Massless propagators: applications in QCD and QED

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SFB TR9

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Phys. Rev. Lett. 88 (2002) 012001
Phys. Rev. D67 (2003) 074026
Phys. Letters. B559 (2003) 245
Phys. Rev. Lett. 95, 012003 (2005)
Phys.Rev.Lett. 96, 012003 (2006)
Phys.Rev.Lett. 97, 061803 (2006)
+ 2 new results $\longrightarrow$ will be for first time reported now

RADCOR 2007

In this talk I will mainly concentrate on the famous R-ratio:

$$\boldsymbol{R(s) = \sigma_{tot}(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)}$$

which is the main theoretical object appearing in precize extraction of α_{s} from inclusive hadronic Z and τ decays

From measurements at Z-peak LEPEWWG arrives at:

$$\alpha_s(M_Z) = 0.1186(27)$$

with predominantly theoretical error from uncalculated

higher orders $\longleftrightarrow$ α_s^4 (massless diagrams!)

$$\boxed{\text{theory error}}$$

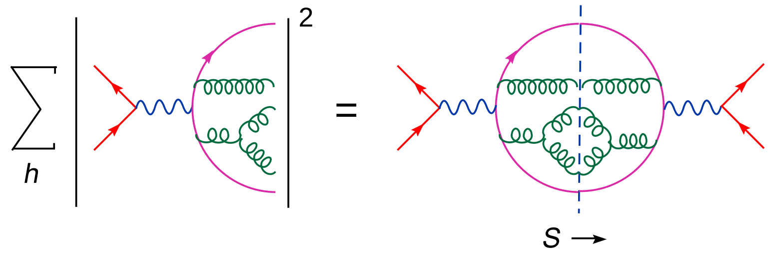
smaller than **present** experimental error (but not much!)

higher QCD corrections are even more important for

$$R_\tau = \Gamma(\tau \to \nu \text{ had})/\Gamma(\tau \to e\nu\nu)$$

Theoretical Framework

R(s) is related (via unitarity) to the correlator of the EM quark currents:



$$R(s) \approx \Im\,\Pi(s - i\delta)$$

$$3Q^2\Pi(q^2 = -Q^2) = \int e^{iqx}\langle 0|T[\ j^v_\mu(x) j^v_\mu(0)\ ]|0\rangle dx$$

To conveniently sum the RG-logs one uses the Adler function:

$$D(Q^2) = Q^2 \frac{d}{d\,Q^2}\Pi(q^2) = Q^2 \int \frac{R(s)}{(s+Q^2)^2}\,ds$$

or $(a_s \equiv \alpha_s/\pi)$

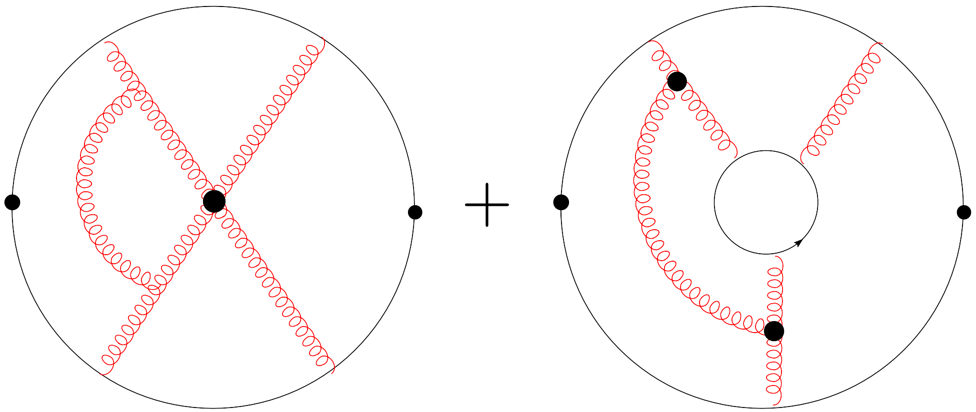
$$R(s) = \frac{1}{2\pi i}\int_{-s-i\delta}^{-s+i\delta} dQ^2 \frac{D(Q^2)}{Q^2} = D(s) - \pi^2\frac{\beta_0^2 d_0}{3} a_s^3 + \dots$$

