Higher-order QCD results on splitting functions and coefficient functions

Andreas Vogt (University of Liverpool)

Collaborations with Sven Moch, Mikhail Rogal and Jos Vermaseren

- NNLO timelike quark-quark, gluon-gluon splitting functions
- Top-mediated Higgs decay into hadrons up to N^3LO
- Towards polarised deeply inelastic scattering at NNLO

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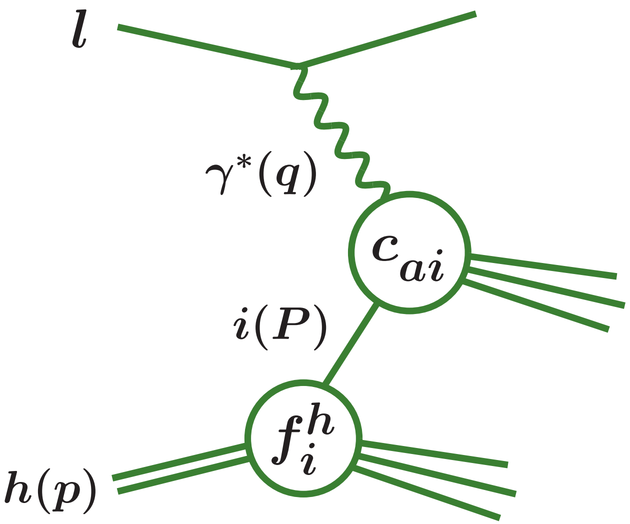
Hard lepton-hadron processes in pQCD (I)

Inclusive deep-inelastic scattering and semi-inclusive l^+l^- annihilation



Left $\longrightarrow$ right: DIS, q spacelike, $Q^2 = -q^2$

$P = \xi p\,,\ f_i^h =$ **parton distributions**

Top $\longrightarrow$ bottom: l^+l^-, q timelike, $Q^2 = q^2$

$p = \xi P\,,$ **fragmentation distributions**

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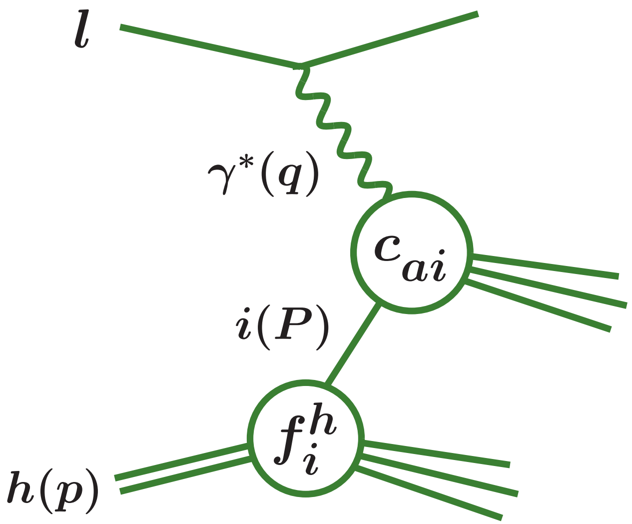

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$p = \xi P\,,$ **fragmentation distributions**

(Un) polarised spacelike / timelike structure functions F_a [up to $\mathcal{O}(1/Q^2)$]

$$F_a^h(x,Q^2) \;=\; \sum_i \left[c_{a,i}(\alpha_s(\mu^2),\mu^2/Q^2) \otimes f_i^h(\mu^2) \right](x)$$

Coefficient fct's: calculation at renormalization / factorization scale $\mu = Q$

Hard lepton-hadron processes in pQCD (II)

Parton / fragmentation distributions f_i : evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}^{S,T}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](\xi)$$

$\otimes$ = Mellin convolution. Initial conditions incalculable in perturbative QCD.

