Analytical calculation of massive Feynman diagrs and the NLO corrections to $gg \to H$ and $H \to \gamma\gamma$

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In collaboration with: U. Aglietti, G. Degrassi and A. Vicini

Plan of the Talk

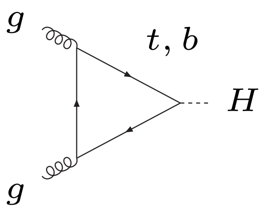
- Introduction
- NLO QCD corrections: analytical expressions for the virtual and real contributions

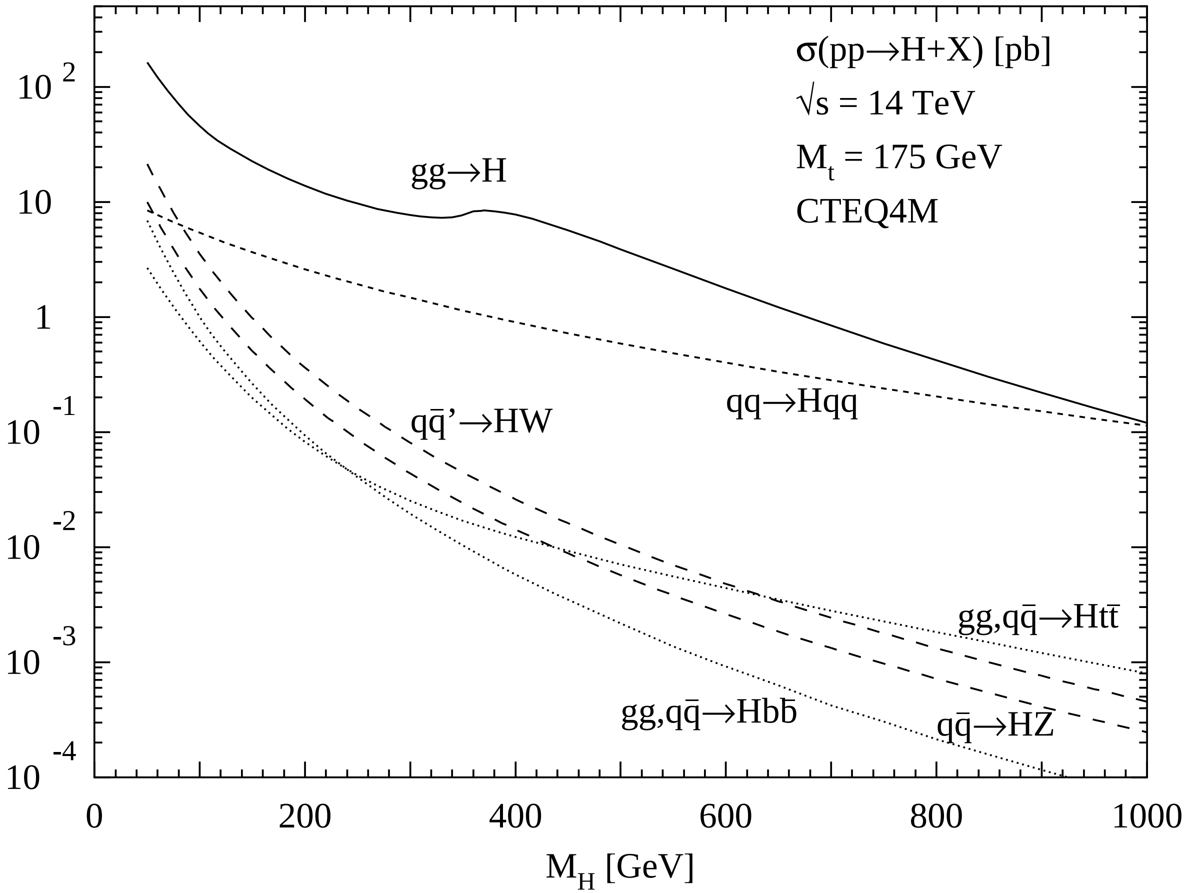
  U. Aglietti, R. B., G. Degrassi and A. Vicini

  R. B., G. Degrassi and A. Vicini
- Applications:
  - Manohar-Wise model
  - MSSM: squark contribution
- Summary

SM Higgs production at the LHC

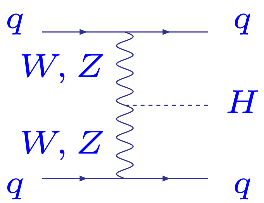
- Large gluon luminosity $\Longrightarrow$ dominant production mech.



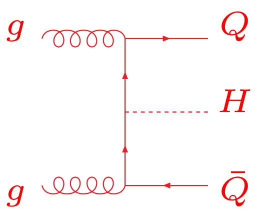


(Djouadi-Spira-Zerwas)

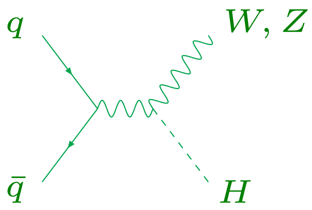
- VBF:



- Associated prod. with $Q\bar{Q}$:

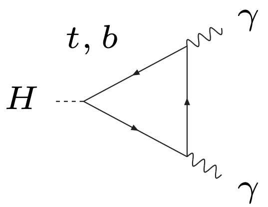


- Associated prod. with W, Z:

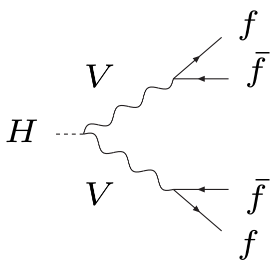


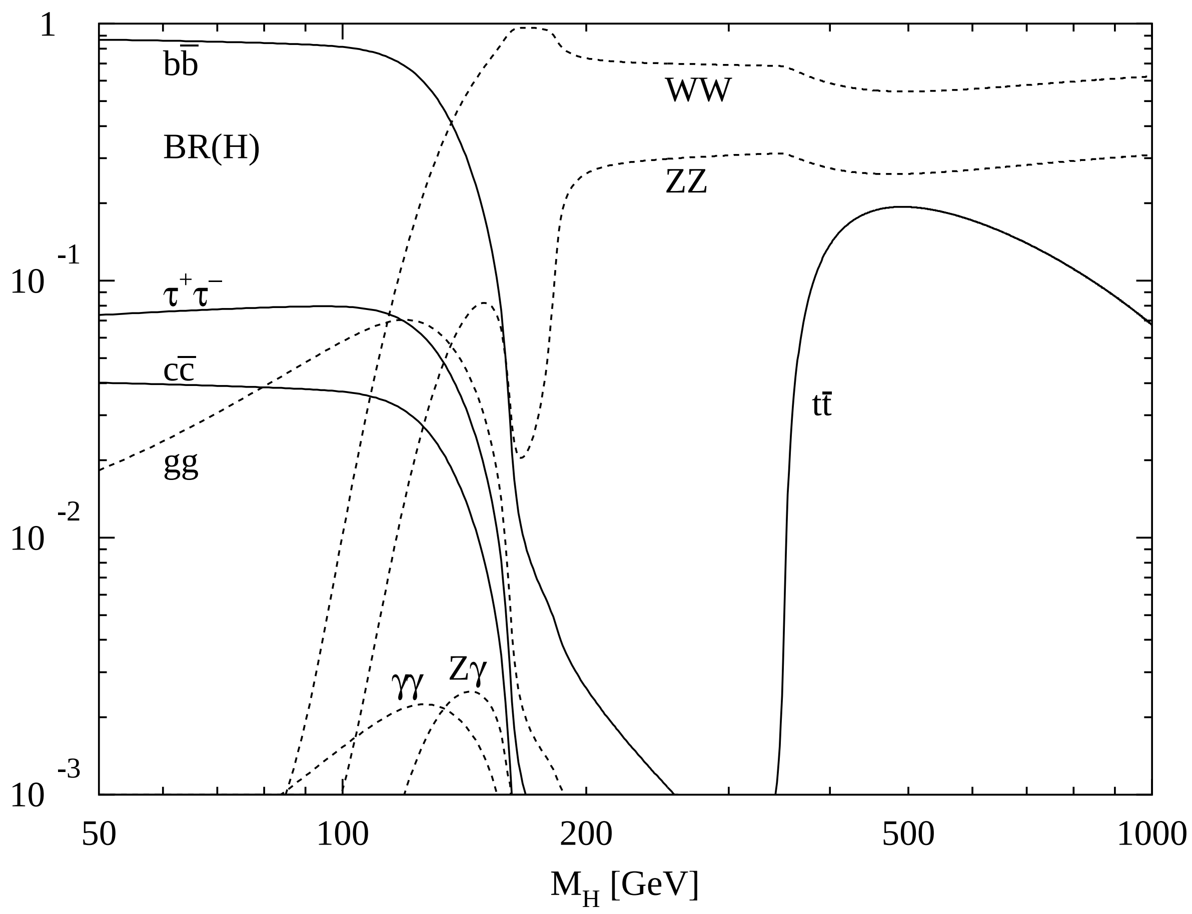
SM Higgs decays (BR)

- $m_H < 140$ GeV $H \to b\bar{b}$ dominant process, but at LHC huge QCD background!
  $H \to \gamma\gamma$ is a rare process ($BR \sim 10^{-3}$), but experimentally clean



- $m_H > 140$ GeV dominant decay channels are $H \to WW, ZZ$





(Djouadi-Spira-Zerwas)

SM predictions for Higgs production

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- LO
  - Georgi-Glashow-Machacek-Nanopoulos ’78

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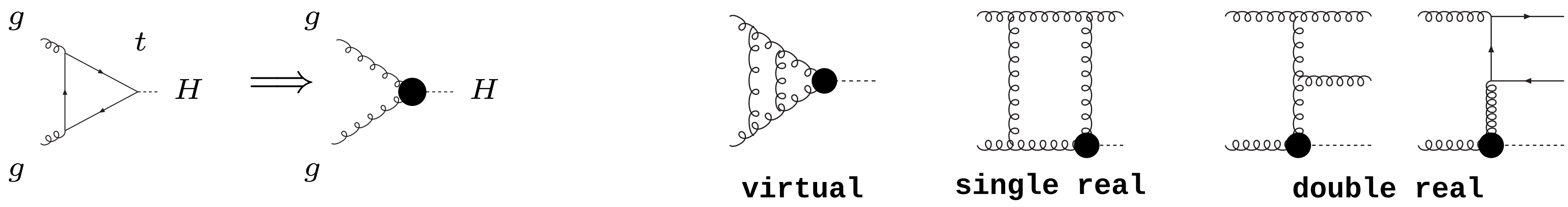