Current Status of R(s):

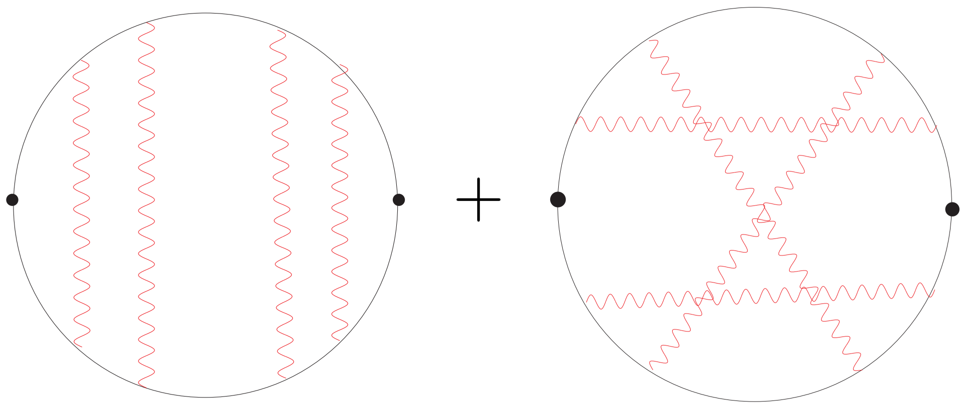
$$\mathbf{R(s) = 1 + \frac{\alpha_s}{\pi} + 1.409\,\frac{\alpha_s^2}{\pi^2} - 12.767\,\frac{\alpha_s^3}{\pi^3} + {?}\,\frac{\alpha_s^4}{\pi^4}}$$

/Gorishnii,Kataev,Larin, (1991)/

R(s) at five loops is conributed by $\approx 17 \cdot 10^3$ of nonabelian or/and non-quenched diagrams like



as well as 2671 purely abelian quenched diagrams like



masslessness $\longleftrightarrow$ simplicity:

5-loop R(s) is reducible*

to 4-loop massless propagators ($\equiv$ p-integrals)

$\longleftarrow$

main object to compute

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* **(i) the same is true for massive corrections like $\mathrm{m_q^2/s}$, etc.**
/J. Kühn, K.Ch (91,94)/

Tool Box *

- **IRR / Vladimirov, (78)/ + IR R^* -operation /K. Ch., Smirnov (1984)/ + resolved combinatorics /K. Ch., (1997)/**
- **reduction to Masters: "direct and automatic" construction of CF's through $1/D$ expansion—made with BAICER—within the Baikov's representation for Feynman integrals**[1]
- **all 4-loop master p-integrals are known analytically /P. Baikov and K.Ch. (2004)/**
- **computing time and required resources: could be huge (the price for full automatization)**; to cope with it we use parallel FORM /Vermaseren, Retey, Fliegner, Tentyukov, ...(2000 – . . .)

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* NO IBP identities are ever used at any step!

[1] Baikov, Phys. Lett. B385 (1996) 403; B474 (2000) 385; Nucl.Phys.Proc.Suppl.116:378-381,2003

$$d_4(N_F = 3) =$$

$$\frac{78631453}{20736} - \frac{1704247}{432}\,\zeta_3 + \frac{4185}{8}\,\zeta_3^2 + \frac{34165}{96}\,\zeta_5 - \frac{1995}{16}\,\zeta_7$$

$$\approx \mathbf{49.0757}$$

and, finally, for the very R(s):

$$1 + a_s + 1.6398\,a_s^2 + 6.3710\,a_s^3 \mathbf{-106.8798}\,a_s^4$$

or with kinematical (trivial!) π^2 terms separated

$$r_4 = 49.0757 - \underline{155.956}$$

Let us compare our exact result with the (12 years old!) PMS/FAQ predictions by Kataev & Starshenko:

$$d_4(FAC/PMS) = 27 \longleftrightarrow d_4(exact) = 49.1$$

$$r_4(FAC/PMS) = (27 - 156) = -129 \longleftrightarrow r_4(exact) = -107.$$

One observes that the quality of the prediction is not especially good but in the R(s) it is getting better due to dominance of (exactly known!) π^2 terms

