Mass effects in 4-particle amplitudes at the 2-loop level of QCD

M. Czakon
Würzburg University

in collaboration with:

A.Mitov and S.Moch
G.Chachamis and D.Eiras

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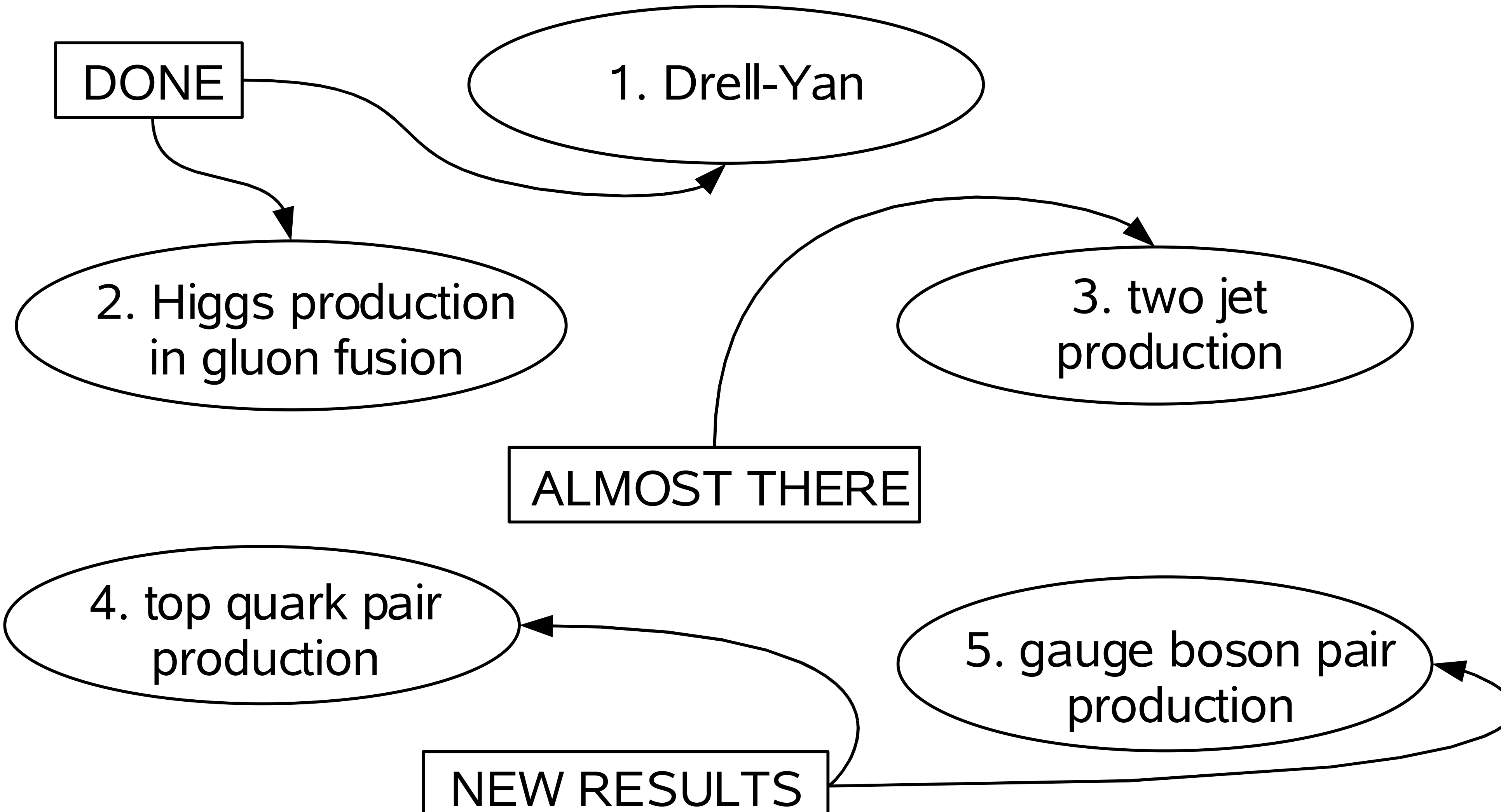
Motivation

- NNLO priority list:



Motivation

CMS study on tops

- What to expect from experiment:

8 million pairs per year

Channel	Selected events for $10 fb^{-1}$
$t\bar{t} \to WbWb \to l\nu bbjj$	70K
$t\bar{t} \to WbWb \to l\nu bbjj$ high P_T sample	3,6K
$t\bar{t} \to WbWb \to l\nu bl\nu b$	20K
$t\bar{t} \to WbWb \to jjbbjj$ high P_T sample	3,4K
Single top t channel	2,5K
Single top Wt channel	1,5K
Single top s channel	0,5K

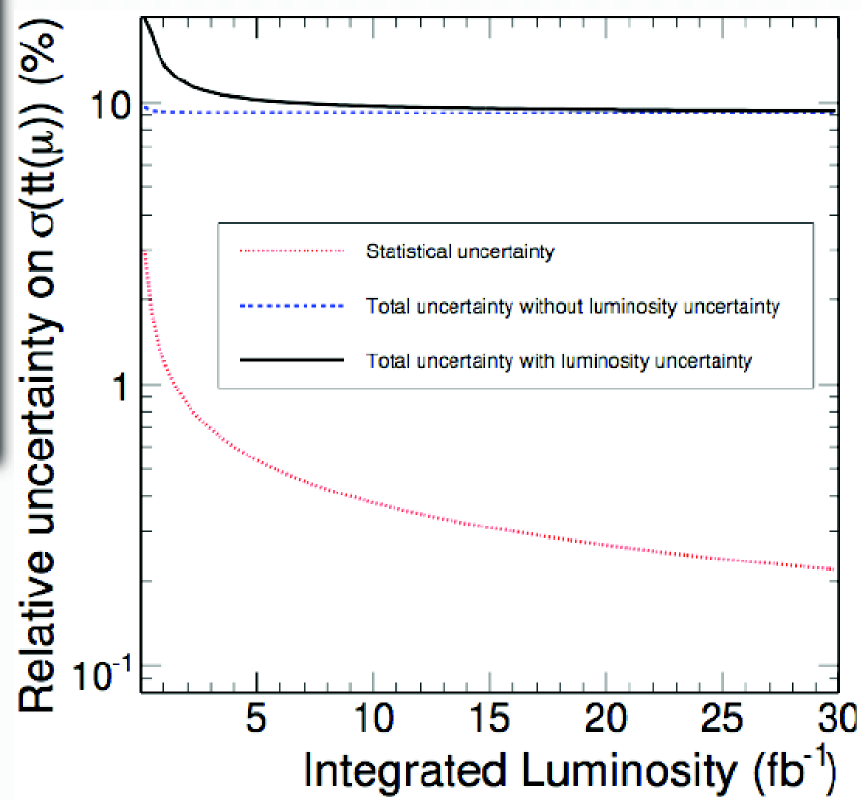
- A Top mass measurement with a precision of the order or below 1 GeV.
- A $\bar{t}t$ production cross section measurement with a precision below 10%.
- Tests of the Top production and decay mechanisms with W polarisations (Top spin correlation) at the level of 1-2% (3-5%).
- Studies on the $\bar{t}t$ invariant mass.

Motivation

	$\Delta\hat{\sigma}_{t\bar{t}(\mu)}/\hat{\sigma}_{t\bar{t}(\mu)}$ 1 fb^{-1}	5 fb^{-1}	10 fb^{-1}
Simulation samples (ϵ_{sim})		0.6%	
Simulation samples (F_{sim})		0.2%	
Pile-Up (30% On-Off)		3.2%	
Underlying Event		0.8%	
Jet Energy Scale (light quarks) (2%)		1.6%	
Jet Energy Scale (heavy quarks) (2%)		1.6%	
Radiation (Λ_{QCD}, Q_0^2)		2.6%	
Fragmentation (Lund b, σ_q)		1.0%	
b-tagging (5%)		7.0%	
Parton Density Functions		3.4%	
Background level		0.9%	
Integrated luminosity	10%	5%	3%
Statistical Uncertainty	1.2%	0.6%	0.4%
Total Systematic Uncertainty	13.6%	10.5%	9.7%
Total Uncertainty	13.7%	10.5%	9.7%

G.Dissertori
at Les Houches '07

CMS PTDR



Status quo of the theory

- Until now:
- NLO QCD corrections with expected precision ~10-15%
- NLO QCD corrections to tt + jet (S.Dittmaier, P.Uwer, S.Weinzierl '07, see P.Uwer's talk)

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$t = (p_1 - p_3)^2 - m_t^2$

- results for NNLO virtual corrections: (A.Mitov, S.Moch, M.C. '07)
- quark-antiquark annihilation (10% contribution to the X-section)
- gluon fusion (90% contribution to the X-section)
- both processes in the high energy limit

arxiv:0707.4139 [hep-ph] soon in NPB

$$\sum_{n=0}^{4} \log^n\left(\frac{m_t^2}{s}\right) f_n\left(-\frac{t}{s}\right)$$

- We computed color and spin averaged amplitudes (can be changed in the future)

- color decomposition for the annihilation channel

from factorization (A.Mitov, S.Moch '06, see S.Moch's talk)

$$2(N^2-1)\left(N^2 A + B + \frac{1}{N^2} C + N n_l D_l + N n_h D_h + \frac{n_l}{N} E_l + \frac{n_h}{N} E_h + (n_l + n_h)^2 F\right)$$

common

direct computation

- and the fusion channel

$$2(N^2-1)\left(N^3 A + N B + \frac{1}{N} C + \frac{1}{N^3} D + N^2 n_l E_l + N^2 n_h E_h + n_l F_l + n_h F_h \right.$$
$$\left. + \frac{n_l}{N^2} G_l + \frac{n_h}{N^2} G_h + N n_l^2 H_l + N n_h n_l H_{lh} + N n_h^2 H_h + \frac{n_l^2}{N} I_l + n_l \frac{n_h}{N} I_{lh} + \frac{n_h^2}{N} I_h\right)$$

