5 = \int d^D x\, Tr(a_\mu a_\nu) Tr(a^\mu a^\nu)\, ,$$

$$\mathcal{I}_6 = \int d^D x\, Tr(a^2)\, .$$

One Loop Invariants

Cohomologically trivial

$$
\begin{aligned}
\mathcal{I}_7 &= \mathcal{S}_0 \int d^Dx\, Tr(\widetilde{\widehat{A}^*}_\mu v^\mu) \\
&= \frac{\Lambda^{D-4}}{g^2} \int d^Dx\, Tr\Big[v^\mu \Big(D^\rho G_{\rho\mu}[a] + M^2 a_\mu\Big)\Big] - \int d^Dx\, Tr(\widetilde{\widehat{A}^*}_\mu \widetilde{\Theta}^\mu) \\
&\qquad + \int d^Dx\, Tr \widetilde{\widehat{A}^*}_\mu (D^\mu[v]\tilde{c})\,, \\
\mathcal{I}_8 &= \Big[\mathcal{S}_0 \int d^Dx\, Tr(\widetilde{\Omega}^*)\Big]^2 = -\int d^Dx\, (Tr(\tilde{c}\,\widetilde{\Omega}^*))^2 + 2i \int d^Dx Tr(\widetilde{K}) Tr(\tilde{c}\,\widetilde{\Omega}^*) \\
&\qquad + \int d^Dx\, (Tr(\widetilde{K}))^2\,. \\
\mathcal{I}_9 &= \mathcal{S}_0 \int d^Dx\, Tr(\widetilde{\Omega}^*) Tr(a^2) \\
&= -i \int d^Dx\, Tr(\tilde{c}\,\widetilde{\Omega}^*) Tr(a^2) - \int d^Dx\, Tr(\widetilde{K}) Tr(a^2)\,, \\
\mathcal{I}_{10} &= \mathcal{S}_0 \int d^Dx\, Tr(\widetilde{c}^* \widetilde{c}) \\
&= \int d^Dx \Big(Tr((D^\mu[a]\widetilde{\widehat{A}^*}_\mu)\widetilde{c}) - \frac{i}{4} Tr((\widetilde{\Omega}^*)^\dagger \widetilde{c}) + \frac{i}{2} Tr(\widetilde{c}^* \{\widetilde{c}, \widetilde{c}\})\Big)\,, \\
\mathcal{I}_{11} &= \mathcal{S}_0 \int d^Dx\, Tr(\widetilde{\Omega}^*) = -i \int d^Dx\, Tr(\tilde{c}\,\widetilde{\Omega}^*) - \int d^Dx\, Tr(\widetilde{K})\,.
\end{aligned}
$$

Perturbative Solution in D dimensions

Only the pole parts are subtracted by adopting the counterterm structure

$$\widehat{\Gamma} = \Gamma^{(0)} + \Lambda_D \sum_{j \geq 1} \int d^D x \mathcal{M}^{(j)}$$

The amplitudes must be normalized as

$$\Lambda_D^{-1} \Gamma^{(n)}$$

Perturbative Solution in *D* dimensions/2

This subtraction scheme is symmetric to all orders in the loop expansion.

Notice that the normalization introduces non-trivial finite parts required for the fulfillment of the functional identities.

Perturbative Solution in *D* dimensions/3

$$\mathcal{I}_1 = \frac{1}{2}\int d^Dx\, \partial_\mu A_{a\nu}\partial^\mu A^\nu_a\,,$$

$$\mathcal{I}_2 = \frac{1}{2}\int d^Dx\, (\partial A_a)^2\,,$$

$$\mathcal{I}_3 = -\frac{1}{2}\int d^Dx\, \epsilon_{abc}\partial_\mu A_{a\nu}A^\mu_b A^\nu_c\,,$$

