Automated Calculation of QED Corrections to Lepton $g-2$

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collaboration with
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“Application of Quantum Field Theory to Phenomenology”
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What's new in electron $g-2$

New measurement by Harvard group:

$$a_e = 1\,159\,652\,180.85\,(0.76) \times 10^{-12}$$

B. Odom, et al., PRL 97, 030802 (2006)

Revised 8th-order QED correction:

$$-1.914\,4\,(35)\,(\alpha/\pi)^4.$$

our collaboration, PRL 99, 110406 (2007)

with a help of automated code-generating system.

Revised Fine structure constant α determined from the experiment and the theory of electron $g-2$:

$$\alpha^{-1}(a_e07) = 137.035\,999\,070\,(98).$$

G. Gabrielse, et al., PRL 99, 039902(E) (2007)

Motivation

- Latest value of fine structure constant α determined by the experiment and the theory of electron $g-2$ is

$$\alpha^{-1}(a_e07) = 137.035\,999\,070\,(12)(37)(90).$$
$$(\alpha^4)(\alpha^5)(\text{expr})$$

- The uncertainty of α^5 term (pure guess) is only 2.5 times smaller than that of experiment.
- The precision of experimental value is expected to be improved further. Then, unknown α^5 term would be the largest source of systematic error.

Reliable estimate of α^5 term is urgently needed.

Theory of Electron $g-2$

- Contributions to electron $g-2$:

$$a_e = a_e(\text{QED}) + a_e(\text{hadronic}) + a_e(\text{weak})$$

QED (mass-independent)	999999996	ppb
QED (mass-dependent)	2.3	ppb
Hadronic	1.4	ppb
Weak	0.03	ppb

(ppb = 10^{-9})

- Electron $g-2$ is explained almost entirely by QED interaction between electron and photons.
It has provided the most stringent test of QED.

- Mass-independent QED correction (A_1) is evaluated by perturbation theory as a power series in terms of α:

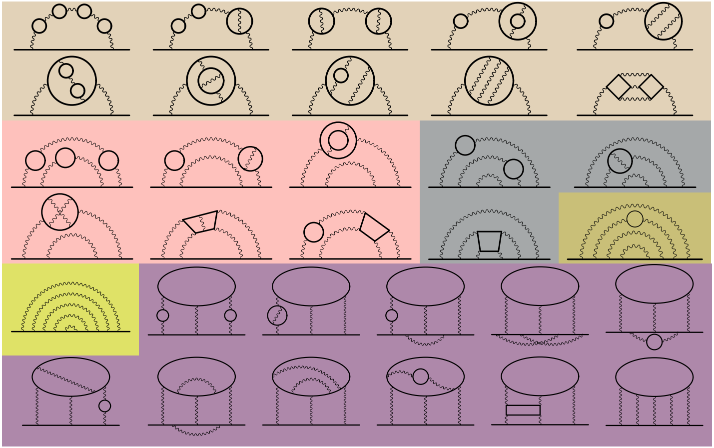
$$A_1 = A_1^{(2)} \left(\frac{\alpha}{\pi}\right) + A_1^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + A_1^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + A_1^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

- Up to 8th-order terms have been obtained analytically and/or numerically:

		year	# diagram
$A_1^{(2)} = \quad 0.5$	Schwinger	1948	1
$A_1^{(4)} = -0.328\,478\ldots$	Sommerfield, Petermann	1957	7
$A_1^{(6)} = \quad 1.181\,241\ldots$	Laporta & Remiddi	1996	72
$A_1^{(8)} = -1.914\,4\,(35)$	our collaboration	2007	891

10th-order Diagrams

- Number of Feynman diagrams is 12672.
- Classified into 32 gauge invariant groups within 6 sets.



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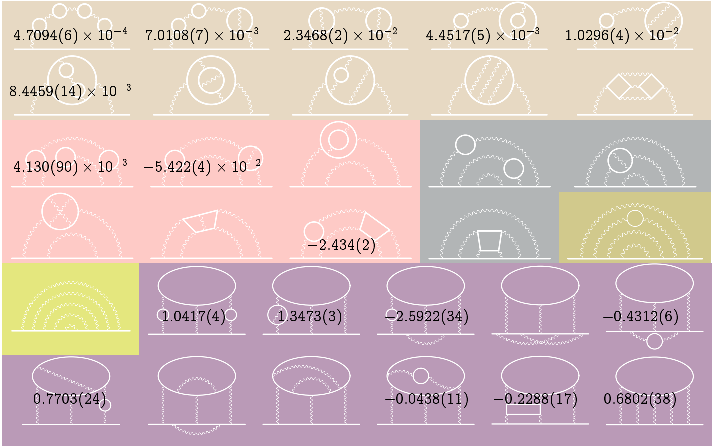

Laporta, PLB328, 522 (1994); Kinoshita and Nio, PRD 73, 053007 (2006)

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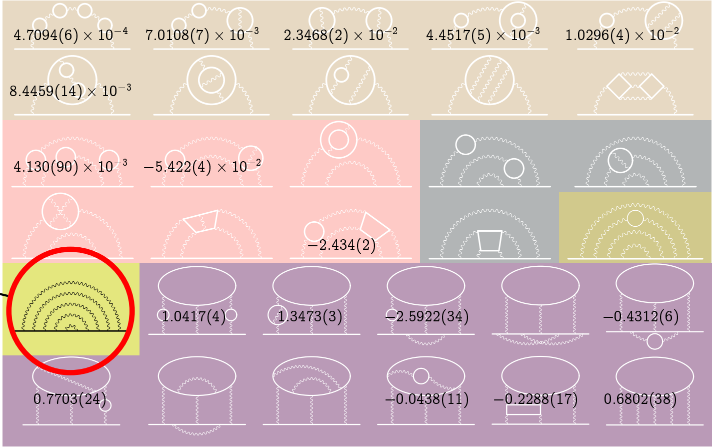
Set V

6354

diagrams

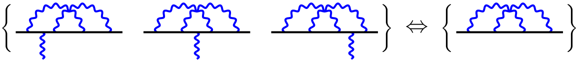


Laporta, PLB328, 522 (1994); Kinoshita and Nio, PRD 73, 053007 (2006)