- The leading color coefficient in quark annihilation

$$
\begin{aligned}
A \;=\;& \frac{1}{\varepsilon^4}\left\{\frac{x^2}{2}-\frac{x}{2}+\frac{1}{4}\right\}+\frac{1}{\varepsilon^3}\Big\{\mathrm{L}_m\left[x^2-x+\frac12\right]+\mathrm{L}_s\left[-x^2+x-\frac12\right]+\frac{21x^2}{4}-\frac{21x}{4}+\mathrm{L}_x(-2x^2\\
&+2x-1)+\frac{19}{8}\Big\}+\frac{1}{\varepsilon^2}\Big\{\mathrm{L}_m\mathrm{L}_s[-2x^2+2x-1]+\mathrm{L}_s^2\left[x^2-x+\frac12\right]+\mathrm{L}_m\Big[\frac{29x^2}{6}-\frac{29x}{6}\\
&+\mathrm{L}_x(-2x^2+2x-1)+\frac{23}{12}\Big]+\mathrm{L}_s\left[-\frac{19x^2}{6}+\frac{19x}{6}+\mathrm{L}_x(4x^2-4x+2)-\frac{13}{12}\right]+\Big(2x^2\\
&-\frac{5x}{2}+\frac54\Big)\mathrm{L}_x^2+\left(-\frac{26x^2}{3}+\frac{55x}{6}-\frac{23}{6}\right)\mathrm{L}_x+\frac{173x^2}{72}-\frac{173x}{72}+\pi^2\left(-\frac{x^2}{6}+\frac{x}{6}-\frac{1}{12}\right)\\
&-\frac{205}{144}\Big\}+\frac{1}{\varepsilon}\Big\{\mathrm{L}_m^3\left[-\frac{x^2}{3}+\frac{x}{3}-\frac16\right]+\mathrm{L}_m\mathrm{L}_s^2[2x^2-2x+1]+\mathrm{L}_s^3\left[-\frac{2x^2}{3}+\frac{2x}{3}-\frac13\right]\\
&+\mathrm{L}_m^2\left[-x^2+x+\mathrm{L}_x\left(x^2-x+\frac12\right)-\frac12\right]+\mathrm{L}_m\mathrm{L}_s\left[-\frac{7x^2}{3}+\frac{7x}{3}+\mathrm{L}_x(4x^2-4x+2)-\frac16\right]\\
&+\mathrm{L}_s^2\left[-\frac{x^2}{2}+\frac{x}{2}+\mathrm{L}_x(-4x^2+4x-2)-\frac34\right]+\mathrm{L}_m\Big[\left(\frac14-\frac{x}{2}\right)\mathrm{L}_x^2+\left(\frac{3x}{2}-x^2\right)\mathrm{L}_x-\frac{47x^2}{12}\\
&+\frac{47x}{12}-\frac{35}{8}\Big]+\mathrm{L}_s\Big[\left(-4x^2+5x-\frac52\right)\mathrm{L}_x^2+\left(\frac{8x^2}{3}-\frac{11x}{3}+\frac13\right)\mathrm{L}_x+\frac{487x^2}{36}-\frac{487x}{36}\\
&+\pi^2\left(\frac{x^2}{3}-\frac{x}{3}+\frac16\right)+\frac{601}{72}\Big]+\left(\frac{x^2}{3}+x-\frac12\right)\mathrm{L}_x^3+\left(-\frac{5x}{2}+\mathrm{L}_y\left(-x^2+\frac{x}{2}-\frac14\right)+\frac34\right)\mathrm{L}_x^2\\
&+\mathrm{Li}_2(x)\left(-2x^2+x-\frac12\right)\mathrm{L}_x+\left(\frac{43x^2}{3}-\frac{151x}{12}+\pi^2\left(\frac{4x^2}{3}-\frac{5x}{6}+\frac{5}{12}\right)+10\right)\mathrm{L}_x\\
&-\frac{9907x^2}{432}+\frac{9907x}{432}+\pi^2\left(-\frac{23x^2}{72}+\frac{5x}{72}+\frac{25}{144}\right)+\mathrm{Li}_3(x)\left(2x^2-x+\frac12\right)+\Big(-\frac{23x^2}{6}\\
&+\frac{17x}{6}-\frac{17}{12}\Big)\zeta_3-\frac{10945}{864}\Big\}+\mathrm{L}_m^4\left[\frac{x^2}{4}-\frac{x}{4}+\frac18\right]+\mathrm{L}_m^3\mathrm{L}_s\left[\frac{2x^2}{3}-\frac{2x}{3}+\frac13\right]+\mathrm{L}_m\mathrm{L}_s^3\Big[-\frac{4x^2}{3}\\
&+\frac{4x}{3}-\frac23\Big]+\mathrm{L}_s^4\left[\frac{x^2}{3}-\frac{x}{3}+\frac16\right]+\mathrm{L}_m^3\left[-\frac{11x^2}{18}+\frac{11x}{18}+\mathrm{L}_x\left(-\frac{x^2}{3}+\frac{x}{3}-\frac16\right)-\frac{5}{36}\right]\\
&+\mathrm{L}_m^2\mathrm{L}_s\left[-\frac{5x^2}{3}+\frac{5x}{3}+\mathrm{L}_x(-2x^2+2x-1)-\frac56\right]+\mathrm{L}_m\mathrm{L}_s^2\Big[-\frac{4x^2}{3}+\frac{4x}{3}+\mathrm{L}_x(-4x^2+4x-2)\\
&-\frac53\Big]+\mathrm{L}_s^3\left[\frac{14x^2}{9}-\frac{14x}{9}+\mathrm{L}_x\left(\frac{8x^2}{3}-\frac{8x}{3}+\frac43\right)+\frac{10}{9}\right]+\mathrm{L}_m^2\Big[\left(\frac{x}{4}-\frac18\right)\mathrm{L}_x^2+\left(\frac{x^2}{2}-\frac{3x}{4}\right)\mathrm{L}_x\\
&+\frac{247x^2}{36}-\frac{247x}{36}+\frac{283}{72}\Big]+\mathrm{L}_m\mathrm{L}_s\Big[\left(x-\frac12\right)\mathrm{L}_x^2+(2x^2-3x)\mathrm{L}_x+\frac{23x^2}{2}-\frac{23x}{2}+\frac{83}{12}\Big]\\
&+\mathrm{L}_s^2\Big[\left(4x^2-5x+\frac52\right)\mathrm{L}_x^2+\left(\frac{14x^2}{3}-\frac{11x}{3}+\frac{10}{3}\right)\mathrm{L}_x-\frac{37x^2}{4}+\frac{37x}{4}+\pi^2\left(-\frac{x^2}{3}+\frac{x}{3}-\frac16\right)\\
&-\frac{35}{8}\Big]+\mathrm{L}_m\Big[\frac{x^2\mathrm{L}_x^3}{3}+\left(-\frac{x}{2}+\mathrm{L}_y\left(-x^2+\frac{x}{2}-\frac14\right)+\frac14\right)\mathrm{L}_x^2+\mathrm{Li}_2(x)\left(-2x^2+x-\frac12\right)\mathrm{L}_x\\
&+\left(-4x^2+\frac{19x}{4}+\pi^2\left(x^2-\frac{x}{2}+\frac14\right)-2\right)\mathrm{L}_x-\frac{781x^2}{72}+\frac{781x}{72}+\pi^2\left(-\frac{7x^2}{12}+\frac{x}{3}-\frac{1}{24}\right)\\
&+\mathrm{Li}_3(x)\left(2x^2-x+\frac12\right)+\left(\frac{7x^2}{3}-\frac{10x}{3}+\frac53\right)\zeta_3-\frac{499}{144}\Big]+\mathrm{L}_s\Big[\left(-\frac{2x^2}{3}-2x+1\right)\mathrm{L}_x^3\\
&+\left(\frac{4x}{3}+\mathrm{L}_y\left(2x^2-x+\frac12\right)+\frac13\right)\mathrm{L}_x^2+\mathrm{Li}_2(x)(4x^2-2x+1)\mathrm{L}_x+\Big(-\frac{86x^2}{3}+\frac{173x}{6}\\
&+\pi^2\left(-\frac{8x^2}{3}+\frac{5x}{3}-\frac56\right)-\frac{49}{3}\Big)\mathrm{L}_x+\frac{2003x^2}{216}-\frac{2003x}{216}+\mathrm{Li}_3(x)(-4x^2+2x-1)\\
&+\pi^2\left(-\frac{43x^2}{36}+\frac{61x}{36}-\frac{91}{72}\right)+\left(\frac{23x^2}{3}-\frac{17x}{3}+\frac{17}{6}\right)\zeta_3-\frac{919}{432}\Big]+\left(-x^2-\frac{x}{24}+\frac{1}{48}\right)\mathrm{L}_x^4\\
&+\left(-\frac{7x^2}{18}+\frac{13x}{12}+\mathrm{L}_y\left(\frac{10x^2}{3}-2x+1\right)-\frac{7}{12}\right)\mathrm{L}_x^3+\Big(\Big(\left(-\frac{x^2}{2}-\frac{5x}{4}+\frac{11}{8}\right)\mathrm{L}_y^2+\Big(\frac{7x^2}{6}\\
&+\frac{13x}{12}+\frac{13}{12}\Big)\mathrm{L}_y+\frac{101x}{72}+\pi^2\left(-\frac{25x^2}{6}+\frac{25x}{12}-\frac{25}{24}\right)+\frac{3}{1-x}-\frac{617}{144}\Big)\mathrm{L}_x^2+\mathrm{S}_{1,2}(x)\Big(-2x^2\\
&-5x+\frac{11}{2}\Big)\mathrm{L}_x+\Big(\frac{260x^2}{9}-\frac{139x}{4}+\pi^2\left(-\frac{25x^2}{9}+\frac{53x}{36}-\frac{47}{18}\right)+\mathrm{L}_y\pi^2\left(-\frac{x^2}{3}+\frac{11x}{6}-\frac{17}{12}\right)\\
&+\left(\frac{52x^2}{3}-\frac{31x}{3}+\frac{31}{6}\right)\zeta_3+\frac{1027}{72}\Big)\mathrm{L}_x+\frac{17845x^2}{2592}-\frac{17845x}{2592}+\pi^4\left(-\frac{11x^2}{80}-\frac{x}{240}+\frac{1}{480}\right)\\
&+\mathrm{Li}_4(x)\left(2x^2+3x-\frac32\right)+\mathrm{S}_{2,2}(x)\left(2x^2+5x-\frac{11}{2}\right)+\pi^2\left(\frac{1009x^2}{432}-\frac{259x}{432}+\frac{403}{864}\right)\\
&+\mathrm{Li}_3(x)\left(-\frac{7x^2}{3}-\frac{13x}{6}+\mathrm{L}_x(-8x^2+2x-1)+\mathrm{L}_y\left(2x^2+5x-\frac{11}{2}\right)-\frac{13}{6}\right)+\mathrm{Li}_2(x)\Big(\Big(7x^2\\
&-\frac{7x}{2}+\frac74\Big)\mathrm{L}_x^2+\left(\frac{7x^2}{3}+\frac{13x}{6}+\mathrm{L}_y\left(-2x^2-5x+\frac{11}{2}\right)+\frac{13}{6}\right)\mathrm{L}_x+\pi^2\left(-\frac{x^2}{3}+\frac{11x}{6}-\frac{17}{12}\right)\Big)\\
&+\mathrm{L}_y\left(-2x^2-5x+\frac{11}{2}\right)\zeta_3+\left(-\frac{14x^2}{9}+\frac{55x}{18}+\frac{149}{36}\right)\zeta_3+\frac{77287}{5184},
\end{aligned}
$$

- Functional basis

$\log(ms), \log(x), \log(1-x), \mathrm{Li}_2(x), \mathrm{Li}_3(x), S_{12}(x), \mathrm{Li}_4(x), S_{22}(x), S_{13}(x)$