$$\mathcal{I}_4 = \frac{1}{4}\int d^Dx\, (A^2)^2\,,$$

$$\mathcal{I}_5 = \frac{1}{4}\int d^Dx\, (A_{a\mu}A^\mu_b)(A_{a\nu}A^\nu_b)\,,$$

$$\mathcal{I}_6 = \frac{1}{2}\int d^Dx\, A^2\,,$$

$$\mathcal{I}_7 = \frac{1}{2}\int d^Dx\, V^\mu_a\Big(D^\rho G_{\rho\mu}[A] + M^2 A_\mu\Big)_a - \frac{1}{2}\int d^Dx\, \widehat{A}^*_{a\mu}\Theta^\mu_a + \frac{1}{2}\int d^Dx\, \widehat{A}^*_{a\mu}(D^\mu[V]c)_a\,,$$

$$\mathcal{I}_8 = \int d^Dx\, (2K_0 - c_a\phi^*_a)^2\,,$$

$$\mathcal{I}_9 = \int d^Dx\, \Big(\frac{1}{2}c_a\phi^*_a A^2 - K_0 A^2\Big)\,,$$

$$\mathcal{I}_{10} = \int d^Dx\, \Big(\frac{1}{2}(D^\mu[A]\widehat{A}^*_\mu)_a c_a - \frac{1}{4}\phi^*_a c_a - \frac{1}{2}c^*_a\epsilon_{abc}c_b c_c\Big)\,,$$

$$\mathcal{I}_{11} = \int d^Dx\, \Big(c_a\phi^*_a - 2K_0\Big)\,.$$

Projections of the one-loop invariants on the ancestor amplitudes

Perturbative Solution in D dimensions/4

The counterterms

$$\widehat{\Gamma}^{(1)} = \frac{\Lambda^{(D-4)}}{(4\pi)^2}\frac{1}{D-4}\Big[\frac{17}{2}(\mathcal{I}_1 - \mathcal{I}_2) - \frac{67}{6}\mathcal{I}_3 + \frac{11}{4}\mathcal{I}_4 - \frac{5}{2}\mathcal{I}_5 + 3M^2\mathcal{I}_6 - 6\mathcal{I}_7 + \frac{3v^2}{128M^4}\mathcal{I}_8 - \frac{v}{8M^2}\mathcal{I}_9\Big].$$

Perturbative Solution in *D* dimensions/5

The self-mass

$$g^2\Sigma_T(M^2)|_{D\sim 4} = g^2\frac{M^2}{(4\pi)^2}\left\{-\frac{23}{4}C_\Lambda + \frac{2}{3} - \frac{33}{4}\int_0^1 dx P(1,x)\right\}$$

with

$$C_\Lambda \equiv \frac{2}{D-4} + \gamma - \ln 4\pi + \ln\left(\frac{M^2}{\Lambda^2}\right)$$

and

$$P(r,x) \equiv x^2 - rx + r.$$

Perturbative Solution in D dimensions/6

Some checks

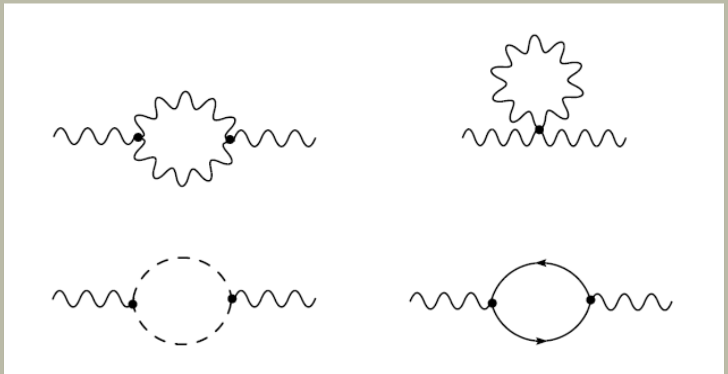
1. $\Sigma_T(0) = \Sigma_L(0)$ is verified for generic D. By this property the pole at $p^2 = 0$ in the 1PI two-point function is avoided. This condition is very important in order to prove physical unitarity in the Landau gauge

