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Recent results on unintegrated parton distributions

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I. Introduction

II. Small-x final states from u-pdf's in Monte-Carlo generators

III. Progress towards precise operator definitions for u-pdf's

I. Introduction

Complex final states with multiple hard scales

$\downarrow$

QCD methods based on parton distributions unintegrated in

both longitudinal and transverse momentum (u-pdf's)

<u>Classic examples:</u>

- Sudakov processes
- small-x physics
- simulation of fully exclusive final states

See J.R. Andersen et al., hep-ph/0604189, Summary of 3rd Lund Workshop;

S. Alekhin et al., hep-ph/0601012, "Hera and the LHC" Workshop Proceedings

♢ For small x, u-pdf's can be introduced in a gauge-invariant manner via high-energy factorization

$\Downarrow$

- resummation of $\ln x$ corrections to QCD evolution equations

  $\hookrightarrow$ including matching with collinear dynamics (ordinary pdf's)

  [see talks by G. Altarelli and M. Ciafaloni]

- Monte Carlo simulation of $x \rightarrow 0$ parton showers

  $\hookrightarrow$ collinear matching yet to be developed

♢ To characterize u-pdf's gauge-invariantly over the whole phase space is more difficult — full framework still missing, much ongoing work

[see talk by T. Rogers]

Outline

▷ Application of u-pdf's to shower Monte-Carlo generators:

- hadronic final states at $x \ll 1$
- multi-jet production
- angular correlations

▷ Progress on unintegrated distributions beyond $x \ll 1$:

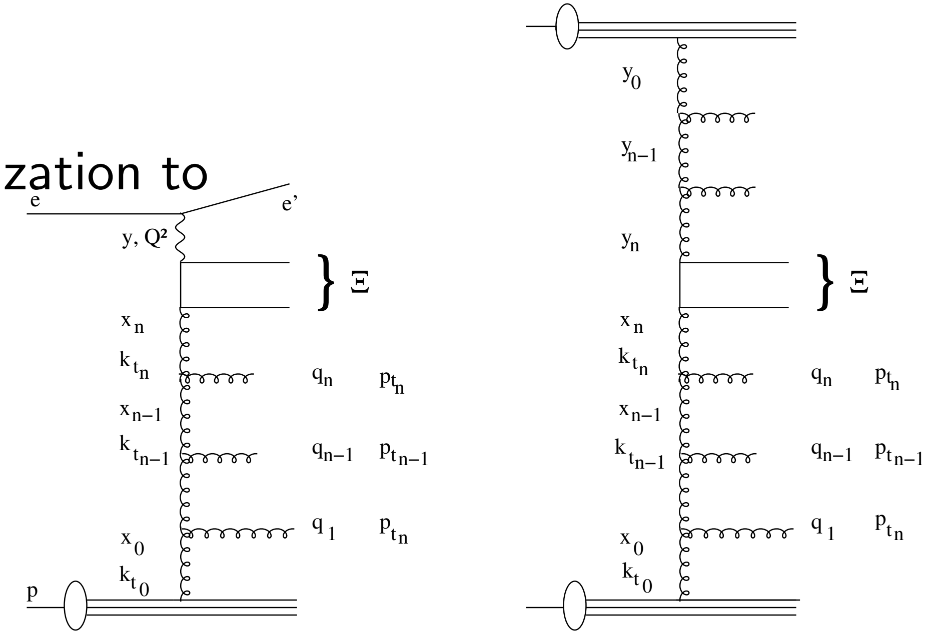
- nonlocal operator matrix elements
- endpoint divergences $x \to 1$
- cut-off vs. subtractive regularization method

II.1 U-pdf's and shower Monte-Carlo generators

◇ All MC's based on u-pdf's rely on $k_\perp$-factorization to

a) generate hard-scattering event

b) couple it to initial-state gluon cascade



(a) (b)

◇ but differ by model for initial-state evolution

(BFKL, CCFM, LDC evolution equations)

• with suitable constraints on angular ordering of gluon emission

$\Rightarrow$ correct leading $\ln x$ behavior

• subleading contributions also important for final states

Implementations:

Höche, Krauss and Teubner, arXiv:0705.4577 (BFKL)

Golec, Jadach, Placzek, Stephens, Skrzypek, hep-ph/0703317 (CCFM)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

SMALLX Marchesini & Webber, 1992 (CCFM)

Advantages over standard Monte-Carlo:

- better treatment of high-energy logarithmic effects
- likely more suitable for simulating underlying event's $k_\perp$

Current limitations:

- collinear radiation associated to $x \sim 1$ not automatically included
- procedure to correct for this not yet systematic

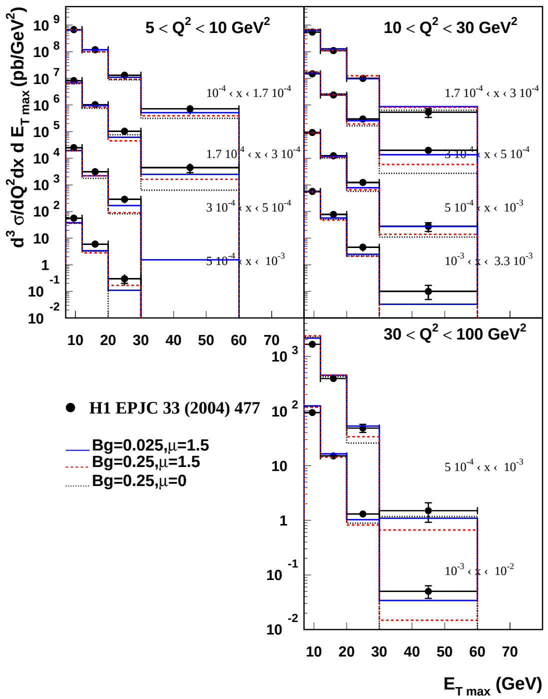
  $\hookrightarrow$ e.g.: LO-DGLAP in Höche et al, 2007
- quark contributions in initial state yet to be implemented

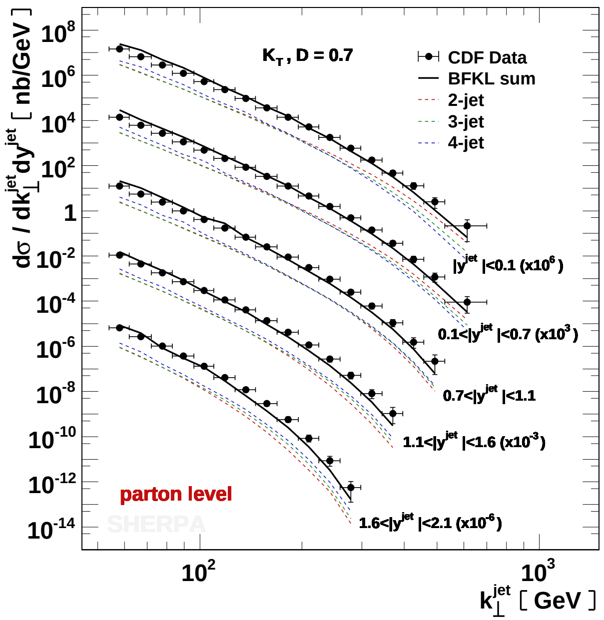
  $\hookrightarrow$ $k_\perp$ kernel for sea-quark evolution [Catani & H, 1994]
- limited knowledge of u-pdf's [Jung et al., arXiv:0706.3793; J. R. Andersen et al., 2006]

II.2 Inclusive examples

- inclusive data used to test model and determine unintegrated gluon

  [$\hookrightarrow$ DIS, jets, heavy flavors]





(left) CASCADE (Jung, 2007) vs. H1 [hep-ex/0310019] jet E_T distribution;

