Fully Unintegrated Parton Correlation Functions and Factorization

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(Work done in collaboration with John Collins and Anna Stasto)

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Overview of PCFs

- Looking at details of final states requires precise kinematics. *(e.g., Watt, Martin, and Ryskin Eur.Phys.J. C31,73 (2003))*

- Exact kinematics forces the use of fully unintegrated parton correlation functions (PCFs) in both initial and final states. *(fully unintegrated PDFs, soft factor, jet factors)*

- Without usual approx., methods of factorization need to be reconsidered.

- Problems even at lowest order.

In This Talk:

- Motivation for deriving a more general factorization formula (using DIS as an example).

- Outline of the main issues involved, and an overview of the basic steps in a derivation.

  (For details, see recent preprint: arXiv:0708:2833)

- Unsolved problems and future work.

Important Distinctions!

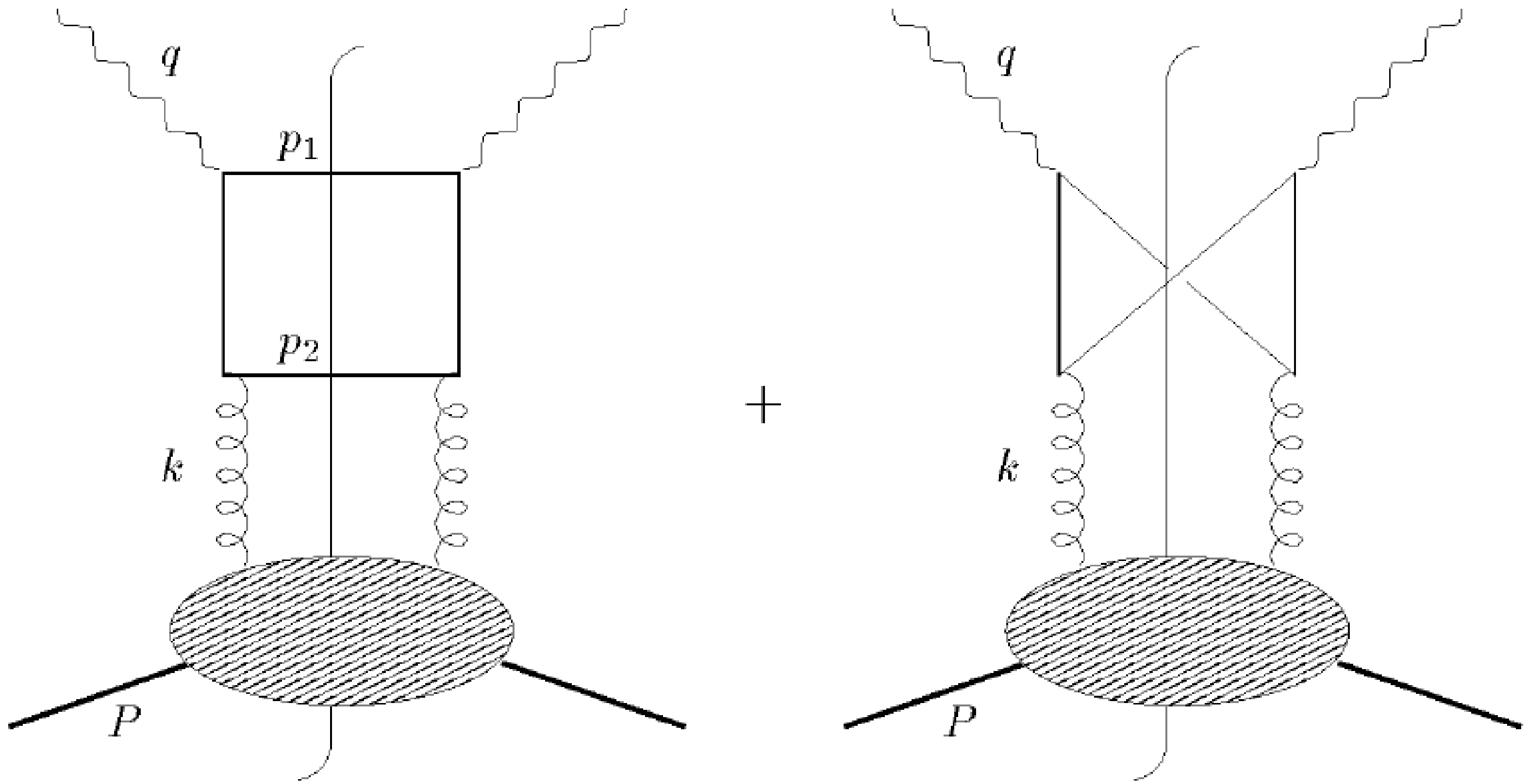
- Integrated PDFs:
  - Standard PDFs of classic LT factorization theorems.
  - Well-known, consistent operator definitions.

- Unintegrated PDFs:
  - Depend on k_T, *but still integrated over invariant energy*.
  - Some consistent operator definitions proposed.

(Collins, 2003 for summary)

- Parton Correlation Functions (Including *Fully Unintegrated* PDFs):
  - Differential in all components of four-momentum.
  - Refers to *fully* unintegrated PDFs as well as jet-factors, and soft factors.