- The leading color coefficient in gluon fusion

$$
\begin{aligned}
A = {} & \frac{1}{\varepsilon^4}\Big\{-8x^2+8x+\frac{2}{1-x}-8+\frac{2}{x}\Big\}+\frac{1}{\varepsilon^3}\Big\{L_m\Big[-8x^2+8x+\frac{2}{1-x}-8+\frac{2}{x}\Big]+L_s\Big[16x^2-16x-\frac{4}{1-x} \\
& +16-\frac{4}{x}\Big]-\frac{154x^2}{3}+\frac{154x}{3}+L_y\Big(8x^2-\frac{4}{1-x}+4\Big)+L_x\Big(8x^2-16x+12-\frac{4}{x}\Big)+\frac{65}{6(1-x)}-\frac{142}{3} \\
& +\frac{65}{6x}\Big\}+\frac{1}{\varepsilon^2}\Big\{L_m^2\Big[2x^2-2x-\frac{1}{2(1-x)}+2-\frac{1}{2x}\Big]+L_mL_s\Big[16x^2-16x-\frac{4}{1-x}+16-\frac{4}{x}\Big] \\
& +L_s^2\Big[-16x^2+16x+\frac{4}{1-x}-16+\frac{4}{x}\Big]+L_m\Big[-22x^2+22x+L_y\Big(4x^2-\frac{2}{1-x}+2\Big) \\
& +L_x\Big(4x^2-8x+6-\frac{2}{x}\Big)+\frac{7}{2(1-x)}-18+\frac{7}{2x}\Big]+L_s\Big[44x^2-44x+L_y\Big(-16x^2+\frac{8}{1-x}-8\Big) \\
& +L_x\Big(-16x^2+32x-24+\frac{8}{x}\Big)-\frac{7}{1-x}+36-\frac{7}{x}\Big]+\Big(-4x^2+12x-9+\frac{3}{x}\Big)L_x^2 \\
& +\Big(18x^2-41x+31-\frac{8}{x}\Big)L_x-\frac{104x^2}{9}+\frac{104x}{9}+L_y^2\Big(-4x^2-4x+\frac{3}{1-x}-1\Big) \\
& +\pi^2\Big(8x^2-8x-\frac{2}{1-x}+8-\frac{2}{x}\Big)+L_y\Big(18x^2+5x-\frac{8}{1-x}+8\Big)-\frac{161}{18(1-x)}-\frac{161}{18x}+\frac{100}{9}\Big\} \\
& +\frac{1}{\varepsilon}\Big\{L_m^3\Big[\frac{2x^2}{3}-\frac{2x}{3}-\frac{1}{6(1-x)}+\frac{2}{3}-\frac{1}{6x}\Big]+L_m^2L_s\Big[-4x^2+4x+\frac{1}{1-x}-4+\frac{1}{x}\Big]+L_mL_s^2\Big[-16x^2 \\
& +16x+\frac{4}{1-x}-16+\frac{4}{x}\Big]+L_s^3\Big[\frac{32x^2}{3}-\frac{32x}{3}-\frac{8}{3(1-x)}+\frac{32}{3}-\frac{8}{3x}\Big]+L_m^2\Big[\frac{22x^2}{3}-\frac{22x}{3} \\
& +L_y\Big(-2x^2+\frac{1}{1-x}-1\Big)+L_x\Big(-2x^2+4x-3+\frac{1}{x}\Big)-\frac{4}{3(1-x)}+\frac{19}{3}-\frac{4}{3x}\Big]+L_mL_s\Big[\frac{44x^2}{3}-\frac{44x}{3} \\
& +L_y\Big(-8x^2+\frac{4}{1-x}-4\Big)+L_x\Big(-8x^2+16x-12+\frac{4}{x}\Big)+\frac{1}{3(1-x)}+\frac{20}{3}+\frac{1}{3x}\Big]+L_s^2\Big[-\frac{44x^2}{3} \\
& +\frac{44x}{3}+L_y\Big(16x^2-\frac{8}{1-x}+8\Big)+L_x\Big(16x^2-32x+24-\frac{8}{x}\Big)-\frac{1}{3(1-x)}-\frac{20}{3}-\frac{1}{3x}\Big] \\
& +L_m\Big[\Big(2x-\frac{3}{2}+\frac{1}{2x}\Big)L_x^2+\Big(-2x^2+\frac{3x}{2}-1+\frac{3}{2x}\Big)L_x+\frac{140x^2}{9}+L_y^2\Big(-2x+\frac{1}{2(1-x)}+\frac{1}{2}\Big) \\
& -\frac{140x}{9}+L_y\Big(-2x^2+\frac{5x}{2}+\frac{3}{2(1-x)}-\frac{3}{2}\Big)+\pi^2\Big(2x^2-2x-\frac{1}{2(1-x)}+2-\frac{1}{2x}\Big)-\frac{80}{9(1-x)}-\frac{80}{9x} \\
& +\frac{451}{18}\Big]+L_s\Big[\Big(8x^2-24x+18-\frac{6}{x}\Big)L_x^2+\Big(-\frac{20x^2}{3}+\frac{70x}{3}-18+\frac{4}{3x}\Big)L_x-\frac{496x^2}{9}+\frac{496x}{9} \\
& +\pi^2\Big(-16x^2+16x+\frac{4}{1-x}-16+\frac{4}{x}\Big)+L_y\Big(-\frac{20x^2}{3}-10x+\frac{4}{3(1-x)}-\frac{4}{3}\Big) \\
& +L_y^2\Big(8x^2+8x-\frac{6}{1-x}+2\Big)+\frac{205}{9(1-x)}+\frac{205}{9x}-\frac{640}{9}\Big]+\Big(-\frac{4x^2}{3}-4x+3-\frac{1}{x}\Big)L_x^3 \\
& +\Big(4x^2+\frac{29x}{6}+L_y\Big(4x^2-4x+3-\frac{1}{x}\Big)-10+\frac{5}{6x}\Big)L_x^2+\mathrm{Li}_2(x)\Big(8x^2-8x+6-\frac{2}{x}\Big)L_x \\
& +\Big(-\frac{332x^2}{9}+\frac{514x}{9}+\pi^2\Big(-\frac{34x^2}{3}+\frac{44x}{3}-11+\frac{11}{3x}\Big)-\frac{116}{3}+\frac{166}{9x}\Big)L_x+\frac{2584x^2}{27}-\frac{2584x}{27} \\
& +\mathrm{Li}_3(x)\Big(-8x^2+8x-6+\frac{2}{x}\Big)+L_y^3\Big(-\frac{4x^2}{3}+\frac{20x}{3}-\frac{1}{1-x}-\frac{7}{3}\Big)+L_y^2\Big(4x^2-\frac{77x}{6}+\frac{5}{6(1-x)}-\frac{7}{6}\Big) \\
& +S_{1,2}(x)\Big(8x^2-8x-\frac{2}{1-x}+6\Big)+\pi^2\Big(19x^2-19x+\frac{1}{4(1-x)}+4+\frac{1}{4x}\Big)+L_y\Big(-\frac{332x^2}{9}+\frac{50x}{3} \\
& +\pi^2\Big(-\frac{34x^2}{3}+8x+\frac{11}{3(1-x)}-\frac{23}{3}\Big)+\frac{166}{9(1-x)}-\frac{166}{9}\Big)+\Big(\frac{184x^2}{3}-\frac{184x}{3}-\frac{40}{3(1-x)}+\frac{178}{3} \\
& -\frac{46}{3x}\Big)\zeta_3-\frac{625}{27(1-x)}-\frac{625}{27x}+\frac{2416}{27}\Big\}+L_m^4\Big[-\frac{5x^2}{6}+\frac{5x}{6}+\frac{5}{24(1-x)}-\frac{5}{6}+\frac{5}{24x}\Big] \\
& +L_m^3L_s\Big[-\frac{4x^2}{3}+\frac{4x}{3}+\frac{1}{3(1-x)}-\frac{4}{3}+\frac{1}{3x}\Big]+L_m^2L_s^2\Big[4x^2-4x-\frac{1}{1-x}+4-\frac{1}{x}\Big]
\end{aligned}
$$

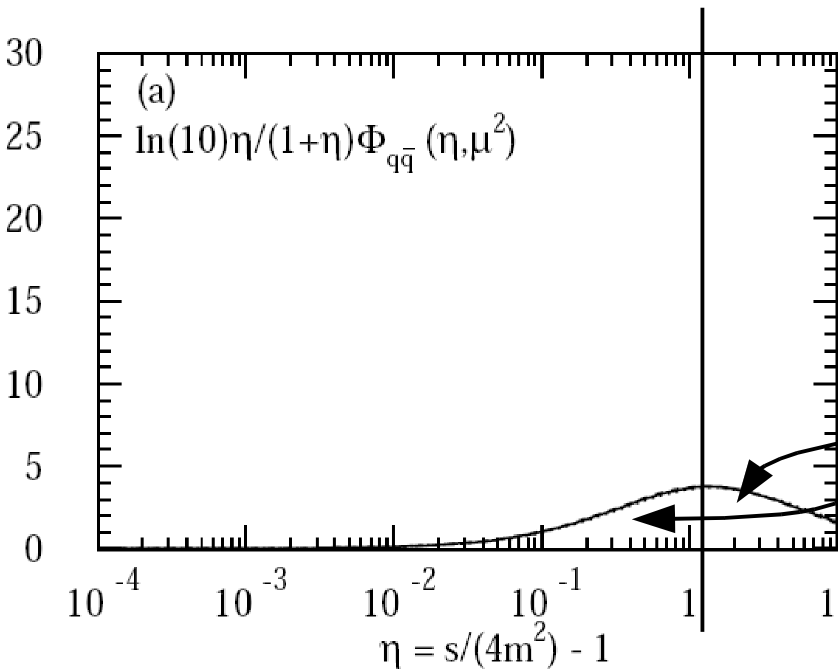
$$
\begin{aligned}
& +L_mL_s^3\Big[\frac{32x^2}{3}-\frac{32x}{3}-\frac{8}{3(1-x)}+\frac{32}{3}-\frac{8}{3x}\Big]+L_s^4\Big[-\frac{16x^2}{3}+\frac{16x}{3}+\frac{4}{3(1-x)}-\frac{16}{3}+\frac{4}{3x}\Big] \\
& +L_m^3\Big[L_y\Big(\frac{2x^2}{3}-\frac{1}{3(1-x)}+\frac{1}{3}\Big)+L_x\Big(\frac{2x^2}{3}-\frac{4x}{3}+1-\frac{1}{3x}\Big)+\frac{1}{6(1-x)}+\frac{1}{6x}-\frac{1}{3}\Big] \\
& +L_m^2L_s\Big[L_y\Big(4x^2-\frac{2}{1-x}+2\Big)+L_x\Big(4x^2-8x+6-\frac{2}{x}\Big)-\frac{1}{1-x}-\frac{1}{x}+2\Big] \\
& +L_mL_s^2\Big[L_y\Big(8x^2-\frac{4}{1-x}+4\Big)+L_x\Big(8x^2-16x+12-\frac{4}{x}\Big)-\frac{4}{1-x}-\frac{4}{x}+8\Big] \\
& +L_s^3\Big[L_y\Big(-\frac{32x^2}{3}+\frac{16}{3(1-x)}-\frac{16}{3}\Big)+L_x\Big(-\frac{32x^2}{3}+\frac{64x}{3}-16+\frac{16}{3x}\Big)+\frac{8}{3(1-x)}+\frac{8}{3x}-\frac{16}{3}\Big] \\
& +L_m^2\Big[\Big(-x+\frac{3}{4}-\frac{1}{4x}\Big)L_x^2+\Big(x^2-\frac{3x}{4}+\frac{1}{2}-\frac{3}{4x}\Big)L_x-\frac{391x^2}{18}+\frac{391x}{18}+L_y^2\Big(x-\frac{1}{4(1-x)}-\frac{1}{4}\Big) \\
& +\pi^2\Big(-x^2+x+\frac{1}{4(1-x)}-1+\frac{1}{4x}\Big)+L_y\Big(x^2-\frac{5x}{4}-\frac{3}{4(1-x)}+\frac{3}{4}\Big)+\frac{505}{72(1-x)}+\frac{505}{72x}-\frac{887}{36}\Big] \\
& +L_mL_s\Big[\Big(-4x+3-\frac{1}{x}\Big)L_x^2+\Big(4x^2-3x+2-\frac{3}{x}\Big)L_x-\frac{148x^2}{9}+\frac{148x}{9}+L_y^2\Big(4x-\frac{1}{1-x}-1\Big) \\
& +\pi^2\Big(-4x^2+4x+\frac{1}{1-x}-4+\frac{1}{x}\Big)+L_y\Big(4x^2-5x-\frac{3}{1-x}+3\Big)+\frac{61}{9(1-x)}+\frac{61}{9x}-\frac{187}{9}\Big] \\
& +L_s^2\Big[\Big(-8x^2+24x-18+\frac{6}{x}\Big)L_x^2+\Big(-8x^2+6x-4+\frac{6}{x}\Big)L_x+\frac{364x^2}{9}-\frac{364x}{9} \\
& +L_y^2\Big(-8x^2-8x+\frac{6}{1-x}-2\Big)+L_y\Big(-8x^2+10x+\frac{6}{1-x}-6\Big)+\pi^2\Big(16x^2-16x-\frac{4}{1-x}+16-\frac{4}{x}\Big) \\
& -\frac{106}{9(1-x)}-\frac{106}{9x}+\frac{376}{9}\Big]+L_m\Big[-\frac{2}{3}x^2L_x^3+\Big(\frac{x}{4}+L_y\Big(2x^2-2x+\frac{3}{2}-\frac{1}{2x}\Big)-\frac{9}{4}\Big)L_x^2 \\
& +\mathrm{Li}_2(x)\Big(4x^2-4x+3-\frac{1}{x}\Big)L_x+\Big(6x^2-\frac{61x}{4}+\pi^2\Big(-2x^2+2x-\frac{3}{2}+\frac{1}{2x}\Big)+\frac{29}{2}-\frac{21}{4x}\Big)L_x-\frac{1165x^2}{54} \\
& +L_y^2\Big(-\frac{x}{4}-2\Big)+\frac{1165x}{54}+\mathrm{Li}_3(x)\Big(-4x^2+4x-3+\frac{1}{x}\Big)+L_y^3\Big(-\frac{2x^2}{3}+\frac{4x}{3}-\frac{2}{3}\Big) \\
& +S_{1,2}(x)\Big(4x^2-4x-\frac{1}{1-x}+3\Big)+\pi^2\Big(\frac{17x^2}{3}-\frac{17x}{3}-\frac{1}{6(1-x)}+\frac{23}{12}-\frac{1}{6x}\Big)+L_y\Big(6x^2+\frac{13x}{4} \\
& +\pi^2\Big(-2x^2+2x+\frac{1}{2(1-x)}-\frac{3}{2}\Big)-\frac{21}{4(1-x)}+\frac{21}{4}\Big)+\Big(-\frac{8x^2}{3}+\frac{8x}{3}+\frac{5}{3(1-x)}-\frac{11}{3}+\frac{2}{3x}\Big)\zeta_3 \\
& +\frac{2203}{216(1-x)}+\frac{2203}{216x}-\frac{3521}{108}\Big]+L_s\Big[\Big(\frac{8x^2}{3}+8x-6+\frac{2}{x}\Big)L_x^3+\Big(-8x^2+5x \\
& +L_y\Big(-8x^2+8x-6+\frac{2}{x}\Big)+9+\frac{2}{x}\Big)L_x^2+\mathrm{Li}_2(x)\Big(-16x^2+16x-12+\frac{4}{x}\Big)L_x+\Big(\frac{400x^2}{9}-\frac{665x}{9} \\
& +\pi^2\Big(\frac{68x^2}{3}-\frac{88x}{3}+22-\frac{22}{3x}\Big)+48-\frac{167}{9x}\Big)L_x-\frac{1040x^2}{27}+\frac{1040x}{27} \\
& +S_{1,2}(x)\Big(-16x^2+16x+\frac{4}{1-x}-12\Big)+\pi^2\Big(-\frac{122x^2}{9}+\frac{122x}{9}-\frac{119}{18(1-x)}+\frac{148}{9}-\frac{119}{18x}\Big) \\
& +L_y^2\Big(-8x^2+11x+\frac{2}{1-x}+6\Big)+L_y^3\Big(\frac{8x^2}{3}-\frac{40x}{3}+\frac{2}{1-x}+\frac{14}{3}\Big)+\mathrm{Li}_3(x)\Big(16x^2-16x+12-\frac{4}{x}\Big)
\end{aligned}
$$