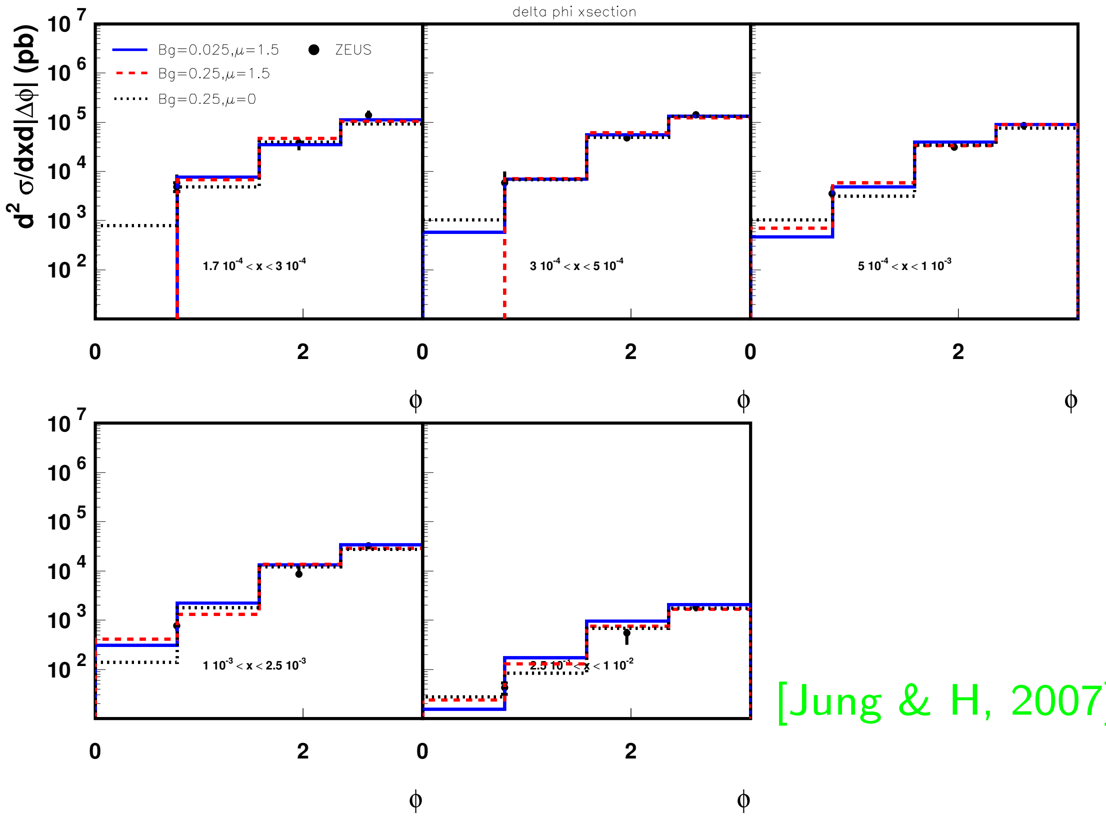
(right) Höche, Krauss and Teubner, 2007 vs. CDF [hep-ex/0701051] jet spectra

▷ sensible results for evolved gluon

▷ poorly constrained at low scales and low x

II.3 Multi-jet correlations

- Zeus [arXiv:0705.1931] measure azimuthal correlations in 3-jet distrib's ($x \sim 10^{-4}$)



[Jung & H, 2007]