It is insructive to compare the vector case to the scalar one*:

$$\widetilde{R} = 1+5.667a_s+a_s^2\left[51.57-\underline{15.63}-\ n_f(1.907-\underline{0.548})\right]$$

$$+\ \ a_s^3\left[648.7-\underline{484.6}-\ n_f(63.74-\underline{37.97})+n_f^2(0.929\ -\underline{0.67})\right]$$

$$+a_s^4\left[9471.-\underline{9431.}-n_f(1454.3-\underline{1233.4})+\ n_f^2(54.78-\underline{45.10})\right.$$

$$\left.-\ \ \ n_f^3(0.454-\underline{0.433})\right]$$

remarkable mutual cancellations in all n_f powers!!!

for $n_f = 3 \longrightarrow a_s^4(5589-6126) = -536.8$

similar cancellations happen for ${\alpha_s}^4 m_a^2/s$ and ${\alpha_s}^4 n_f^2$ terms in R(s)

As a result the PMS/FAC predictions are very-very good for the dynamical (euclidean) terms but fail miserably after adding π^2 terms!

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* /P. Baikov, K. Ch, J.Kühn, PLR 96, 012003 (2006)/

Phenomenological Applications

B.Contour Improvement[1] PT (CIPT) versus Fixed Order PT (FOPT)

in extracting α_s from R_τ

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \text{hadrons})}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} \sim \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + 2\frac{s}{M_\tau^2}\right) R(s)$$

typical results look (using only $O(\alpha_s^3)$ approx. for R(s)!)

$$\alpha_s^{\text{FOPT}}(M_Z) = 0.1204 \pm 0.0036$$

$$\alpha_s^{\text{CIPT}}(M_Z) = 0.1223 \pm 0.002$$

[1] A.A. Pivovarov (1991,1992); F. Le Diberder and A. Pich (1992)/

Phenomenological Applications

R_τ: the dependence on d_4 was thoroughly investigated in
(P. Baikov and K.Ch., Phys. Rev. D67 (2003) 074026)

with $d_4 = 0$

$\alpha_s(M_\tau) = 0.34 \pm 0.035$ (FOPT) and $= 0.358 \pm 0.021$ (CIPT)

with old (guessed!) value of $d_4 = 26$

$\alpha_s(M_\tau) = 0.327 \pm 0.02$ (FOPT) and $= 0.351 \pm 0.01$ (CIPT)

with new (exact) value of $d_4 = 49.1$

$\alpha_s(M_\tau) = 0.326 \pm 0.02$ (FOPT) and $= 0.347 \pm 0.01$ (CIPT)

the difference (FOPT) - (CIPT) survives: π^2 terms seems to be overestimated in CIPT approach?

Finally, using only FOPT we get (preliminary; theoretical uncertainty from the scale variation):

$$\alpha_s^{\mathrm{FOPT}}(M_Z) = 0.1183 \quad \pm \quad 0.0007_{ex} \quad \pm \quad 0.001_{th}$$

Consider more attentively the old result for the 4-loop R(s) and note that the "pure" C_F terms look astonishingly simple:

$$R(s) = r_0 + A_s(\mu) r_1 + A_s^2(\mu)\left[r_2 + \ln\frac{\mu^2}{s}\,(r_1\,\beta_0)\right]$$
$$+A_s^3(\mu)\left[r_3 + \ln\frac{\mu^2}{s}\,(2r_2\,\beta_0 + r_1\,\beta_1) + \ln^2\frac{\mu^2}{s}\left(r_1\,\beta_0^2\right)\right] + a_s^4(\mu)\,[\ldots]$$