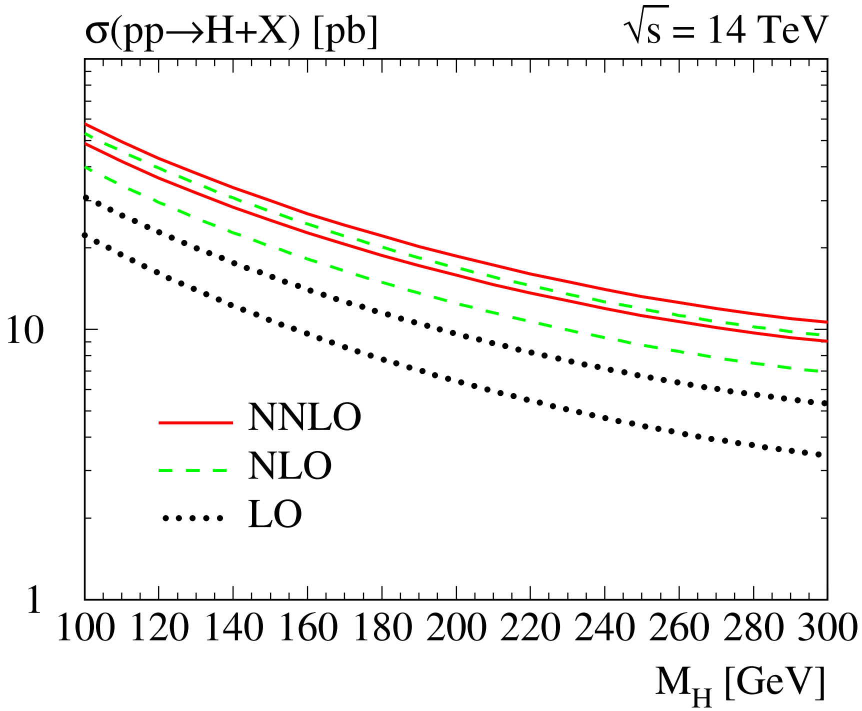

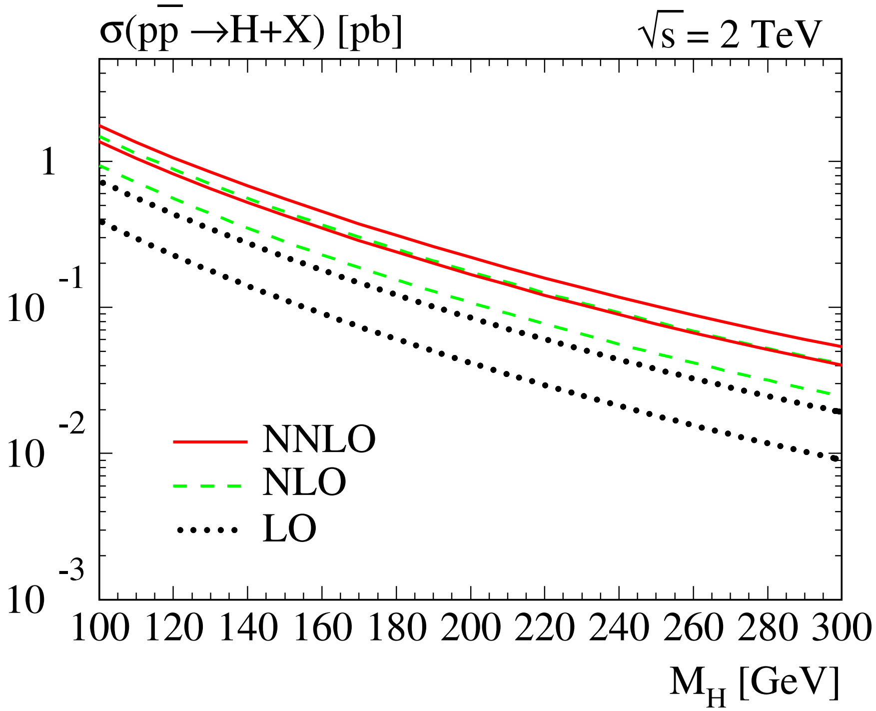

Gluon-fusion production cross section for a Standard Model Higgs boson at the LHC (14 TeV) and at the Tevatron (2 TeV) at leading, next-to-leading, and next-to-next-to-leading order.

Increase of 15-20% of the cross section.

(R. Harlander)

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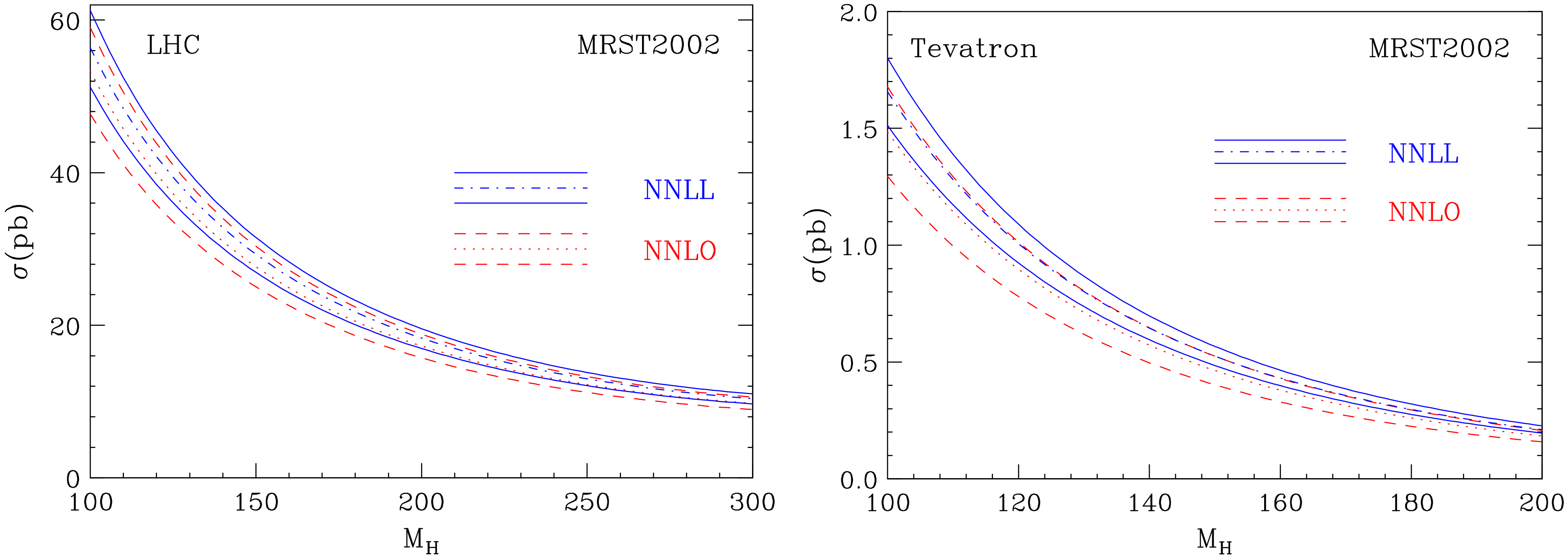
  - Catani-De Florian-Grazzini-Nason '03

SM predictions for Higgs production



NNLL and NNLO cross-sections at the LHC (left) and Tevatron (right) using MRST2002 parton densities.

- Additional increase of the cross section $\sim 6\%$.
- Decrease in the scale dependence $\Longrightarrow$ Theoretical uncertainty $< 10\%$ (confirmed by Moch-Vogt '05).

(Catani, de Florian, Grazzini and Nason)

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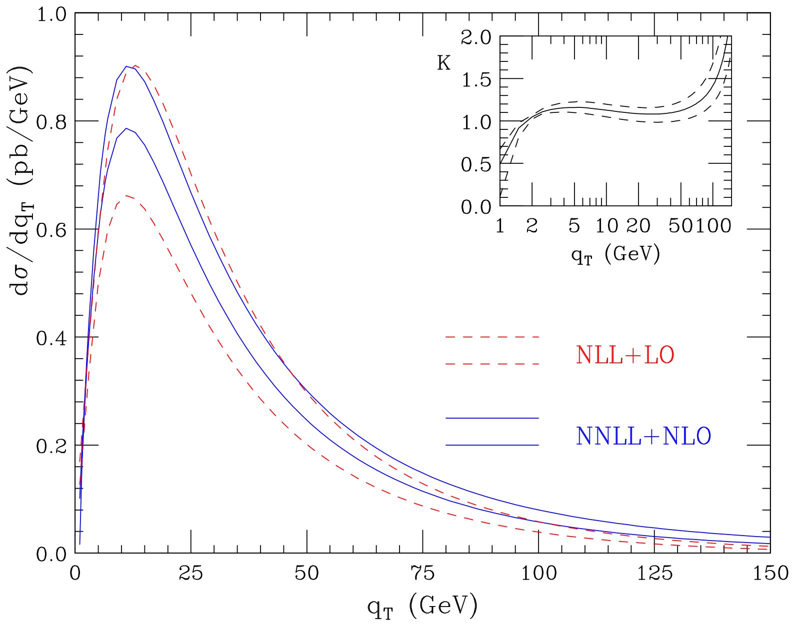
- Higher order p_T distribution ($m_t \to \infty$); Rapidity distribution
  - De Florian-Grazzini-Kunst '99, Del Duca-Kilgore-Oleari-Schmidt-Zeppenfeld '01, Bozzi-Catani-De Florian-Grazzini '03, '06, '07, Anastasiou-Dixon-Melnikov '03

SM predictions for Higgs production

- For small transverse momentum ($q_T \ll m_H$) the q_T-spectrum is affected by large logarithms of the form $\alpha_S^n \ln^{2n}(m_H^2/q_T^2)$.
- They spoil the reliability of the perturbative series and they must be resummed.

LO+NLL and NLO+NNLL q_T-spectra for $m_H = 125$ GeV



(Bozzi, Catani, de Florian, Grazzini)

- Note that the NLO+NNLL band lies in the one of LO+NLL
- Enhancement of central value and reduction of the scale dependence

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- Differential distributions
  - Anastasiou-Melnikov-Petriello '04-'05, Catani-Grazzini '07

MSSM predictions for Higgs production

MSSM predictions for Higgs production

- NLO QCD corrections
  - fermionic corrections to A (Spira-Djouadi-Graudenz-Zerwas '93)
  - squark corrections to h, H, $m_0 \to \infty$ (Dawson-Djouadi-Spira '96)
  - full set of corr h, H and A, $m_0 \to \infty$ (Harlander-Steinhauser '03/'04, Harlander-Hofmann '06)
  - squark contrib to h, H retaining the full dependence on m_0 (Muhlleitner-Spira '06)

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- H+jet
  - complete one-loop MSSM calculation for the production of the lighter neutral Higgs boson in association with a high-p_T hadronic jet, in hadronic collisions (Brein-Hollik '03)
  - fermionic one-loop contributions h, H plus one jet (Field-Dawson-Smith '04)
  - The NLO QCD corrections to A plus one jet ($m_0 \to \infty$) (Field-Smith-Tejeda-Yeomans-van Neerven '03)

SM predictions for $H \to \gamma\gamma$

- LO
  - Ellis-Gaillard-Nanopoulos '76, Shifman-Vainshtein-Voloshin-Zakharov '79,
- NLO QCD corrections
  - Zheng-Wu '90, Djouadi-Spira-van der Bij-Zerwas '91, Dawson-Kauffman '93, Djouadi-Spira-Zerwas '93, Melnikov-Yakovlev '93, Inoue-Najima-Oka-Saito '94, Steinhauser '96
  - Fleischer-Tarasov-Tarasov '04, Harlander-Kant '05, Anastasiou-Beerli-Bucherer-Daleo-Kunst '06, Aglietti-B.-Degrassi-Vicini '06, Passarino-Sturm-Uccirati '07
- NLO EW corrections
  - corrections at $\mathcal{O}(G_\mu m_t^2)$ (Liao-Li '97)
  - corrections at $\mathcal{O}(G_\mu m_H^2)$ (Korner-Melnikov-Yakovlev '96)
  - exact light-fermion contribution (Aglietti-B.-Degrassi-Vicini '04)
  - contributions involving top and weak bosons below W thr. (Degrassi-Maltoni '05)
  - full EW contributions (Passarino-Sturm-Uccirati '07)

Decay Width

Decay Width

The Decay width can be expressed as follows:

$$\Gamma(H \to \gamma\gamma) = \frac{G_\mu \alpha^2 m_H^3}{128\sqrt{2}\pi^3} |\mathcal{F}|^2$$

- G_μ, α and m_H are respectively the Fermi constant, fine-structure constant and mass of the Higgs boson
- $T^{\mu\nu} = \left[(q_1 \cdot q_2)\, g^{\mu\nu} - q_1^\nu\, q_2^\mu \right] \mathcal{F}$
- For the extraction of $\mathcal{F}$ we use the projector $P^{\mu\nu} = \frac{1}{(D-2) q_1 \cdot q_2} \left\{ g_{\mu\nu} - \frac{q_1^\mu q_2^\nu + q_1^\nu q_2^\mu}{q_1 \cdot q_2} \right\}$

We consider: $HVV = g\,\lambda_1\, m_W\,,$ $\quad HFF = g\,\lambda_{1/2}\,\frac{m_{1/2}}{2\,m_W}\,,$ $\quad HSS = g\,\lambda_0\,\frac{A^2}{m_W}$

$$\mathcal{F} = \lambda_1\, Q_1^2\, N_1\, \mathcal{F}_1 + \lambda_{1/2}\, Q_{1/2}^2\, N_{1/2} \mathcal{F}_{1/2} + \lambda_0\, Q_0^2\, N_0\, \frac{A^2}{m_0^2}\, \mathcal{F}_0\,,$$

The form factors $\mathcal{F}_i$, $i = 1, 1/2, 0$ can be calculated in perturbation theory:

$$\mathcal{F}_i = \mathcal{F}_i^{(1l)} + \mathcal{F}_i^{(2l)} + \ldots$$

Decay Width

Once the form factor T_5 is known, the Decay width can be expressed as follows:

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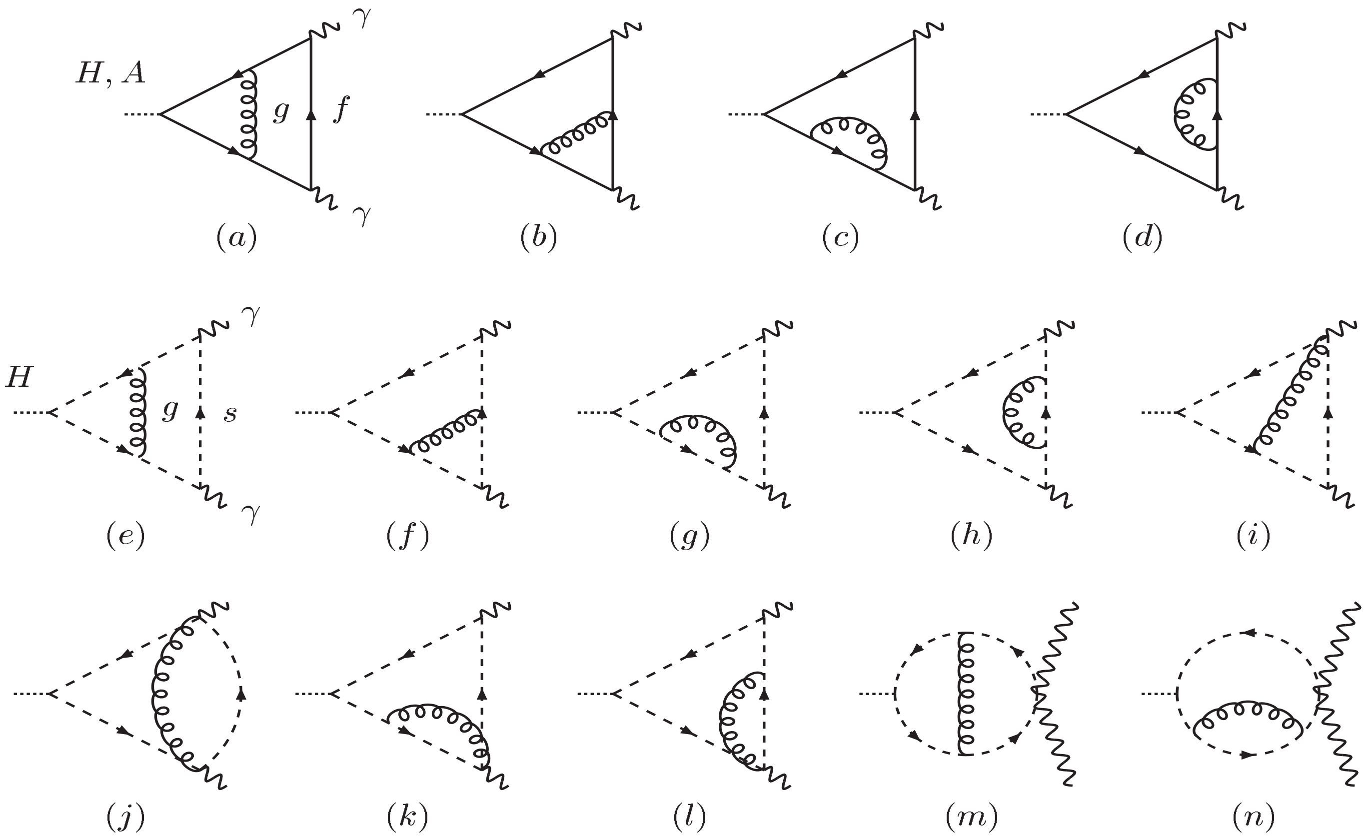
$$\begin{aligned}
\mathcal{F}_1^{(1l)} &= 2(1+6y_1) - 12y_1(1-2y_1)\, H(0,0,x_1) \\
\mathcal{F}_{1/2}^{(1l)} &= -4y_{1/2}\left[2 - \left(1-4y_{1/2}\right)\, H(0,0,x_{1/2})\right] \\
\mathcal{F}_0^{(1l)} &= 4y_0\left[1 + 2\, y_0\, H(0,0,x_0)\right]
\end{aligned}$$

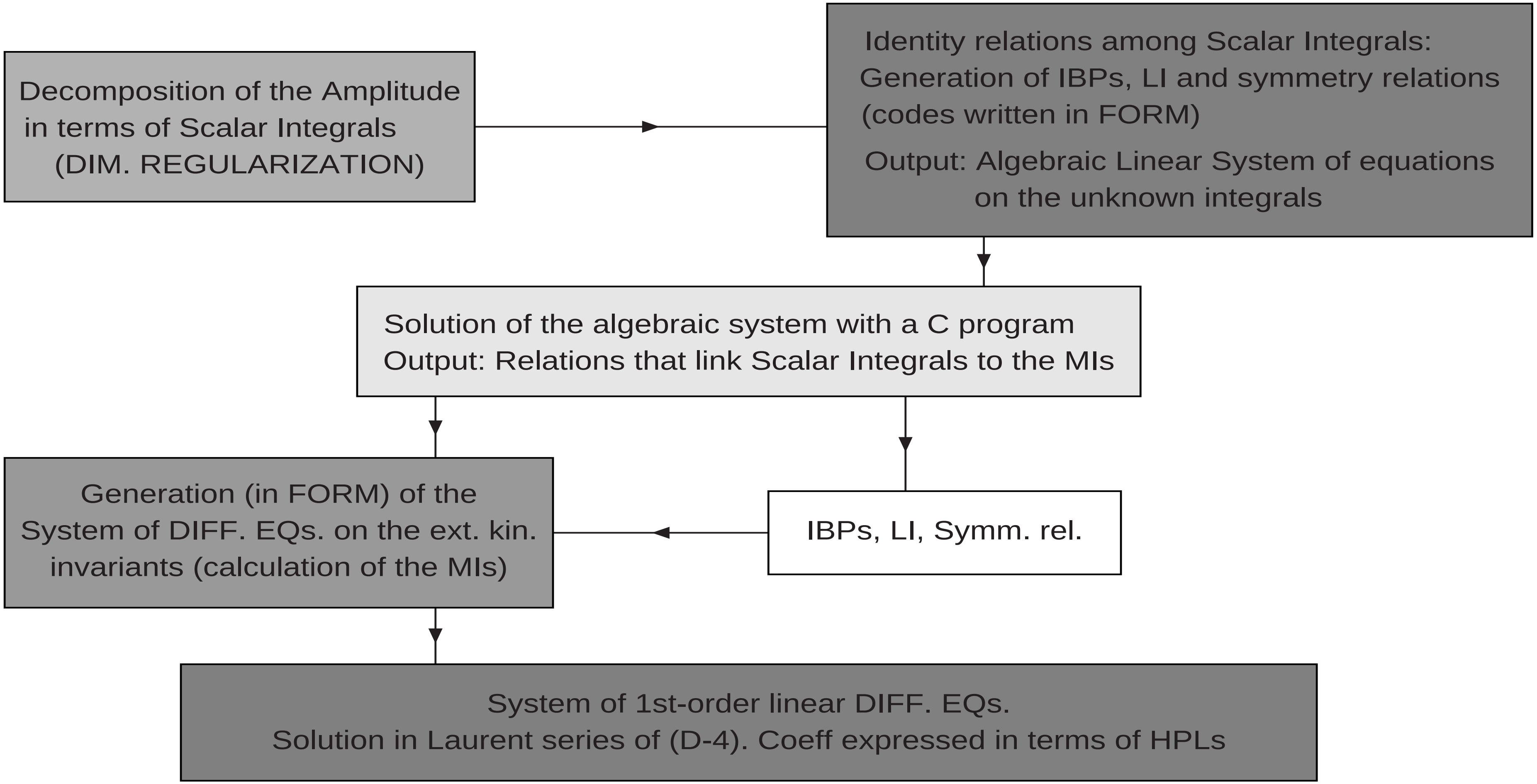
$$y_i \equiv \frac{m_i^2}{m_H^2}, \qquad x_i \equiv \frac{\sqrt{1-4y_i}-1}{\sqrt{1-4y_i}+1}$$

Two-Loop QCD Contributions

$$\mathcal{F}_{QCD}^{(2l)} = \frac{\alpha_S}{\pi} \sum_{i=(0,1/2)} C(R_i)\, \mathcal{F}_i^{(2l)}$$

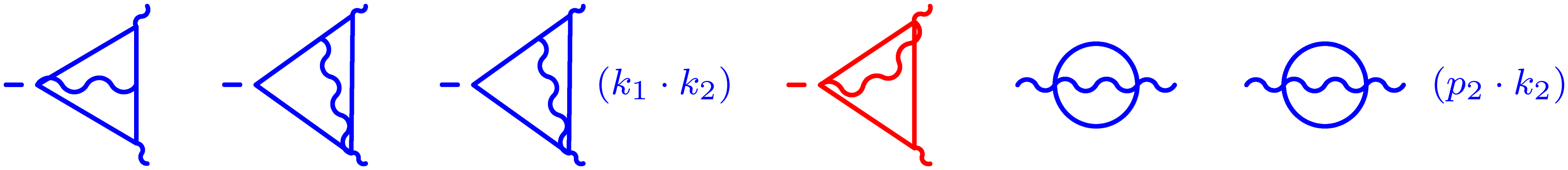


Laporta Algorithm and Diff. Equations



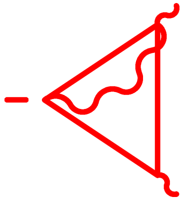
The Master Integrals

The calculation of the contributions due to the two-loop QCD Feynman diagrams can be reduced to the calculation of the following six two-loop scalar integrals (evaluated in D dimensions):

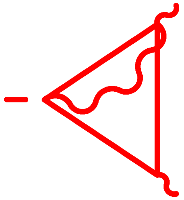


For the 4-denominator MI we have the following Differential Equation:

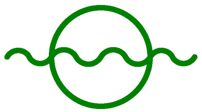
$$\frac{d}{ds}$$



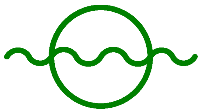
$$= -\frac{1}{s}$$



$$-\frac{1}{4a}\left\{\frac{(D-3)}{s}+\frac{(3D-5)}{(s-4a)}\right\}$$



$$+\frac{3(D-2)}{2a^2}\left\{\frac{1}{s}-\frac{1}{(s-4a)}\right\}$$

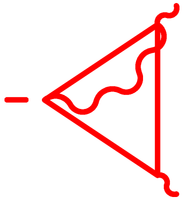


$(p_2 \cdot k_2)$

$$+\frac{(D-4)}{8a^2}\left\{\frac{1}{s}-\frac{1}{(s-4a)}\right\}$$

Anastasiou, Beerli, Bucherer, Daleo and Kunszt, JHEP 0701 (2007) 082;
Aglietti, B., Degrassi and Vicini, JHEP 0701 (2007) 021.

The Master Integrals



$$= \left(\frac{\mu^2}{a}\right)^{2\epsilon} \sum_{i=-2}^{1} \epsilon^i F_i + \mathcal{O}\left(\epsilon^2\right) , \qquad x = \frac{\sqrt{p^2+4m_t^2} - \sqrt{p^2}}{\sqrt{p^2+4m_t^2} + \sqrt{p^2}}$$

$$F_{-2} = \frac{1}{2} \qquad F_{-1} = \frac{1}{2}$$

$$\begin{aligned} F_0 = & -\frac{5}{2} - \frac{4\zeta(3)}{(1-x)^2} + \frac{4\zeta(3)}{(1-x)} + \left[2 - \frac{4}{(1-x)}\right] H(0;x) - H(0,0;x) \\ & + \left[\frac{2}{(1-x)^2} - \frac{2}{(1-x)}\right] H(0,0,0;x) + \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(1,0,0;x) \end{aligned}$$

$$\begin{aligned} F_1 = & -\frac{35}{2} + \frac{8\zeta^2(2)}{5(1-x)^2} - \frac{4\zeta(3)}{(1-x)^2} + \frac{4\zeta(2)}{(1-x)} - \frac{8\zeta^2(2)}{5(1-x)} + \frac{4\zeta(3)}{(1-x)} - 2\zeta(2) + 3\zeta(3) - \left[12 + \frac{24}{(1-x)}\right] H(-1,0;x) \\ & + \left[12 - \frac{6\zeta(3)}{(1-x)^2} + \frac{6\zeta(3)}{(1-x)} - \frac{24}{(1-x)} + \zeta(2)\right] H(0;x) + 6H(0,-1,0;x) + \left[9 - \frac{2\zeta(2)}{(1-x)^2} + \frac{4}{(1-x)^2} + \frac{2\zeta(2)}{(1-x)} \right. \\ & \left. - \frac{20}{(1-x)}\right] H(0,0;x) - \left[\frac{12}{(1-x)^2} - \frac{12}{(1-x)}\right] H(0,0,-1,0;x) - \left[3 - \frac{2}{(1-x)^2} + \frac{2}{(1-x)}\right] H(0,0,0;x) + \left[\frac{6}{(1-x)^2} \right. \\ & \left. - \frac{6}{(1-x)}\right] H(0,0,0,0;x) + \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(0,0,1,0;x) - 2H(0,1,0;x) - \left[\frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(0,1,0,0;x) \\ & - \left[\frac{12\zeta(3)}{(1-x)^2} - \frac{12\zeta(3)}{(1-x)}\right] H(1;x) + \left[4 - \frac{4\zeta(2)}{(1-x)^2} + \frac{4\zeta(2)}{(1-x)} - \frac{8}{(1-x)}\right] H(1,0;x) - \left[\frac{24}{(1-x)^2} - \frac{24}{(1-x)}\right] H(1,0,-1,0;x) \\ & + \left[2 + \frac{4}{(1-x)^2} - \frac{4}{(1-x)}\right] H(1,0,0;x) + \left[\frac{12}{(1-x)^2} - \frac{12}{(1-x)}\right] H(1,0,0,0;x) + \left[\frac{8}{(1-x)^2} - \frac{8}{(1-x)}\right] H(1,0,1,0;x) \\ & - \left[\frac{8}{(1-x)^2} - \frac{8}{(1-x)}\right] H(1,1,0,0;x) \end{aligned}$$