$\Rightarrow$ predictions: fit-analyses of reference processes, universality of $f_i(\xi,\mu^2)$

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Expansion in α_s : splitting functions P, coefficient functions c_a

$$\begin{aligned} P &= \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots \\ c_a &= \underbrace{\alpha_s^{n_a}\Big[c_a^{(0)}}\, + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots \Big] \end{aligned}$$

LO: approximate shape, rough estimate of rate

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Expansion in α_s : splitting functions P, coefficient functions c_a

$$P \;=\; \alpha_s\, P^{(0)} + \alpha_s^2\, P^{(1)} + \alpha_s^3\, P^{(2)} + \ldots$$

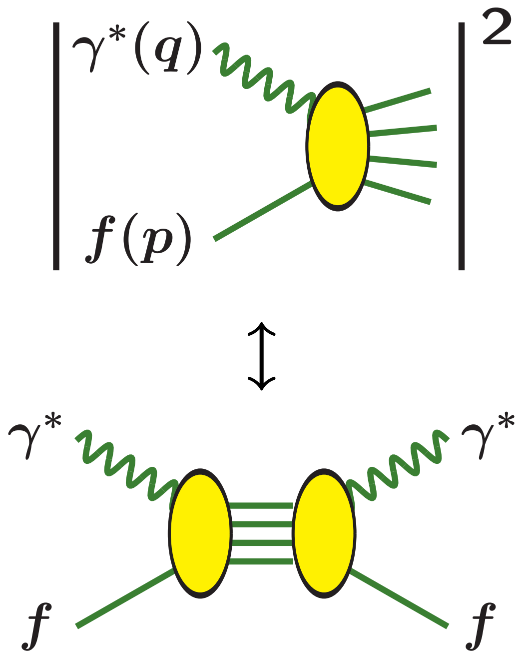
$$c_a \;=\; \underbrace{\alpha_s^{\,n_a}\Big[\, c_a^{(0)} + \alpha_s\, c_a^{(1)}}\; + \alpha_s^2\, c_a^{(2)} + \ldots \Big]$$

NLO: first real prediction of size of cross sections

NNLO, $P^{(2)}$, $c_a^{(2)}$: first serious error estimate of pQCD predictions

Three-loop calculation of unpolarised DIS

Optical theorem: $\gamma^* f$ **total cross sections** $\longleftrightarrow$ **forward amplitudes**



Coefficient of $(2p\cdot q)^N$ $\longleftrightarrow$ N**-th moment** $A^N = \int_0^1 dx\, x^{N-1} A(x)$

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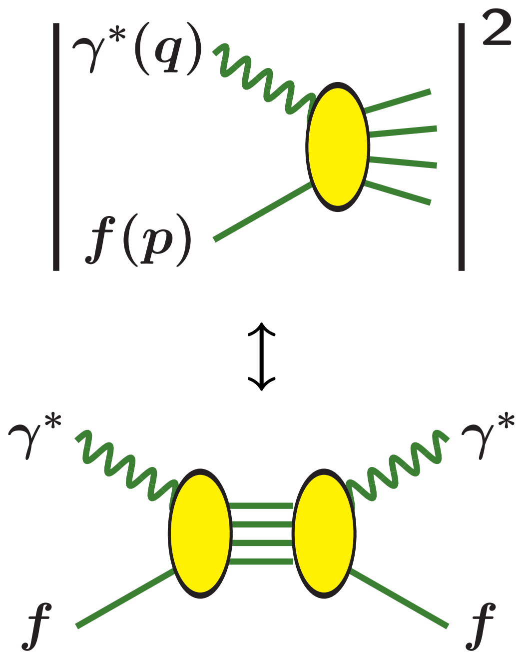

	tree	1-loop	2-loop	3-loop
qγ	1	3	25	359
gγ		2	17	345
hγ			2	56
qW	1	3	32	589
qH		1	23	696
gH	1	8	218	6378
hH		1	33	1184
sum	3	18	350	9607

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$P_{\rm gg},\ P_{\rm gq}$: **DIS by Higgs exchange in heavy-top limit** ($G^a_{\mu\nu} G_a^{\mu\nu}$ **coupling**)

Gluon polarisation sum $\longleftrightarrow$ **diagrams with external ghost** h

From spacelike to timelike quantities (I)

DIS $\longrightarrow$ semi-incl. l^+l^-: crossing, $x \longrightarrow 1/x$ relation for bare tree diagrams