$$
\begin{aligned}
& +L_y\Big(\frac{400x^2}{9}-15x+\pi^2\Big(\frac{68x^2}{3}-16x-\frac{22}{3(1-x)}+\frac{46}{3}\Big)-\frac{167}{9(1-x)}+\frac{167}{9}\Big) \\
& +\Big(-\frac{368x^2}{3}+\frac{368x}{3}+\frac{80}{3(1-x)}-\frac{356}{3}+\frac{92}{3x}\Big)\zeta_3-\frac{211}{27(1-x)}-\frac{211}{27x}+\frac{55}{27}\Big] \\
& +\Big(\frac{13x^2}{6}+\frac{x}{6}-\frac{1}{8}+\frac{1}{24x}\Big)L_x^4+\Big(\frac{2x^2}{3}-\frac{17x}{12}+L_y\Big(-\frac{22x^2}{3}+8x-6+\frac{2}{x}\Big)+\frac{26}{3}-\frac{3}{4x}\Big)L_x^3 \\
& +\Big(\Big(2x^2+2x-\frac{15}{2}+\frac{9}{2x}\Big)L_y^2+\Big(-2x^2-\frac{59x}{12}+1+\frac{31}{12x}\Big)L_y-\frac{265x}{72}+\pi^2\Big(12x^2-\frac{38x}{3}+\frac{19}{2}-\frac{19}{6x}\Big) \\
& -\frac{5}{1-x}-\frac{91}{72x}-\frac{17}{12}\Big)L_x^2+\Big(-\frac{262x^2}{27}+\frac{3379x}{216}+L_y\pi^2\Big(-\frac{20x^2}{3}+\frac{8x}{3}+2-\frac{2}{x}\Big) \\
& +\pi^2\Big(-\frac{17x^2}{9}+4x+10-\frac{16}{3x}\Big)+\Big(-\frac{172x^2}{3}+\frac{272x}{3}-68+\frac{68}{3x}\Big)\zeta_3-\frac{97}{18}-\frac{2279}{216x}\Big)L_x-\frac{1246x^2}{81} \\
& +S_{2,2}(x)\Big(-32x+\frac{18}{1-x}+16-\frac{18}{x}\Big)+\frac{1246x}{81}+\pi^2\Big(-\frac{422x^2}{27}+\frac{422x}{27}+\frac{589}{54(1-x)}-\frac{3581}{108}+\frac{589}{54x}\Big) \\
& +S_{1,3}(x)\Big(-4x^2-4x+\frac{3}{1-x}-1\Big)+\pi^4\Big(\frac{17x^2}{30}+\frac{7x}{30}-\frac{173}{360(1-x)}+\frac{121}{180}-\frac{23}{360x}\Big) \\
& +L_y^3\Big(\frac{2x^2}{3}+\frac{x}{12}-\frac{3}{4(1-x)}+\frac{95}{12}\Big)+L_y^4\Big(\frac{13x^2}{6}-\frac{9x}{2}+\frac{1}{24(1-x)}+\frac{53}{24}\Big)+\mathrm{Li}_4(x)\Big(4x^2-12x+9-\frac{3}{x}\Big) \\
& +S_{1,2}(x)\Big(-4x^2+\frac{107x}{6}+L_y\Big(-16x^2+12x+\frac{5}{1-x}-11\Big)+L_x\Big(8x^2+8x-30+\frac{18}{x}\Big)+\frac{31}{6(1-x)} \\
& -\frac{71}{6}\Big)+L_y^2\Big(\frac{265x}{72}+\pi^2\Big(12x^2-\frac{34x}{3}-\frac{19}{6(1-x)}+\frac{53}{6}\Big)-\frac{91}{72(1-x)}-\frac{367}{72}-\frac{5}{x}\Big) \\
& +\mathrm{Li}_3(x)\Big(4x^2+\frac{59x}{6}+L_y\Big(-8x^2-8x+30-\frac{18}{x}\Big)+L_x\Big(12x^2-8x+6-\frac{2}{x}\Big)-2-\frac{31}{6x}\Big) \\
& +\mathrm{Li}_2(x)\Big(\Big(\Big(-14x^2+14x-\frac{21}{2}+\frac{7}{2x}\Big)L_x^2+\Big(-4x^2-\frac{59x}{6}+L_y\Big(8x^2+8x-30+\frac{18}{x}\Big)+2+\frac{31}{6x}\Big)L_x \\
& +\pi^2\Big(-8x+\frac{2}{1-x}+4-\frac{2}{x}\Big)\Big)\Big)+\Big(52x^2-\frac{395x}{6}+\frac{23}{6(1-x)}+\frac{89}{6}+\frac{9}{x}\Big)\zeta_3+L_y\Big(-\frac{262x^2}{27}+\frac{271x}{72} \\
& +\pi^2\Big(-\frac{17x^2}{9}-\frac{2x}{9}-\frac{16}{3(1-x)}+\frac{109}{9}\Big)+\Big(-\frac{100x^2}{3}+20x+\frac{53}{3(1-x)}-\frac{161}{3}+\frac{18}{x}\Big)\zeta_3 \\
& -\frac{2279}{216(1-x)}+\frac{119}{216}\Big)+\frac{16313}{648(1-x)}+\frac{16313}{648x}-\frac{39113}{648},
\end{aligned}
$$

- Functional basis

$$\log(ms), \log(x), \log(1-x), \mathrm{Li}_2(x), \mathrm{Li}_3(x), S_{12}(x), \mathrm{Li}_4(x), S_{22}(x), S_{13}(x)$$

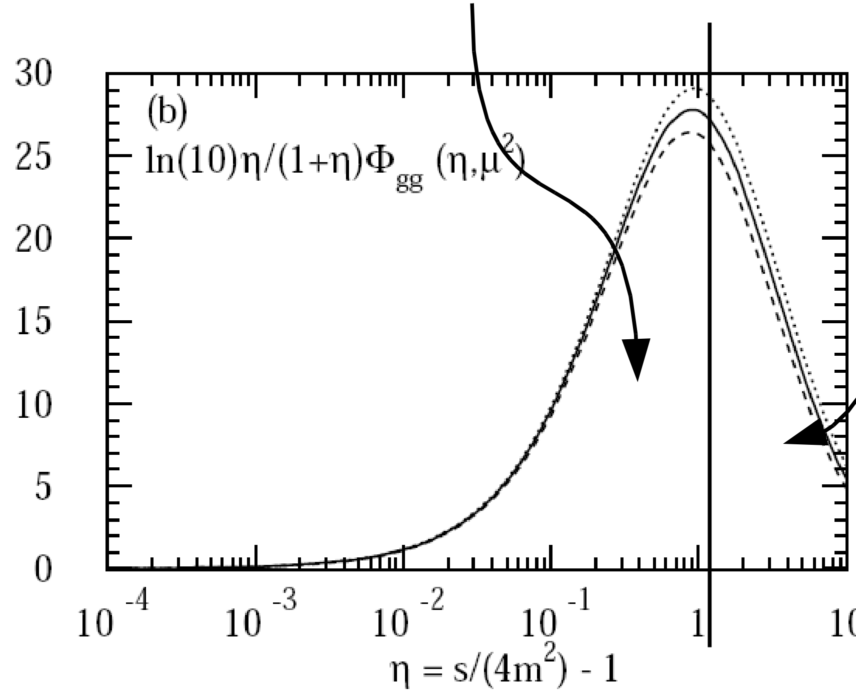
Particle fluxes at the LHC



convergent high-energy expansion

in $ms \equiv \frac{m_t^2}{s}$?

what about the rest of events ?



what about the angular variation ?

$$x \equiv -\frac{t}{s} \in \left[1/2(1-\beta), 1/2(1+\beta)\right]$$

$$\beta \equiv \sqrt{1-\frac{4m_t^2}{s}}$$

Direct computation

- Statistics
- annihilation channel

  - 190 diagrams expressed through 2812 integrals

  - 145 master integrals in the full result

  - 69 master integrals needed in the present calculation
- fusion channel

  - 726 diagrams expressed through 8676 integrals

  - 422 master integrals in the full result

  - 174 master integrals needed in the present calculation

Direct computation

- Reduction to masters as a first step (S.Laporta '00)
- The real problem is how to compute the masters
- We know the functions in the solution after expansion, but that **doesn't help much**
- We even have an idea of the functions in the full result, but that **doesn't help much either**
- One of the methods to proceed:

Mellin-Barnes representations

Steps

- construct representations (**MBrepresentation.m,** G.Chachamis, M.C.)
- perform an analytic continuation in ε to the vicinity of 0 (**MB.m,** M.C.)
- expand in the mass by recursively closing the contours in the multifold integrals (**MBasymptotics,** M.C.)
- perform as much of the integrations with the help of the Barnes lemmas
- resum the remaining integrals with non-trivial kinematic dependence by transforming into harmonic series (**XSummer,** S.Moch, P.Uwer)
- resum the remaining constants by high-precision numerical evaluation (~ 80 digits) and subsequent fit to a transcendental basis (**PSLQ,** D.Bailey)
- as a last resort expand in x and resum by fitting to a basis

Non-planar graphs

- on-shell graphs have no Euclidean domain of definition

- need to extend the integral by using U as an independent parameter to have a properly defined MB representation

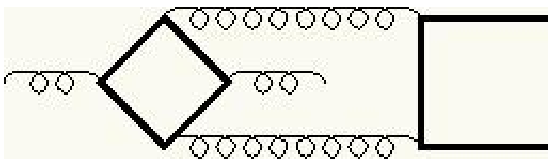
- loop-by-loop integration produces more compact representations in the case of massive integrals, but the expansion falls back on the massless case

- the massless loop-by-loop is six-dimensional and the U parameter regulates part of the divergence

$$A\frac{1}{\epsilon^4}+B\frac{1}{\epsilon^3}\log(S+T+U)+...$$

- necessary to construct representations by directly integrating the two-loop Feynman parameter integral