- We focus on Set V that consists of 6354 vertex diagrams forming one gauge-invariant set, which have no lepton loops (called *q-type*).
- 9 vertex diagrams are related to 1 self-energy-like diagram by Ward-Takahashi identity,

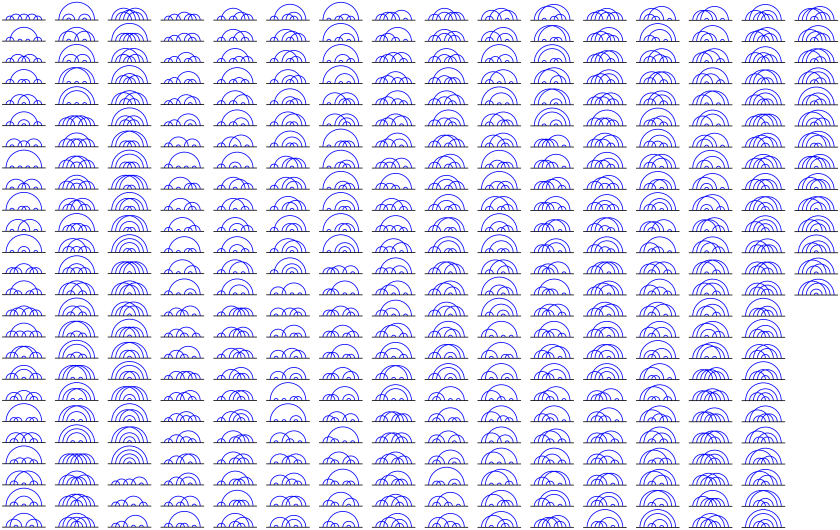
$$\Lambda^{\nu}(p,q) \simeq -q_{\mu} \left. \frac{\partial \Lambda^{\mu}(p,q)}{\partial q_{\nu}} \right|_{q\to 0} - \frac{\partial \Sigma(p)}{\partial p_{\nu}}.$$

e.g. 4th-order case:



- The number of diagrams reduces to 706 by WT sum.
- Time-reversal symmetry reduces the number to 389.

389 independent integrals of 10th order



Numerical Approach

Our strategy:

- We evaluate diagrams by numerical means.
- Diagrams are calculated separately.
- Amplitude of each diagram must be a finite value.

Calculation Steps:

(1) Construct (unrenormalized) amplitude.
$\Rightarrow$ Feynman parametric integral.

(2) Construct subtraction terms for UV and IR divergences.
$\Rightarrow$ Point-by-point subtraction.

Construction of Amplitude

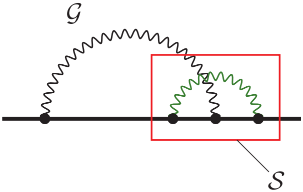
- Amplitude is given by an integral over loop momenta.
- It is converted into Feynman parametric integral over $\{z_i\}$. Momentum integration is carried out analytically, and it yields

$$M_{\mathcal{G}}^{(2n)} = \left(-\frac{1}{4}\right)^n \Gamma(n-1) \int (dz)_{\mathcal{G}} \left[\frac{F_0}{U^2 V^{n-1}} + \frac{F_1}{U^3 V^{n-2}} + \cdots \right]$$

- Integrand is expressed by a rational function of *building blocks*, B_{ij}, A_j, U, C_{ij} and V.
- Building blocks are given by functions of $\{z_i\}$, reflecting the topology of diagram, flow of momenta, etc.

Subtraction of UV Divergences

- UV divergence occurs when loop momenta in a subdiagram go to infinity.
- It corresponds to the region of Feynman parameter space $z_i \sim \mathcal{O}(\epsilon)$ for $i \in \mathcal{S}$.
- We adopt subtractive approach by appropriate subtraction terms.
- In order to carry it out numerically, the singularities are cancelled point-by-point on Feynman parameter space.
- The whole UV-divergences are dealt with Zimmermann's forests.



$$M_{\mathcal{G}} - L_{\mathcal{S}} M_{\mathcal{G}/\mathcal{S}}.$$

$$\downarrow$$

$$\int (dz)_{\mathcal{G}} \left[m_{\mathcal{G}} - \mathbb{K}_{\mathcal{S}} m_{\mathcal{G}} \right]$$

K-operation

Cvitanović and Kinoshita, 1974

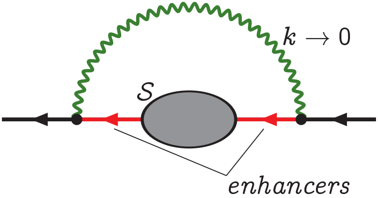
- K-operation produces $\mathbb{K}_{\mathcal{S}} m_{\mathcal{G}}$ from $m_{\mathcal{G}}$ by simple power-counting rule.
- It cancels the singularity of $m_{\mathcal{G}}$ associated with the subdiagram $\mathcal{S}$ point-by-point.
- The subtraction integral factorizes by construction into a lower-order amplitude $M_{\mathcal{G}/\mathcal{S}}$ exactly and counter term $L_{\mathcal{S}}^{\mathrm{UV}}$ by

$$L_{\mathcal{S}}^{\mathrm{UV}} \times M_{\mathcal{G}/\mathcal{S}}.$$

- $L_{\mathcal{S}}^{\mathrm{UV}}$ is the leading UV-divergent part of $L_{\mathcal{S}}$.
  $(L_{\mathcal{S}} - L_{\mathcal{S}}^{\mathrm{UV}})$ must be supplemented later step (called Residual renormalization).

Subtraction of IR Divergences

- A diagram may have IR divergence when some momenta of photon go to zero. It is really divergent by “enhancer” leptons that are close to on-shell by kinematical constraint.



- We adopt subtractive approach for those divergences point-by-point on the Feynman parameter space.

Old Scheme for IR Subtraction

- In the Feynman parametric integration, IR divergences correspond to vanishing of V-function in the region

$$z_i = \begin{cases} \mathcal{O}(\delta) & \text{if } i \text{ is a lepton line in } \mathcal{R}, \\ \mathcal{O}(1) & \text{if } i \text{ is a photon line in } \mathcal{R}, \\ \mathcal{O}(\epsilon), \epsilon \sim \delta^2 & i \in \mathcal{S}. \end{cases}$$

- The IR subtraction term by old I-operation is constructed based on power-counting in this region. It takes factorized form by construction:

$$L_{\mathcal{R}(k)}[F_0]\, M_{\mathcal{S}} + M_{\mathcal{R}^\star}[I]\, \Delta\delta\tilde{m}_{\mathcal{S}}.$$

- Linear IR divergences have been handled by close examination of the integrand.
  $\Longrightarrow$ Unfeasible for automated treatment.