Unrenormalized spacelike Hg structure function $F^{\rm b}_{H,g}$ for $D = 4 - 2\varepsilon$

$$F^{\rm b}_{H,g}(a_{\rm s}^{\rm b}, Q^2) \;=\; \delta(1-x) \,+\, \sum_{n=1} (a_{\rm s}^{\rm b})^n \left(Q^2/\mu^2\right)^{-n\varepsilon} F^{\rm b}_{H,n}$$

Iterative decomposition in Hgg form factors and real-emission parts $\mathcal{R}_n$

$$F^{\rm b}_{H,1} \;=\; 2\,\mathcal{F}_1\,\delta(1-x) + \mathcal{R}_1$$

$$F^{\rm b}_{H,2} \;=\; (2\,\mathcal{F}_2 + \mathcal{F}_1^{\,2})\delta(1-x) + 2\,\mathcal{F}_1\mathcal{R}_1 + \mathcal{R}_2$$

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Analytic cont. of $\mathcal{R}_n$: q^2 [$\rightarrow (i\pi)^k$], phase-space factor $x^{1-2\varepsilon}$, $x \longrightarrow 1/x$

$$\ln(1-x) \;\rightarrow\; \ln(1-x) - \ln x + i\,\pi$$

Curci et al. (80); Floratos et al. (81); Stratmann, Vogelsang (96); . . .

Only $\mathcal{R}_1$ from trees only (same $\mathcal{R}_1^T$ from '$i = 0$'): $\longrightarrow$ 'small' 3-loop problem

From spacelike to timelike quantities (II)

Reassemble for timelike case ($\mathcal{F}_n^T$ known), α_s and $G^a_{\mu\nu} G_a^{\mu\nu}$ renormalization:

Timelike splitting and coefficient functions from mass-factorization relation

$$F_{H,g}^{(1)T} = -\frac{1}{\varepsilon} P_{\rm gg}^{(0)} + c_{H,\rm g}^{(1)T} + \varepsilon\, a_{H,\rm g}^{(1)T} + \varepsilon^2\, b_{H,\rm g}^{(1)T} + \ldots$$

$$F_{H,g}^{(2)T} = \frac{1}{2\varepsilon^2} \left\{ \left(P_{\rm gi}^{(0)} + \beta_0 \delta_{\rm gi} \right) P_{\rm ig}^{(0)} \right\} - \frac{1}{2\varepsilon} \left\{ P_{\rm gg}^{(1)T} + 2 P_{\rm gi}^{(0)} c_{H,\rm i}^{(1)T} \right\} + c_{H,\rm g}^{(2)T} - P_{\rm gi}^{(0)} a_{H,\rm i}^{(1)T} + \varepsilon \left\{ a_{H,\rm g}^{(2)T} - P_{\rm gi}^{(0)} b_{H,\rm i}^{(1)T} \right\} + \ldots$$

$$F_{H,g}^{(3)T} = -\frac{1}{6\varepsilon^3} \left\{ P_{\rm gi}^{(0)} P_{\rm ij}^{(0)} P_{\rm jg}^{(0)} + \ldots \right\} + \frac{1}{6\varepsilon^2} \left\{ 2 P_{\rm gi}^{(0)} P_{\rm ig}^{(1)T} + \ldots \right\} - \frac{1}{6\varepsilon} \left\{ 2 P_{\rm gg}^{(2)T} + 3 P_{\rm gi}^{(1)T} c_{H,\rm i}^{(1)T} + 6 P_{\rm gi}^{(0)} c_{H,\rm i}^{(2)T} - 3 P_{\rm gi}^{(0)} \left(P_{\rm ij}^{(0)} + \beta_0 \delta_{\rm ij} \right) a_{H,\rm j}^{(1)T} \right\} + c_{H,\rm g}^{(3)T} - \frac{1}{2} P_{\rm gi}^{(1)T} a_{H,\rm i}^{(1)T} - P_{\rm gi}^{(0)} a_{H,\rm i}^{(2)T} + \frac{1}{2} P_{\rm gi}^{(0)} \left(P_{\rm ij}^{(0)} + \beta_0 \delta_{\rm ij} \right) b_{H,\rm j}^{(1)T} + \ldots$$