- Example non-planar integral



$$-\frac{\pi^2\,(\pi^2 + \mathrm{IPi}\,H[1,\ x] - 4\,\mathrm{IPi}\,\mathrm{Log}[2] + \mathrm{IPi}\,\mathrm{Log}[\mathrm{ms}] - \mathrm{IPi}\,\mathrm{Log}[x])}{\sqrt{\mathrm{ms}}\ \sqrt{(1-x)\,x}}$$

5 masters

square root singularities

scalar:

$$-\frac{101\,\pi^4}{90\,(1-x)} + \frac{64}{(1-x)\,x} + \frac{4\,\pi^2}{(1-x)\,x} + \frac{47\,\pi^4}{180\,(1-x)\,x} - \frac{48\,H[1,\ x]}{1-x} + \frac{14\,\pi^2\,H[1,\ x]}{3\,(1-x)} + \frac{16\,H[1,\ x]}{(1-x)\,x} - \frac{6\,\pi^2\,H[1,\ x]}{(1-x)\,x} + \frac{28\,\pi^2\,H[0,\ 1,\ x]}{3\,(1-x)} - \frac{14\,\pi^2\,H[0,\ 1,\ x]}{3\,(1-x)\,x} + \frac{16\,H[1,\ 1,\ x]}{1-x} +$$

$$\frac{14\,\pi^2\,H[1,\ 1,\ x]}{3\,(1-x)} - \frac{2\,\pi^2\,H[1,\ 1,\ x]}{(1-x)\,x} + \frac{4\,H[0,\ 0,\ 1,\ x]}{1-x} - \frac{16\,H[0,\ 0,\ 1,\ x]}{(1-x)\,x} + \frac{4\,H[0,\ 1,\ 1,\ x]}{1-x} + \frac{12\,H[0,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{4\,H[1,\ 1,\ 1,\ x]}{1-x} - \frac{12\,H[1,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{36\,H[0,\ 0,\ 0,\ 1,\ x]}{1-x} -$$

$$\frac{24\,H[0,\ 0,\ 0,\ 1,\ x]}{(1-x)\,x} - \frac{16\,H[0,\ 0,\ 1,\ 1,\ x]}{1-x} + \frac{8\,H[0,\ 0,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{12\,H[0,\ 1,\ 0,\ 1,\ x]}{1-x} - \frac{6\,H[0,\ 1,\ 0,\ 1,\ x]}{(1-x)\,x} + \frac{12\,H[1,\ 0,\ 0,\ 1,\ x]}{1-x} - \frac{6\,H[1,\ 0,\ 0,\ 1,\ x]}{(1-x)\,x} - \frac{12\,H[1,\ 0,\ 1,\ 1,\ x]}{1-x} +$$

$$\frac{4\,H[1,\ 0,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{8\,H[1,\ 1,\ 1,\ 1,\ x]}{1-x} - \frac{10\,H[1,\ 1,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{7\,\mathrm{Log}[\mathrm{ms}]^4}{12\,(1-x)\,x} - \frac{48\,\mathrm{Log}[x]}{1-x} + \frac{14\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)} + \frac{32\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{4\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)\,x} - \frac{14\,\pi^2\,H[1,\ x]\,\mathrm{Log}[x]}{3\,(1-x)} +$$

$$\frac{16\,H[1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{11\,\pi^2\,H[1,\ x]\,\mathrm{Log}[x]}{3\,(1-x)\,x} - \frac{4\,H[0,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} + \frac{16\,H[0,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{12\,H[1,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{24\,H[0,\ 0,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} + \frac{16\,H[0,\ 0,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} +$$

$$\frac{4\,H[0,\ 1,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} - \frac{2\,H[0,\ 1,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{12\,H[1,\ 0,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} + \frac{6\,H[1,\ 0,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{8\,H[1,\ 1,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} - \frac{4\,H[1,\ 1,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{8\,\mathrm{Log}[x]^2}{1-x} -$$

$$\frac{7\,\pi^2\,\mathrm{Log}[x]^2}{3\,(1-x)} + \frac{8\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{4\,\pi^2\,\mathrm{Log}[x]^2}{3\,(1-x)\,x} + \frac{2\,H[1,\ x]\,\mathrm{Log}[x]^2}{1-x} - \frac{2\,H[1,\ x]\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{6\,H[0,\ 1,\ x]\,\mathrm{Log}[x]^2}{1-x} - \frac{4\,H[0,\ 1,\ x]\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{2\,H[1,\ 1,\ x]\,\mathrm{Log}[x]^2}{1-x} + \frac{H[1,\ 1,\ x]\,\mathrm{Log}[x]^2}{(1-x)\,x} +$$

$$\frac{2\,\mathrm{Log}[x]^3}{3\,(1-x)} + \frac{4\,\mathrm{Log}[x]^3}{3\,(1-x)\,x} - \frac{4\,H[1,\ x]\,\mathrm{Log}[x]^3}{3\,(1-x)} + \frac{2\,H[1,\ x]\,\mathrm{Log}[x]^3}{3\,(1-x)\,x} - \frac{\mathrm{Log}[x]^4}{3\,(1-x)} - \frac{\mathrm{Log}[x]^4}{12\,(1-x)\,x} + \mathrm{Log}[\mathrm{ms}]^3\left(-\frac{4}{3\,(1-x)\,x} - \frac{2\,H[1,\ x]}{3\,(1-x)} + \frac{7\,H[1,\ x]}{3\,(1-x)\,x} - \frac{2\,\mathrm{Log}[x]}{3\,(1-x)} - \frac{5\,\mathrm{Log}[x]}{3\,(1-x)\,x}\right) +$$

$$\mathrm{Log}[\mathrm{ms}]^2\left(\frac{8}{(1-x)\,x} + \frac{2\,\pi^2}{(1-x)\,x} - \frac{10\,H[1,\ x]}{1-x} + \frac{6\,H[1,\ x]}{(1-x)\,x} + \frac{3\,H[1,\ 1,\ x]}{(1-x)\,x} - \frac{10\,\mathrm{Log}[x]}{1-x} + \frac{4\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{4\,H[1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{3\,\mathrm{Log}[x]^2}{2\,(1-x)\,x}\right) +$$

$$\frac{-\frac{\pi^4\,\sqrt{(1-x)\,x}}{(1-x)\,x} + \mathrm{IPi}\left(-\frac{\pi^2\,\sqrt{(1-x)\,x}\,H[1,\ x]}{(1-x)\,x} + \frac{4\,\pi^2\,\sqrt{(1-x)\,x}\,\mathrm{Log}[2]}{(1-x)\,x} - \frac{\pi^2\,\sqrt{(1-x)\,x}\,\mathrm{Log}[\mathrm{ms}]}{(1-x)\,x} + \frac{\pi^2\,\sqrt{(1-x)\,x}\,\mathrm{Log}[x]}{(1-x)\,x}\right)}{\sqrt{\mathrm{ms}}} + \frac{4\,\mathrm{Zeta}[3]}{1-x} - \frac{4\,\mathrm{Zeta}[3]}{(1-x)\,x} + \frac{28\,H[1,\ x]\,\mathrm{Zeta}[3]}{1-x} - \frac{8\,H[1,\ x]\,\mathrm{Zeta}[3]}{(1-x)\,x} +$$

$$\frac{28\,\mathrm{Log}[x]\,\mathrm{Zeta}[3]}{1-x} - \frac{18\,\mathrm{Log}[x]\,\mathrm{Zeta}[3]}{(1-x)\,x} + \mathrm{IPi}\left(\frac{2\,\pi^2}{3\,(1-x)} - \frac{16}{(1-x)\,x} + \frac{16\,H[1,\ x]}{1-x} + \frac{10\,\pi^2\,H[1,\ x]}{3\,(1-x)} - \frac{\pi^2\,H[1,\ x]}{3\,(1-x)\,x} - \frac{8\,H[0,\ 1,\ x]}{1-x} + \frac{4\,H[0,\ 1,\ x]}{(1-x)\,x} - \frac{12\,H[1,\ 1,\ x]}{1-x} + \frac{4\,H[0,\ 0,\ 1,\ x]}{1-x} +\right.$$

$$\frac{2\,H[0,\ 0,\ 1,\ x]}{(1-x)\,x} + \frac{4\,H[0,\ 1,\ 1,\ x]}{1-x} - \frac{2\,H[0,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{2\,H[1,\ 0,\ 1,\ x]}{(1-x)\,x} + \frac{4\,H[1,\ 1,\ 1,\ x]}{1-x} + \frac{4\,H[1,\ 1,\ 1,\ x]}{(1-x)\,x} - \frac{5\,\mathrm{Log}[\mathrm{ms}]^3}{3\,(1-x)\,x} + \frac{16\,\mathrm{Log}[x]}{1-x} + \frac{10\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)} - \frac{16\,\mathrm{Log}[x]}{(1-x)\,x} -$$

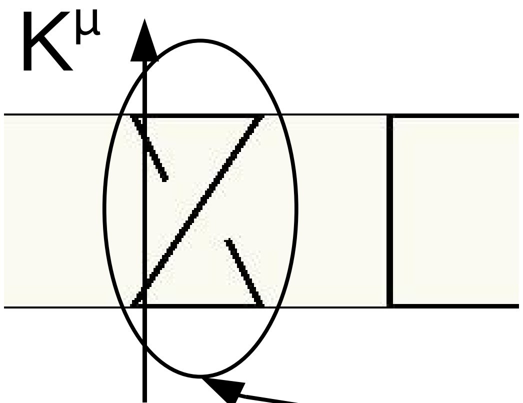
$$\frac{10\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)\,x} + \frac{4\,H[1,\ x]\,\mathrm{Log}[x]}{1-x} - \frac{4\,H[1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} - \frac{4\,H[0,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} + \frac{6\,\mathrm{Log}[x]^2}{1-x} - \frac{6\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{2\,H[1,\ x]\,\mathrm{Log}[x]^2}{1-x} - \frac{H[1,\ x]\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{2\,\mathrm{Log}[x]^3}{3\,(1-x)} - \frac{4\,\mathrm{Log}[x]^3}{3\,(1-x)\,x} +$$

$$\mathrm{Log}[\mathrm{ms}]^2\left(-\frac{6}{(1-x)\,x} - \frac{2\,H[1,\ x]}{1-x} - \frac{2\,\mathrm{Log}[x]}{1-x} + \frac{2\,\mathrm{Log}[x]}{(1-x)\,x}\right) + \mathrm{Log}[\mathrm{ms}]\left(\frac{16}{(1-x)\,x} + \frac{10\,\pi^2}{3\,(1-x)\,x} - \frac{12\,H[1,\ x]}{1-x} + \frac{2\,H[1,\ 1,\ x]}{(1-x)\,x} - \frac{12\,\mathrm{Log}[x]}{1-x} + \frac{12\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{\mathrm{Log}[x]^2}{(1-x)\,x}\right) - \frac{4\,\mathrm{Zeta}[3]}{1-x} + \left.\frac{18\,\mathrm{Zeta}[3]}{(1-x)\,x}\right) +$$

$$\mathrm{Log}[\mathrm{ms}]\left(-\frac{32}{(1-x)\,x} - \frac{4\,\pi^2}{3\,(1-x)\,x} + \frac{32\,H[1,\ x]}{1-x} + \frac{16\,\pi^2\,H[1,\ x]}{3\,(1-x)} - \frac{16\,H[1,\ x]}{(1-x)\,x} - \frac{2\,\pi^2\,H[1,\ x]}{(1-x)\,x} - \frac{8\,H[1,\ 1,\ x]}{1-x} + \frac{4\,H[1,\ 1,\ 1,\ x]}{1-x} - \frac{2\,H[1,\ 1,\ 1,\ x]}{(1-x)\,x} + \frac{32\,\mathrm{Log}[x]}{1-x} + \frac{16\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)} - \frac{16\,\mathrm{Log}[x]}{(1-x)\,x} -\right.$$

$$\left.\frac{10\,\pi^2\,\mathrm{Log}[x]}{3\,(1-x)\,x} - \frac{12\,H[1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{4\,H[1,\ 1,\ x]\,\mathrm{Log}[x]}{1-x} - \frac{6\,H[1,\ 1,\ x]\,\mathrm{Log}[x]}{(1-x)\,x} + \frac{4\,\mathrm{Log}[x]^2}{1-x} - \frac{4\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{2\,H[1,\ x]\,\mathrm{Log}[x]^2}{1-x} + \frac{H[1,\ x]\,\mathrm{Log}[x]^2}{(1-x)\,x} + \frac{2\,\mathrm{Log}[x]^3}{3\,(1-x)} - \frac{\mathrm{Log}[x]^3}{3\,(1-x)\,x} + \frac{20\,\mathrm{Zeta}[3]}{(1-x)\,x}\right)$$