Photon-Gluon Fusion

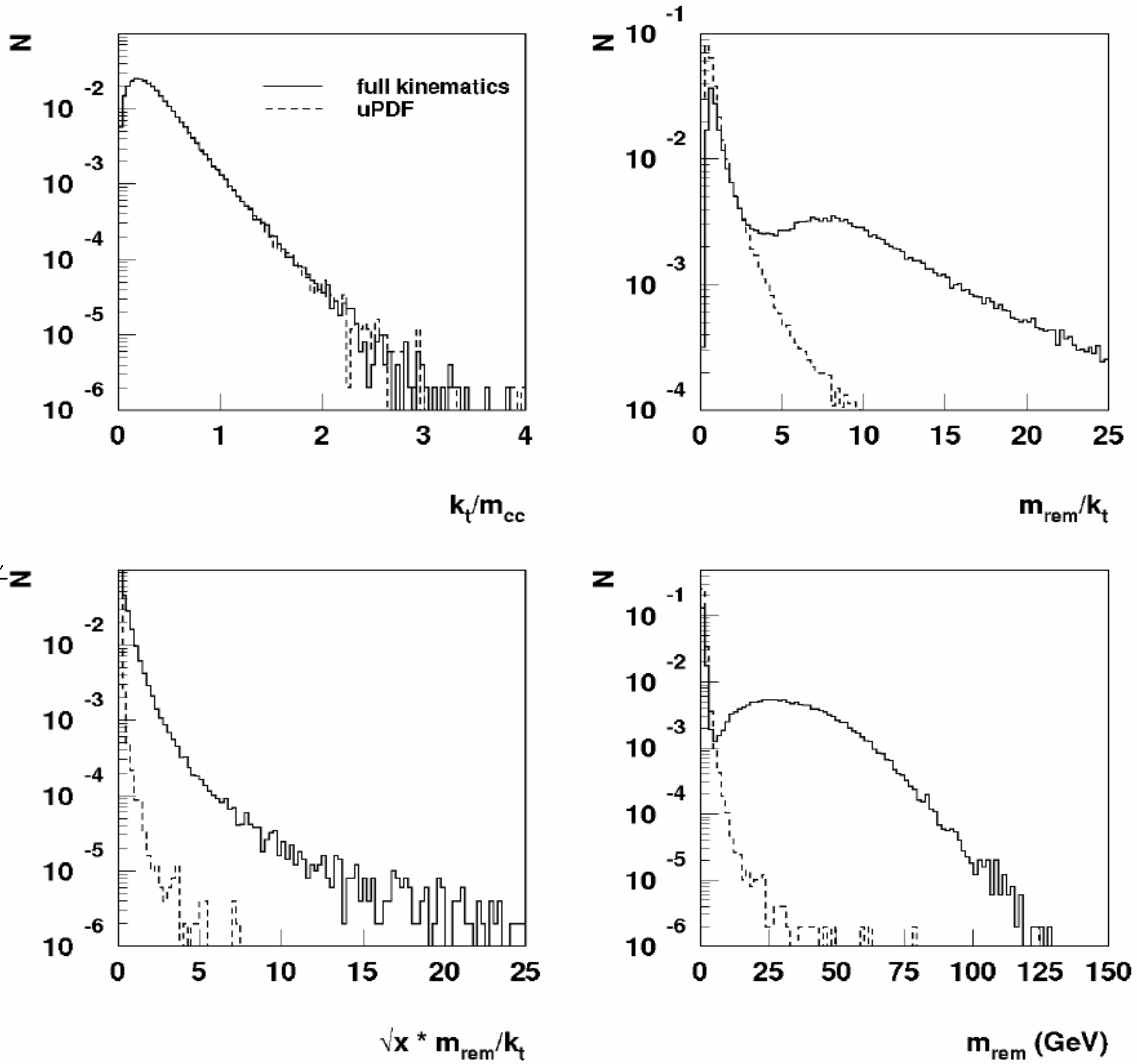


Errors in final state kinematics

$c\bar{c}$

Pair-production

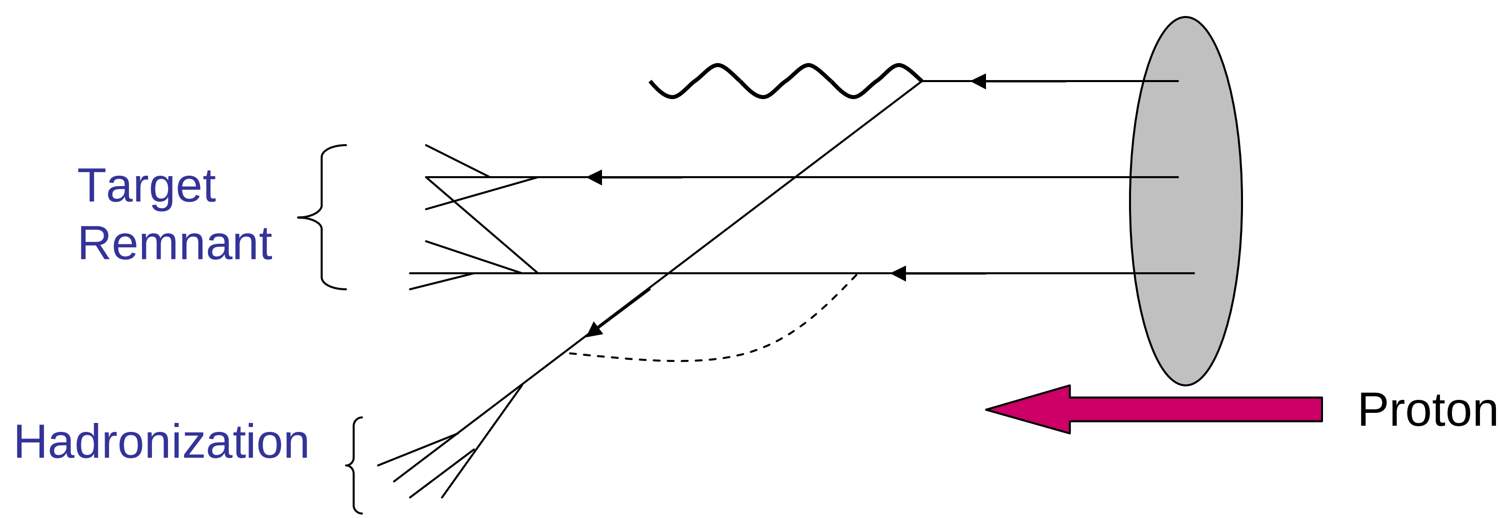
$$k^2 = -\frac{k_T^2 + x m_{rem}^2}{1-x}$$



(Collins and Jung: hep-ph/0508280)

Focus on Leading Order DIS:

Conventional Parton Model Intuition



How to transition to field theory?

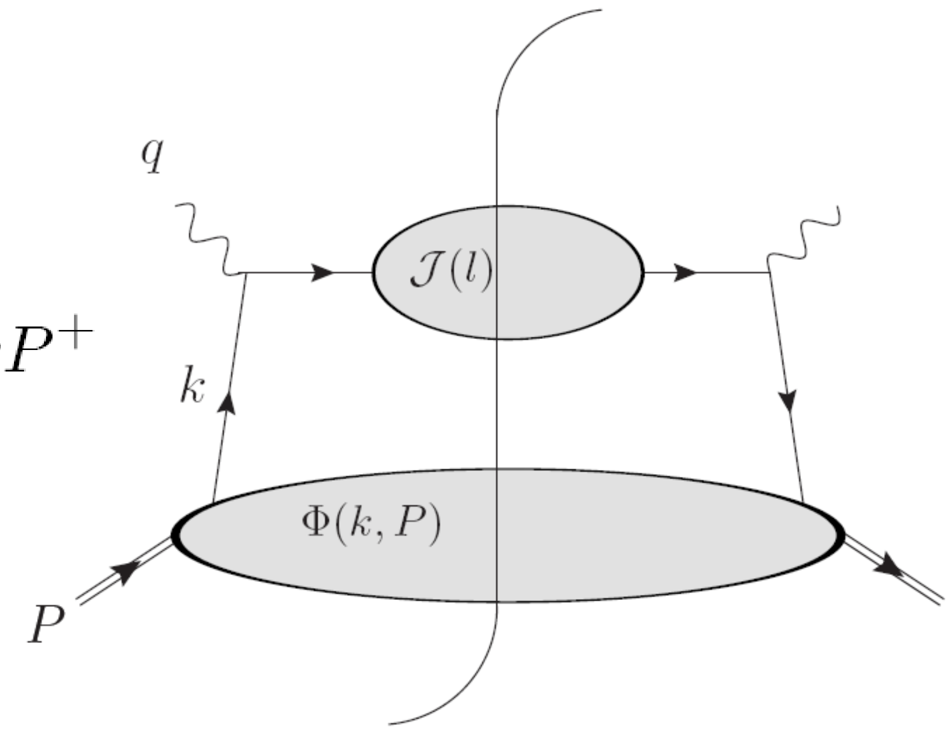
Unapproximated LO graph

$$W^{\mu\nu}(q,P) = \sum_j \frac{e_j^2}{4\pi} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\gamma^\mu \mathcal{J}(k+q)\gamma^\nu \Phi(k,P)]$$

In general, struck quark will hadronize (at least).