Two-Loop QCD Contributions

$$\mathcal{F}^{(2l)}_{QCD} = \frac{\alpha_S}{\pi} \sum_{i=(0,1/2)} C(R_i)\, \mathcal{F}^{(2l)}_i$$

For instance in the case of on-shell quark masses the fermion contribution is:

$$\mathcal{F}^{(2l,OS)}_{1/2} = \mathcal{F}^{(2l,a)}_{1/2}(x_{1/2}) + \frac{4}{3}\mathcal{F}^{(2l,b)}_{1/2}(x_{1/2})$$

$$\begin{aligned}
\mathcal{F}^{(2l,a)}_{1/2}(x) = & \frac{36x}{(x-1)^2} - \frac{4x\left(1-14x+x^2\right)}{(x-1)^4}\zeta_3 - \frac{4x(1+x)}{(x-1)^3} H(0,x) - \frac{8x\left(1+9x+x^2\right)}{(x-1)^4} H(0,0,x) \\
& + \frac{2x\left(3+25x-7x^2+3x^3\right)}{(x-1)^5} H(0,0,0,x) + \frac{4x\left(1+2x+x^2\right)}{(x-1)^4} \big[\zeta_2 H(0,x) + 4H(0,-1,0,x) \\
& - H(0,1,0,x)\big] + \frac{4x\left(5-6x+5x^2\right)}{(x-1)^4} H(1,0,0,x) - \frac{8x\left(1+x+x^2+x^3\right)}{(x-1)^5}\Big[\frac{9}{10}\zeta_2{}^2 + 2\zeta_3 H(0,x) \\
& + \zeta_2 H(0,0,x) + \frac{1}{4}H(0,0,0,0,x) + \frac{7}{2}H(0,1,0,0,x) - 2H(0,-1,0,0,x) + 4H(0,0,-1,0,x) \\
& - H(0,0,1,0,x)\Big]
\end{aligned}$$

$$\mathcal{F}^{(2l,b)}_{1/2}(x) = -\frac{12x}{(x-1)^2} - \frac{6x(1+x)}{(x-1)^3} H(0,x) + \frac{6x\left(1+6x+x^2\right)}{(x-1)^4} H(0,0,x)$$

Two-Loop QCD Contributions

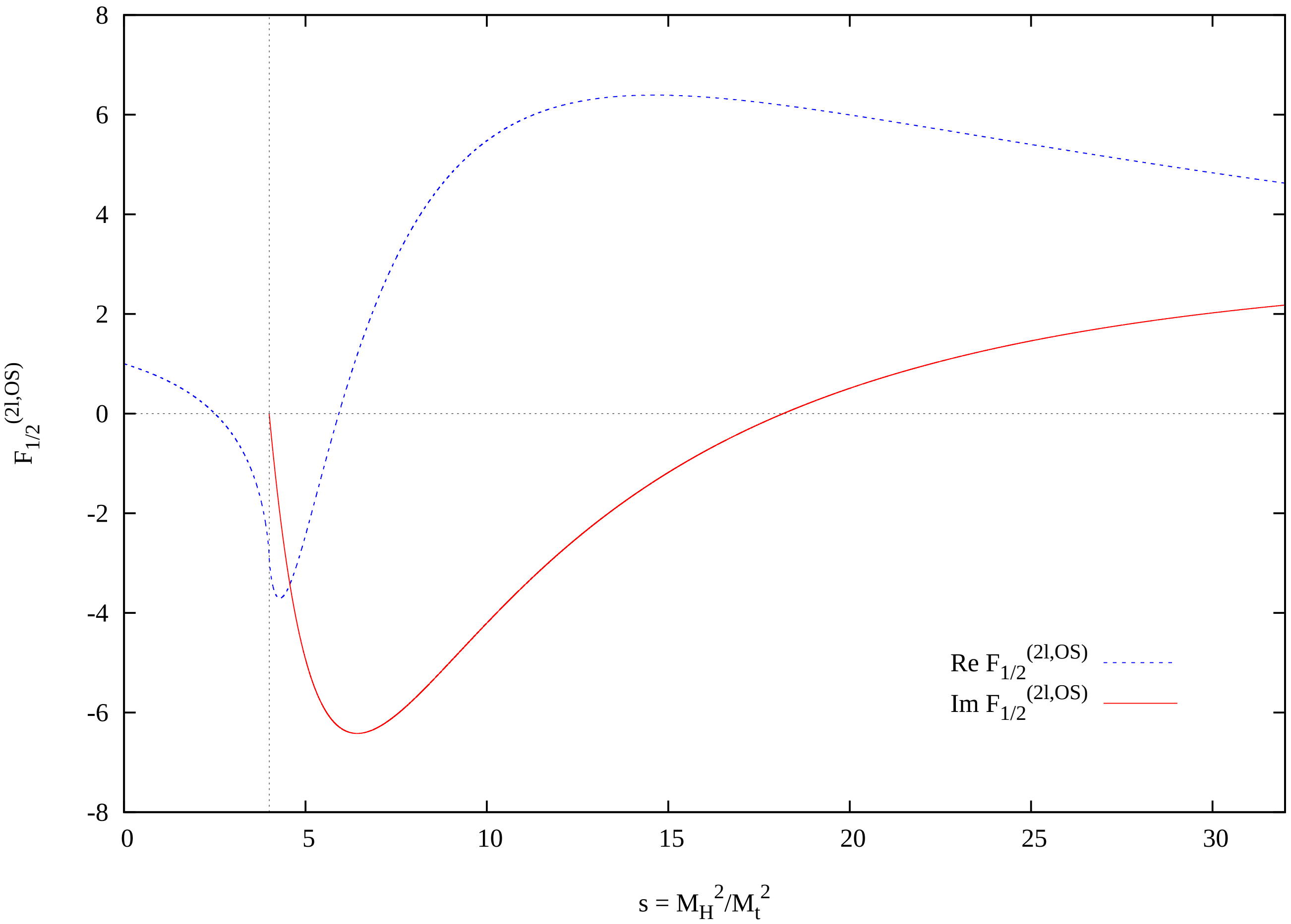
$$\mathcal{F}^{(2l)}_{QCD} = \frac{\alpha_S}{\pi} \sum_{i=(0,1/2)} C(R_i)\, \mathcal{F}^{(2l)}_i$$

For instance in the case of on-shell quark masses the fermion contribution is:

$$\mathcal{F}^{(2l)}_0 = \mathcal{F}^{(2l,a)}_0(x_0) + \frac{7}{3}\,\mathcal{F}^{(2l,b)}_0(x_0) + \mathcal{F}^{(2l,c)}_0(x_0) \ln\left(\frac{m_0^2}{\mu^2}\right)$$

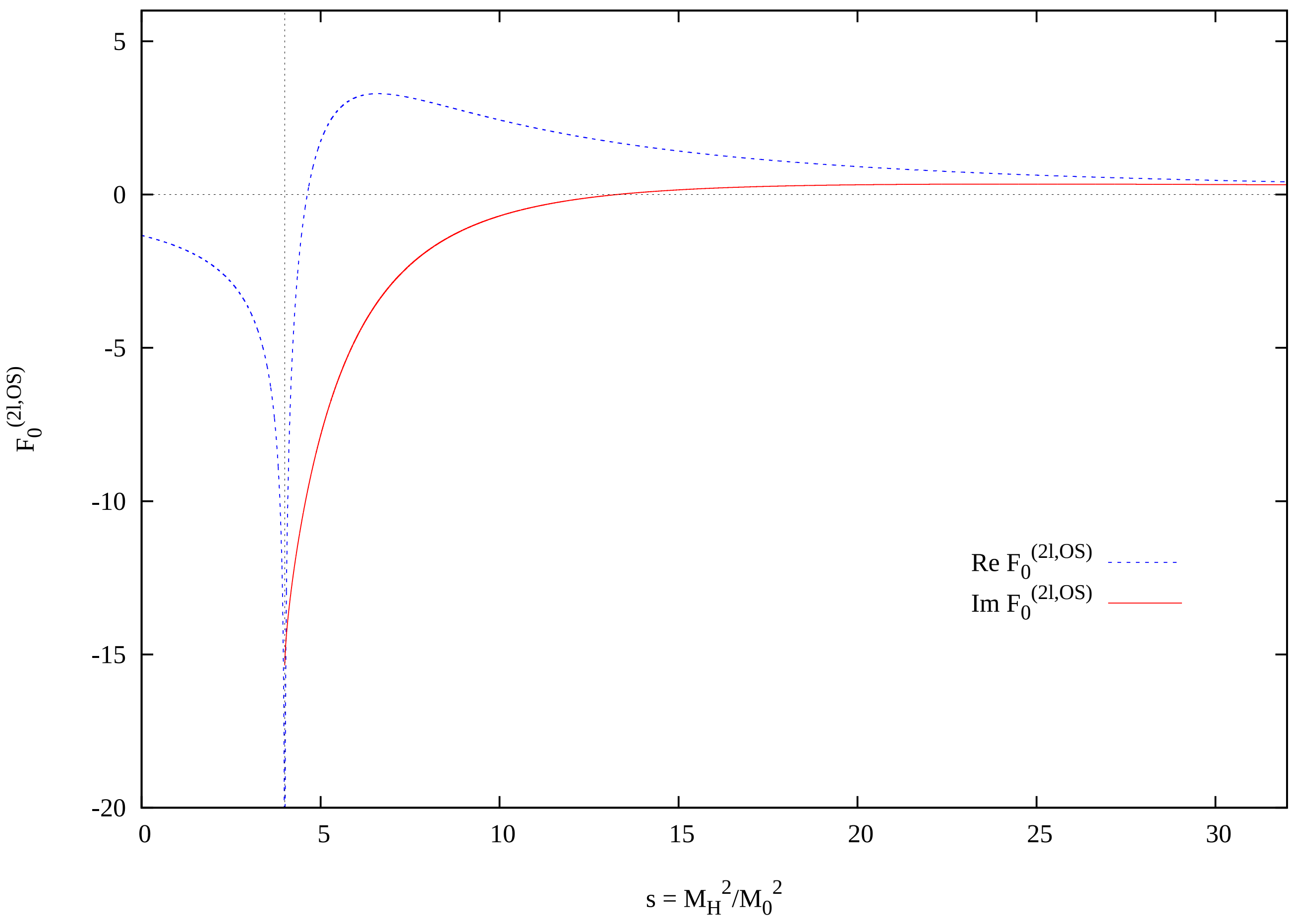
$$\begin{aligned}
\mathcal{F}^{(2l,a)}_0(x) &= -\frac{14x}{(x-1)^2} - \frac{24x^2}{(x-1)^4}\zeta_3 + \frac{x\left(3-8x+3x^2\right)}{(x-1)^3(x+1)} H(0,x) + \frac{34x^2}{(x-1)^4} H(0,0,x) \\
&\quad - \frac{8x^2}{(x-1)^4}\left[\zeta_2 H(0,x) + 4H(0,-1,0,x) - H(0,1,0,x) + H(1,0,0,x)\right] \\
&\quad - \frac{2x^2(5-11x)}{(x-1)^5} H(0,0,0,x) + \frac{16x^2\left(1+x^2\right)}{(x-1)^5(x+1)} \left[\frac{9}{10}{\zeta_2}^2 + 2\zeta_3 H(0,x) + \zeta_2 H(0,0,x)\right. \\
&\quad \left. + \frac{1}{4} H(0,0,0,0,x) + \frac{7}{2} H(0,1,0,0,x) - 2H(0,-1,0,0,x) + 4H(0,0,-1,0,x) - H(0,0,1,0,x)\right] \\
\mathcal{F}^{(2l,b)}_0(x) &= \frac{6x^2}{(x-1)^3(x+1)} H(0,x) - \frac{6x^2}{(x-1)^4} H(0,0,x) \\
\mathcal{F}^{(2l,c)}_0(x) &= -\frac{3}{4}\,\mathcal{F}^{(1l)}_0
\end{aligned}$$