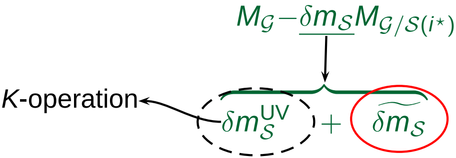
New IR subtraction Scheme

our collaboration, arXiv:0709.1568

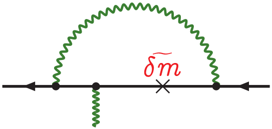
- There are two types of sources of IR divergence in $M_{\mathcal{G}}$ associated with a self-energy subdiagram.
- To identify these terms, introduce two subtraction operations called *I*-subtraction and *R*-subtraction. The entire IR subtraction terms are given by combinations of them.
- The forms of subtraction terms can be recognized from the structure of the diagram, not the explicit form of integrand. $\Longrightarrow$ Feasible for automation.

R-subtraction

- In two-step renormalization by *K*-operation, self-mass is subtracted only for the leading UV divergent part.



- This scheme effectively introduces spurious two-point vertex which causes linear or worse IR divergences.



- We subtract this contribution away by the term

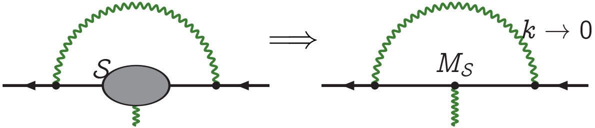
$$\widetilde{\delta m_{\mathcal{S}}} M_{\mathcal{G}/\mathcal{S}(i^\star)}$$

It is called *Residual self-mass subtraction*.

I-subtraction

- In the IR limit, a vertex diagram behaves in a factorized form

$$\widetilde{L}_{\mathcal{R}(k)} M_{\mathcal{S}}.$$



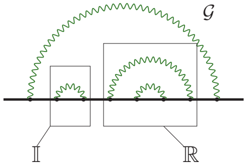
$\widetilde{L}_{\mathcal{R}(k)}$ accounts for logarithmic IR divergence.

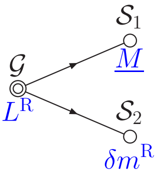
- To cancel out this divergence, we prepare the subtraction term of the form $\widetilde{L}_{\mathcal{R}(k)} M_{\mathcal{S}}$.
  It is called *I*-subtraction.

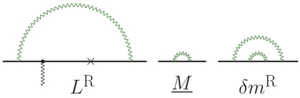
Nested IR singularities

- Nested IR divergences emerge when there are more than one self-energy subdiagrams account for IR singularities.
- These divergences are dealt with the combinations of *I*-/*R*-subtractions.
- The combinations are organized in a forest-like structure called

  “annotated forest”.
- The entire set of IR subtraction terms can be recognized from the form of the diagram, and feasible by automation.







Finite Amplitude

- Finite amplitude $\Delta M_{\mathcal{G}}$ free from both UV and IR divergences is thus given by

$$\Delta M_{\mathcal{G}} = M_{\mathcal{G}}$$

← unrenormalized amplitude

$$+ \sum_{f} \prod_{\mathcal{S} \in f} (-\mathbb{K}_{\mathcal{S}}) M_{\mathcal{G}}$$

← UV subtraction terms

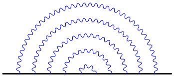
f: Zimmermann's forests

$$+ \sum_{\tilde{f}} (-\mathbb{I}_{\mathcal{S}_i}) \cdots (-\mathbb{R}_{\mathcal{S}_j}) \cdots M_{\mathcal{G}}$$

← IR subtraction terms

$\tilde{f}$: "annotated forests"