- Massless, Collinear approx:

$$k^+ = xP^+ + \frac{m_J^2 + k_T^2}{2(q^- + k^-)} \longrightarrow xP^+$$



Steps to Reproduce Parton Model:

- For performing integrals substitute large components into the bubbles:

$$k \longrightarrow (x_{\rm Bj} P^+, k^-, \mathbf{k}_T)$$
$$l \longrightarrow \left(l^+, \frac{Q^2}{2x_{\rm Bj}P^+}, \mathbf{0}_T\right)$$

- Integrate over small components:

$$W^{\mu\nu}(q,P) \simeq \frac{e_j^2}{4\pi}\underbrace{\left\{\int \frac{dk^- d^2\mathbf{k}_T}{(2\pi)^4}\Phi_j^+(x_{Bj}P^+, k^-, \mathbf{k}_T)\right\}}_{\text{Parton Distribution???}} \underbrace{\mathrm{Tr}\left[\gamma^\mu\gamma^+\gamma^\nu\gamma^-\right]}_{\text{Reproduces Partonic LO structure functions.}} \overbrace{\underbrace{\left\{\int dl^+ \mathcal{J}_j^-(l^+, q^-, \mathbf{0}_T)\right\}}_{\text{Set to one by unitarity argument}}}^{\text{Explicit final state bubble drops out}}$$

Explicit final state bubble drops out

Parton Distribution???

UV divergent – requires renormalization.
Not gauge invariant.

Reproduces Partonic LO structure functions.

Set to one by unitarity argument

Note shift in final state kinematics before and after approx!

The Standard PDFs

Operator definition:

(Reproduces integral form up to c.t.)

$$f_j(x_{\rm Bj},\mu) \;=\; \int \frac{dw^-}{4\pi} e^{-ix_{\rm Bj}p^+w^-} \langle p|\bar{\psi}(0,w^-,\mathbf{0}_T) V_w^\dagger(u_{\rm J})\gamma^+ V_0(u_{\rm J})\psi(0)|p\rangle_R$$

Light-like Wilson lines for gauge invariance:

$$V_w(n) = P\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(w+\lambda n)\right)$$

$$u_J = (0,1,\mathbf{0}_T)$$ → *Light-like!*

$$V_w^\dagger(u_{\rm J})V_0(u_{\rm J}) = P\exp\left(-ig\int_0^{w^-} d\lambda\, u_{\rm J}\cdot A(\lambda u_{\rm J})\right)$$

Complications With Unintegrated (k_T-dependent) PDFs

- Generalization:

$$P(x, \mathbf{k}_T, \mu) \overset{??}{=} \int \frac{dw^- d\mathbf{w}_T}{16\pi^3} e^{-ixp^+ w^- + i\mathbf{k}_T \cdot \mathbf{w}_T} \times \langle p | \bar{\psi}(0, w^-, \mathbf{w}_T) \underbrace{V_w^\dagger(n) I_{n;w,0} \gamma^+ V_0(n)}_{\text{light-like Wilson lines.}} \psi(0) | p \rangle$$

- Wilson line needed to link points at infinity.
  (Belitsky et. al Nucl. Phys.B 656, 165 2003)

- Light-cone divergence in outgoing quark direction!

- Divergence persists when gluon mass and UV cutoff are included.

- Divergence is regulated when non light-like lines are used (but involves practical complications.)

Summary of LO Deeply Inelastic Scattering in the Conventional Treatment:

- There is a re-assignment of final state kinematics.
- Can produce large errors (even with k_T-dependent PDFs).
- Complications with operator definitions of k_T-dependent PDFs.
- Standard kinematical approximations are necessary for reproduction of standard LO DIS expression (parton model).

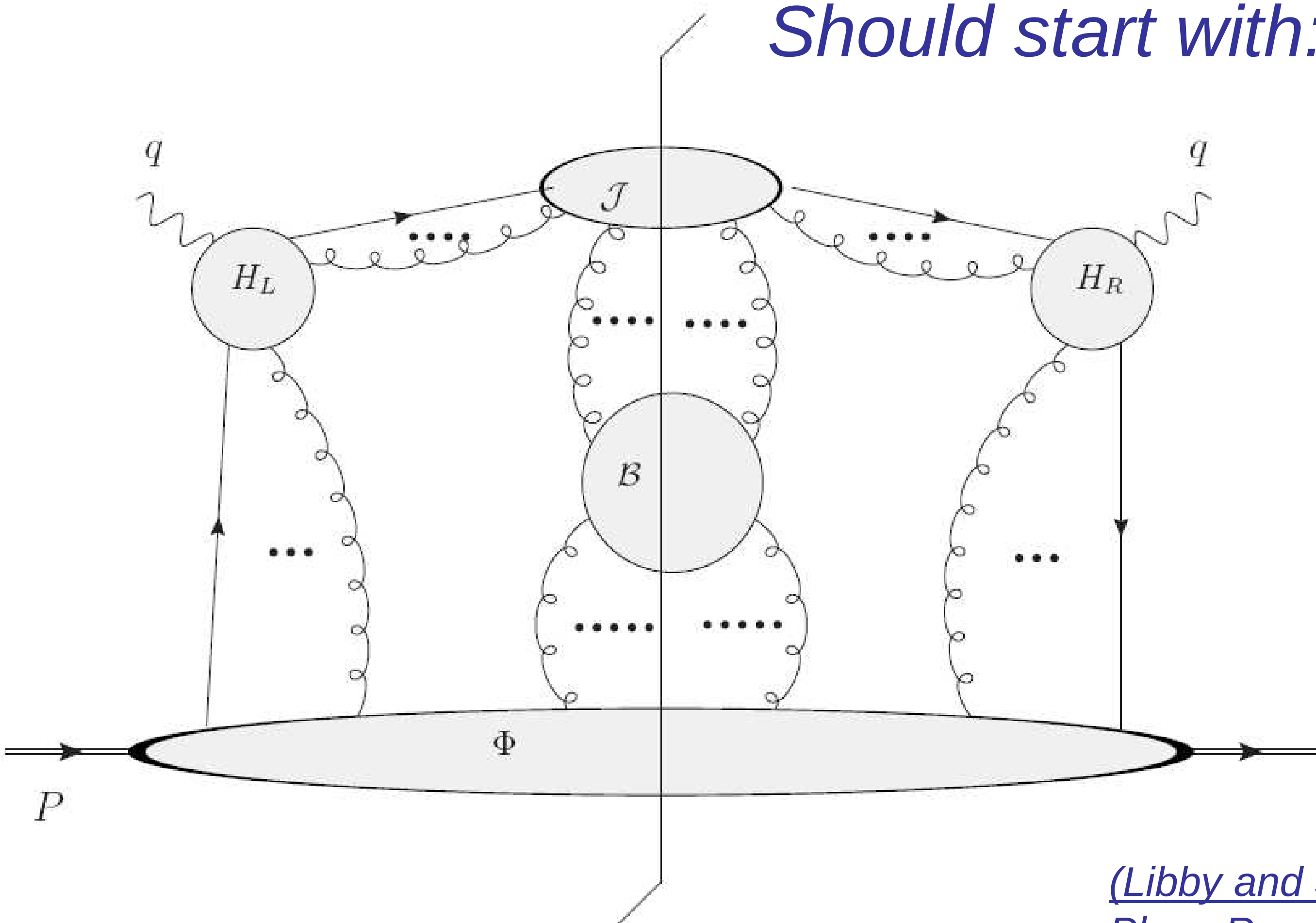
What is Needed?