• Working with non-planar tensor integrals



tensors rank 2 unavoidable

it's wiser to replace tensors
by inverse denominators (K^4)

- generates an 8-fold representation, but double analytic continuation

- reduces the number of 4-folds at the end (~ 400 → ~ 10)

- simpler structure of the result

$$
\begin{aligned}
&\frac{1}{180\,(1-x)}\\
&(-68\,\pi^4 - 45\,\pi^4\,x + 480\,\pi^2\,\mathrm{H}[1,1,x] + 120\,\pi^2\,x\,\mathrm{H}[1,1,x] + 1440\,\mathrm{IPi}\,\mathrm{H}[0,0,1,x] - 360\,\mathrm{IPi}\,x\,\mathrm{H}[0,0,1,x] - 720\,\mathrm{IPi}\,\mathrm{H}[0,1,1,x] + 1080\,\mathrm{IPi}\,x\,\mathrm{H}[0,1,1,x] + 360\,\mathrm{IPi}\,x\,\mathrm{H}[1,0,1,x] +\\
&\quad 1440\,\mathrm{IPi}\,\mathrm{H}[1,1,1,x] + 1440\,\mathrm{IPi}\,x\,\mathrm{H}[1,1,1,x] + 720\,x\,\mathrm{H}[0,0,0,1,x] + 1440\,\mathrm{H}[0,0,1,1,x] + 720\,x\,\mathrm{H}[0,0,1,1,x] + 2880\,\mathrm{H}[0,1,0,1,x] + 360\,x\,\mathrm{H}[0,1,0,1,x] -\\
&\quad 1440\,\mathrm{H}[0,1,1,1,x] - 2880\,x\,\mathrm{H}[0,1,1,1,x] + 2880\,\mathrm{H}[1,0,0,1,x] + 360\,x\,\mathrm{H}[1,0,0,1,x] - 2880\,\mathrm{H}[1,0,1,1,x] - 720\,x\,\mathrm{H}[1,0,1,1,x] - 1080\,x\,\mathrm{H}[1,1,1,1,x] +\\
&\quad 240\,\mathrm{IPi}\,\pi^2\,\mathrm{Log}[\mathrm{ms}] + 240\,\mathrm{IPi}\,\pi^2\,x\,\mathrm{Log}[\mathrm{ms}] + 720\,\mathrm{IPi}\,\mathrm{H}[1,1,x]\,\mathrm{Log}[\mathrm{ms}] + 1080\,\mathrm{IPi}\,x\,\mathrm{H}[1,1,x]\,\mathrm{Log}[\mathrm{ms}] + 1440\,x\,\mathrm{H}[0,0,1,x]\,\mathrm{Log}[\mathrm{ms}] - 720\,\mathrm{H}[0,1,1,x]\,\mathrm{Log}[\mathrm{ms}] -\\
&\quad 1440\,x\,\mathrm{H}[0,1,1,x]\,\mathrm{Log}[\mathrm{ms}] - 360\,x\,\mathrm{H}[1,1,1,x]\,\mathrm{Log}[\mathrm{ms}] + 180\,\pi^2\,\mathrm{Log}[\mathrm{ms}]^2 + 240\,\pi^2\,x\,\mathrm{Log}[\mathrm{ms}]^2 + 180\,x\,\mathrm{H}[1,1,x]\,\mathrm{Log}[\mathrm{ms}]^2 - 60\,\mathrm{IPi}\,\mathrm{Log}[\mathrm{ms}]^3 - 15\,\mathrm{Log}[\mathrm{ms}]^4 +\\
&\quad 90\,x\,\mathrm{Log}[\mathrm{ms}]^4 + 720\,\mathrm{IPi}\,\mathrm{H}[1,1,x]\,\mathrm{Log}[x] - 1440\,x\,\mathrm{H}[0,0,1,x]\,\mathrm{Log}[x] - 1440\,\mathrm{H}[0,1,1,x]\,\mathrm{Log}[x] - 360\,x\,\mathrm{H}[0,1,1,x]\,\mathrm{Log}[x] - 2880\,\mathrm{H}[1,0,1,x]\,\mathrm{Log}[x] -\\
&\quad 360\,x\,\mathrm{H}[1,0,1,x]\,\mathrm{Log}[x] + 1440\,\mathrm{H}[1,1,1,x]\,\mathrm{Log}[x] + 720\,x\,\mathrm{H}[1,1,1,x]\,\mathrm{Log}[x] - 120\,\pi^2\,x\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Log}[x] + 720\,\mathrm{H}[1,1,x]\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Log}[x] + 360\,x\,\mathrm{H}[1,1,x]\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Log}[x] -\\
&\quad 360\,\mathrm{IPi}\,x\,\mathrm{Log}[\mathrm{ms}]^2\,\mathrm{Log}[x] - 180\,x\,\mathrm{Log}[\mathrm{ms}]^3\,\mathrm{Log}[x] + 60\,\pi^2\,x\,\mathrm{Log}[x]^2 + 720\,\mathrm{H}[1,1,x]\,\mathrm{Log}[x]^2 + 180\,x\,\mathrm{H}[1,1,x]\,\mathrm{Log}[x]^2 + 540\,\mathrm{IPi}\,x\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Log}[x]^2 + 90\,x\,\mathrm{Log}[\mathrm{ms}]^2\,\mathrm{Log}[x]^2 -\\
&\quad 240\,\mathrm{IPi}\,x\,\mathrm{Log}[x]^3 + 60\,x\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Log}[x]^3 - 45\,x\,\mathrm{Log}[x]^4 + 120\,\mathrm{H}[0,1,x]\,(4\,\pi^2 - 6\,\mathrm{IPi}\,(2-x)\,\mathrm{Log}[x] + 9\,x\,\mathrm{Log}[x]^2 + 6\,\mathrm{Log}[\mathrm{ms}]\,(\mathrm{IPi} - 2\,x\,\mathrm{Log}[x])) + 2880\,\mathrm{IPi}\,\mathrm{Zeta}[3] -\\
&\quad 1800\,\mathrm{IPi}\,x\,\mathrm{Zeta}[3] + 2880\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Zeta}[3] + 2880\,x\,\mathrm{Log}[\mathrm{ms}]\,\mathrm{Zeta}[3] - 1800\,x\,\mathrm{Log}[x]\,\mathrm{Zeta}[3] + 60\,\mathrm{H}[1,x]\,(10\,\mathrm{IPi}\,\pi^2 - \mathrm{IPi}\,\pi^2\,x + 6\,\mathrm{IPi}\,x\,\mathrm{Log}[\mathrm{ms}]^2 + 3\,x\,\mathrm{Log}[\mathrm{ms}]^3 +\\
&\qquad \pi^2\,(-8-5\,x)\,\mathrm{Log}[x] + 3\,\mathrm{IPi}\,(4-3\,x)\,\mathrm{Log}[x]^2 - 6\,x\,\mathrm{Log}[x]^3 - \mathrm{Log}[\mathrm{ms}]\,(2\,\pi^2\,(-5-x) + 12\,\mathrm{IPi}\,(1-x)\,\mathrm{Log}[x] - 9\,x\,\mathrm{Log}[x]^2) + 48\,\mathrm{Zeta}[3] + 36\,x\,\mathrm{Zeta}[3]))
\end{aligned}
$$

Power corrections

- Let's consider the expansion in ms of the bare 2-loop leading color contribution to quark annihilation

$$\langle M^{(0)}|M^{(2)}\rangle_{\text{bare}} = N^4 \left(\frac{\alpha_s^{(0)}}{2\pi}\right)^2 A_0^{(0)} + \ldots$$