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&\quad + c_{H,\rm g}^{(2)T} - P_{\rm gi}^{(0)} a_{H,\rm i}^{(1)T} + \varepsilon \left\{ a_{H,\rm g}^{(2)T} - P_{\rm gi}^{(0)} b_{H,\rm i}^{(1)T} \right\} + \ldots \\
F_{H,g}^{(3)T} &= -\frac{1}{6\varepsilon^3} \left\{ P_{\rm gi}^{(0)} P_{\rm ij}^{(0)} P_{\rm jg}^{(0)} + \ldots \right\} + \frac{1}{6\varepsilon^2} \left\{ 2 P_{\rm gi}^{(0)} P_{\rm ig}^{(1)T} + \ldots \right\} \\
&\quad - \frac{1}{6\varepsilon} \left\{ 2 P_{\rm gg}^{(2)T} + 3 P_{\rm gi}^{(1)T} c_{H,\rm i}^{(1)T} + 6 P_{\rm gi}^{(0)} c_{H,\rm i}^{(2)T} - 3 P_{\rm gi}^{(0)} \left(P_{\rm ij}^{(0)} + \beta_0 \delta_{\rm ij} \right) a_{H,\rm j}^{(1)T} \right\} \\
&\quad + c_{H,\rm g}^{(3)T} - \frac{1}{2} P_{\rm gi}^{(1)T} a_{H,\rm i}^{(1)T} - P_{\rm gi}^{(0)} a_{H,\rm i}^{(2)T} + \frac{1}{2} P_{\rm gi}^{(0)} \left(P_{\rm ij}^{(0)} + \beta_0 \delta_{\rm ij} \right) b_{H,\rm j}^{(1)T} + \ldots
\end{aligned}
$$

Products = convolutions, performed via N-space using FORM   Vermaseren

Two-loop 'off-diagonal' quantities like $c_{H,\rm q}^{(2)T}$: direct cont. of $F_{H,q}^{\rm b}$ [with i]

Timelike results and checks

Second order including ε^2 terms, all cases: agreement with known results

$\gamma^* q, g$: Rijken, van Neerven (96, ε^0); Mitov, Moch (06, ε^1). $H q, g$: see below

Three-loop splitting functions: $P_{\rm ps}^{(2)T}$; $P_{\rm gg}^{(2)T}$ up to coeff. of $C_A^3 \zeta_2 \ln^2 x \, p_{\rm gg}(x)$

(expected, cf. non-singlet case). Fixed by $n_f = 0$ momentum sum rule (MSR)

Confirmed by extending NS approach of Dokshitzer, Marchesini, Salam (05)

$$P_{\rm gg}^{(2)T-S}\Big|_{C_A^k n_f^{3-k}} \; = \; 2 \left[\left\{ \ln x \cdot P_{\rm av.}^{(1)} \right\} \otimes P_{\rm gg}^{(0)} + \left\{ \ln x \cdot P_{\rm gg}^{(0)} \right\} \otimes P_{\rm av.}^{(1)} \right]_{C_A^k n_f^{3-k}}$$