A generalization of the basic QCD factorization theorem that includes…

- Exact overall kinematics of initial and final states.
- Explicit factors representing final states.
- Accounts for target collinear, jet collinear, and soft gluons.
- NP factors differential in all components of four-momentum.
- Hard scattering calculated with on-shell Feynman graphs.
- Factorization formula with power suppressed corrections.

General Graphical Structure

Should start with:



(Libby and Sterman, Phys. Rev. D18, 4337 (1978))

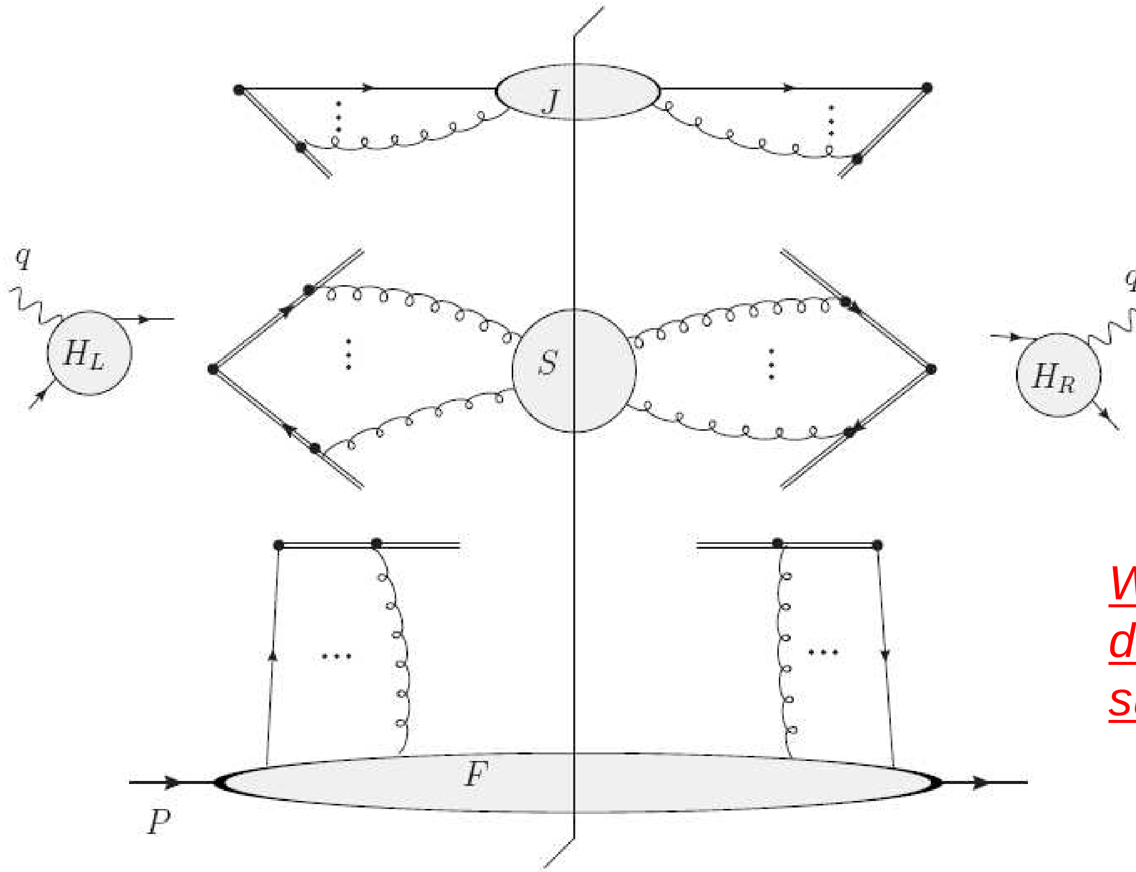
Must disentangle soft and collinear gluons to get topological factorization…

Strategy Overview

- Propose gauge invariant operator definitions of PCFs.
- Consider extra soft/collinear gluon attachments that contribute to the leading region.
  - Characterize regions, R, of gluon momentum.
- Apply *consistent* soft, target collinear, and jet-collinear approximations.
  - No approximations in any the initial or final state bubbles!
- Sum over graphs, Γ, and apply Ward identities to obtain topological factorization.
- Apply double counting subtractions to larger regions.
- Identify PCFs and obtain factorization formula:

$$\sigma = \sum_R \sum_\Gamma C_R \Gamma + _{power\ suppressed\ corrections}$$

Topological Factorization:



We also need double counting subtractions.

A formula of this type is the goal.

$$\sigma = C \otimes F \otimes J \otimes S + \mathcal{O}((\Lambda/Q)^a|\sigma|), \qquad a > 0$$

Defining PCFs

- ❑ Need universal, operator definitions for the PCFs.

- ❑ What are the required features?

- Depend on all components of momentum. (No approximations on kinematics.)

- Wilson lines to enforce gauge invariance should be non-light-like:

$$n_{\rm T} = (1, -e^{-2y_{\rm T}}, \mathbf{0}_T) \qquad n_{\rm J} = (e^{-|2y_J|}, 1, \mathbf{0}_T) \qquad n_s = (-e^{y_s}, e^{-y_s}, \mathbf{0}_T)$$

$$y_{\rm T} >> 0 \qquad\qquad y_{\rm J} << 0$$

- Rapidity variables, y_T and y_J, effectively cut-off rapidity divergences.

- Soft rapidity y_s characterizes boundary between left and right moving partons.

Defining PCFs

Example: Soft factor (coordinate space)

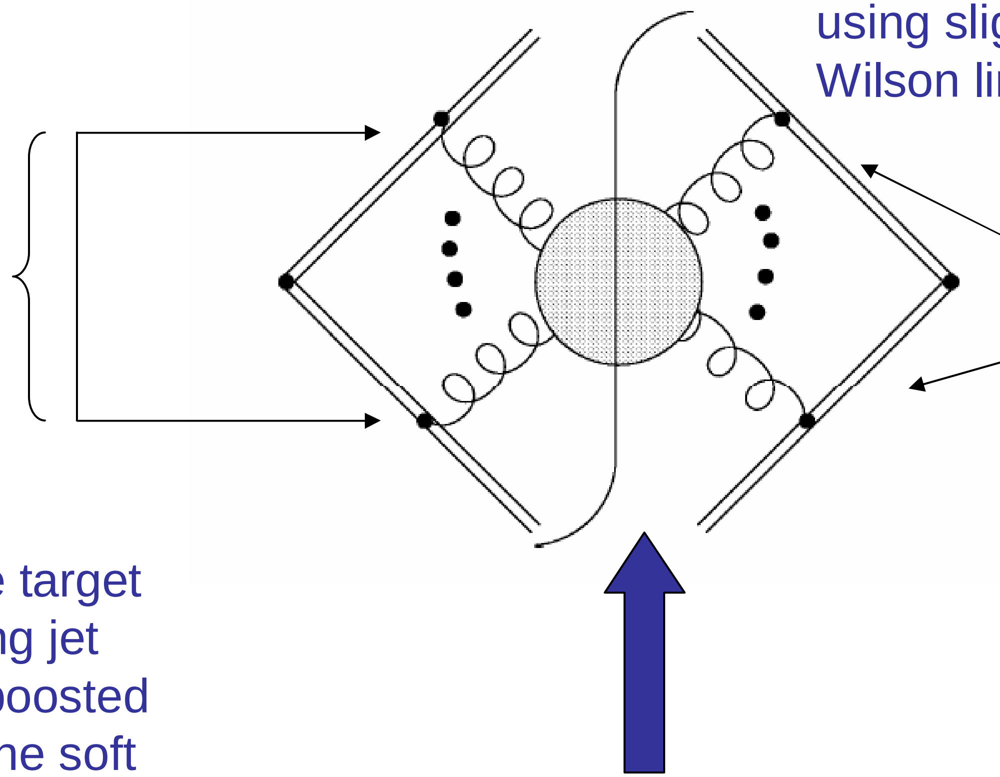
$$\tilde{S}(w, y_{\mathrm{T}}, y_{\mathrm{J}}, \mu) = \langle 0 | I^{\dagger}_{n_{\mathrm{T}};w,0} V_w(n_{\mathrm{T}}) V_w^{\dagger}(n_{\mathrm{J}}) I_{n_{\mathrm{J}};w,0} V_0(n_{\mathrm{J}}) V_0^{\dagger}(n_{\mathrm{T}}) | 0 \rangle_R$$