Real and Imaginary parts of $\mathcal{F}_{1/2}^{(2l,OS)}$



In full numerical agreement with Spira-Djouadi-Graudenz-Zerwas and analytical agreement with Harlander-Kant

Real and Imaginary parts of $\mathcal{F}_0^{(2l,OS)}$



In full numerical agreement with Mühlleitner-Spira

Production Cross Section

$$\sigma(h_1 + h_2 \to H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{a,h_1}(x_1, \mu_F^2) f_{b,h_2}(x_2, \mu_F^2) \int_0^1 dz\, \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z)$$

$$\hat{\sigma}_{ab}(z) = \sigma^{(0)}\, z\, G_{ab}(z)$$

$$\sigma^{(0)} = \frac{G_\mu \alpha_S^2(\mu_R^2)}{128\sqrt{2}\,\pi} \left| \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\, \mathcal{G}_i^{(1l)} \right|^2$$

is the Born-level contribution with $\mathcal{G}_i^{(1l)} = \mathcal{F}_i^{(1l)}$

$$\mathcal{G}_{1/2}^{(1l)} = -4y_{1/2}\left[2 - \left(1 - 4y_{1/2}\right) H(0,0,x_{1/2})\right]$$

$$\mathcal{G}_0^{(1l)} = 4y_0\left[1 + 2\, y_0\, H(0,0,x_0)\right]$$

Production Cross Section

$$G_{ab}(z) = G^{(0)}_{ab}(z) + \frac{\alpha_s(\mu_R^2)}{\pi} G^{(1)}_{a,b}(z)$$

$$\begin{aligned}
G^{(0)}_{ab}(z) &= \delta(1-z)\,\delta_{ag}\,\delta_{bg} \\
G^{(1)}_{gg}(z) &= \delta(1-z)\left[C_A\,\frac{\pi^2}{3} + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_F^2}\right) + \sum_{i=0,1/2} \mathcal{G}_i^{(2l)}\right] \\
&\quad + P_{gg}(z)\ln\left(\frac{\hat{s}}{\mu_F^2}\right) + C_A\,\frac{4}{z}\,(1-z+z^2)^2\,\mathcal{D}_1(z) + C_A\,\mathcal{R}_{gg} \\
G^{(1)}_{q\bar{q}}(z) &= \mathcal{R}_{q\bar{q}} \\
G^{(1)}_{qg}(z) &= P_{gq}(z)\left[\ln(1-z) + \frac{1}{2}\ln\left(\frac{\hat{s}}{\mu_F^2}\right)\right] + \mathcal{R}_{qg}
\end{aligned}$$

$$P_{gg}(z) = 2\,C_A\left[\mathcal{D}_0(z) + \frac{1}{z} - 2 + z(1-z)\right] \qquad P_{gq}(z) = C_F\,\frac{1+(1-z)^2}{z} \qquad \mathcal{D}_i(z) = \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

Production Cross Section

$$G_{ab}(z) = G_{ab}^{(0)}(z) + \frac{\alpha_s(\mu_R^2)}{\pi}\, G_{a,b}^{(1)}(z)$$

$$\begin{aligned}
G_{ab}^{(0)}(z) &= \delta(1-z)\,\delta_{ag}\,\delta_{bg} \\
G_{gg}^{(1)}(z) &= \delta(1-z)\left[C_A\,\frac{\pi^2}{3} + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_F^2}\right) + \sum_{i=0,1/2} \mathcal{G}_i^{(2l)}\right] \\
&\quad + P_{gg}(z)\ln\left(\frac{\hat{s}}{\mu_F^2}\right) + C_A\,\frac{4}{z}\,(1-z+z^2)^2\,\mathcal{D}_1(z) + C_A\,\mathcal{R}_{gg} \\
G_{q\bar{q}}^{(1)}(z) &= \mathcal{R}_{q\bar{q}} \\
G_{qg}^{(1)}(z) &= P_{gq}(z)\left[\ln(1-z) + \frac{1}{2}\ln\left(\frac{\hat{s}}{\mu_F^2}\right)\right] + \mathcal{R}_{qg}
\end{aligned}$$

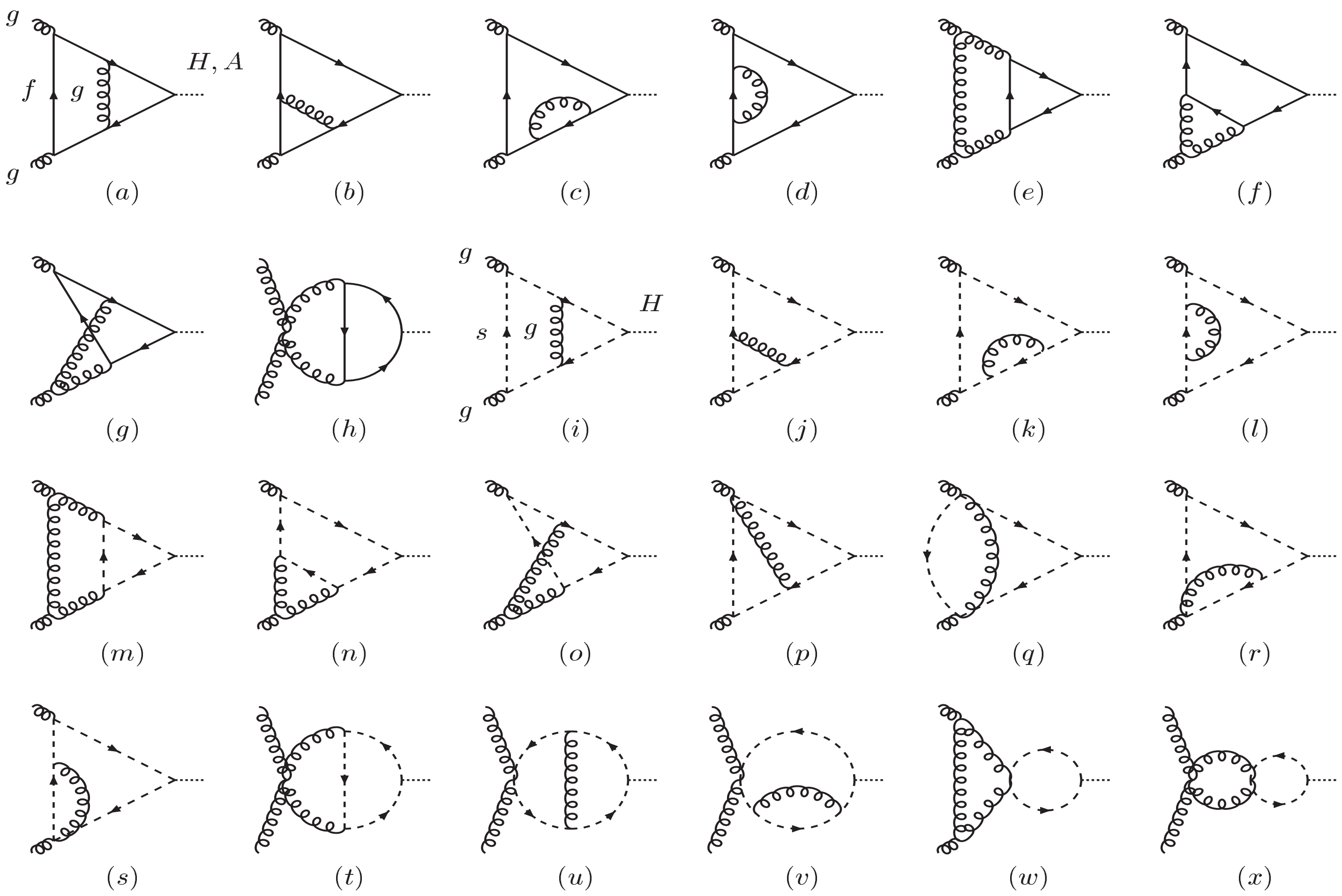
$$P_{gg}(z) = 2\,C_A\left[\mathcal{D}_0(z) + \frac{1}{z} - 2 + z(1-z)\right] \qquad P_{gq}(z) = C_F\,\frac{1+(1-z)^2}{z} \qquad \mathcal{D}_i(z) = \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

Production Cross Section

The function $\mathcal{G}_i^{(2l)}$ can be cast in the following form:

$$\begin{aligned}
\mathcal{G}_i^{(2l)} &= \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i) \Big(C(R_i)\, \mathcal{G}_i^{(2l,C_R)}(x_i) + C_A\, \mathcal{G}_i^{(2l,C_A)}(x_i) \Big) \\
& \quad \times \left(\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\, \mathcal{G}_j^{(1l)} \right)^{-1} + h.c.
\end{aligned}$$

Feynman Diags for the $2 \to 1$ part



QCD Contribution

$$\mathcal{G}_i^{(2l,C_R)} = \mathcal{F}_i^{(2l)}$$

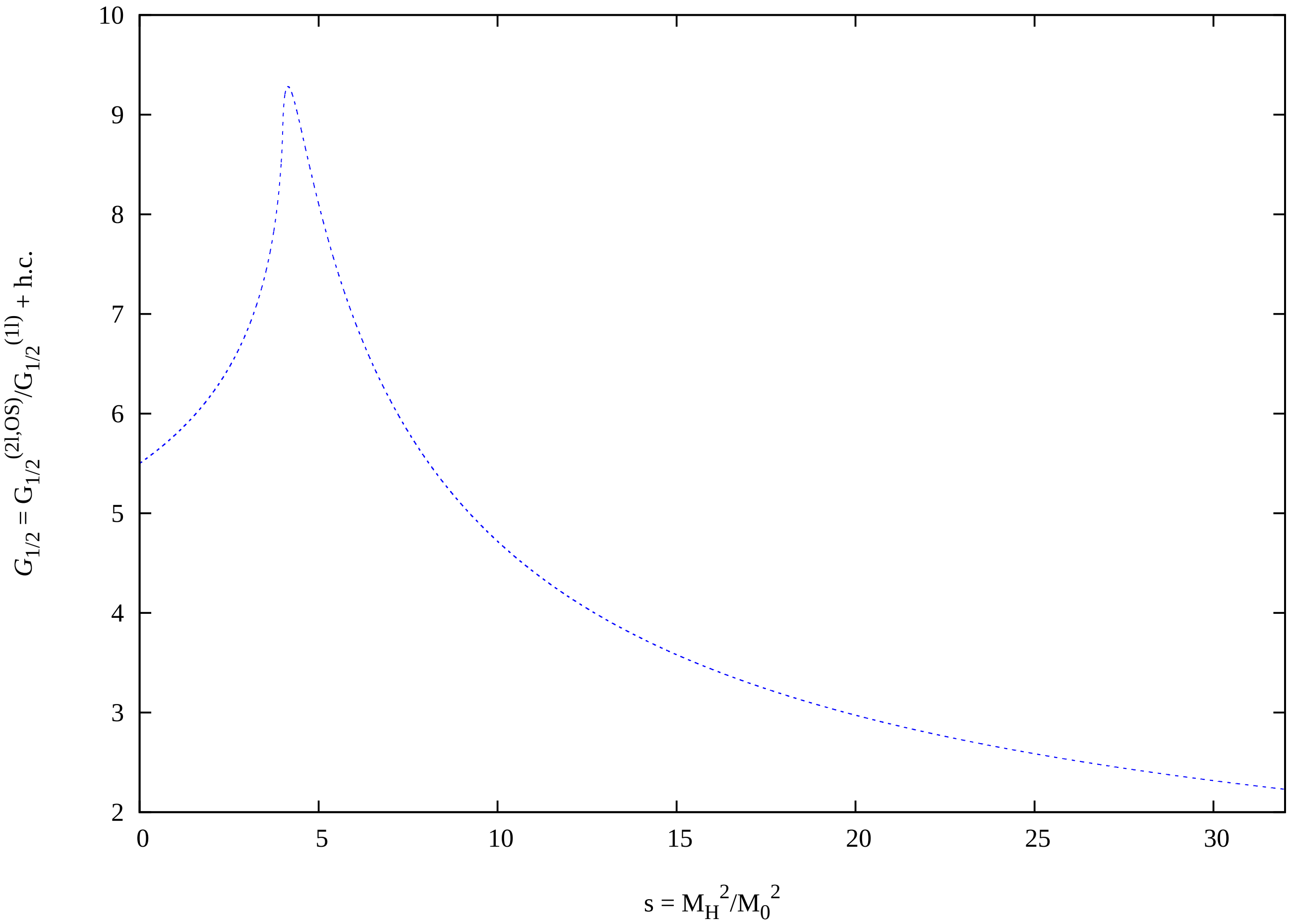
$$\mathcal{G}_{1/2}^{(2l,C_A)}(x) = \frac{4x}{(x-1)^2}\left[3 + \frac{x(1+8x+3x^2)}{(x-1)^3}\,H(0,0,0,x) - \frac{2(1+x)^2}{(x-1)^2}\,\mathcal{H}_2(x) + \zeta_3 - H(1,0,0,x)\right]$$