Example of UV and IR subtraction terms



X072

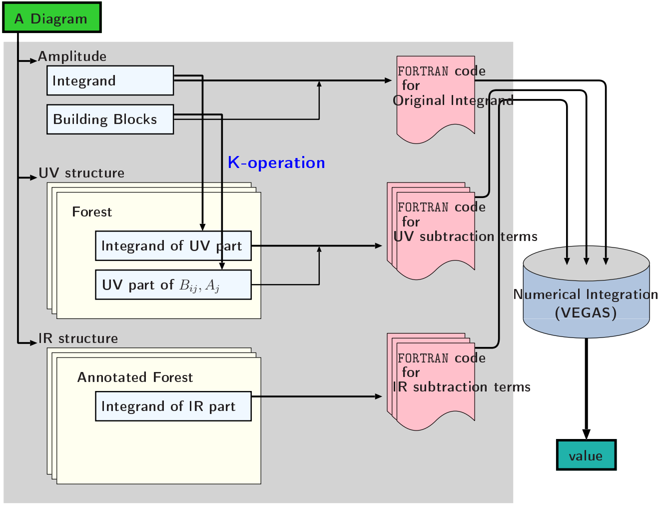
```
DMX072 = MX072 - dmm47uv*M21s - Bm47uv*M2 - dm6buv*M4b2s - B6buv*M4b - dm4buv*M6b3s - B4buv*M6b - dm2uv*Mm474s - B2uv*Mm47 + dm6buv*
dm21suv*M21s + B6buv*dm2uv*M21s + B6buv*B2uv*M2 + dm4buv*dm4b2suv*M21s + B4buv*dm4buv*M21s + B4buv*B4buv*M2 + dm2uv*dm6b3suv*M21s +
B2uv*dm6buv*M21s + B2uv*B6buv*M2 + dm4buv*dm21suv*M4b2s + B4buv*dm2uv*M4b2s + B4buv*B2uv*M4b + dm2uv*dm4b2suv*M4b2s + B2uv*dm4buv*M4
b2s + B2uv*B4buv*M4b + dm2uv*dm21suv*M6b3s + B2uv*dm2uv*M6b3s + B2uv*B2uv*M6b - dm4buv*dm21suv*dm21suv*M21s - B4buv*dm2uv*dm21suv*M2
1s - B4buv*B2uv*dm2uv*M21s - B4buv*B2uv*B2uv*M2 - dm2uv*dm4b2suv*dm21suv*M21s - B2uv*dm4buv*dm21suv*M21s - B2uv*B4buv*dm2uv*M21s - B
2uv*B4buv*B2uv*M2 - dm2uv*dm21suv*dm4b2suv*M21s - B2uv*dm2uv*dm4b2suv*M21s - B2uv*B2uv*dm4buv*M21s - B2uv*B2uv*B4buv*M2 - dm2uv*dm21
suv*dm21suv*M4b2s - B2uv*dm2uv*dm21suv*M4b2s - B2uv*B2uv*dm2uv*M4b2s - B2uv*B2uv*B2uv*M4b + dm2uv*dm21suv*dm21suv*dm21suv*M21s + B2u
v*dm2uv*dm21suv*dm21suv*M21s + B2uv*B2uv*dm2uv*dm21suv*M21s + B2uv*B2uv*B2uv*dm2uv*M21s + B2uv*B2uv*B2uv*B2uv*M2 - dmm47ir*M21s + dm
6buv*dm21sir*M21s + B6buv*dm2ir*M21s + dm4buv*dm4b2sir*M21s + B4buv*dm4bir*M21s + dm2uv*dm6b3sir*M21s + B2uv*dm6bir*M21s - dm4buv*dm
21suv*dm21sir*M21s - B4buv*dm2uv*dm21sir*M21s - B4buv*B2uv*dm2ir*M21s - dm2uv*dm4b2suv*dm21sir*M21s - B2uv*dm4buv*dm21sir*M21s - B2u
v*B4buv*dm2ir*M21s - dm2uv*dm21suv*dm4b2sir*M21s - B2uv*dm2uv*dm4b2sir*M21s - B2uv*B2uv*dm4bir*M21s + dm2uv*dm21suv*dm21suv*dm21sir*
M21s + B2uv*dm2uv*dm21suv*dm21sir*M21s + B2uv*B2uv*dm2uv*dm21sir*M21s + B2uv*B2uv*B2uv*dm2ir*M21s - Mm47*L2vir + dm6buv*M21s*L2vir +
 B6buv*M2*L2vir + dm4buv*M4b2s*L2vir + B4buv*M4b*L2vir + dm2uv*M6b3s*L2vir + B2uv*M6b*L2vir - dm4buv*dm21suv*M21s*L2vir - B4buv*dm2u
v*M21s*L2vir - B4buv*B2uv*M2*L2vir - dm2uv*dm4b2suv*M21s*L2vir - B2uv*dm4buv*M21s*L2vir - B2uv*B4buv*M2*L2vir - dm2uv*dm21suv*M4b2s*
L2vir - B2uv*dm2uv*M4b2s*L2vir - B2uv*B2uv*M4b*L2vir + dm2uv*dm21suv*dm21suv*M21s*L2vir + B2uv*dm2uv*dm21suv*M21s*L2vir + B2uv*B2uv*
dm2uv*M21s*L2vir + B2uv*B2uv*B2uv*M2*L2vir - dm6bir*M4b2s + dm6bir*dm21suv*M21s + dm4buv*dm21sir*M4b2s + B4buv*dm2ir*M4b2s + dm2uv*d
m4b2sir*M4b2s + B2uv*dm4bir*M4b2s - dm4buv*dm21sir*dm21suv*M21s - B4buv*dm2ir*dm21suv*M21s - dm2uv*dm4b2sir*dm21suv*M21s - B2uv*dm4b
ir*dm21suv*M21s - dm2uv*dm21suv*dm21sir*M4b2s - B2uv*dm2uv*dm21sir*M4b2s - B2uv*B2uv*dm2ir*M4b2s + dm2uv*dm21suv*dm21sir*dm21suv*M21
s + B2uv*dm2uv*dm21sir*dm21suv*M21s + B2uv*B2uv*dm2ir*dm21suv*M21s - M6b*L4b2vir + M6b*L2vuv*L2vir + dm4buv*M21s*L4b2vir + B4buv*M2*
L4b2vir + dm2uv*M4b2s*L4b2vir + B2uv*M4b*L4b2vir - dm4buv*M21s*L2vuv*L2vir - B4buv*M2*L2vuv*L2vir - dm2uv*M4b2s*L2vuv*L2vir - B2uv*M
4b*L2vuv*L2vir - dm2uv*dm21suv*M21s*L4b2vir - B2uv*dm2uv*M21s*L4b2vir - B2uv*B2uv*M2*L4b2vir + dm2uv*dm21suv*M21s*L2vuv*L2vir + B2uv
*dm2uv*M21s*L2vuv*L2vir + B2uv*B2uv*M2*L2vuv*L2vir - dm4bir*M6b3s + dm4bir*dm4b2suv*M21s + dm4bir*dm21suv*M4b2s + dm2uv*dm21sir*M6b3
s + B2uv*dm2ir*M6b3s - dm4bir*dm21suv*dm21suv*M21s - dm2uv*dm21sir*dm4b2suv*M21s - B2uv*dm2ir*dm4b2suv*M21s - dm2uv*dm21sir*dm21suv*