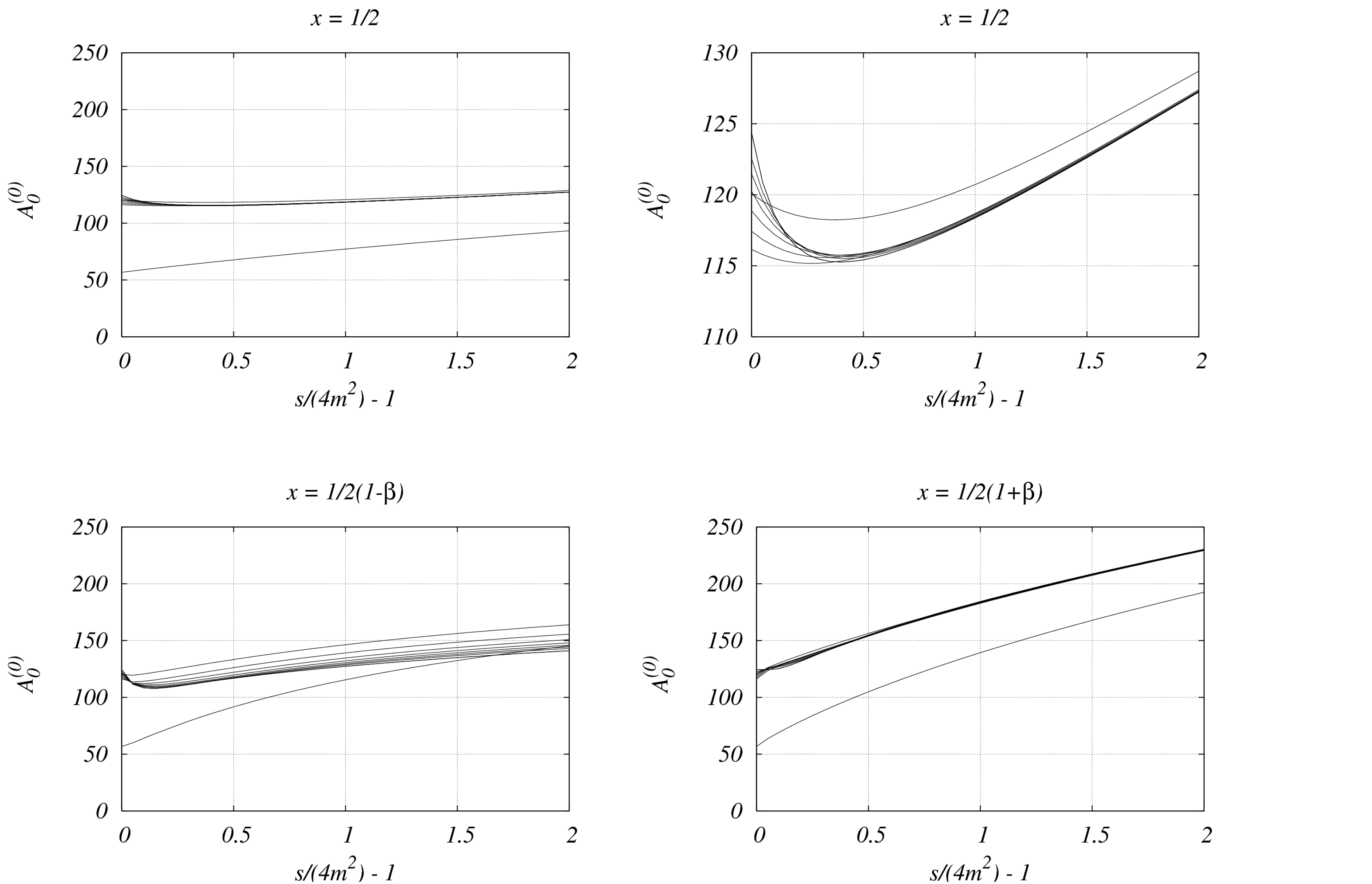
- at x=1/2 and with r = 4 ms

11 terms up to ms^{10}

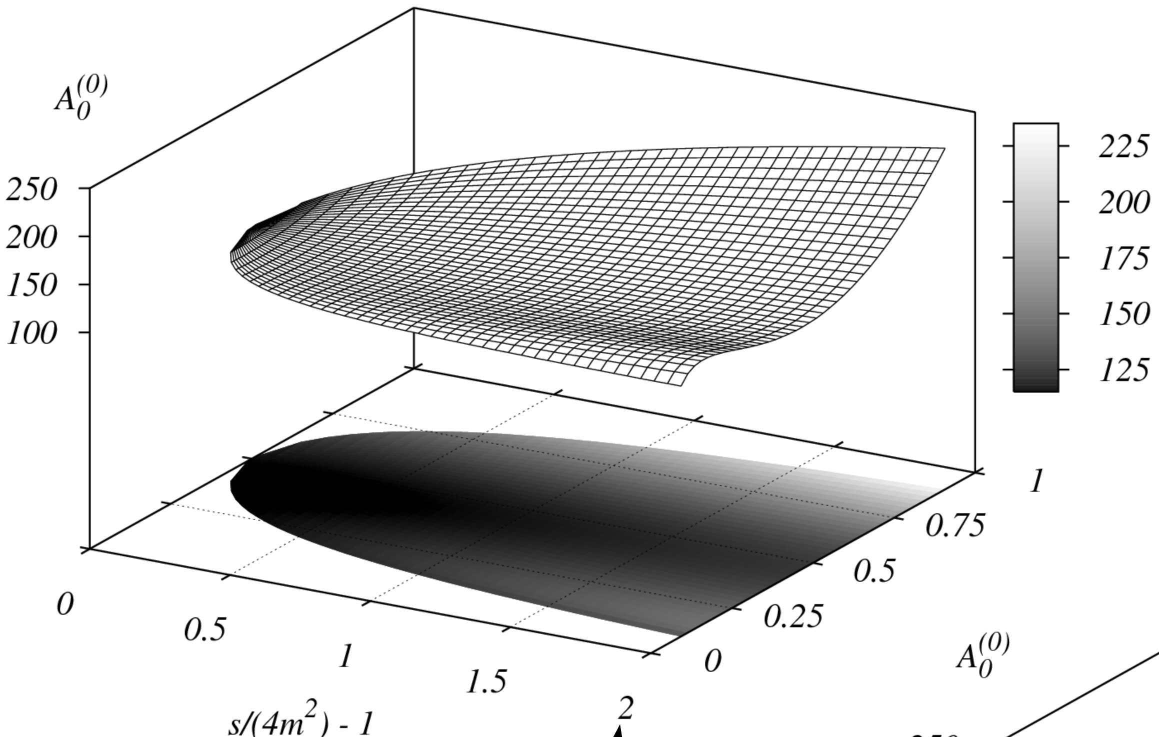
$$\begin{aligned}
&36.4466 + 36.6376\, r - 2.11948\, r^2 + 0.318695\, r^3 + 1.8244\, r^4 + \\
&3.25332\, r^5 + 4.54712\, r^6 + 5.73099\, r^7 + 6.82737\, r^8 + 7.85347\, r^9 + 8.82223\, r^{10} + \\
&(-7.13933 - 13.2047\, r - 1.679\, r^2 - 1.79336\, r^3 - 0.682462\, r^4 + 0.344393\, r^5 + \\
&\quad 1.31353\, r^6 + 2.23685\, r^7 + 3.12317\, r^8 + 3.97884\, r^9 + 4.80856\, r^{10})\, \mathrm{Log}[ms] + \\
&(4.44113 + 2.67214\, r - 1.97207\, r^2 - 0.387011\, r^3 - 0.0835894\, r^4 + 0.0520038\, r^5 + \\
&\quad 0.164006\, r^6 + 0.287566\, r^7 + 0.428835\, r^8 + 0.5859\, r^9 + 0.755209\, r^{10})\, \mathrm{Log}[ms]^2 + \\
&(-0.608904 - 1.10606\, r + 0.155183\, r^2 + 0.298532\, r^3 + 0.425373\, r^4 + 0.543219\, r^5 + \\
&\quad 0.651184\, r^6 + 0.75001\, r^7 + 0.840853\, r^8 + 0.924882\, r^9 + 1.00314\, r^{10})\, \mathrm{Log}[ms]^3 + \\
&(0.0625 + 0.0833333\, r + 0.0208333\, r^2 + 0.00260417\, r^3 - 0.0078125\, r^4 - 0.0157064\, r^5 - \\
&\quad 0.0222168\, r^6 - 0.027832\, r^7 - 0.032814\, r^8 - 0.0373214\, r^9 - 0.0414581\, r^{10})\, \mathrm{Log}[ms]^4
\end{aligned}$$

- The series is derived under the assumption that $m_t^2 \ll s, |t|, |u|$. what happens at the edge of the phase space ?

Power corrections



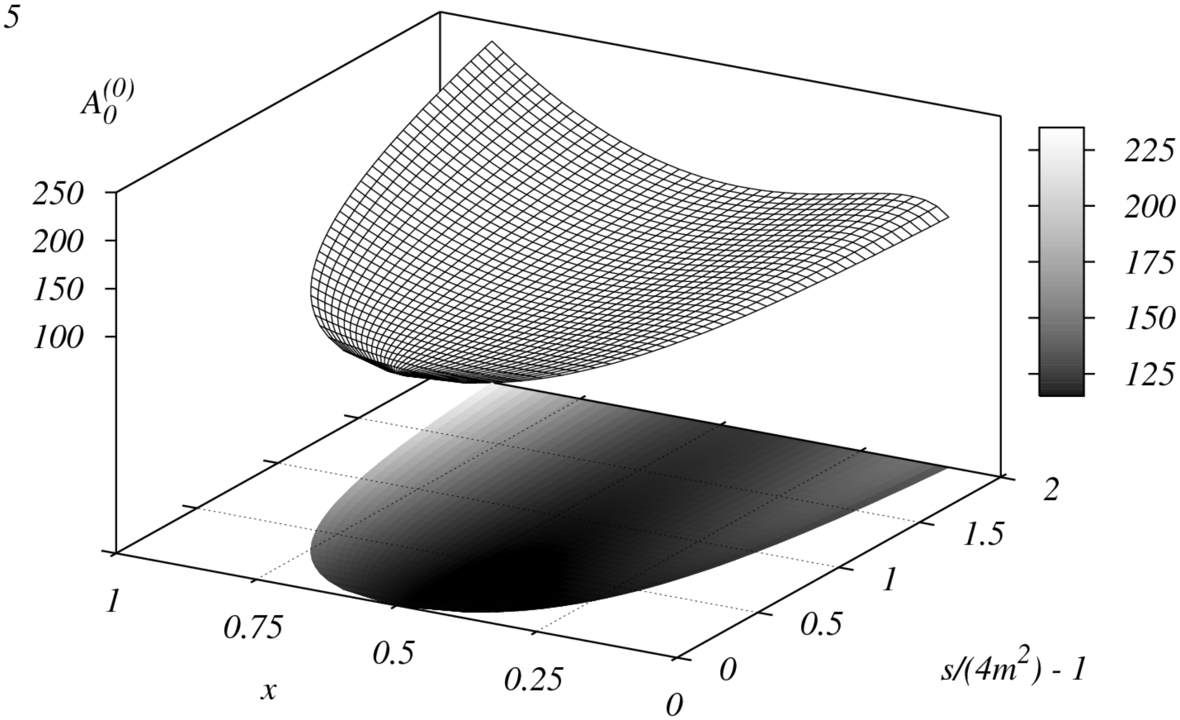
Full mass dependence



$\sqrt{s} \approx 590\,\mathrm{GeV}$

- A practical prescription for implementation in MC:

- construct a dense grid
- use interpolation
- renormalize at the end



Merging ideas for workable numerics

sector decomposition
T.Binoth, G.Heinrich '00

integration by contour deformation
D.Soper '98, Z.Nagy, D.Soper '06

K.Melnikov, A.Lazopoulos '07, Ch.Anastasiou, A.Daleo '07

see also R.Boughezal's talk

MB representations
E.Boos,A.Davydychev '91
V.Smirnov '99, B.Tausk '99

expansions from DEQs
M.Caffo, H.Czyz, E.Remiddi '98

numerical integration of DEQs
M.Caffo, H.Czyz, E.Remiddi '98

M.C. '07

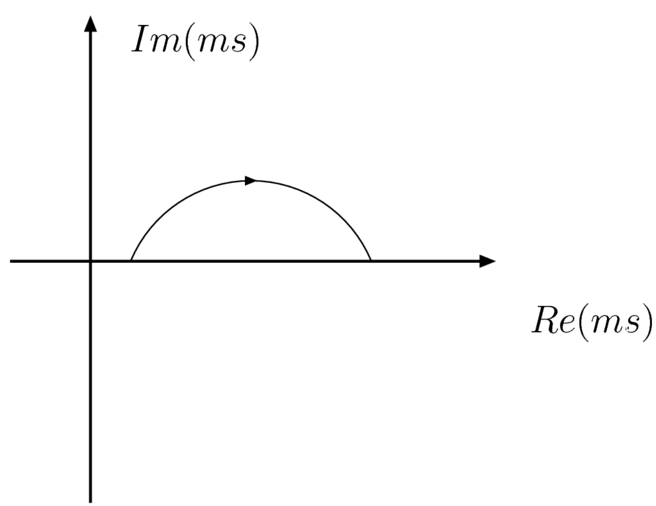
Steps

- Compute the high energy asymptotics of the master integrals obtaining the leading behaviour of the amplitude
- Determine the coefficients of the mass expansions using differential equations in ms obtaining the power corrections

$$ms \frac{d}{d\,ms} M_i(ms, x, \epsilon) = \sum_j C_{ij}(ms, x, \epsilon) M_j(ms, x, \epsilon)$$

- Evaluate the expansions for $ms \ll 1$ to obtain the desired numerical precision of the boundaries

- Evolve the functions from the boundary point with differential equations first in ms and then in x (**ODEPACK**)



Efficiency

- starting point

  $ms = 0.002,\ \ x = 0.25$

- singularities of coefficients of DEQs:

  $\beta = 0$

  $x = 1/2(1 \pm \beta)$

  $ms = x$

  $x = 1/2\ \ \ldots$

- interpolation necessary

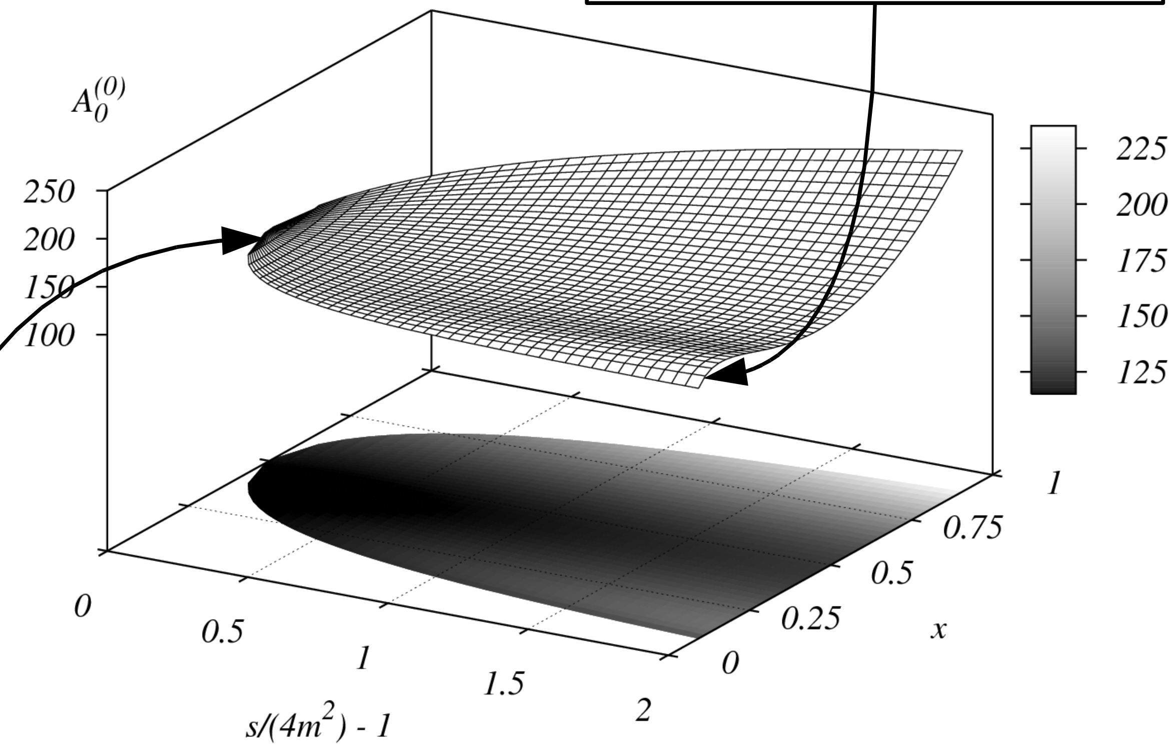
- relative errors required

  $10^{-14}\ \ (ms),\ \ 10^{-12}\ \ (x)$

- contour deformation

  $\Delta_{ms} = 0.01,\ \ \Delta_{x} = 0.01$

time: 1 s
precision: 10 digits
stability: 12 digits



$\beta = 10^{-1}$
$x = 1/2(1 - \beta) + 10^{-4}$
time: 22 s
precision: 8 digits
stability: 8 digits