- jet clustering free of nonglobal logs

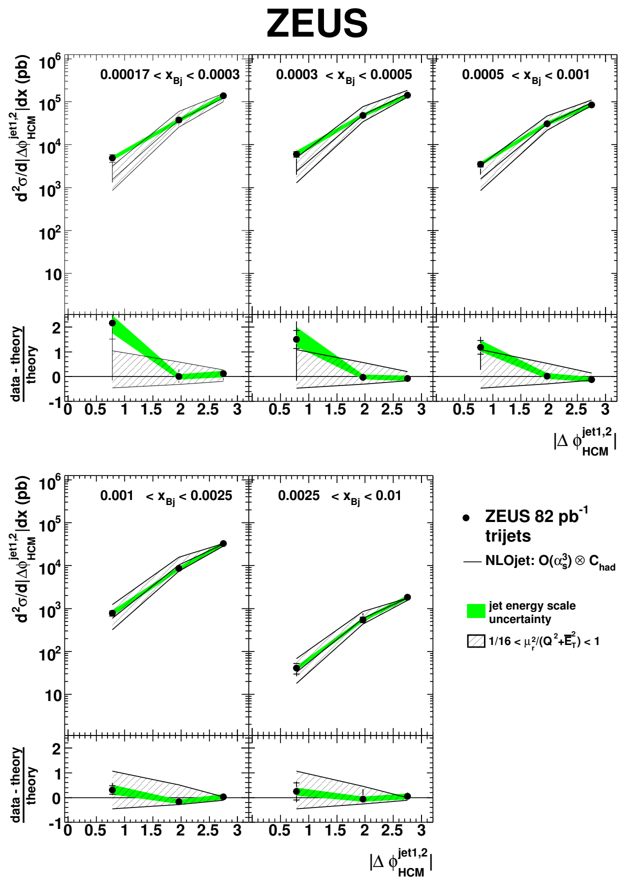


Figure 11: *Trijet cross sections as functions of $|\Delta\phi^{\rm jet1,2}_{\rm HCM}|$. The measurements are compared to* NLOJET *calculations at $\mathcal{O}(\alpha_s^3)$. The boundaries for the bins in $|\Delta\phi^{\rm jet1,2}_{\rm HCM}|$ are given in Table 5. Other details as in the caption to Fig. 1.*

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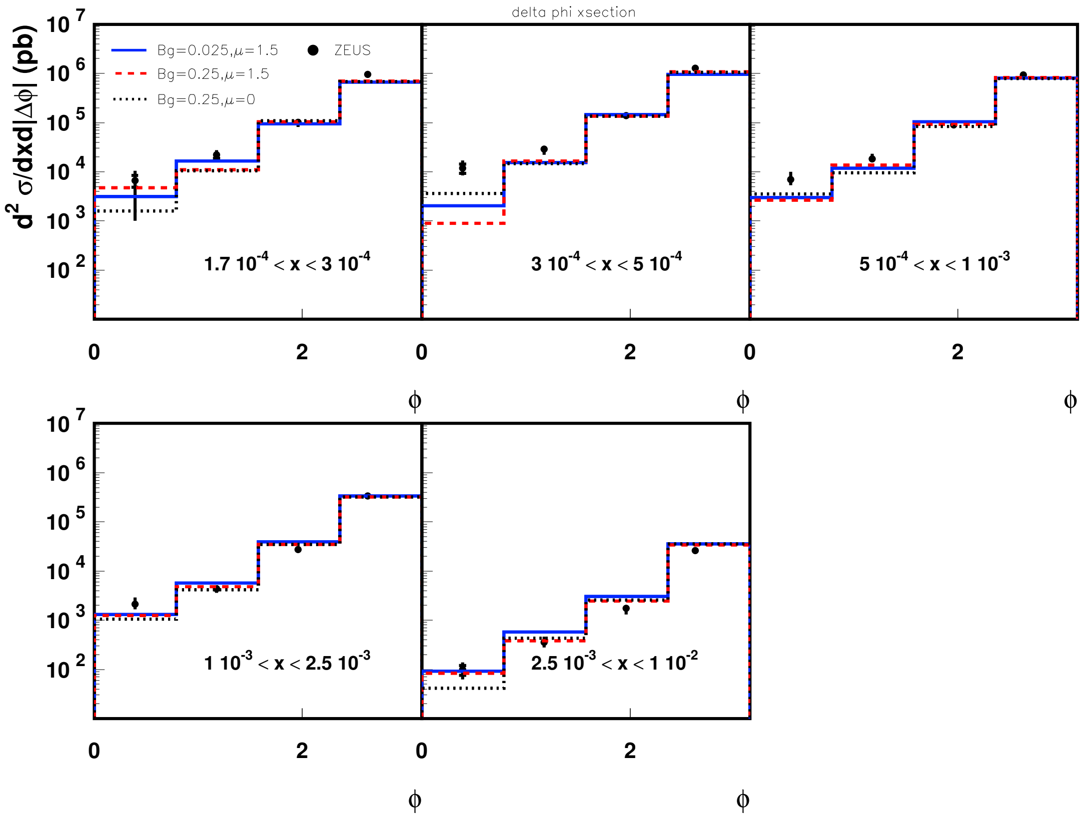
smallest x and smallest ϕ:

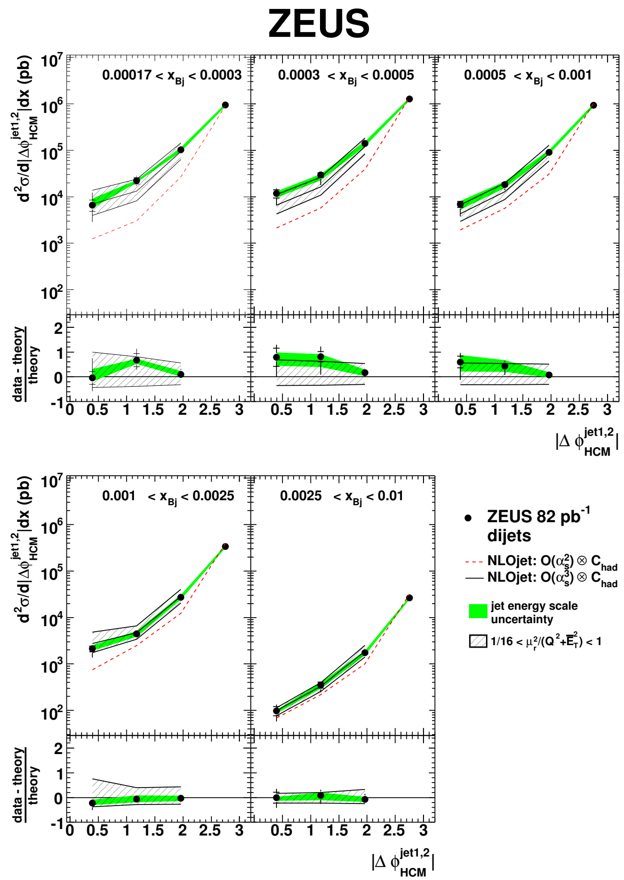
▷ potential effect of truncating multi-gluon emission

▷ reduced contribution from back-to-back events [Banfi, Dasgupta & Delenda, 2007]

▷ non-negligible $k_\perp$ enhances $x \to 0$ shower effects

Similar dynamical effects observed in di-jet distributions:

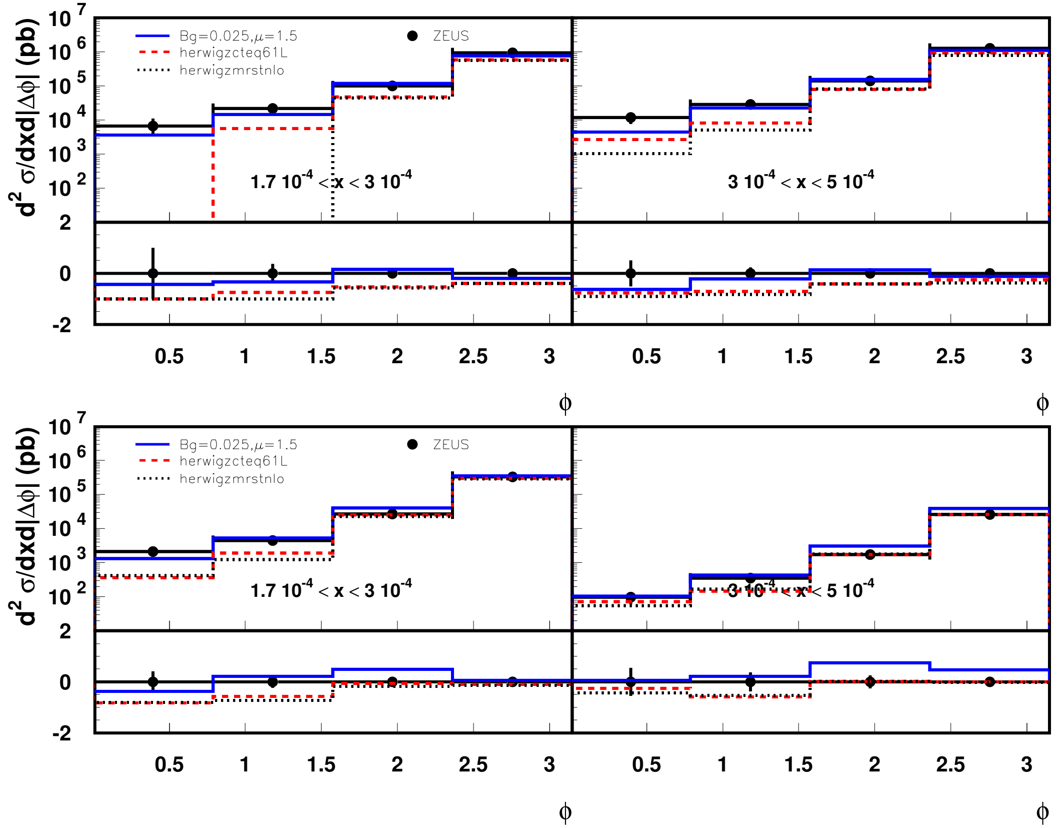




- Large correction from order-α_s^2 to order-α_s^3 for decreasing x and decreasing ϕ

- Different shapes than from standard shower MC's, e.g. HERWIG

  $\hookrightarrow$ soft/collinear radiation but no $x \to 0$ effects



[Jung & H, 2007]

II.4 Further developments

♢ Measurements in the forward region

- MC results depend strongly on evolution model
- not quite accessible yet
- deeper understanding of u-pdf's likely to be needed

♢ Relevant glue-glue applications:

- production of b, c

  large NLO uncertainties at LHC energies

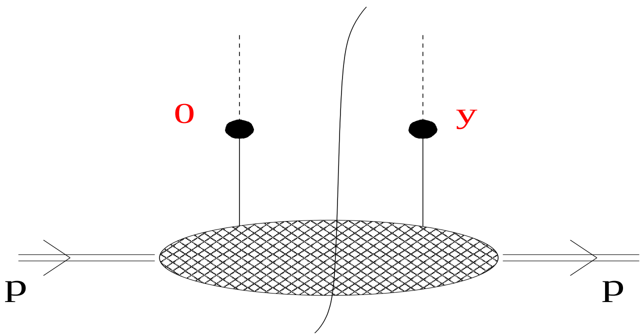
  [Nason et al. 2004]

- final states with Higgs

  possibly 15 $\%$ in p_t spectrum from $x \ll 1$ terms

  [Kulesza, Sterman & Vogelsang, 2004]

III. Towards precise characterizations of u-pdf's



$p = (p^+, m^2/2p^+, 0_\perp)$

Gauge-invariant matrix element:

$$\widetilde{f}(y) = \langle\ P \mid \overline{\psi}(y)\ V_y^\dagger(n)\ \gamma^+\ V_0(n)\ \psi(0) \mid P\ \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$$V_y(n) = \mathcal{P}\exp\left(ig_s \int_0^\infty d\tau\ n\cdot A(y+\tau\ n)\right)$$

Boer & Mulders, 1998

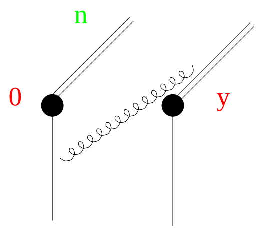
Belitsky, Ji & Yuan, 2003

Collins, 2003

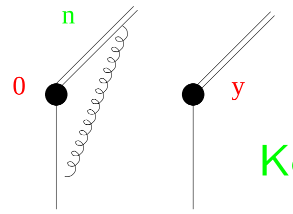
Ok at tree level, but more subtle at the level of radiative corrections:

- incomplete KLN cancellations near $x = 1$ — Brodsky et al., 2001; Collins & Soper, 1981
- UV divergences / relation with ordinary pdf's — Collins & Zu, 2005 $(\lambda\phi^3)_6$; Catani, Ciafaloni & H, 1993 $(x\rightarrow 0)$; Balitsky & Braun, 1991 (OPE)