- Scale μ for renormalization of standard UV divergences.
- Non-light like directions n_T and n_J Wilson lines – y_T and y_J are effectively regulators of the light-cone divergences.
- Fourier transform to momentum space.
- Evolution with y_T and y_J?
- Similar issues involved target PCF and jet PCF. (More complications due to double counting subtractions.)

Anticipate the graphical structure of the soft PCF:

- Cutoff high rapidity gluons by using slightly non-light-like Wilson lines

Eikonal lines near the target and jet directions



$$n_{\rm T} = (1, -e^{-2y_{\rm T}}, \mathbf{0}_T)$$

$$n_{\rm J} = (e^{-|2y_J|}, 1, \mathbf{0}_T)$$

$$y_{\rm T} >> 0,\ y_{\rm J} << 0$$

- Both the target and outgoing jet are highly boosted relative to the soft gluons.

Soft gluons and final state bubble

Full Factorization

$$P_{\mu\nu}W^{\mu\nu} = \int \frac{d^4k_{\rm T}}{(2\pi)^4}\frac{d^4k_{\rm J}}{(2\pi)^4}\frac{d^4k_S}{(2\pi)^4}(2\pi)^4\delta^{(4)}(q+P-k_{\rm T}-k_{\rm J}-k_S)\times$$

$$\times\,|H(Q,\mu)|^2\; S(k_S,y_{\rm T},y_{\rm J},\mu)\; F_{\rm mod}(k_{\rm T},y_p,y_{\rm T},y_s,\mu)\; J_{\rm mod}(k_{\rm J},y_{\rm J},y_s,\mu)$$

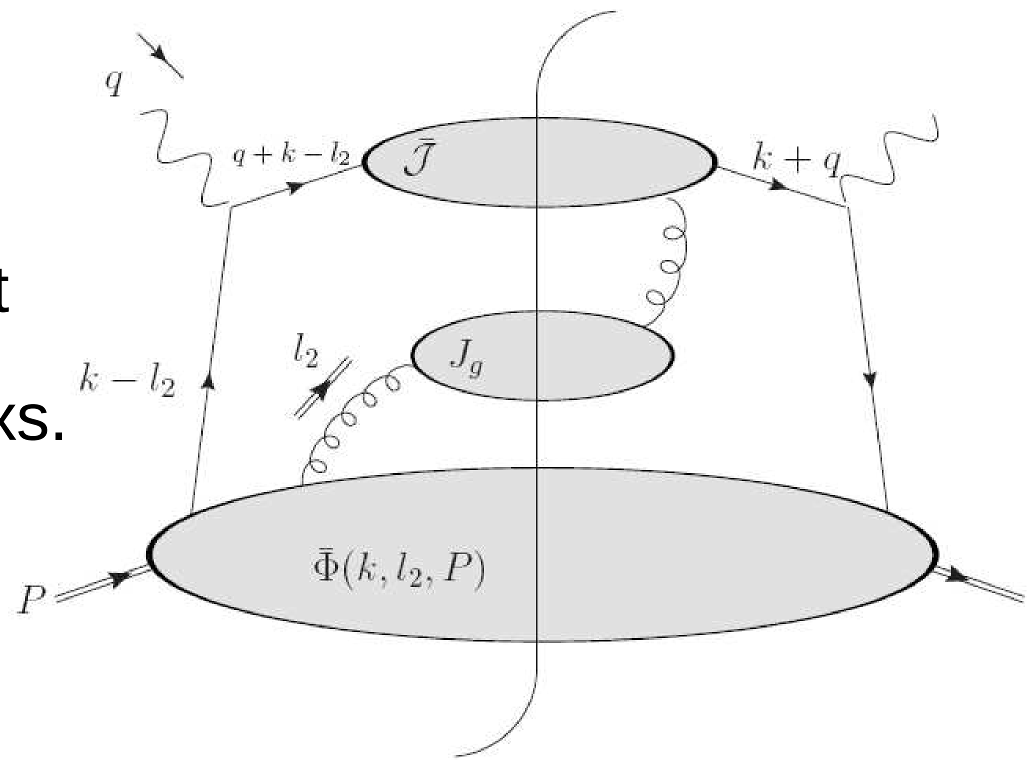
Fourier Transform

$$\tilde{F}_{\rm mod}(w,y_p,y_{\rm T},y_s,\mu) = \frac{\langle p|\bar{\psi}(w)V_w^\dagger(n_s)I_{n_s;w,0}\frac{\gamma^+}{2}V_0(n_s)\psi(0)|p\rangle_R}{\langle 0|I^\dagger_{n_{\rm T};w,0}V_w(n_{\rm T})V_w^\dagger(n_s)I_{n_s;w,0}V_0(n_s)V_0^\dagger(n_{\rm T})|0\rangle_R}$$

$$\tilde{J}_{\rm mod}(w,y_{\rm J},y_s,\mu) = \frac{\langle 0|\bar{\psi}(w)V_w^\dagger(-n_s)I_{-n_s;w,0}\gamma^-V_0(-n_s)\psi(0)|0\rangle_R}{\langle 0|I^\dagger_{-n_s;w,0}V_w(-n_s)V_w^\dagger(n_{\rm J})I_{n_{\rm J};w,0}V_0(n_{\rm J})V_0^\dagger(-n_s)|0\rangle_R}$$

Graphical Example of Factorization

- Analyze soft region -
  make soft approx.
- Analyze target and jet regions -
  make collinear approxs.
- Perform subtractions for target and jet regions.
- Identify contributions to PCFs