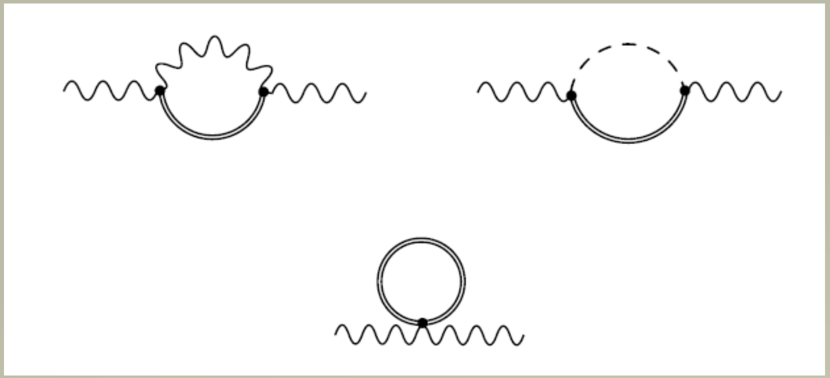
2. For $p^2 = M^2$, Σ_T contains only $H_2(M^2, M^2)$ which is the only Feynman integral with a physical discontinuity across the real positive p^2 axis.

3. As a check on $\Sigma_L(p^2)$ the relevant Slavnov-Taylor identity is explicitly evaluated

Perturbative Solution in *D* dimensions/7

The self-mass





This separation between Feynman diagrams of the linear and the nonlinear theory does not hold in general.

Uniqueness of the tree-level vertex functional

The Stückelberg action is the only one fulfilling the weak power-counting formula.

The invariants $I_1, \ldots, I_5$ contains vertices with two phi's, two A's and two derivatives.
They give rise to one-loop diagrams with degree of divergence equal to 4 and any number of external legs.

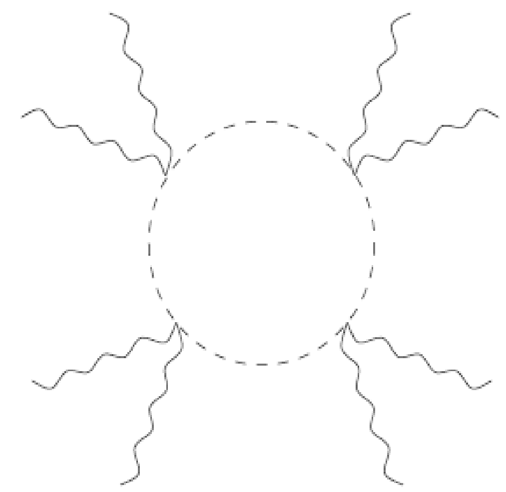


Figure 1: A weak power-counting violating graph.

Stability?

The removal of the divergences can be implemented through a canonical transformation on the classical action order by order in the $\hbar$ expansion.

In this sense (see Weinberg & Gomis 1996) this is a stable theory.

The number of physical parameters

Are the coefficients of the invariants I_j compatible with the weak power-counting bound additional *bona fide* parameters?

They are not, since they cannot be inserted back into the tree-level vertex functional without violating either the symmetries or the weak power-counting theorem.

The number of physical parameters/1

The physical parameters are the mass M and the gauge coupling constant g.

Since the scale of radiative corrections Λ cannot be reabsorbed by a change in M and g, Λ must also be considered as a further physical parameter.

The number of physical parameters/2

Lessons from the nonlinear sigma model

The most general action compatible with the defining local functional equation and the weak power-counting theorem is

$$\Gamma^{(0)}_{NLSM} = \Lambda^{D-4} \int d^D x \left(\frac{v^2}{8} (J_{a\mu} - F_{a\mu})^2 + K_0 \phi_0 + \mathcal{P}[J] \right)$$

under the assumption that $$\frac{\delta \Gamma^{(0)}_{NLSM}}{\delta K_0(x)} = \phi_0(x)$$

Gauge-invariant local function depending only on J

Conclusions and Outlook

- Nonlinearly realized massive Yang-Mills theory can be symmetrically subtracted to all orders in the $\hbar$ expansion

- The tools: hierarchy,
  weak power-counting,
  functional equations

Conclusions and Outlook

- The number of physical parameters is finite. Hence the model can be tested against experiments.

- Is there a renormalization group equation in the proposed subtraction scheme?

- Extension to SU(2) x U(1)