Intel(R) Core(TM) 2 Duo, 2GHz
Intel Fortran compiler

W pair production

- Accurate knowledge needed

  - signal: to study the gauge structure of the Standard Model

  - background: for Higgs boson production and decay in the mass range $M_H \in [140, 175]$ GeV

- Large enhancements at the NLO level in the dominant quark annihilation channel

  - 70% with general LHC cuts

  - 20 – 30% with Higgs boson search cuts

- Necessity to study scale variation in the case of gluon fusion (leading order error determination)

  - 30% enhancement with Higgs boson search cuts, otherwise 5%

W pair production

- Leading color contribution at 2-loops in t-channel production in quark annihilation

$$
\begin{aligned}
&\frac{\pi^4\,(88-307x)}{2880\,(-1+x)}+\frac{51863x-1800x^2}{10368\,(-1+x)}+\frac{\frac{3}{32\,ms}-\frac{x}{8\,(-1+x)}-\frac{(-1+x)\,x}{32\,ms^2}}{\varepsilon^4}+\\
&\frac{(-72+432x+499x^2-228x^3)\,H[1,x]}{288\,(-1+x)\,x}+\frac{(18-54x+63x^2-79x^3-9x^4)\,H[1,1,x]}{36\,(-1+x)\,x^2}+\\
&\frac{(-1+x^2-2x^3-x^4)\,H[0,1,1,x]}{4\,(-1+x)\,x^2}+\frac{(3-22x^3-3x^4)\,H[1,1,1,x]}{12\,(-1+x)\,x^2}+\\
&\frac{(-4+3x)\,H[0,0,1,1,x]}{4\,(-1+x)}+\frac{x\,H[0,1,1,1,x]}{2\,(-1+x)}+\frac{x\,H[1,0,1,1,x]}{2\,(-1+x)}+\\
&\frac{(-1+x+x^2)\,\mathrm{Log}[ms]^3}{24\,(-1+x)}+\left(\frac{-251x-720x^2}{864\,(-1+x)}+\frac{(-3+x)\,H[1,x]}{6\,(-1+x)}+\right.\\
&\quad\left.\frac{(6+11x)\,H[1,1,x]}{12\,(-1+x)}-\frac{x\,H[0,1,1,x]}{2\,(-1+x)}-\frac{x\,H[1,1,1,x]}{-1+x}\right)\mathrm{Log}[s]+\\
&\left(\frac{-8x+9x^2}{18\,(-1+x)}-\frac{x\,H[1,x]}{4\,(-1+x)}+\frac{x\,H[1,1,x]}{2\,(-1+x)}\right)\mathrm{Log}[s]^2+\frac{x\,\mathrm{Log}[s]^3}{72\,(-1+x)}-\frac{x\,\mathrm{Log}[s]^4}{12\,(-1+x)}+\\
&\mathrm{Log}[ms]^2\left(\frac{7-10x-5x^2}{16\,(-1+x)}+\frac{(-1+x+x^2)\,\mathrm{Log}[s]}{4\,(-1+x)}\right)+\\
&\pi^2\left(\frac{216+132x+1007x^2-780x^3}{1728\,(-1+x)\,x}+\frac{(-18-66x^2-17x^3+6x^4)\,H[1,x]}{144\,(-1+x)\,x^2}+\right.\\
&\quad\frac{(-4+11x)\,H[0,1,x]}{24\,(-1+x)}+\frac{13x\,H[1,1,x]}{24\,(-1+x)}-\frac{13\,(-1+x+x^2)\,\mathrm{Log}[ms]}{24\,(-1+x)}+\\
&\quad\left.\left(\frac{13x}{72\,(-1+x)}-\frac{3x\,H[1,x]}{4\,(-1+x)}\right)\mathrm{Log}[s]+\frac{x\,\mathrm{Log}[s]^2}{2\,(-1+x)}\right)+\\
&\mathrm{Log}[ms]\left(\frac{115-43x-37x^2}{144\,(-1+x)}+\frac{(-6+19x-16x^2-13x^3)\,H[1,x]}{24\,(-1+x)\,x}+\right.\\
&\quad\left.\frac{(1-x^3-x^4)\,H[1,1,x]}{4\,(-1+x)\,x^2}+\frac{(5-8x-2x^2)\,\mathrm{Log}[s]}{6\,(-1+x)}+\frac{(-1+x+x^2)\,\mathrm{Log}[s]^2}{2\,(-1+x)}\right)+\\
&\frac{-\frac{13x}{16\,(-1+x)}+\frac{\frac{43}{64}-\frac{3\,\mathrm{Log}[s]}{16}}{ms}+\frac{x\,\mathrm{Log}[s]}{4\,(-1+x)}+\frac{-\frac{17}{64}\,(-1+x)\,x+\frac{1}{16}\,(-1+x)\,x\,\mathrm{Log}[s]}{ms^2}}{\varepsilon^3}+\frac{1}{\varepsilon^2}\\
&\left(\frac{\pi^2\,x}{4\,(-1+x)}+\frac{35x+72x^2}{288\,(-1+x)}-\frac{x\,H[1,x]}{8\,(-1+x)}+\frac{x\,H[1,1,x]}{4\,(-1+x)}+\right.\\
&\quad\frac{(-1+x+x^2)\,\mathrm{Log}[ms]}{4\,(-1+x)}+\frac{17x\,\mathrm{Log}[s]}{24\,(-1+x)}-\frac{x\,\mathrm{Log}[s]^2}{4\,(-1+x)}+\frac{\frac{25}{384}-\frac{3\pi^2}{16}-\frac{21\,\mathrm{Log}[s]}{32}+\frac{3\,\mathrm{Log}[s]^2}{16}}{ms}+\\
&\quad\left.\frac{-\frac{433\,(-1+x)\,x}{1152}+\frac{1}{16}\,\pi^2\,(-1+x)\,x+\frac{29}{96}\,(-1+x)\,x\,\mathrm{Log}[s]-\frac{1}{16}\,(-1+x)\,x\,\mathrm{Log}[s]^2}{ms^2}\right)+\\
&\left(\frac{17x}{72\,(-1+x)}-\frac{3x\,H[1,x]}{4\,(-1+x)}+\frac{7x\,\mathrm{Log}[s]}{12\,(-1+x)}\right)\mathrm{Zeta}[3]+\frac{1}{ms}\\
&\left(-\frac{41443}{13824}+\frac{263\,\pi^4}{3840}+\frac{1003\,\mathrm{Log}[s]}{1152}-\frac{\mathrm{Log}[s]^2}{24}-\frac{3\,\mathrm{Log}[s]^3}{32}+\frac{\mathrm{Log}[s]^4}{16}+\right.\\
&\quad\left.\pi^2\left(-\frac{161}{2304}+\frac{19\,\mathrm{Log}[s]}{48}-\frac{3\,\mathrm{Log}[s]^2}{8}\right)+\left(\frac{1}{4}-\frac{7\,\mathrm{Log}[s]}{16}\right)\mathrm{Zeta}[3]\right)+\\
&\frac{1}{ms^2}\left(\frac{9083\,(-1+x)\,x}{41472}-\frac{263\,\pi^4\,(-1+x)\,x}{11520}+\frac{877\,(-1+x)\,x\,\mathrm{Log}[s]}{3456}-\right.\\
&\quad\frac{17}{72}\,(-1+x)\,x\,\mathrm{Log}[s]^2+\frac{25}{288}\,(-1+x)\,x\,\mathrm{Log}[s]^3-\frac{1}{48}\,(-1+x)\,x\,\mathrm{Log}[s]^4+\\
&\quad\pi^2\left(\frac{2153\,(-1+x)\,x}{6912}-\frac{43}{144}\,(-1+x)\,x\,\mathrm{Log}[s]+\frac{1}{8}\,(-1+x)\,x\,\mathrm{Log}[s]^2\right)+\\
&\quad\left.\left(-\frac{13}{72}\,(-1+x)\,x+\frac{7}{48}\,(-1+x)\,x\,\mathrm{Log}[s]\right)\mathrm{Zeta}[3]\right)+\frac{1}{\varepsilon}\\
&\left(\frac{-2521x+1512x^2}{1728\,(-1+x)}+\frac{(4-5x)\,H[1,x]}{16\,(-1+x)}-\frac{H[1,1,x]}{4\,(-1+x)}+\frac{x\,H[0,1,1,x]}{4\,(-1+x)}+\frac{x\,H[1,1,1,x]}{2\,(-1+x)}+\right.\\
&\quad\frac{(1-x-x^2)\,\mathrm{Log}[ms]^2}{8\,(-1+x)}+\left(\frac{31x-72x^2}{144\,(-1+x)}+\frac{x\,H[1,x]}{4\,(-1+x)}-\frac{x\,H[1,1,x]}{2\,(-1+x)}\right)\mathrm{Log}[s]-\\
&\quad\frac{x\,\mathrm{Log}[s]^2}{4\,(-1+x)}+\frac{x\,\mathrm{Log}[s]^3}{6\,(-1+x)}+\pi^2\left(\frac{17x}{96\,(-1+x)}+\frac{3x\,H[1,x]}{8\,(-1+x)}-\frac{x\,\mathrm{Log}[s]}{2\,(-1+x)}\right)+\\
&\quad\mathrm{Log}[ms]\left(\frac{-7+9x+5x^2}{8\,(-1+x)}+\frac{(1-x-x^2)\,\mathrm{Log}[s]}{2\,(-1+x)}\right)+\\
&\quad\frac{\frac{591}{2304}+\pi^2\left(-\frac{51}{128}+\frac{3\,\mathrm{Log}[s]}{8}\right)-\frac{\mathrm{Log}[s]}{64}+\frac{5\,\mathrm{Log}[s]^2}{16}-\frac{\mathrm{Log}[s]^3}{8}+\frac{7\,\mathrm{Zeta}[3]}{32}}{ms}-\frac{7x\,\mathrm{Zeta}[3]}{24\,(-1+x)}+\frac{1}{ms^2}\\
&\quad\left(-\frac{4045\,(-1+x)\,x}{6912}+\frac{235}{576}\,(-1+x)\,x\,\mathrm{Log}[s]-\frac{3}{16}\,(-1+x)\,x\,\mathrm{Log}[s]^2+\frac{1}{24}\,(-1+x)\,x\,\mathrm{Log}[s]^3+\right.\\
&\quad\left.\left.\pi^2\left(\frac{83}{384}\,(-1+x)\,x-\frac{1}{8}\,(-1+x)\,x\,\mathrm{Log}[s]\right)-\frac{7}{96}\,(-1+x)\,x\,\mathrm{Zeta}[3]\right)\right)
\end{aligned}
$$

- full result soon to follow (G.Chachamis, M.C., D.Eiras)

Conclusions

- Leading high energy behaviour of the 2-loop virtual corrections available for both production channels in the case of top pairs

- Leading high energy behaviour of the 2-loop virtual corrections almost complete in the quark annihilation channel for W pairs

- Power corrections and full mass dependence in both cases are feasible

- Lots to be done from here...