One-loop analysis
(coordinate space)



(a)



(b)

H, hep-ph/0702196

Korchemsky & Marchesini, 1993

$$\widetilde{f}_{(a)+(b)}(y) = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left[e^{ip\cdot yv}\ 2^{d/2-1} \left(\frac{\rho^2}{\mu^2}\right)^{d/4-1} \right.$$

$$\left. \times \quad \frac{1}{(-y^2\mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip\cdot y}\ \Gamma(2-\frac{d}{2})\ (\frac{\mu^2}{\rho^2})^{2-d/2} \right]$$

where K = modified Bessel function, Γ = Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v\lambda^2$$

$$\simeq \frac{\alpha_s C_F}{\pi}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left\{ \left[e^{ip\cdot yv} - e^{ip\cdot y} \right]\ \Gamma(2-\frac{d}{2})\ (\frac{4\pi\mu^2}{\rho^2})^{2-d/2} \right.$$

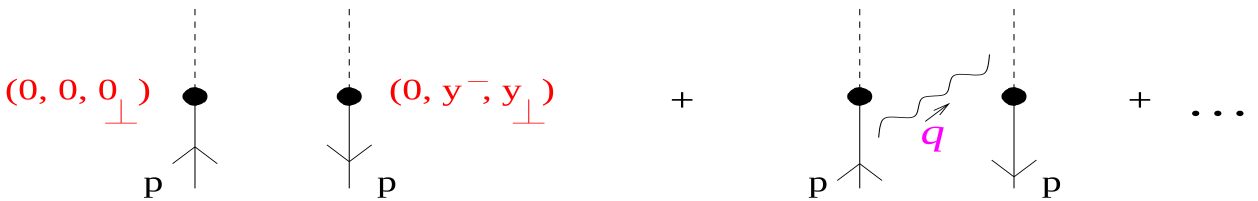
$$\left. + \quad e^{ip\cdot yv}\ \pi^{2-d/2}\ \Gamma(\frac{d}{2}-2)\ (-y^2\mu^2)^{2-d/2} + \cdots \right\} ,$$

▷ $v \to 1$: endpoint singularity

- cancels for ordinary pdf (first term in rhs)
- present, even at $d \neq 4$ and finite ρ, in subsequent terms

◇ Suppose a gluon is absorbed or emitted by eikonal line:

n = (0, 1, $0_\perp$)



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\, \delta(k_\perp) \int dx' dk'_\perp P_R(x', k'_\perp)$$

where $$P_R = \frac{\alpha_s\, C_F}{\pi^2} \left[\frac{1}{1-x}\; \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right]$$ ρ=IR regulator

↑ *endpoint singularity* $(q^+ \longrightarrow 0,\ \forall\ k_\perp)$

◇ Physical observables:

$$\mathcal{O} = \int dx\ dk_\perp\ f_{(1)}(x, k_\perp)\ \varphi(x, k_\perp)$$

$$= \int dx\ dk_\perp\ [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

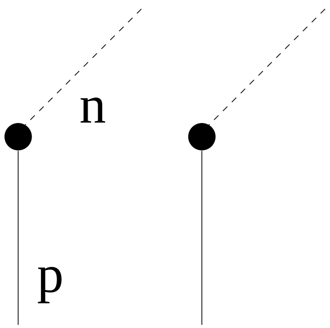
inclusive case: φ independent of $k_\perp$ $\Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences from incomplete KLN cancellation

CUT-OFF REGULARIZATION

▷ cut-off from gauge link in non-lightlike direction n

$\Rightarrow$ analysis of factorization in LO: Collins, Rogers & Stasto, arXiv:0708.2833



$\eta = (p.n)^2 / n^2$

Chen, Idilbi & Ji, 2007

Ji, Ma & Yuan, 2005

Korchemsky & Radyushkin, 1992

Collins, 1989

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim k_\perp / \sqrt{4\eta}$

Drawbacks:

- good for leading accuracy, but makes it difficult to go beyond
- lightcone limits $y^2 \to 0$ and $n^2 \to 0$ do not commute $\Rightarrow$

$$\Rightarrow \int dk_\perp \; f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq \text{ ordinary pdf}$$

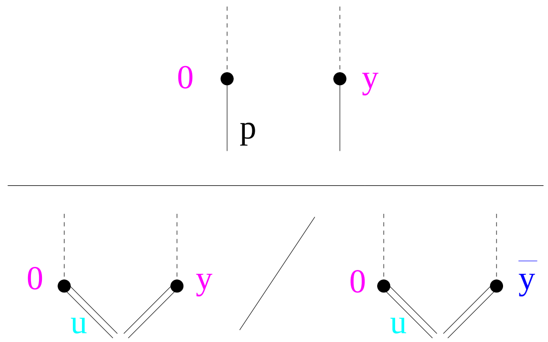
UPDF'S WITH SUBTRACTIVE REGULARIZATION

- subtractive method more systematic than cut-off
- formulation for eikonal-line matrix elements: Collins & H, 2001.

▷ gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) =$

$$\frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \; / \; \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; $u =$ auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$ (drops out of integrated f)

- denominator cancels the endpoint divergence

  (explicit form at one loop: H, hep-ph/0702196)

- counterterms from gauge-invariant operator matrix elements

  Collins, hep-ph/0304122

One loop expansion: $[\zeta = (p^{+2}/2)u^-/u^+]$

$$f^{(\mathrm{subtr})}_{(1)}(x,k_\perp) = P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp) \quad (\leftarrow\text{from numerator})$$

$$- \quad W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta) \quad (\leftarrow\text{from vev's})$$

with $P_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + m^2(1-x)^2)] + \ldots\right\}$ = real emission prob.

$W_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots\right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$

$$\Rightarrow \mathcal{O} = \int dx\ dk_\perp\ f^{(\mathrm{subtr})}_{(1)}(x,k_\perp)\ \varphi(x,k_\perp)$$

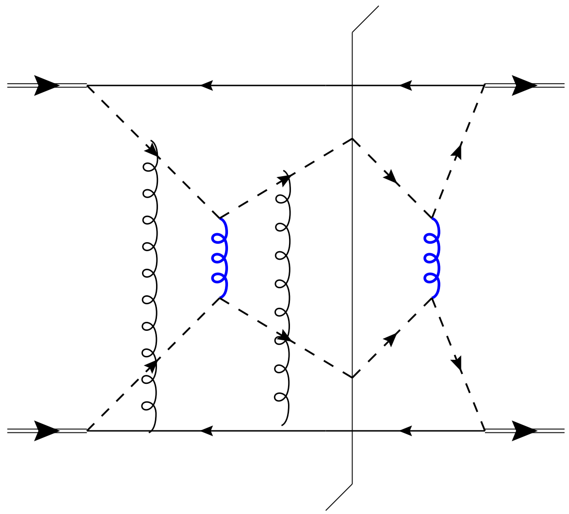
$$= \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp) - \varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp) - \varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

FURTHER ISSUES AT HIGHER ORDER

- soft gluon exchange with “spectator” partons

  ⇒ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

♢ should appear at N^3LO (2 soft, 1 collinear partons)

♢ does it survive destructive interference from soft-color coherence?

- evolution equations for u-pdf's

  (Regge/Sudakov matching, target fragmentation, ...)

Ceccopieri and Trentadue, 2007

Collins and Qiu, 2007

IV. Conclusions

- U-pdf's being proved to be useful tool for simulation of $x \to 0$ parton showers

  ▷ Results from $k_\perp$ shower Monte-Carlo's for small-x multi-jet final states

- Extension of u-pdf's over whole phase space important to turn these Monte-Carlo's into general-purpose tools

- Open issues on factorization, lack of complete KLN cancellation

  $\Rightarrow$ need to address new problems compared to ordinary pdf's

  ▷ subtractive regularization $(x \to 1)$

  - likely more suitable than cut-off for sub-leading issues
  - more transparent relation with OPE and standard pdf's