M4b2s - B2uv*dm2ir*dm21suv*M4b2s + dm2uv*dm21sir*dm21suv*dm21suv*M21s + B2uv*dm2ir*dm21suv*dm21suv*M21s - M4b*L6b3vir + M4b*L4b2vuv*
L2vir + M4b*L2vuv*L4b2vir + dm2uv*M21s*L6b3vir + B2uv*M2*L6b3vir - M4b*L2vuv*L2vuv*L2vir - dm2uv*M21s*L4b2vuv*L2vir - B2uv*M2*L4b2vu
v*L2vir - dm2uv*M21s*L2vuv*L4b2vir - B2uv*M2*L2vuv*L4b2vir + dm2uv*M21s*L2vuv*L2vuv*L2vir + B2uv*M2*L2vuv*L2vuv*L2vir - M2*Lm474vir
+ M2*L6b3vuv*L2vir + M2*L4b2vuv*L4b2vir + M2*L2vuv*L6b3vir - M2*L4b2vuv*L2vuv*L2vir - M2*L2vuv*L4b2vuv*L2vir - M2*L2vuv*L2vuv*L4b2vi
r + M2*L2vuv*L2vuv*L2vuv*L2vir + dm6bir*dm21sir*M21s - dm4buv*dm21sir*dm21sir*M21s - B4buv*dm2ir*dm21sir*M21s - dm2uv*dm4b2sir*dm21s
ir*M21s - B2uv*dm4bir*dm21sir*M21s + dm2uv*dm21suv*dm21sir*dm21sir*M21s + B2uv*dm2uv*dm21sir*dm21sir*M21s + B2uv*B2uv*dm2ir*dm21sir*
M21s + dm6bir*M21s*L2vir - dm4buv*dm21sir*M21s*L2vir - B4buv*dm2ir*M21s*L2vir - dm2uv*dm4b2sir*M21s*L2vir - B2uv*dm4bir*M21s*L2vir +
 dm2uv*dm21suv*dm21sir*M21s*L2vir + B2uv*dm2uv*dm21sir*M21s*L2vir + B2uv*B2uv*dm2ir*M21s*L2vir + M6b*L2vir*L2vir - dm4buv*M21s*L2vir
*L2vir - B4buv*M2*L2vir*L2vir - dm2uv*M4b2s*L2vir*L2vir - B2uv*M4b*L2vir*L2vir + dm2uv*dm21suv*M21s*L2vir*L2vir + B2uv*dm2uv*M21s*L2
vir*L2vir + B2uv*B2uv*M2*L2vir*L2vir + dm4bir*dm4b2sir*M21s - dm4bir*dm21suv*dm21sir*M21s - dm2uv*dm21sir*dm4b2sir*M21s - B2uv*dm2ir
*dm4b2sir*M21s + dm2uv*dm21sir*dm21suv*dm21sir*M21s + B2uv*dm2ir*dm21suv*dm21sir*M21s + dm4bir*M4b2s*L2vir - dm4bir*dm21suv*M21s*L2v
ir - dm2uv*dm21sir*M4b2s*L2vir - B2uv*dm2ir*M4b2s*L2vir + dm2uv*dm21sir*dm21suv*M21s*L2vir + B2uv*dm2ir*dm21suv*M21s*L2vir + M4b*L4b
2vir*L2vir - M4b*L2vuv*L2vir*L2vir - dm2uv*M21s*L4b2vir*L2vir - B2uv*M2*L4b2vir*L2vir + dm2uv*M21s*L2vuv*L2vir*L2vir + B2uv*M2*L2vuv
*L2vir*L2vir + M2*L6b3vir*L2vir - M2*L4b2vuv*L2vir*L2vir - M2*L2vuv*L4b2vir*L2vir + M2*L2vuv*L2vuv*L2vir*L2vir + dm4bir*dm21sir*M4b2
s - dm4bir*dm21sir*dm21suv*M21s - dm2uv*dm21sir*dm21sir*M4b2s - B2uv*dm2ir*dm21sir*M4b2s + dm2uv*dm21sir*dm21sir*dm21suv*M21s + B2uv
*dm2ir*dm21sir*dm21suv*M21s + dm4bir*M21s*L4b2vir - dm4bir*M21s*L2vuv*L2vir - dm2uv*dm21sir*M21s*L4b2vir - B2uv*dm2ir*M21s*L4b2vir +
 dm2uv*dm21sir*M21s*L2vuv*L2vir + B2uv*dm2ir*M21s*L2vuv*L2vir + M4b*L2vir*L4b2vir - M4b*L2vir*L2vuv*L2vir - dm2uv*M21s*L2vir*L4b2vir
 - B2uv*M2*L2vir*L4b2vir + dm2uv*M21s*L2vir*L2vuv*L2vir + B2uv*M2*L2vir*L2vuv*L2vir + M2*L4b2vir*L4b2vir - M2*L4b2vir*L2vuv*L2vir -
M2*L2vuv*L2vir*L4b2vir + M2*L2vuv*L2vir*L2vuv*L2vir + M2*L2vir*L6b3vir - M2*L2vir*L4b2vuv*L2vir - M2*L2vir*L2vuv*L4b2vir + M2*L2vir*
L2vuv*L2vuv*L2vir - dm4bir*dm21sir*dm21sir*M21s + dm2uv*dm21sir*dm21sir*dm21sir*M21s + B2uv*dm2ir*dm21sir*dm21sir*M21s - dm4bir*dm21
sir*M21s*L2vir + dm2uv*dm21sir*dm21sir*M21s*L2vir + B2uv*dm2ir*dm21sir*M21s*L2vir - dm4bir*M21s*L2vir*L2vir + dm2uv*dm21sir*M21s*L2v
ir*L2vir + B2uv*dm2ir*M21s*L2vir*L2vir - M4b*L2vir*L2vir*L2vir + dm2uv*M21s*L2vir*L2vir*L2vir + B2uv*M2*L2vir*L2vir*L2vir - M2*L4b2v
ir*L2vir*L2vir + M2*L2vuv*L2vir*L2vir*L2vir - M2*L2vir*L4b2vir*L2vir + M2*L2vir*L2vuv*L2vir*L2vir - M2*L2vir*L2vir*L4b2vir + M2*L2vi
r*L2vir*L2vuv*L2vir + M2*L2vir*L2vir*L2vir*L2vir
```