$$\mathcal{G}_0^{(2l,C_A)}(x) = \frac{4x}{(x-1)^2}\left[-\frac{3}{2} + \frac{x(1-7x)}{(x-1)^3}H(0,0,0,x) + \frac{4x}{(x-1)^2}\mathcal{H}_2(x)\right]$$

with

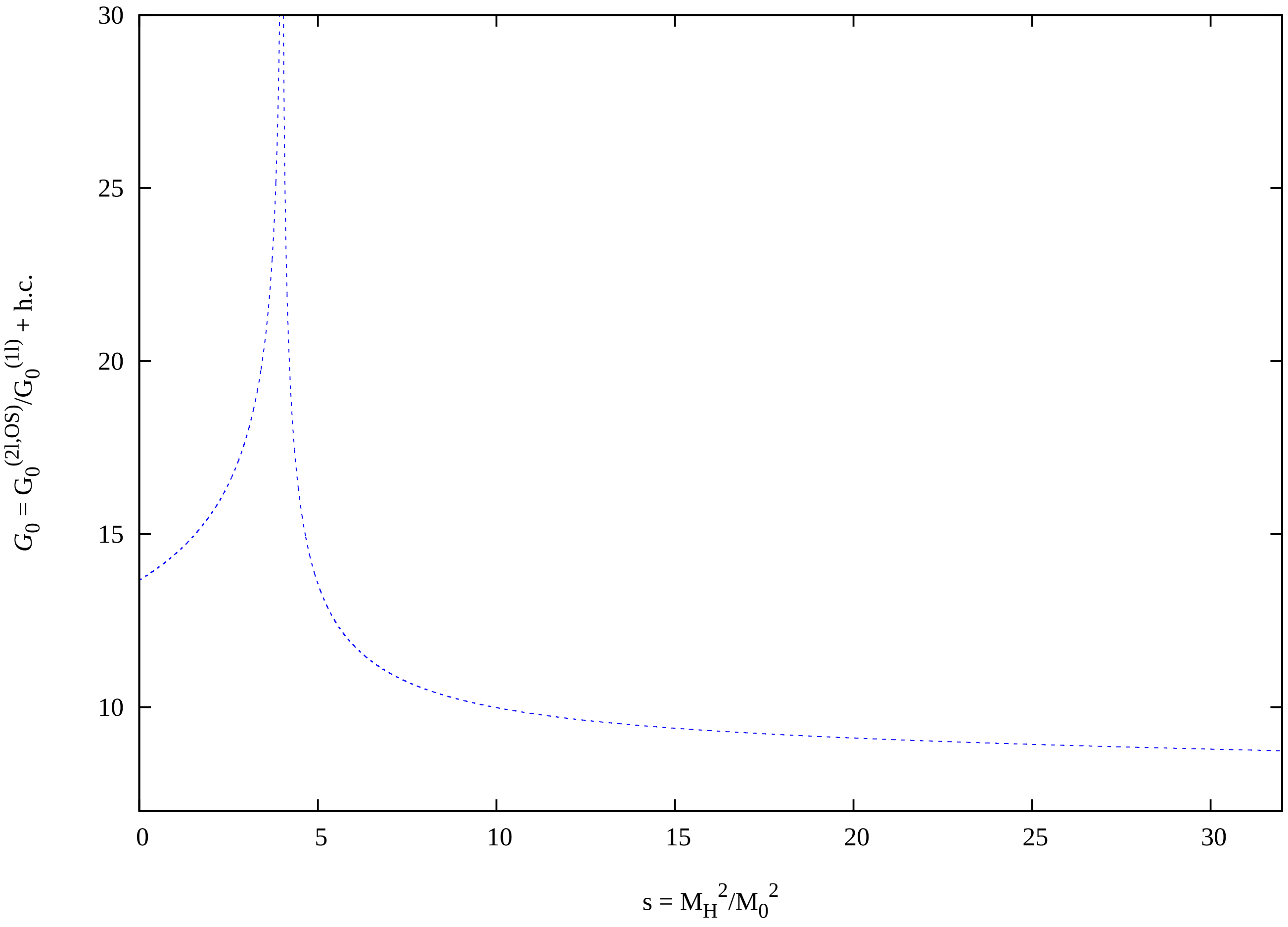
$$\mathcal{H}_2(x) = \frac{4}{5}{\zeta_2}^2 + 2\zeta_3 + \frac{3\zeta_3}{2}\,H(0,x) + 3\zeta_3 H(1,x) + \zeta_2 H(1,0,x) + \frac{1}{4}\,(1+2\zeta_2)\,H(0,0,x) -2\,H(1,0,0,x) + H(0,0,-1,0,x) + \frac{1}{4}H(0,0,0,0,x) + 2\,H(1,0,-1,0,x) - H(1,0,0,0,x)$$

The Ratio $\mathcal{G}_{1/2}$



In full numerical agreement with Spira-Djouadi-Graudenz-Zerwas and analytical agreement with Harlander-Kant

The Ratio $\mathcal{G}_0$



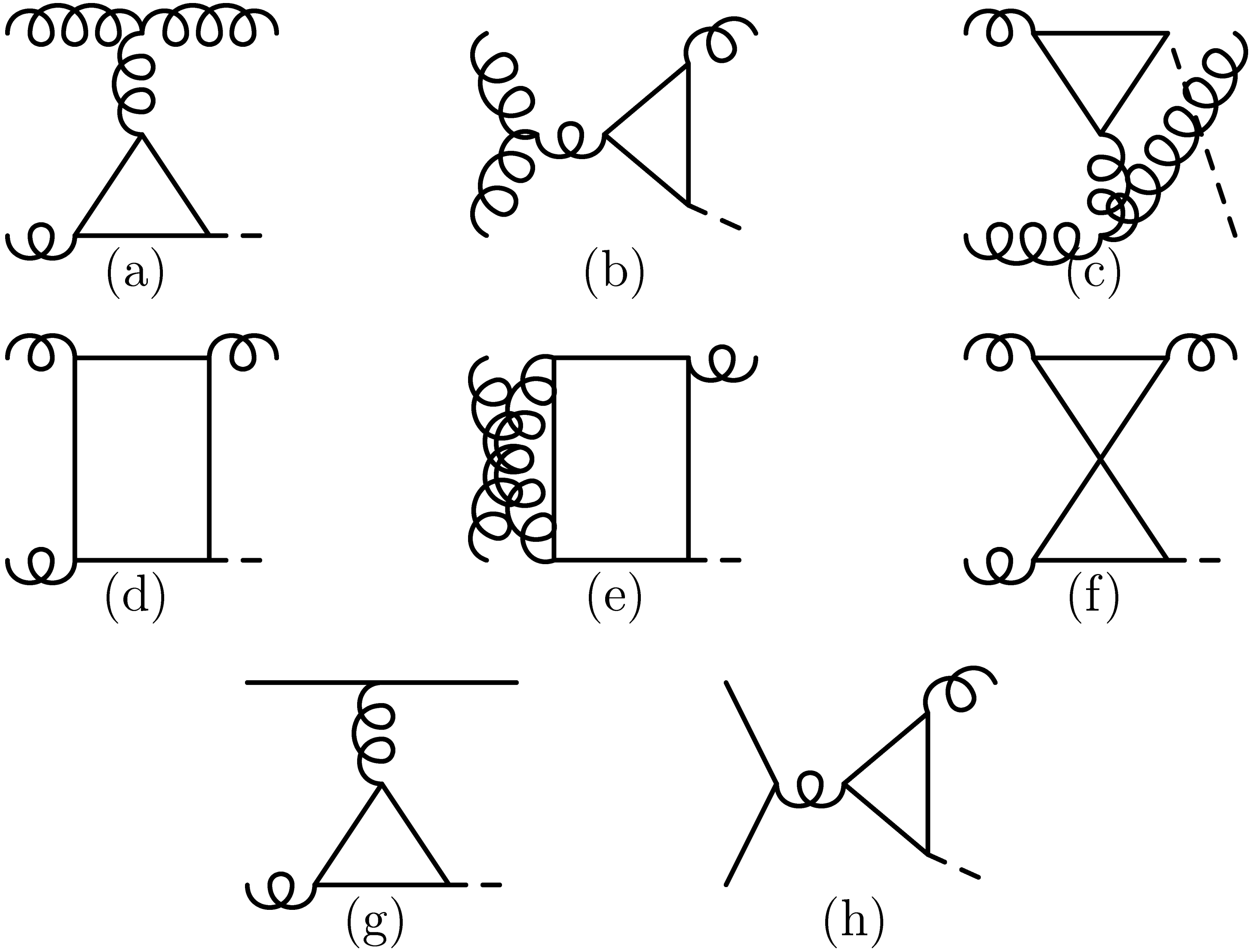
In full numerical agreement with Mühlleitner-Spira

Production Cross Section

$$G_{ab}(z) = G_{ab}^{(0)}(z) + \frac{\alpha_s(\mu_R^2)}{\pi}\, G_{a,b}^{(1)}(z)$$

$$\begin{aligned}
G_{ab}^{(0)}(z) &= \delta(1-z)\,\delta_{ag}\,\delta_{bg} \\
G_{gg}^{(1)}(z) &= \delta(1-z)\left[C_A\,\frac{\pi^2}{3} + \beta_0 \ln\left(\frac{\mu_R^2}{\mu_F^2}\right) + \sum_{i=0,1/2} \mathcal{G}_i^{(2l)}\right] \\
&\quad + P_{gg}(z)\ln\left(\frac{\hat{s}}{\mu_F^2}\right) + C_A\,\frac{4}{z}\,(1-z+z^2)^2\,\mathcal{D}_1(z) + C_A\,\mathcal{R}_{gg} \\
G_{q\bar{q}}^{(1)}(z) &= \mathcal{R}_{q\bar{q}} \\
G_{qg}^{(1)}(z) &= P_{gq}(z)\left[\ln(1-z) + \frac{1}{2}\ln\left(\frac{\hat{s}}{\mu_F^2}\right)\right] + \mathcal{R}_{qg}
\end{aligned}$$

Feynman Diags for the $2 \to 2$ part



Real Radiation

$$\mathcal{R}_{gg} = \frac{1}{z(1-z)} \int_0^1 \frac{dv}{v(1-v)} \left\{ \frac{8\, z^4 \left|\mathcal{A}_{gg}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\, \mathcal{G}_j^{(1l)}\right|^2} - (1-z+z^2)^2 \right\}$$

$$\hat{t} = -\hat{s}(1-z)(1-v) \qquad\qquad \hat{u} = -\hat{s}(1-z)v$$

with

$$\left|\mathcal{A}_{gg}(s,t,u)\right|^2 = |A_2(s,t,u)|^2 + |A_2(u,s,t)|^2 + |A_2(t,u,s)|^2 + |A_4(s,t,u)|^2$$

$$A_2(s,t,u) = \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\, y_i^2\ \left[b_i(s_i,t_i,u_i) + b_i(s_i,u_i,t_i)\right]$$

$$A_4(s,t,u) = \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\, y_i^2\ \left[c_i(s_i,t_i,u_i) + c_i(t_i,u_i,s_i) + c_i(u_i,s_i,t_i)\right]$$

with

$$s_i \equiv \frac{s}{m_i^2}, \qquad t_i \equiv \frac{t}{m_i^2}, \qquad u_i \equiv \frac{u}{m_i^2}.$$