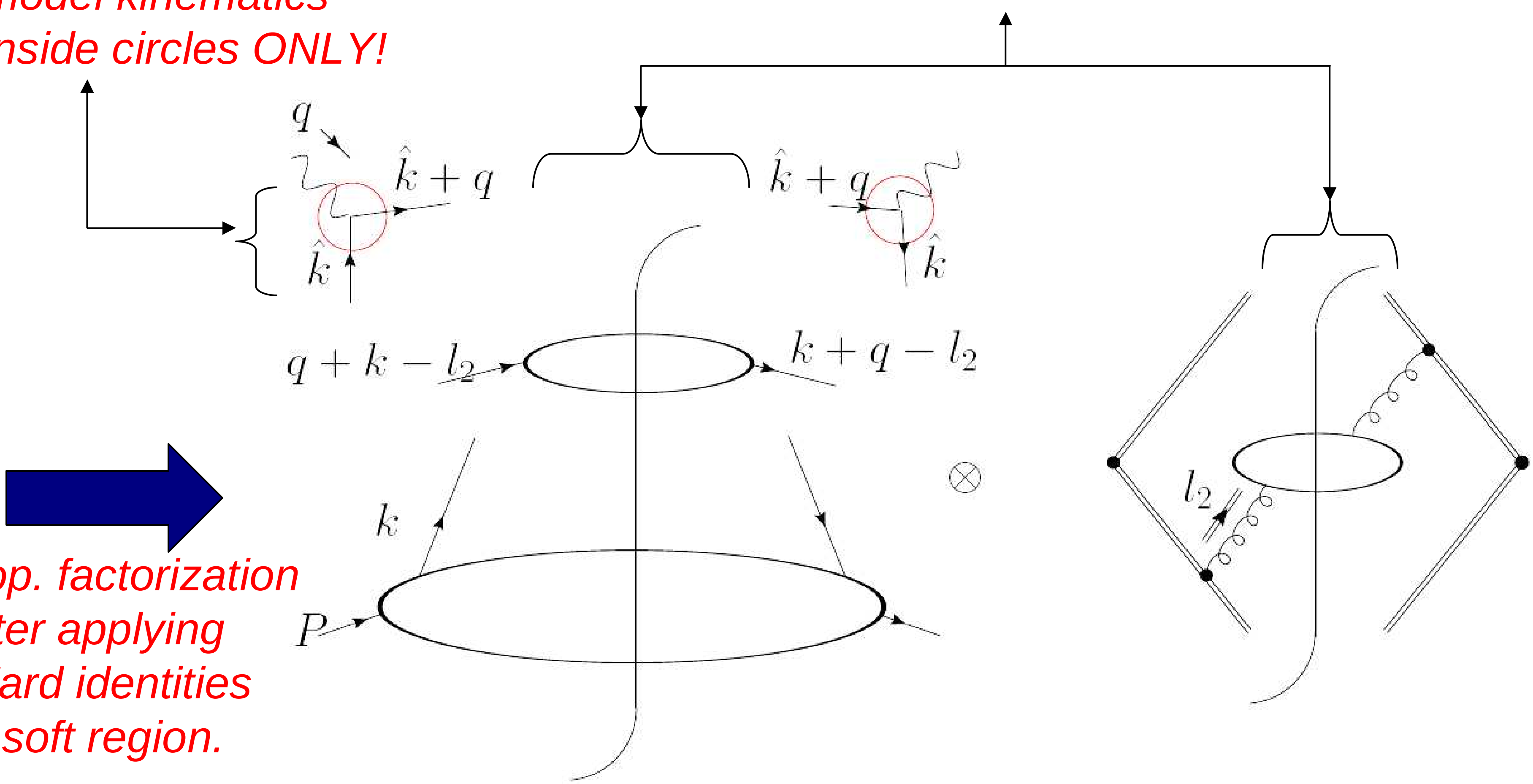


Example: sum graphs with this topology.

Graphical Example (Cont.): Soft Region

Apply parton model kinematics inside circles ONLY!

All bubbles evaluated with exact kinematics.

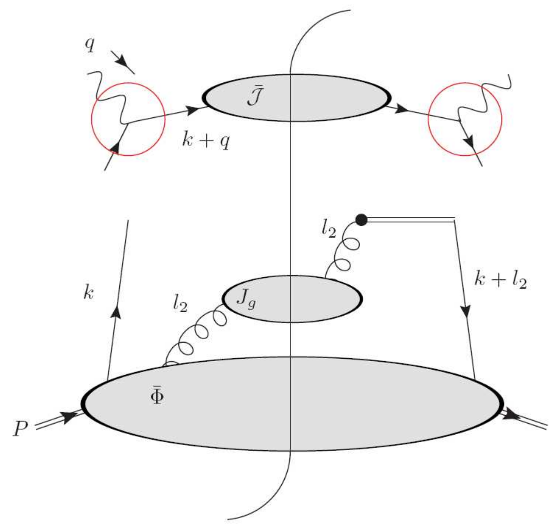


Top. factorization after applying Ward identities in soft region.

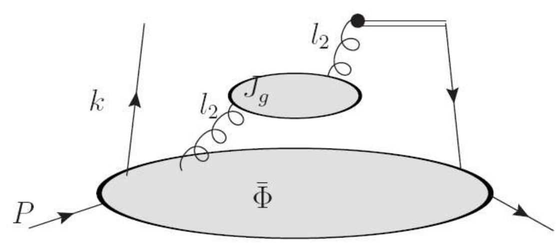
Graphical Example (Cont.): Subtractions

Consider target-collinear region

After applying Ward identities.



$F(k, P) =$



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The contribution to the PDF (target PCF) requires double counting subtractions.

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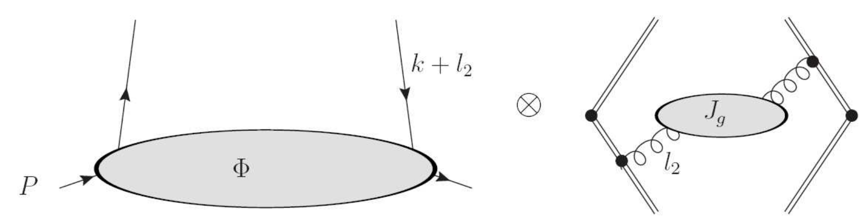


Outlook

- Evolution equations?
  - Relation to CSS formalism?
- Recovery of other approaches in appropriate limits? (e.g. BFKL, CCFM, etc…)
- Factorization for higher order hard scattering?
- Ward identity arguments in non-Abelian case?

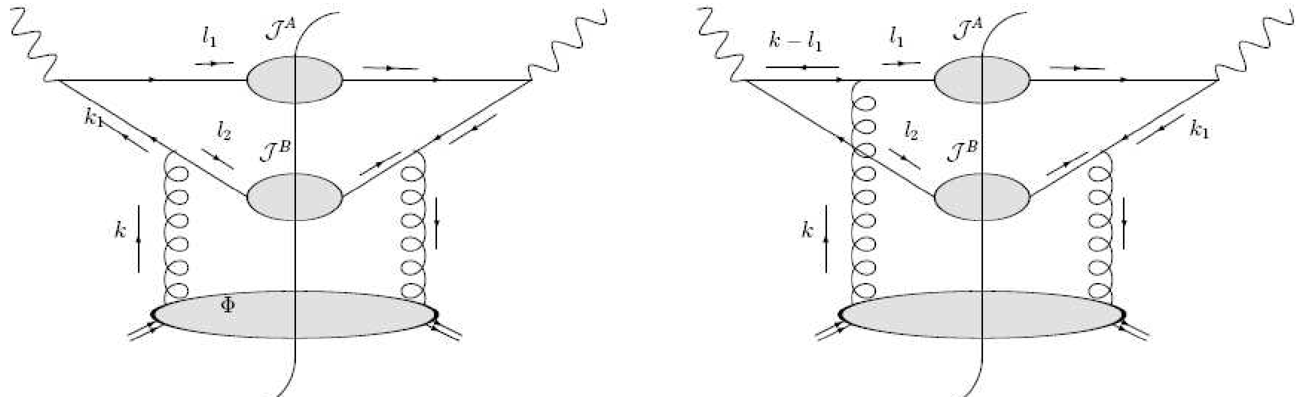
Beyond Leading Order Hard Scattering- Connection to Phenomenology

- We now have well-defined steps for determining higher-order hard scattering.
- Ex: Gluon Induced DIS.