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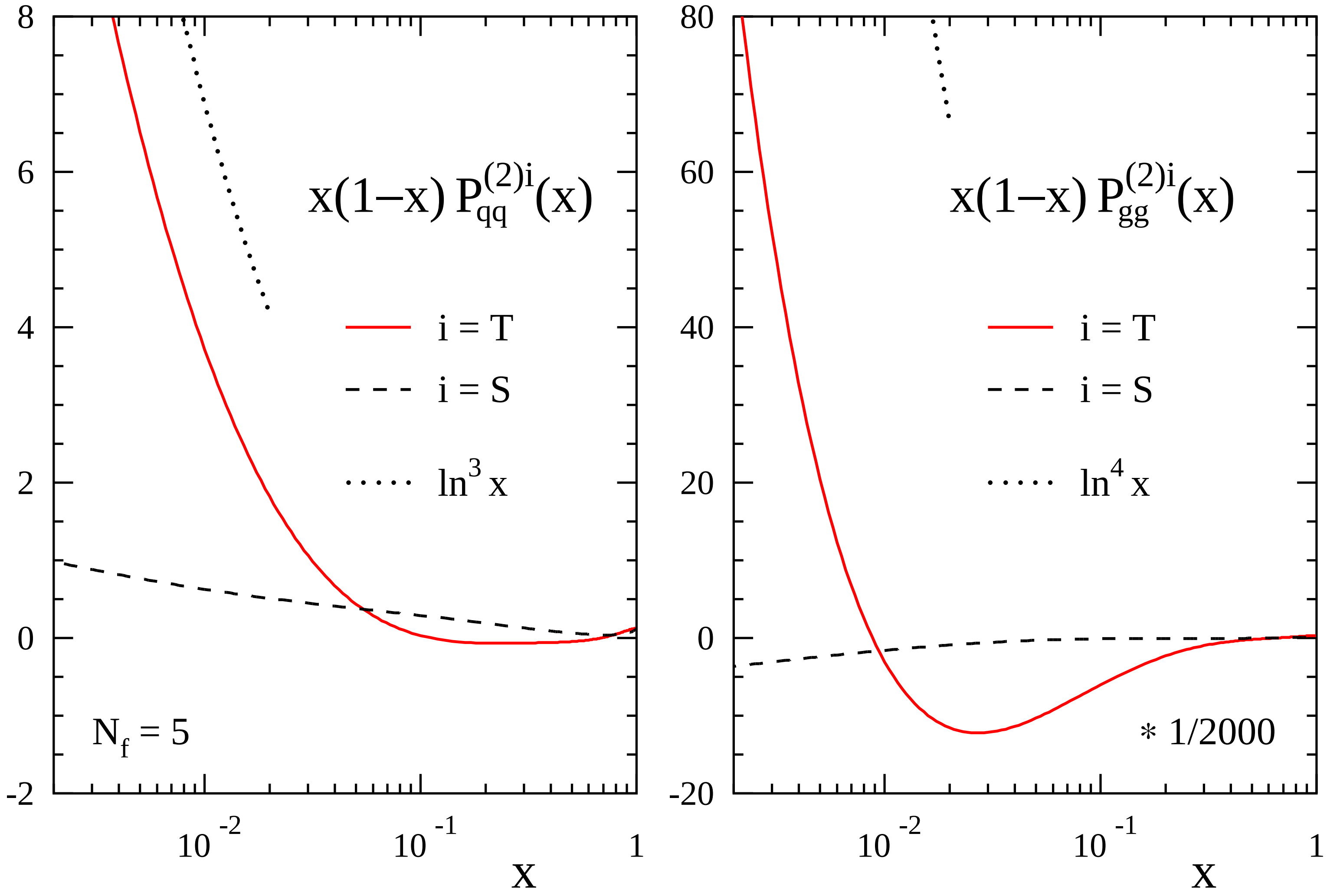
$P_{\rm qg}^{(2)T}$ and $P_{\rm gq}^{(2)T}$ except for ζ_2 terms $\Rightarrow$ checks by non-ζ_2 parts of MSR

MSR fixes ζ_2 in second moments: $P_{\rm qg}^{(2)T}$, $P_{\rm gq}^{(2)T}(N=2)$ completely known

Leading double logs $x^{-1} \ln^4 x$ agree with Mueller (81); Bassetto et al. (82)