Automated Calculation

our collaboration, NPB740, 138 (2006) and arXiv:0709.1568

- We developed an automated code-generating system for *q-type* diagrams.
- It takes a single-line information that specifies a diagram, and generates numerical integration code in FORTRAN, readily integrated by VEGAS, adaptive integration routine.
- The symbolic manipulations are carried out by FORM and Maple.

Flow of Process



Tests of Automated Code-generator

- For 4th- and 6th-order cases, the numerical integration of codes generated by automated system, after taking account of residual renormalization, reproduces the known exact results satisfactorily.

	4th order	6th order
Numerical	$-0.343\,95(53)$	$0.905\,2(11)$
Exact result	$-0.344\,166\cdots$	$0.904\,979\cdots$

- For 8th-order case, the new result is compared with the previous numerical result.
  It revealed an inconsistency in the old calculation for the treatment of IR subtraction terms.

Revised 8th-order correction

- After correcting the error, the results become:

$A_1^{(8)}$(q-type)$^{\text{corrected}}$	$-2.179\,16\,(343)$
$A_1^{(8)}$(q-type)$^{\text{new}}$	$-2.205\,(54)$ (*tentative*)

  They agree within the present numerical precision.
- The revised eighth-order term is

$$A_1^{(8)} = -1.914\,4\,(35).$$

  It is now firmly established by two independent calculation.
- As a direct consequence, the new value of α is determined from the experiment and the theory of electron $g-2$:

$$\alpha^{-1}(a_e07) = 137.035\,999\,070\,(98)$$

Toward 10th-order correction of q-type diagrams

- Now the numerical integration codes for finite amplitudes of all 389 integrals are generated.
- Statistics (per diagram):
  - 10–20 min. for code generation on *hp*'s Alpha.
  - $100,000$ lines of FORTRAN code.
  - 10^8 sampling points $\times$ 1 iteration takes 4–6 hours on 64 CPU Xeon cluster.
- Crude estimate of those diagrams shows that the integrals are all finite. The subtraction scheme works as expected. (presented in the next slide)
- Residual renormalization step is in progress.
- To evaluate Set V diagrams within a few percent of accuracy, it will take a year or more.

X001	-0.29807 (0.03272)	**X116**	1.80588 (0.00496)	**X209**	0.14436 (0.00400)	**X322**	0.92052 (0.00321)
X003	-0.11416 (0.00944)	**X117**	0.32320 (0.00447)	**X210**	0.76527 (0.00492)	**X343**	3.88046 (0.00290)
X013	-1.35399 (0.00384)	**X118**	-3.22253 (0.01057)	**X225**	0.29281 (0.00983)	**X344**	3.41470 (0.00367)
X014	0.78332 (0.01410)	**X119**	-0.10546 (0.01128)	**X231**	-0.74672 (0.00582)	**X345**	-1.00146 (0.00241)
X015	2.10198 (0.00195)	**X120**	1.79134 (0.01585)	**X232**	0.40097 (0.01157)	**X346**	0.28443 (0.00367)
X016	-0.96093 (0.00192)	**X121**	-0.86296 (0.00439)	**X235**	0.70401 (0.00998)	**X347**	-2.67922 (0.00280)
X019	1.21827 (0.01399)	**X122**	-0.74143 (0.00417)	**X259**	0.01603 (0.00485)	**X348**	-0.48587 (0.00376)
X021	-0.29674 (0.00489)	**X123**	-3.33388 (0.00748)	**X260**	-0.40072 (0.00358)	**X349**	2.08161 (0.00430)
X031	2.29316 (0.00288)	**X125**	0.74806 (0.01889)	**X265**	-0.67414 (0.00337)	**X350**	1.45479 (0.00230)
X032	-0.24265 (0.00127)	**X127**	1.13490 (0.00585)	**X266**	0.11786 (0.00481)	**X351**	0.24490 (0.00340)
X033	-1.37714 (0.00143)	**X128**	0.59155 (0.01287)	**X271**	0.24149 (0.00534)	**X352**	-0.13189 (0.00252)
X034	1.25388 (0.00205)	**X129**	1.43123 (0.01235)	**X272**	-0.73389 (0.00926)	**X353**	0.18836 (0.00252)
X035	-0.58384 (0.00142)	**X165**	-2.13804 (0.01138)	**X275**	-0.74340 (0.00445)	**X354**	-2.03746 (0.00248)
X037	-0.74165 (0.00199)	**X166**	-2.28564 (0.01211)	**X276**	-0.55445 (0.00283)	**X355**	-1.06375 (0.00308)
X039	0.31638 (0.00441)	**X172**	1.43007 (0.02248)	**X277**	2.78432 (0.00151)	**X356**	2.07081 (0.00486)
X047	-4.45507 (0.00326)	**X178**	0.70789 (0.00378)	**X278**	-0.15593 (0.00442)	**X357**	0.36337 (0.00367)
X048	-0.80512 (0.00160)	**X179**	-0.43781 (0.00341)	**X279**	0.82314 (0.00380)	**X358**	0.03325 (0.00425)
X049	-0.02951 (0.00133)	**X180**	0.02424 (0.00439)	**X280**	-1.00961 (0.00464)	**X359**	-0.15150 (0.00457)
X050	-1.22223 (0.00176)	**X185**	-0.13128 (0.00497)	**X281**	-1.37236 (0.00407)	**X360**	-0.47088 (0.00419)
X051	-0.17333 (0.00202)	**X186**	1.16339 (0.00492)	**X282**	0.48407 (0.00338)	**X361**	2.53192 (0.00641)
X053	0.36460 (0.00153)	**X195**	-1.06649 (0.00450)	**X283**	-0.05049 (0.00420)	**X362**	-0.56599 (0.00358)
X055	-0.36339 (0.00142)	**X196**	-2.03753 (0.00288)	**X284**	-0.27114 (0.00320)	**X363**	-2.34163 (0.00220)
X076	-5.24240 (0.02298)	**X197**	-0.38704 (0.00222)	**X285**	0.01690 (0.00389)	**X364**	2.38995 (0.00212)
X077	3.26159 (0.04430)	**X198**	-2.34519 (0.00271)	**X286**	0.77754 (0.00377)	**X367**	-0.71804 (0.00490)
X078	0.94031 (0.04534)	**X199**	1.04927 (0.00382)	**X287**	0.18741 (0.00680)	**X370**	-1.47907 (0.00453)
X091	-1.81680 (0.04856)	**X200**	0.00925 (0.00425)	**X296**	0.54479 (0.00457)	**X371**	-0.00744 (0.00415)
X093	-1.76036 (0.00496)	**X201**	-0.48774 (0.00369)	**X297**	-0.47919 (0.00468)	**X372**	-1.28753 (0.00254)
X094	-1.04602 (0.00991)	**X202**	1.92431 (0.00297)	**X303**	0.32133 (0.00246)	**X373**	0.56838 (0.00393)
X095	0.57914 (0.00434)	**X203**	0.90371 (0.00233)	**X304**	-0.34223 (0.00489)	**X376**	1.03692 (0.00335)
X096	1.28491 (0.01790)	**X204**	-1.93238 (0.00384)	**X305**	0.46192 (0.00397)	**X377**	0.41915 (0.00361)
X101	-0.26254 (0.00930)	**X205**	-0.90380 (0.00489)	**X313**	0.95129 (0.00427)	**X378**	1.30815 (0.00338)
X102	-1.39119 (0.03119)	**X206**	1.64469 (0.00651)	**X314**	0.79917 (0.00704)	**X379**	-0.34017 (0.00523)
X103	0.82288 (0.01934)	**X207**	0.28937 (0.00418)	**X320**	0.55847 (0.00452)	**X381**	1.06769 (0.00379)
X115	-0.59474 (0.00646)	**X208**	0.52154 (0.00402)	**X321**	-0.91541 (0.00784)		