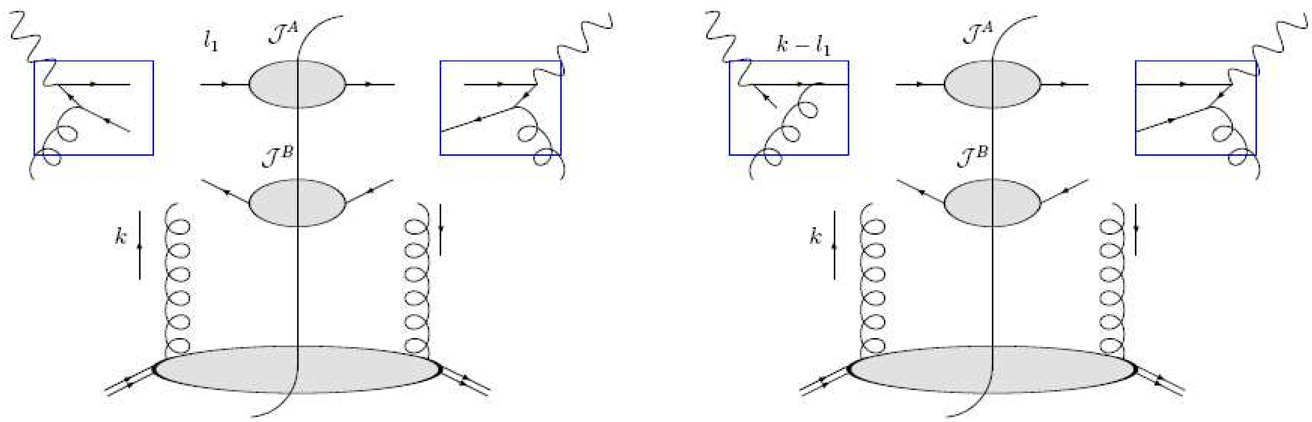
(Work In Progress…)

- Need to define a target gluon PCF.

Gluon Induced DIS: Basic Steps



(Neglecting extra soft and collinear lines for now.)



+ subtraction terms.

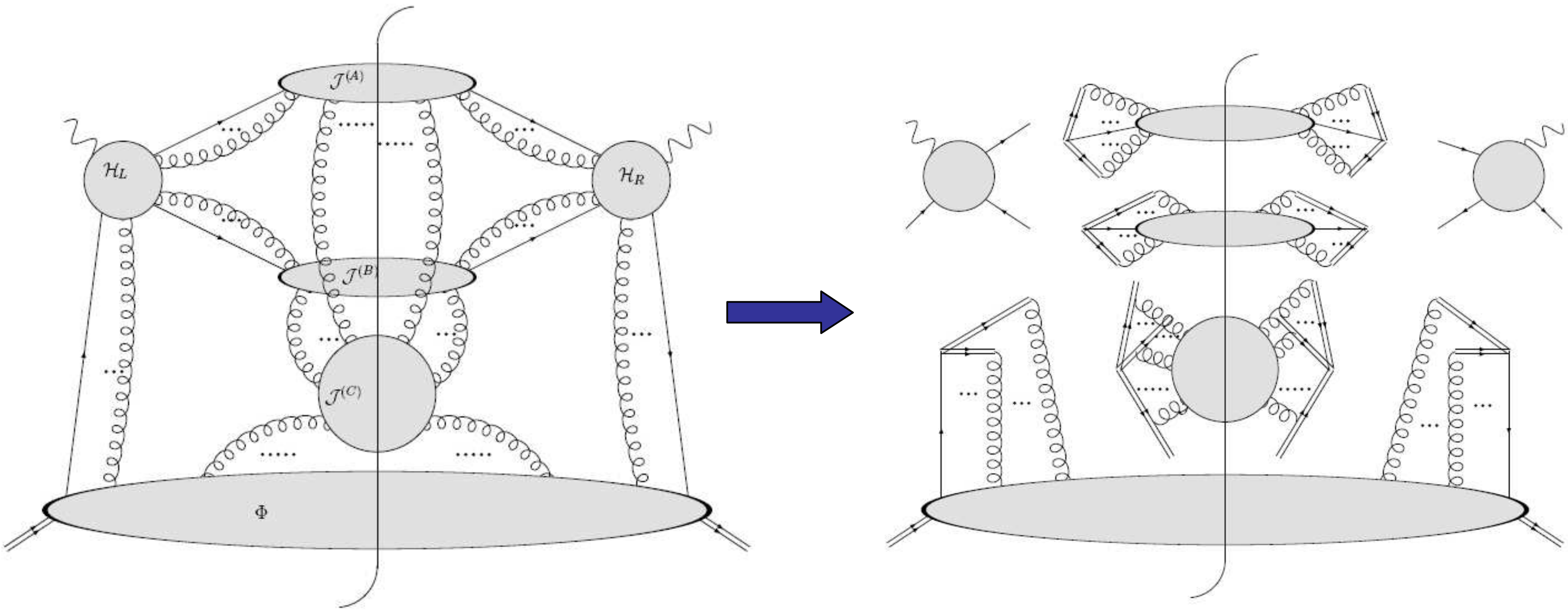
Extract Hard Scattering…

Conclusions:

- Exact kinematics needed. (Unintegrated PDFs not enough.)
  (Basic program outlined for scalar theory by Collins and Zu (2005))
- Requires exact definitions for parton correlation functions.
- We have defined parton correlation functions and derived a factorization formula for the case of an abelian gauge theory.
  (Strongly suggestive of a structure for the non-abelian case.)