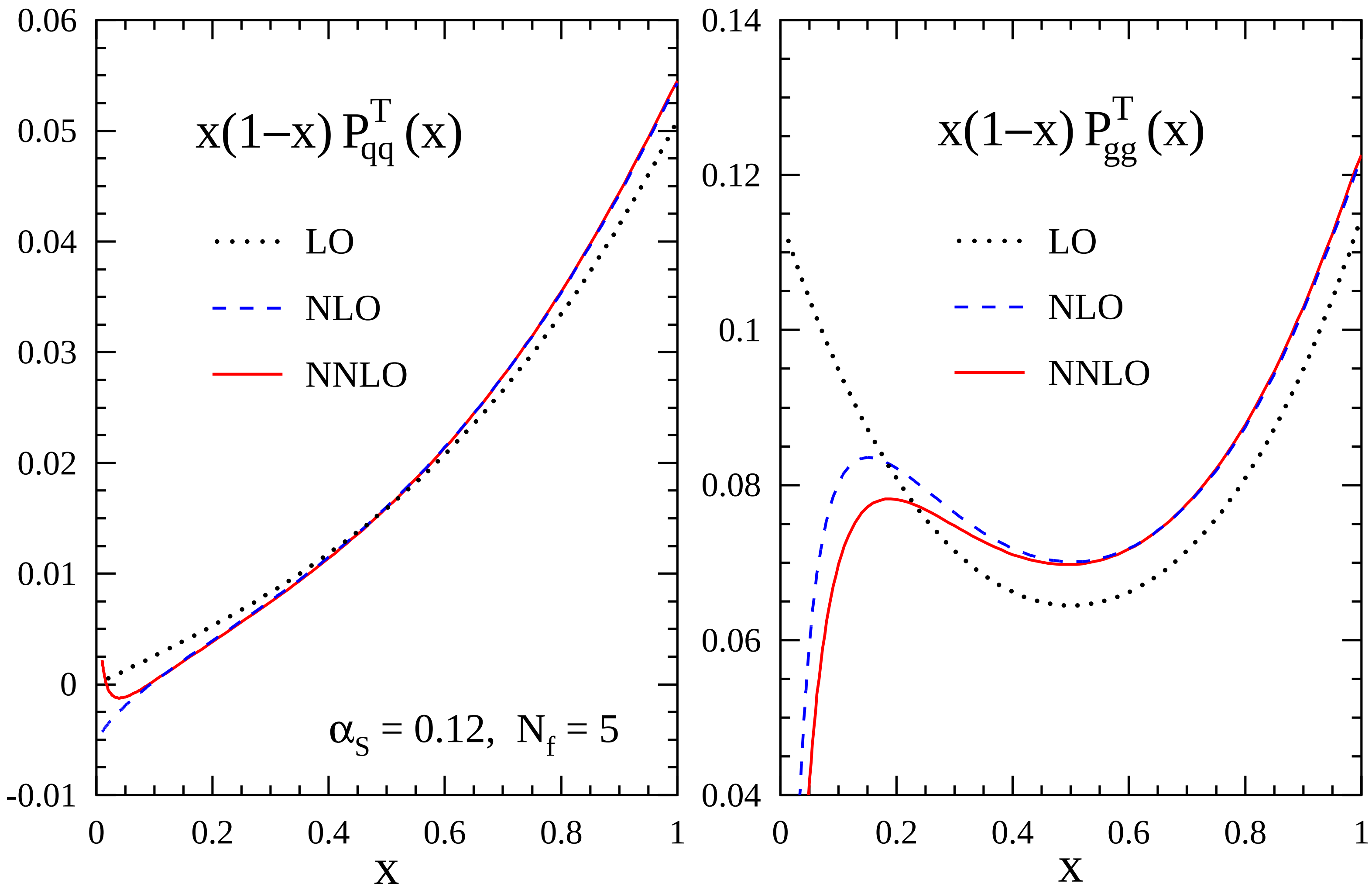
Three-loop coefficient functions: $c_{H\,\rm q,g}^{(3)T}$ except for ζ_2 terms, more below

Third-order diagonal splitting functions



T : extreme small-x rise from $x \approx 10^{-2}$, in gg despite huge cancellations

Perturbative expansion for the timelike case



P^T_{ff} stable for $x \gtrsim 0.1$, Mellin convolutions: wider range of applicability