$$\text{where} \quad A_s \equiv \frac{\alpha_s}{4\pi} \quad \text{and}$$

$$r_0 = 1, \qquad r_1 = 3\,C_F, \quad r_2 = -\frac{3}{2}C_F^2 + \ldots, \quad r_3 = -\frac{69}{2}C_F^3 + \ldots$$

$$r_0 = 1, \qquad r_1 = 3\, C_F, \quad r_2 = -\frac{3}{2}\, C_F^2, \quad r_3 = -\frac{69}{2}\, C_F^3$$

cmp. r_i to the β-function of quenched QED:

$$\beta^{\text{qQED}} = \frac{4}{3}\, A\, \left\{ 1 + 3\, A - \frac{3}{2}\, A^2 - \frac{69}{2}\, A^3 \right\} \quad \text{with} \quad A = \frac{\alpha}{4\pi}$$

It is not by chance, but well-known fact:

the C_F-only part of $R(s)$ is given essentially by

the quenched QED β-function after replacement

$$C_F\, \alpha_s \to \alpha$$

PUZZLE of β^{qQED}

- it is scheme independent in all orders
- the coefficients are simple rational numbers at 1,2,3 and four loops: (4/3, 4,-2,-46)
- if

$$\beta^{\mathrm{qQED}}(\alpha_0) \equiv 0$$

tnen $\alpha = \alpha_0$ leads to self-consistent **finite** solution of (massless) QED

/K.Johnson and M. Baker, (1973)/

some people undestand the observed rationality of β^{qQED} at 1,2,3 and 4 loops and hope that it is not a pure coincidence.

For instance: David Broadhurst: (in hep-th/9909185)

"Noting the profound work of Alain Connes and Dirk Kreimer [1], one arrives at the nub of the rationality of quenched QED: dimensional regularization of the *derivative* of the scheme-independent single-fermion-loop Gell-Mann-Low function, via Fock-Feynman-Schwinger formalism"

We have computed the $\beta^{\mathbf{qQED}}$ at 5 loops.

Our results reads:

$$\frac{4}{3}, \quad 4, \quad -2, \quad -46, \quad \frac{4157}{6} + 128\,\zeta_3$$

Two comments:

1. no chance for finite QED (at this order)

2. It gives (numerically small) $\mathcal{O}(C_F^4 a_s^4)$ contribution to R(s)

2005 - 2007: Three $\mathcal{O}(\alpha_{\mathbf{s}}^4)$ Calculations

used CPU time (measured in terms of a one 3 GH PC)

- **QCD: scalar $R^{SS}(s)$: ($\approx$ 5 years)**
- **Quenched QED: β-function ($\approx$ 25 years)**
- **QCD: $R^{VV}(s)$ at $N_F = 3$ ($\approx$ 40 years)**

Reality Check

calculations of $R^{VV}(s)$ with $N_F = 3$ were made mainly on HP XC4000 supercomputer of the Karlsruhe University (claster of Dual Core AMD Opteron 2.6 GH). It took about 40 CPU years, but only 3 calendar months (up to 160 processors were simultaneously used).

To compare: scalar $R^{SS}(s)$ took about 5 CPU years and QQED about 25 CPU years, with calendar time about 1 year for each (but with older processors and less developed software).

Summary: Results

- **complete results on $R^{SS}(s)$ (with full N_F dependence) and $R^{VV}(s)$ (for $N_F = 3$ at the moment) at $\mathcal{O}({\alpha_s}^4)$ order are available**

- **full N_F dependence for $R^{VV}(s)$ is under way and expected soon**

- **the C_F^4 term in $R^{VV}(s)$ ($\equiv$ the five-loop qQED β-function) is finished and** ***ceases to be purely rational number any more!***

- **higher order terms in these (and others quantities too !!) massless correlators display interesting cancellations between kinematical ($\sim \pi^2$) and dynamical contributions**

Summary: Puzzles (irrelevant for physics?)

- **Could one ever understand the mysterious absence of ζ_4 in massless physical (that is scale-invariant quantities) like D^{VV}?**

- **Could one ever understand the mysterious structure of irrationalities in the quenched QED β-function?**