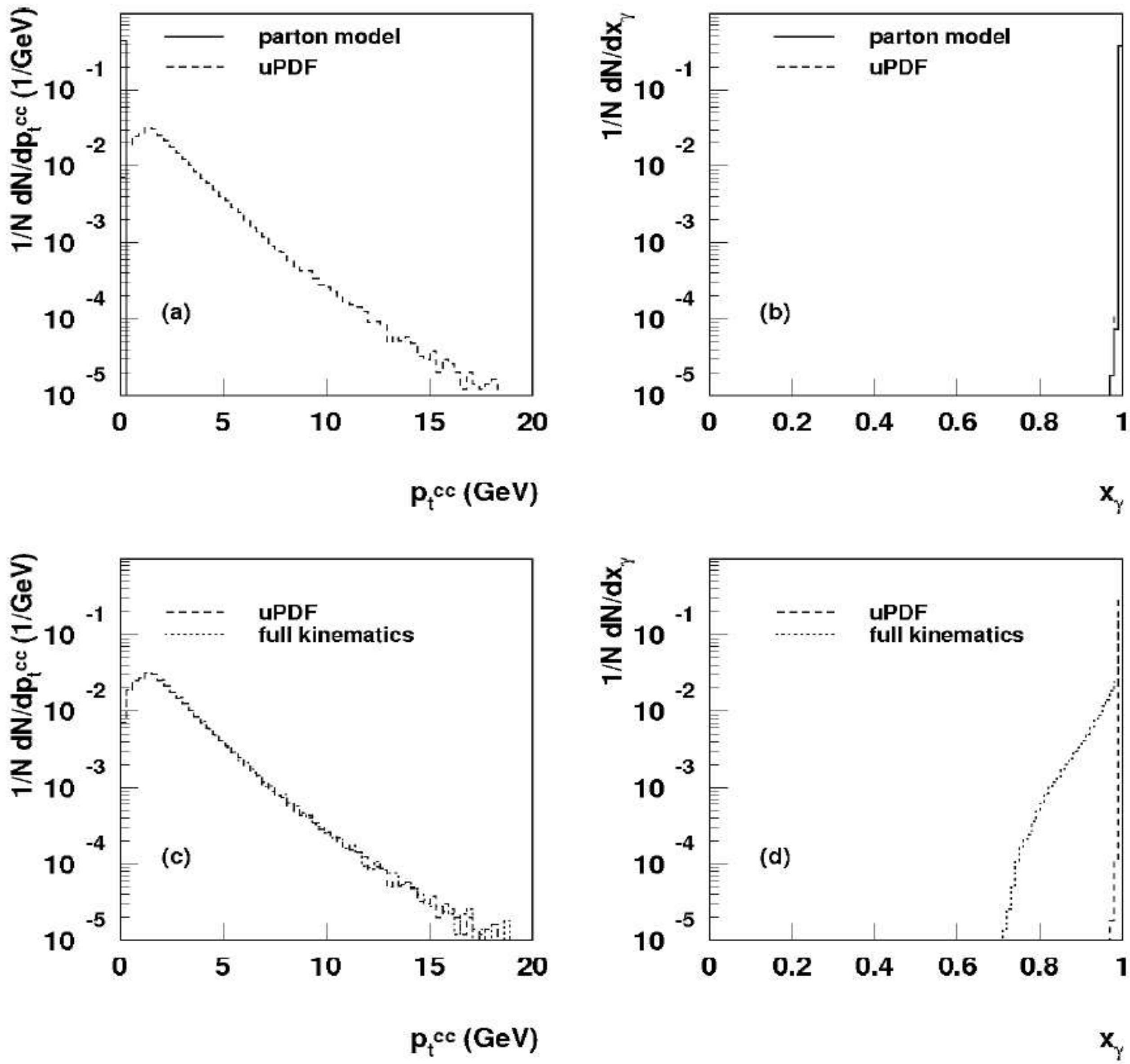
Backup slides

Factorization Beyond LO



Errors in final state kinematics

$c\bar{c}$
Pair-production
In DIS.

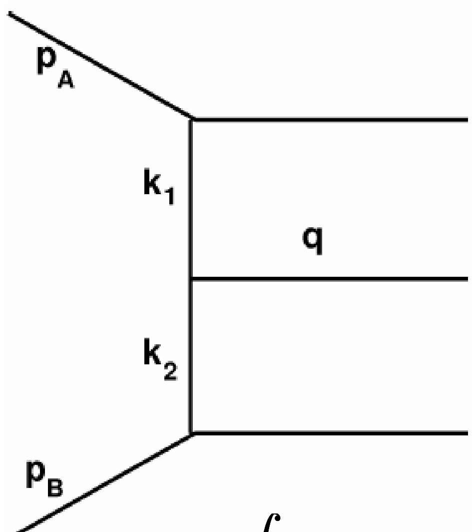


(Collins and Jung: hep-ph/0508280)

Kinematics and Final States

Small-x: Multi-Regge kinematics

(Need for doubly integrated PDFs: Martin, Watt, Ryskin Eur.Phys.J.C31,73(2003))



$$k_1 = (\alpha_1 p_A, \beta_1 p_B, k_{1T}) \;,\; k_2 = (\alpha_2 p_A, \beta_2 p_B, k_{2T})$$

$$0 \ll \alpha_2 \ll \alpha_1 \ll 1, \quad 0 \ll |\beta_1| \ll |\beta_2| \ll 1$$

$$\int d(P.S.) = \frac{1}{4(2\pi)^8 s} \int d^2k_{1T}\, d^2k_{2T} \ln \frac{s}{k_{2T}^2}$$

$$\int d(P.S.) = \frac{1}{4(2\pi)^8 s} \ln \frac{s}{m^2} \int d^2k_{1T}\, d^2k_{2T}$$

Hadronic Mass Scale

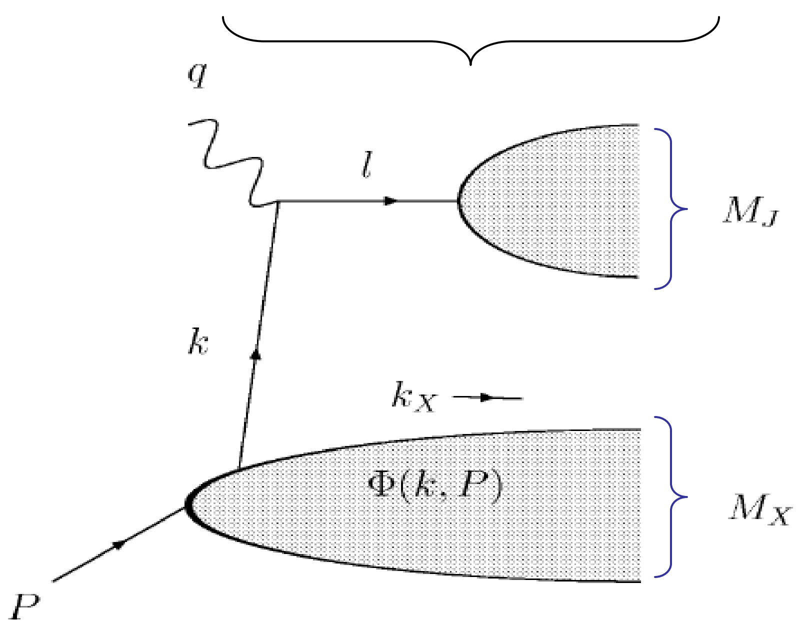
Standard Approx.

Poor for large k_{2T}.

Kinematics and Final States

Large-x

Two jets with fixed invariant energy in final state:



$$s = (1-x)M_p^2 + \frac{Q^2}{x}(1-x)$$

$$\Rightarrow \quad k_T^2 < \frac{(1-x)}{4}M_p^2 + \frac{Q^2}{4x}(1-x)$$

But k_T runs to order Q^2 in the def. of the PDF!

Complications with typical unintegrated PDFs

$$\mathcal{F}(x, k_T^2) \stackrel{??}{=} Q^2 \frac{\partial}{\partial Q^2} xg(x, Q^2) \qquad \text{or,}$$

$$xg(x, Q^2) \stackrel{??}{=} \int_0^{Q^2} \frac{dk^2}{k^2} \mathcal{F}(x, k_T^2)$$

- Positivity: $\mathcal{F}(x, k_T^2) > 0\,?$
- Scale dependence in $\mathcal{F}(x, k_T^2)$?

  (See, e.g., Kimber, Martin Ryskin)
- Consistent operator definitions of UI PDFs?