NNLO timelike splitting functions at $N=2$

$$P_{\mathrm{qq}}^{(2)T}(N=2) \;=\; -P_{\mathrm{gq}}^{(2)T}(N=2) \;=\; -\,C_F^3\left(\frac{54556}{243}-\frac{7264}{27}\,\zeta_2-320\,\zeta_3+256\,\zeta_2^2\right)$$
$$-\,C_F^2C_A\left(\frac{6608}{243}-\frac{2432}{9}\,\zeta_2+\frac{2464}{9}\,\zeta_3-\frac{128}{3}\,\zeta_2^2\right)-\,C_FC_A^2\left(\frac{20920}{243}+\frac{64}{3}\,\zeta_3\right)$$
$$-\,C_FC_An_f\left(\frac{55}{81}+\frac{296}{27}\,\zeta_2-\frac{512}{9}\,\zeta_3\right)-\,C_F^2n_f\left(\frac{2281}{81}-\frac{32}{9}\,\zeta_2+\frac{64}{9}\,\zeta_3\right)$$
$$P_{\mathrm{gg}}^{(2)T}(N=2) \;=\; -P_{\mathrm{qg}}^{(2)T}(N=2) \;=\; -\,C_A^2n_f\left(\frac{6232}{243}-\frac{2132}{27}\,\zeta_2-\frac{128}{9}\,\zeta_3+\frac{160}{3}\,\zeta_2^2\right)$$
$$+\,C_An_f^2\left(\frac{2}{27}-\frac{160}{27}\,\zeta_2+\frac{64}{9}\,\zeta_3\right)-\,C_AC_Fn_f\left(\frac{2681}{243}-\frac{760}{27}\,\zeta_2+\frac{56}{9}\,\zeta_3\right)$$
$$-\,C_F^2n_f\left(\frac{10570}{243}-\frac{352}{27}\,\zeta_2-\frac{32}{9}\,\zeta_3\right)-\,C_Fn_f^2\left(\frac{41}{9}-\frac{128}{27}\,\zeta_2\right)$$

Numerical for QCD with $n_f = 5$ flavours: benign perturbative expansions

$$P_{\mathrm{gq}}^{T}(N=2,\ n_f=5) \;\simeq\; 8\alpha_s/(9\pi)\ \big(1\,-\,0.687\,\alpha_s\,+\,0.447\,\alpha_s^2\,+\,\ldots\big)$$
$$P_{\mathrm{qg}}^{T}(N=2,\ n_f=5) \;\simeq\; 5\alpha_s/(6\pi)\ \big(1\,-\,1.049\,\alpha_s\,+\,1.163\,\alpha_s^2\,+\,\ldots\big)$$

NNLO complete for $N=2$ analyses of incl. single-hadron production in e^+e^-

Top-mediated Higgs decay into hadrons

$c_{H\mathrm{q,g}}^{(n)T}$: N^{n}LO coeff. for single-hadron inclusive Higgs decay (large m_t limit)

$$\Rightarrow \quad (\,C^{\,T}_{\phi,q} + C^{\,T}_{\phi,g}\,)(N=2) \;=\; 1 + a_{\mathrm{s}}\,c_{\phi}^{(1)} + a_{\mathrm{s}}^{2}c_{\phi}^{(2)} + a_{\mathrm{s}}^{3}c_{\phi}^{(3)} + \ldots$$

enters decay rate, with $\mathcal{L}_{\mathrm{eff}}$ prefactor known to N^{3}LO Chetyrkin et al. (97)

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Two- and three-loop expansion coefficients from analytic continuation

$$\begin{aligned} c^{(2)}_{\phi} \;&=\; C_A^2 \left(\frac{37631}{54} - \frac{242}{3}\,\zeta_2 - 110\,\zeta_3 \right) - C_A n_f \left(\frac{6665}{27} - \frac{88}{3}\,\zeta_2 + 4\,\zeta_3 \right) \\ &\quad - C_F n_f \left(\frac{131}{3} - 24\,\zeta_3 \right) + n_f^2 \left(\frac{508}{27} - \frac{8}{3}\,\zeta_2 \right) \\ c^{(3)}_{\phi} \;&=\; f(\zeta_2) + C_A^3 \left(\frac{15420961}{729} - \frac{178156}{27}\,\zeta_3 + \frac{3080}{3}\,\zeta_5 \right) + C_A n_f^2 \left(\frac{413308}{243} + \frac{56}{9}\,\zeta_3 \right) \\ &\quad - C_A^2 n_f \left(\frac{2670508}{243} - \frac{9772}{9}\,\zeta_3 + \frac{80}{3}\,\zeta_5 \right) - C_F C_A n_f \left(\frac{23221}{9} - 1364\,\zeta_3 - 160\,\zeta_5 \right) \\ &\quad + C_F^2 n_f \left(\frac{221}{3} - 320\,\zeta_5 + 192\,\zeta_3 \right) + C_F n_f^2 \,(440 - 240\,\zeta_3) - n_f^3 \left(\frac{57016}{729} - \frac{64}{27}\,\zeta_3 \right) \end{aligned}$$