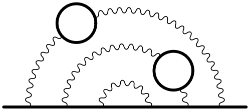
X002	-6.74983 (0.39490)	**X066**	-3.52588 (0.04225)	**X133**	2.62632 (0.04747)	**X182**	1.29226 (0.01852)	**X247**	14.55949 (0.47190)	**X316**	0.11380 (0.03768)
X004	5.42330 (0.20110)	**X067**	-1.71922 (0.07167)	**X134**	-0.48709 (0.07433)	**X183**	-0.01213 (0.02080)	**X248**	-2.03791 (0.18100)	**X317**	0.51989 (0.22780)
X005	1.44465 (0.11480)	**X068**	3.42313 (0.18930)	**X135**	1.13331 (0.07438)	**X184**	0.48849 (0.07146)	**X249**	3.99477 (0.01911)	**X318**	-9.42684 (0.30460)
X006	-5.28071 (0.25800)	**X069**	-1.14158 (0.02901)	**X136**	-7.41325 (0.27500)	**X187**	1.27317 (0.01505)	**X250**	-0.86129 (0.04801)	**X319**	0.83524 (0.08676)
X007	-3.69572 (0.20410)	**X070**	3.13049 (0.12130)	**X137**	-2.62601 (0.19760)	**X188**	1.81376 (0.02836)	**X251**	-1.26684 (0.03699)	**X323**	0.11279 (0.03775)
X008	-14.86438 (0.99850)	**X071**	3.42958 (0.12900)	**X138**	9.89781 (0.50510)	**X189**	-3.54277 (0.11080)	**X252**	-10.83749 (0.27410)	**X324**	-8.70649 (0.20750)
X009	-2.87034 (0.06121)	**X072**	-5.97834 (0.26510)	**X139**	14.99130 (0.67410)	**X190**	-2.47643 (0.17010)	**X253**	17.10443 (0.70420)	**X325**	12.07292 (0.64700)
X010	11.36626 (0.49360)	**X073**	4.49682 (0.24860)	**X140**	-2.40568 (0.11300)	**X191**	0.18020 (0.02842)	**X254**	1.89744 (0.10180)	**X326**	-8.73850 (0.08422)
X011	5.38370 (0.29990)	**X074**	5.77774 (0.33330)	**X141**	-12.66350 (0.48160)	**X192**	2.49358 (0.05016)	**X255**	7.58262 (0.27250)	**X327**	1.46898 (0.12240)
X012	-9.82913 (0.56980)	**X075**	-8.40657 (0.48890)	**X142**	-1.74841 (0.25360)	**X193**	-4.21865 (0.08833)	**X256**	-3.21349 (0.96440)	**X328**	-0.27350 (0.02181)
X017	0.51591 (0.00738)	**X079**	5.94260 (0.30260)	**X143**	10.46136 (0.49870)	**X194**	-0.96093 (0.11210)	**X257**	5.59169 (0.03085)	**X329**	-0.87052 (0.02919)
X018	0.06642 (0.00811)	**X080**	1.37278 (0.20370)	**X144**	24.80308 (1.01900)	**X211**	5.10597 (0.04039)	**X258**	-0.42818 (0.01867)	**X330**	-4.87408 (0.05888)
X020	-8.36211 (0.30140)	**X081**	-6.29318 (0.34600)	**X145**	-16.08794 (1.10400)	**X212**	-0.31324 (0.06238)	**X261**	6.30684 (0.02059)	**X331**	4.31695 (0.09189)
X022	1.49070 (0.12380)	**X082**	-9.13289 (0.59930)	**X146**	-2.75812 (0.25300)	**X213**	-2.40804 (0.01369)	**X262**	-2.27267 (0.01635)	**X332**	2.75429 (0.13310)
X023	0.63324 (0.04549)	**X083**	19.48474 (1.22100)	**X147**	1.14177 (0.02589)	**X214**	0.66484 (0.01634)	**X263**	-2.75563 (0.01780)	**X333**	7.53241 (0.42750)
X024	-7.81060 (0.38210)	**X084**	8.48221 (0.09961)	**X148**	-1.28773 (0.04202)	**X215**	0.12323 (0.01428)	**X264**	4.75755 (0.03976)	**X334**	4.81919 (0.29100)
X025	-0.88268 (0.13320)	**X085**	-1.36167 (0.23340)	**X149**	-8.19282 (0.09061)	**X216**	-1.18046 (0.02671)	**X267**	-0.63526 (0.01164)	**X335**	-2.18030 (0.09550)
X026	-7.55013 (0.34490)	**X086**	1.96596 (0.23270)	**X150**	2.67103 (0.10900)	**X217**	-2.16701 (0.06159)	**X268**	0.11468 (0.02309)	**X336**	-0.74829 (0.00789)
X027	-2.31507 (0.03479)	**X087**	-17.36256 (0.69260)	**X151**	-12.12307 (0.46550)	**X218**	-1.72741 (0.07359)	**X269**	-0.63950 (0.03454)	**X337**	-1.18947 (0.01503)
X028	5.47590 (0.30830)	**X088**	-5.65762 (0.48140)	**X152**	14.85216 (0.68870)	**X219**	0.24134 (0.35050)	**X270**	-1.56223 (0.06899)	**X338**	-1.83821 (0.02199)
X029	6.50129 (0.18910)	**X089**	12.76092 (0.68140)	**X153**	15.60121 (0.68420)	**X220**	-2.01615 (0.18380)	**X273**	-2.00243 (0.02835)	**X339**	0.49361 (0.02978)
X030	-12.56070 (0.66010)	**X090**	1.34631 (0.08945)	**X154**	-21.36901 (0.97680)	**X221**	0.99233 (0.11860)	**X274**	0.92572 (0.04953)	**X340**	-2.48978 (0.16990)
X036	0.24903 (0.00745)	**X092**	2.36196 (0.22300)	**X155**	4.94023 (0.02549)	**X222**	-0.05994 (0.23670)	**X288**	4.15701 (0.01856)	**X341**	1.80547 (0.01413)
X038	-0.27625 (0.00594)	**X097**	4.69268 (0.07112)	**X156**	-0.70982 (0.05046)	**X223**	18.01181 (0.68050)	**X289**	-1.50091 (0.01561)	**X342**	2.33120 (0.08021)