Real Radiation

$$b_{1/2}(s,t,u) = B_{1/2}(s,t,u) + \frac{s}{4}\Big[H(0,0,x_{1/2}) - H(0,0,x_s)\Big] - \left(\frac{s}{2} - \frac{s^2}{s+u}\right)\Big[H(0,0,x_{1/2}) - H(0,0,x_t)\Big]$$
$$- \frac{s}{8}\mathrm{H}_3(s,u,t) + \frac{s}{4}\,\mathrm{H}_3(t,s,u)$$

$$c_{1/2}(s,t,u) = C_{1/2}(s,t,u) + \frac{1}{2\,y_{1/2}}\Big[H(0,0,x_{1/2}) - H(0,0,x_s)\Big] + \frac{1}{4\,y_{1/2}}\mathrm{H}_3(s,u,t)$$

$$b_0(s,t,u) = -\frac{1}{2}B_0(s,t,u) \qquad c_0(s,t,u) = -\frac{1}{2}C_0(s,t,u)$$

$$y_i = \frac{m_i^2}{m_H^2}, \qquad x_i = \frac{\sqrt{1-4y_i}-1}{\sqrt{1-4y_i}+1} \qquad (i = 0, 1/2)\ ; \qquad x_a = \frac{\sqrt{1-4/a}-1}{\sqrt{1-4/a}+1} \qquad (a = s,t,u)$$

$$B_i(s,t,u) = \frac{s(t-s)}{s+t} + \frac{2\left(tu^2+2stu\right)}{(s+u)^2}\Big[\sqrt{1-4y_i}H(0,x_i) - \sqrt{1-4/t}H(0,x_t)\Big] - \left(1+\frac{tu}{s}\right)H(0,0,x_i)$$
$$+H(0,0,x_s) - 2\left(\frac{2s^2}{(s+u)^2} - 1 - \frac{tu}{s}\right)[H(0,0,x_i) - H(0,0,x_t)] + \frac{1}{2}\left(\frac{tu}{s}+3\right)\mathrm{H}_3(s,u,t) - \mathrm{H}_3(t,s,$$

$$C_i(s,t,u) = -2s - 2\,[H(0,0,x_i) - H(0,0,x_s)] - \mathrm{H}_3(u,s,t)$$

$$\mathrm{H}_3(a,b,c) = \int_0^1 dx \frac{1}{x(1-x)+a/(b\,c)}\{\ln[1-bx(1-x)] + \ln[1-cx(1-x)] - \ln[1-(a+b+c)x(1-x)]\}$$

Real Radiation

$$\mathcal{R}_{q\bar{q}} = \frac{128}{27} \frac{z\,(1-z)\,\left|\mathcal{A}_{q\bar{q}}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\,\mathcal{G}_j^{(1l)}\right|^2}$$

$$\mathcal{R}_{qg} = C_F \int_0^1 \frac{dv}{(1-v)} \left\{ \frac{1+(1-z)^2 v^2}{[1-(1-z)v]^2} \frac{2\,z\left|\mathcal{A}_{qg}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2} \lambda_j \left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\,\mathcal{G}_j^{(1l)}\right|^2} - \frac{1+(1-z)^2}{2z} \right\} + \frac{1}{2} C_F\, z$$

$$\mathcal{A}_{qg}(\hat{s},\hat{t},\hat{u}) = \mathcal{A}_{qq}(\hat{t},\hat{s},\hat{u})$$

$$\mathcal{A}_{q\bar{q}}(s,t,u) = \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\, y_i\, d_i(s_i,t_i,u_i)$$

$$d_{1/2}(s,t,u) = D_{1/2}(s,t,u) - 2\left[H(0,0,x_{1/2}) - H(0,0,x_s)\right]$$

$$d_0(s,t,u) = -\frac{1}{2} D_0(s,t,u)$$

$$D_i(s,t,u) = 4 + \frac{4\,s}{(t+u)}\left[\sqrt{1-4y_i}\,H(0,x_i) - \sqrt{1-4/s}\,H(0,x_s)\right] + \frac{8}{t+u}\left[H(0,0,x_i) - H(0,0,x_s)\right]$$

Manohar-Wise Model

Manohar-Wise Model

- Additional colore scalar weak doublet $S^a = \begin{pmatrix} S^a_+ \\ S^a_0 \end{pmatrix} = \begin{pmatrix} S^a_+ \\ \frac{S^a_{0R} + i S^a_{0I}}{\sqrt{2}} \end{pmatrix}$ in the $SU(N_c)$ adjoint representation.
- Potential:

$$V = \frac{\lambda}{4}\left(H^{\dagger i}H_i - \frac{v^2}{2}\right)^2 + 2m_S^2\, TrS^{\dagger i}S_i + \lambda_1 H^{\dagger i}H_i TrS^{\dagger j}S_j + \lambda_2 H^{\dagger i}H_j TrS^{\dagger j}S_i + \left(\lambda_3 H^{\dagger i}H^{\dagger j}TrS_iS_j \;+\; h.c.\right) + \cdots$$

- Mass spectrum:

$$m^2_{S_+} = m_S^2 + \lambda_1 \frac{v^2}{4}$$

$$m^2_{S_{0R}} = m_S^2 + (\lambda_1 + \lambda_2 + 2\lambda_3)\frac{v^2}{4}$$

$$m^2_{S_{0I}} = m_S^2 + (\lambda_1 + \lambda_2 - 2\lambda_3)\frac{v^2}{4}$$

Manohar-Wise Model

- Additional colore scalar weak doublet $S^a = \begin{pmatrix} S^a_+ \\ S^a_0 \end{pmatrix} = \begin{pmatrix} S^a_+ \\ \frac{S^a_{0R}+iS^a_{OI}}{\sqrt{2}} \end{pmatrix}$ in the $SU(N_c)$ adjoint representation.
- Potential:

$$V = \frac{\lambda}{4}\left(H^{\dagger i}H_i - \frac{v^2}{2}\right)^2 + 2m_S^2\, TrS^{\dagger i}S_i + \lambda_1 H^{\dagger i}H_i TrS^{\dagger j}S_j + \lambda_2 H^{\dagger i}H_j TrS^{\dagger j}S_i + \left(\lambda_3 H^{\dagger i}H^{\dagger j}TrS_iS_j \;+\; h.c.\right) + \cdots$$

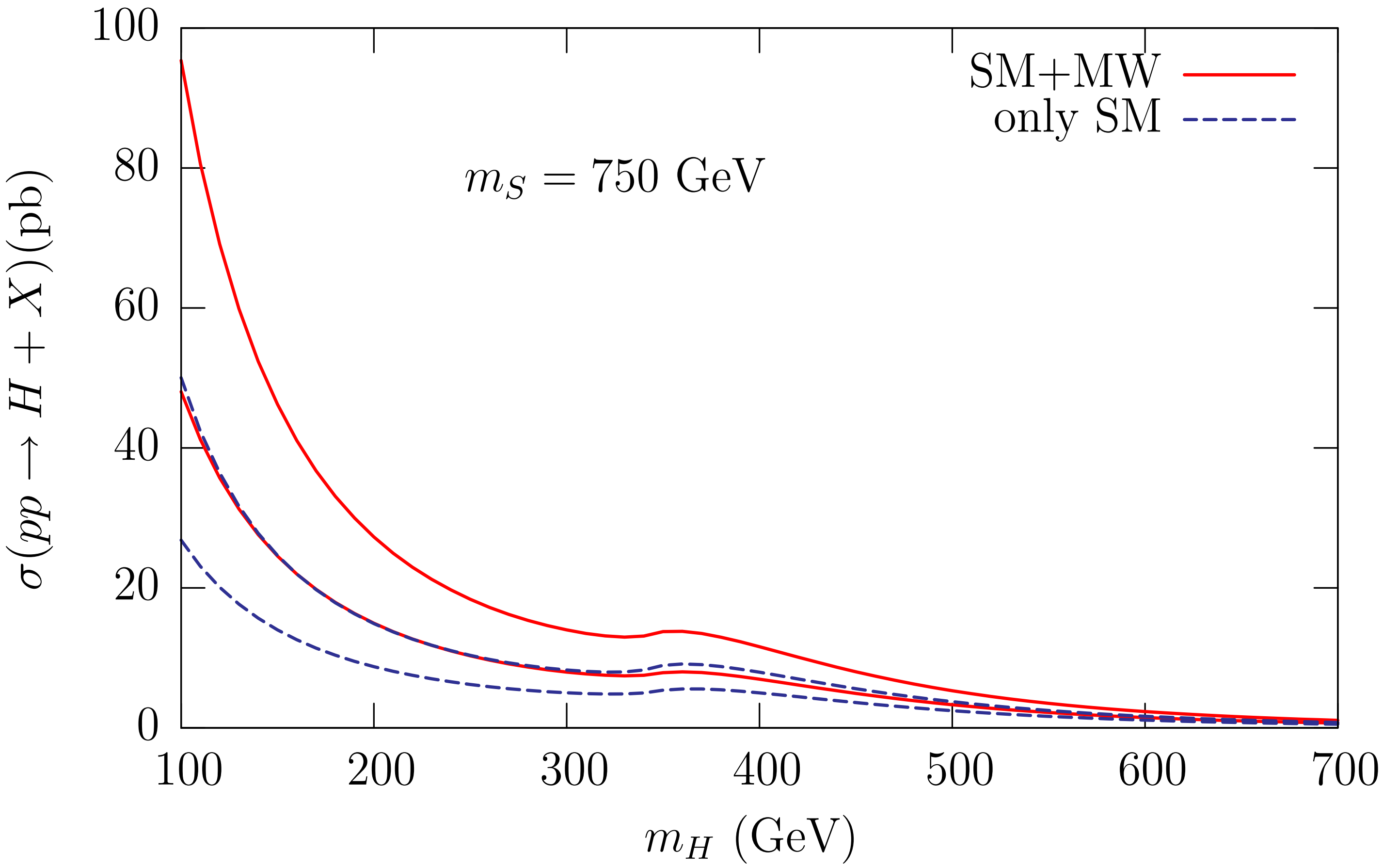
- Couplings to the standard Higgs :

$$HS^a_+S^b_- = g\,\frac{\lambda_1}{4}\,\frac{v^2}{m_W}\delta^{ab}$$

$$HS^a_{0R}S^b_{OR} = g\,\frac{\lambda_1+\lambda_2+2\lambda_3}{8}\,\frac{v^2}{m_W}\delta^{ab}$$

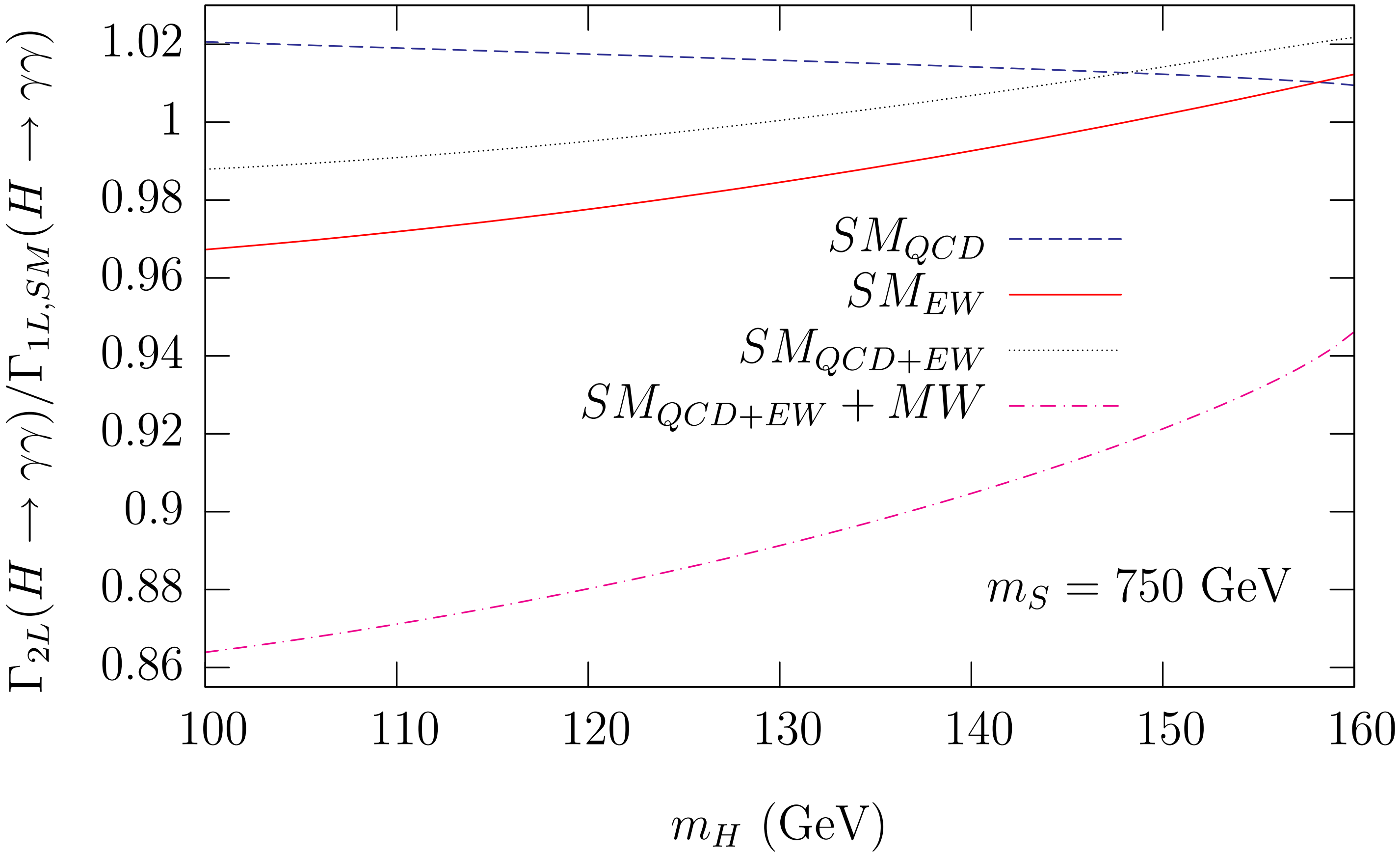
$$HS^a_{0I}S^b_{0I} = g\,\frac{\lambda_1+\lambda_2-2\lambda_3}{8}\,\frac{v^2}{m_W}\delta^{ab}$$