Non-trivial check of Chetyrkin et al. (97’); Baikov, Chetyrkin (06) despite $f(\zeta_2)$

Two-loop calculations of polarised DIS

Q : fractions of proton spin carried by quarks, gluons, angular momentum?

Helicity-dependent splitting functions ΔP and coefficient functions for g_1

- **Structure function g_1 analogous to $F_{2,3,L}$: $\Delta P^{(1)}_{\rm qq}, \ \Delta P^{(1)}_{\rm qg}, \ c^{(2)}_{g_1,\,{\rm q/g}}$**

  Zijlstra, van Neerven (93) [Errata 97, 07]

  γ_5: Larin scheme $\Leftrightarrow$ 't Hooft, Veltman (72); Breitenlohner, Maison (77)

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- All NLO splitting functions $\Delta P^{(1)}_{\mathrm{ff'}}$ using the OPE

  Mertig, van Neerven (95) [beware of hep-ph version]

  γ_5: reading-point method, Kreimer [et al.] (90 - 94)

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- All NLO splitting functions $\Delta P^{(1)}_{\mathrm{ff'}}$ using axial gauge   Vogelsang (95/6)

  γ_5: direct HVBM scheme, checks with Larin and reading point

Usually add. renormalization/factorization required, cf. Matiounine et al. (98)

Towards the polarised α_s^3 splitting functions

Programme: follow our three-loop calculation of unpolarised case but with

- partly more complicated external-line projectors, e.g., $g\gamma$: $\varepsilon_{\kappa\lambda pq}\varepsilon_{\mu\nu pq}$
- more diagrams (cf. qW vs. $q\gamma$ above), 900 three-loop diagrams for $g\gamma$
- new integrals, despite the existing database with $\mathcal{O}(10^5)$ entries
- a pseudoscalar (Higgs) coupling to $\widetilde{G}^a_{\mu\nu} G^{\mu\nu}_a$ for $\Delta P^{(2)}_{gq}$, $\Delta P^{(2)}_{gg}$

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Present status: so far running $q\gamma$ and $g\gamma$: 2 loops done, 3 loops in progress

- NLO splitting functions $\Delta P^{(1)}_{\rm qq,qg}$ correct (Larin γ_5 + Vogelsang fact. trf.)
- NNLO coefficient functions $c^{(2)}_{g_1,\,\rm q/g}$ confirm final form of results of ZvN
- n_f part of $\Delta P^{(2)+}_{\rm ns}$ identical to unpol. $P^{(2)-}_{\rm ns}$ after factorization transf. $\surd$

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Present status: so far running $q\gamma$ **and** $g\gamma$**: 2 loops done, 3 loops in progress**

- **NLO splitting functions** $\Delta P^{(1)}_{\rm qq,qg}$ **correct (Larin** γ_5 **+ Vogelsang fact. trf.)**
- **NNLO coefficient functions** $c^{(2)}_{g_1,\,\rm q/g}$ **confirm final form of results of ZvN**
- n_f **part of** $\Delta P^{(2)+}_{\rm ns}$ **identical to unpol.** $P^{(2)-}_{\rm ns}$ **after factorization transf.** $\surd$

Next step (soon ⇒ proceedings): $C_F n_f^2$ **part of** $\Delta P^{(2)}_{\rm qg}$

Predictions: $N=1$, **Altarelli, Lampe (90);** $\ln^4 x$ **term, Blümlein, A.V. (96)**