X040	1.52803 (0.03750)	**X098**	-1.37816 (0.10690)	**X157**	-11.63190 (0.10140)	**X224**	2.63787 (0.11370)	**X290**	-3.71770 (0.01356)	**X365**	0.48951 (0.01234)
X041	3.13093 (0.06537)	**X099**	3.57877 (0.19970)	**X158**	-0.04370 (0.21420)	**X226**	1.05441 (0.02548)	**X291**	1.59204 (0.02210)	**X366**	5.58143 (0.02656)
X042	-4.90767 (0.17930)	**X100**	-15.70832 (0.61720)	**X159**	-0.12305 (0.17620)	**X227**	0.68692 (0.04568)	**X292**	0.91294 (0.01786)	**X368**	-0.27639 (0.01948)
X043	-2.88667 (0.03857)	**X104**	6.61063 (0.27570)	**X160**	15.11707 (0.62180)	**X228**	-8.02600 (0.40570)	**X293**	-1.15402 (0.03069)	**X369**	-3.17332 (0.04674)
X044	4.70044 (0.18270)	**X105**	3.11256 (0.15780)	**X161**	7.87908 (0.46670)	**X229**	-2.07427 (0.23300)	**X294**	-3.30574 (0.02041)	**X374**	0.95616 (0.06075)
X045	3.65553 (0.19020)	**X106**	-12.30700 (0.50140)	**X162**	-13.00517 (0.69050)	**X230**	16.17237 (0.72770)	**X295**	1.78380 (0.02148)	**X375**	0.53272 (0.15240)
X046	-8.43409 (0.44160)	**X107**	-4.77374 (0.45450)	**X163**	6.60935 (0.08159)	**X233**	8.53194 (0.05344)	**X298**	-1.88115 (0.01427)	**X380**	-0.90541 (0.04037)
X052	0.98170 (0.01090)	**X108**	11.67745 (0.69610)	**X164**	-7.56679 (0.42070)	**X234**	-2.11021 (0.11820)	**X299**	-0.25993 (0.01453)	**X382**	-1.62987 (0.04181)
X054	-0.48439 (0.00907)	**X109**	-0.00314 (0.05379)	**X167**	11.39373 (0.11060)	**X236**	2.18729 (0.11670)	**X300**	-9.24989 (0.12570)	**X383**	-4.70311 (0.01437)
X056	-0.23670 (0.00661)	**X110**	1.99153 (0.14100)	**X168**	3.31845 (0.06875)	**X237**	-13.47859 (0.44710)	**X301**	-1.05003 (0.07236)	**X384**	1.92295 (0.01935)
X057	2.62978 (0.01887)	**X111**	3.46085 (0.08017)	**X169**	-6.53674 (0.12350)	**X238**	1.41396 (0.04538)	**X302**	-2.75386 (0.25660)	**X385**	-0.69486 (0.01472)
X058	-5.14612 (0.04058)	**X112**	-12.63455 (0.54790)	**X170**	0.96093 (0.16880)	**X239**	-3.83715 (0.27470)	**X306**	0.17591 (0.02784)	**X386**	0.73425 (0.02575)
X059	2.18800 (0.02090)	**X113**	-4.34018 (0.31240)	**X171**	-3.26863 (0.33420)	**X240**	11.97385 (0.56570)	**X307**	-0.05337 (0.04651)	**X387**	1.68891 (0.10780)
X060	-3.48740 (0.10830)	**X114**	12.18633 (0.77740)	**X173**	0.47944 (0.16790)	**X241**	12.37148 (0.56080)	**X308**	1.83691 (0.01492)	**X388**	-0.38759 (0.02049)
X061	-3.76650 (0.09860)	**X124**	11.14372 (0.07170)	**X174**	2.01581 (0.11180)	**X242**	-10.69600 (0.39640)	**X309**	-4.03196 (0.08822)	**X389**	-0.26005 (0.06155)
X062	6.33536 (0.22350)	**X126**	-1.02468 (0.14100)	**X175**	-2.28728 (0.22500)	**X243**	3.73653 (0.42200)	**X310**	-0.20307 (0.11510)		
X063	3.33628 (0.01019)	**X130**	-1.23915 (0.10990)	**X176**	0.76582 (0.02062)	**X244**	-3.26093 (0.19300)	**X311**	-0.41601 (0.03111)		
X064	-0.27480 (0.00753)	**X131**	3.08498 (0.16830)	**X177**	-0.03351 (0.04017)	**X245**	0.10100 (0.03949)	**X312**	-1.07487 (0.05985)		
X065	0.17962 (0.00626)	**X132**	-9.27577 (0.39330)	**X181**	-4.33995 (0.01622)	**X246**	-0.80472 (0.11260)	**X315**	-1.28150 (0.02460)		

Application to other Sets

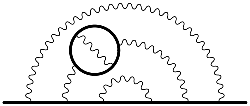
- Our automated code-generator for q-type diagrams can be extended by small modifications to diagrams with vacuum polarizations inserted.
- Numerical evaluations of those diagrams have been carried out, and the results are now being verified.

$$A_1^{(10)}(\text{Set III}(a)) = 2.126\,80\,(54)$$
$$A_1^{(10)}(\text{Set III}(b)) = 3.327\,43\,(70)$$

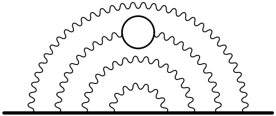
preliminary. do not quote until published.



Set III(a)



Set III(b)



Set IV

Summary

- A new subtraction scheme is developed for IR divergences of the amplitude of electron $g-2$. It is based only on diagramatic structure, and suitable for automated treatment.
- We have developed the automated code-generating system for numerical calculation of diagrams without lepton loops (q-type).
- It revealed an inconsistency in 8th-order correction. The revised $A_1^{(8)}$ term is obtained.
- Numerical integration of 10th-order q-type diagrams is now in production run. The preliminary result will be presented in near future.