Manohar-Wise Model



$\lambda_1(m_S) = 4$, $\lambda_2(m_S) = 1$, $\lambda_3(m_S) = 1/2$,

Manohar-Wise Model



$\lambda_1(m_S) = 4,\ \lambda_2(m_S) = 1,\ \lambda_3(m_S) = 1/2,$

MSSM: squark contributions

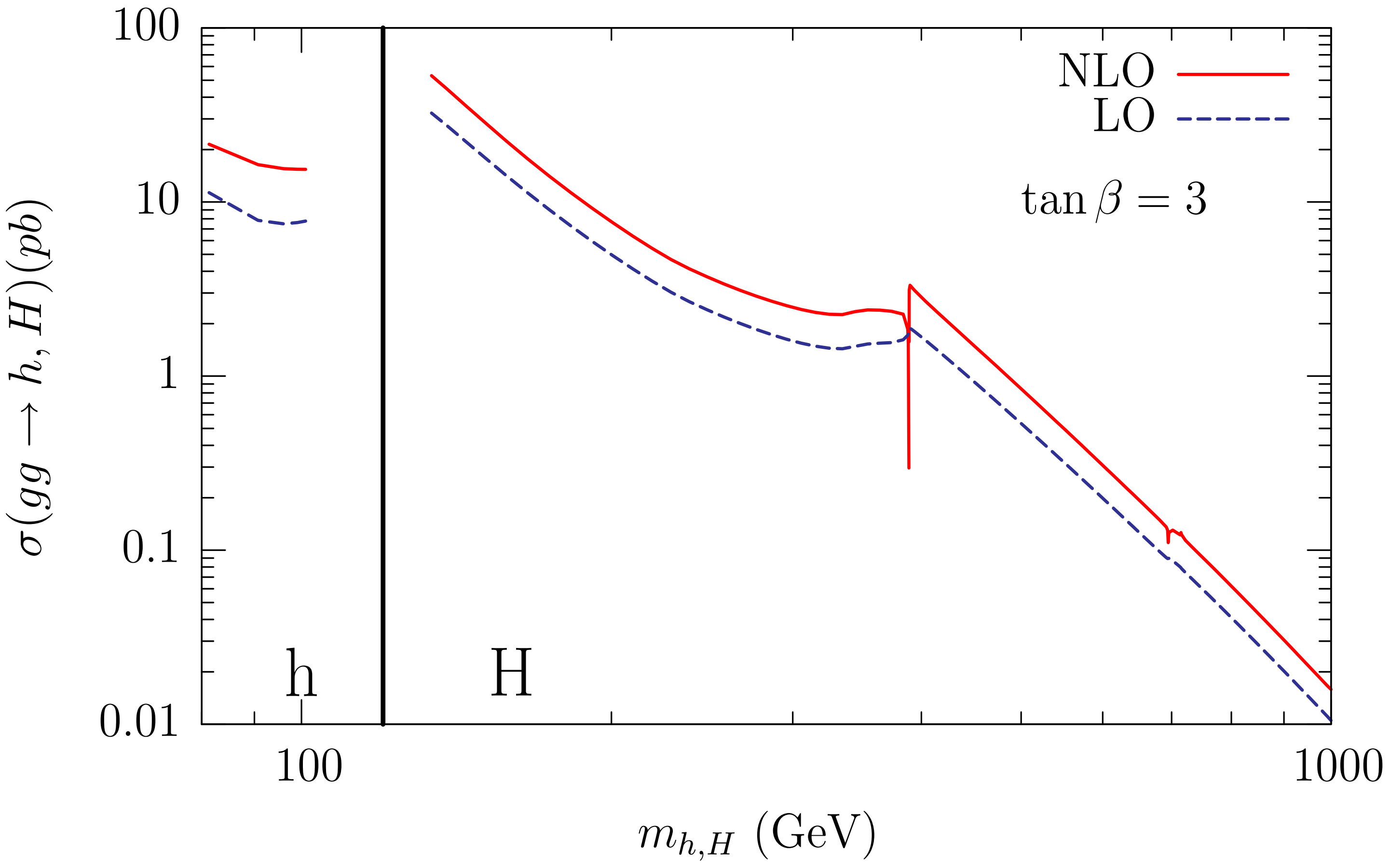
- The Higgs sector of the MSSM containes 5 physical states: two CP-even neutral bosons, h and H, one CP-odd neutral one, A and two charged Higgs bosons, $H^{\pm}$.
- At the lowest order the MSSM Higgs sector can be specified in terms of m_A and $\tan\beta = v_2/v_1$.
- We evaluated the production cross section for two values of $\tan\beta$: $\tan\beta = 30$ and $\tan\beta = 3$.
- We need the mass spectrum of the MSSM particles:

$$m_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}_L}^2 + m_q^2 + m_Z^2 (I_q^3 - e_q \sin^2\theta_W)\cos 2\beta & m_q (A_q - \mu\,(\cot\beta)^{2I_q^3}) \\ m_q (A_q - \mu\,(\cot\beta)^{2I_q^3}) & m_{\tilde{q}_R}^2 + m_q^2 + m_Z^2\, e_q \sin^2\theta_W \cos 2\beta \end{pmatrix}$$

$$m_q^{(\overline{\text{DR}})} = m_q^{(\overline{\text{MS}})} - \frac{g_s^2}{16\pi^2} C_F\, m_q$$

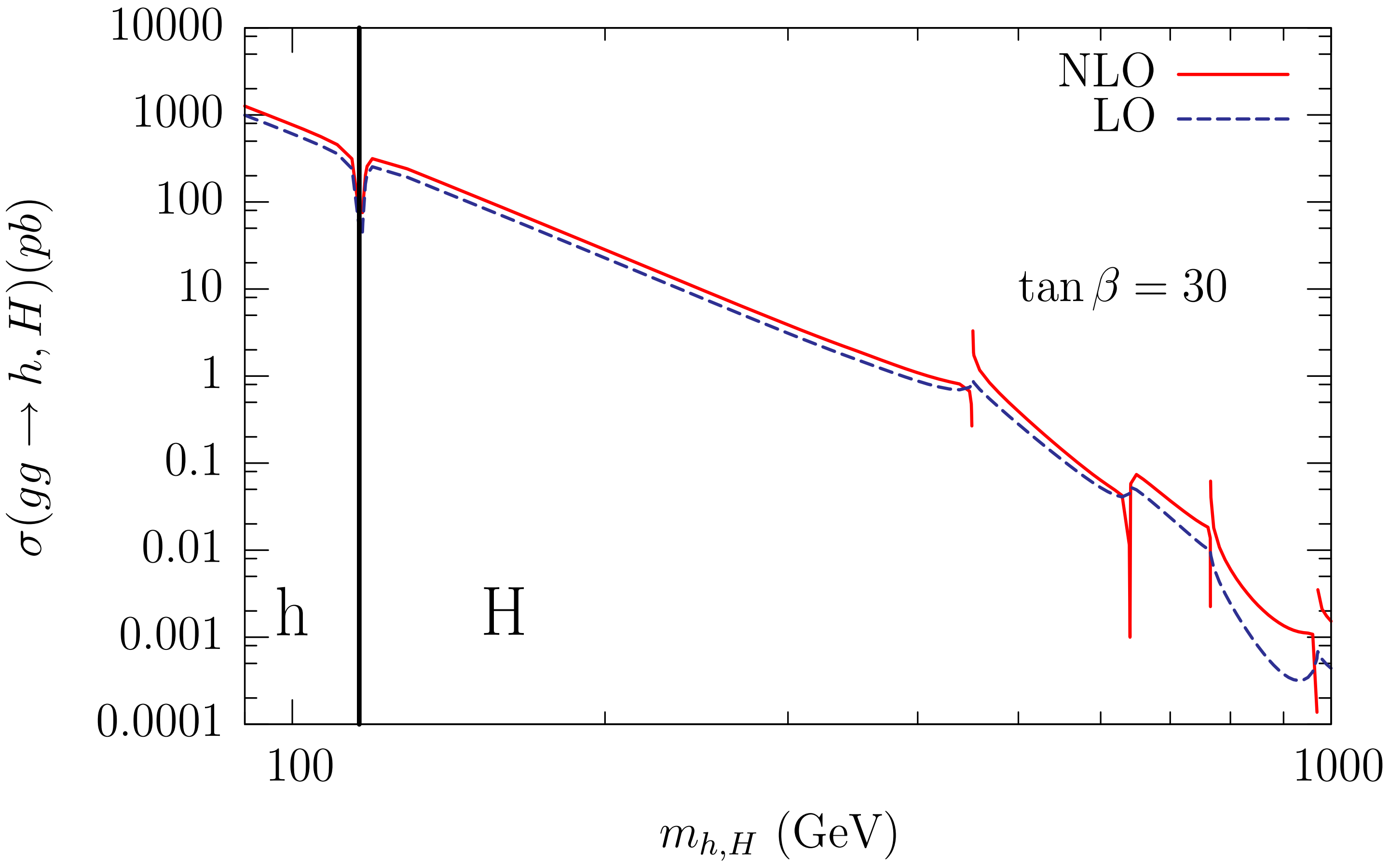
- Input parameters for the squark mass matrix at $\mu_{EWSB} = 300$ GeV chosen: $m_{\tilde{q}_L}^2 = m_{\tilde{t}_R}^2 = m_{\tilde{b}_R}^2 = 350$ GeV $A_t = A_b = -600$ GeV, $\mu = 300$ GeV, $m_t^{\overline{\text{MS}}}(\mu_{EWSB}) = 153$ GeV, $m_b^{\overline{\text{MS}}}(\mu_{EWSB}) = 2.3$ GeV

MSSM: squark contributions



The $\overline{\text{MS}}$ squark mass eigenvalues are: $m_{\tilde{t}_1} = 190$ GeV, $m_{\tilde{t}_2} = 500$ GeV, $m_{\tilde{b}_1} = 350$ GeV, $m_{\tilde{b}_2} = 360$ GeV. m_h and m_H from Suspect.

MSSM: squark contributions



The $\overline{\text{MS}}$ squark mass eigenvalues are: $m_{\tilde{t}_1} = 230$ GeV, $m_{\tilde{t}_2} = 490$ GeV, $m_{\tilde{b}_1} = 320$ GeV, $m_{\tilde{b}_2} = 380$ GeV. m_h and m_H from Suspect.

Summary

- We presented analytic formulas for the NLO QCD corrections to the Higgs production in gluon fusion and to its decay in two photons, in the cases in which a heavy fermion or scalar particle runs in the loops.
- The two-loop virtual corrections were calculated using the Laporta algorithm for the reduction to the MIs and the differential equations for their analytical evaluation. The real part is a standard one-loop calculation of $2 \to 2$ amplitudes, that can be written in terms of B_0, C_0 and D_0 functions, very well known in the literature.
- The formulas are written in a general way, in terms of harmonic and Nielsen's polylogarithms and they are easy to be evaluated numerically.
- Our results for the NLO QCD corrections with fermions are in analytical and numerical agreement with results already present in the literature (for instance with HIGLU). For the scalars we found analytical and numerical agreement for the virtual corrections (we did not check yet the full CS).
- As applications of our formulas, we considered the NLO QCD corrections in the Manohar-Wise model and the squark contribution in the MSSM.