Results for Charged-Current Deep-Inelastic Scattering at three loops

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Content

- Introduction to the Deep-Inelastic Scattering

▲ Charged-current deep-inelastic scattering at three loops.
S. Moch and M. R. Nucl. Phys. B **782**, 51 (2007)
▲ Differences between CC coefficient functions
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- Introduction to the Deep-Inelastic Scattering
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- Results: its analysis and applications

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- Deep-inelastic lepton-hadron scattering ($e^{\pm}p$, $e^{\pm}n$, νp, $\bar{\nu}p$, ... - collisions)

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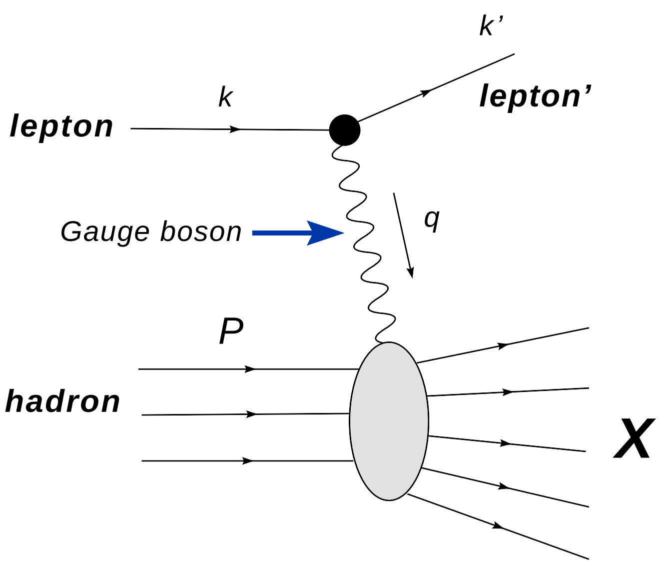

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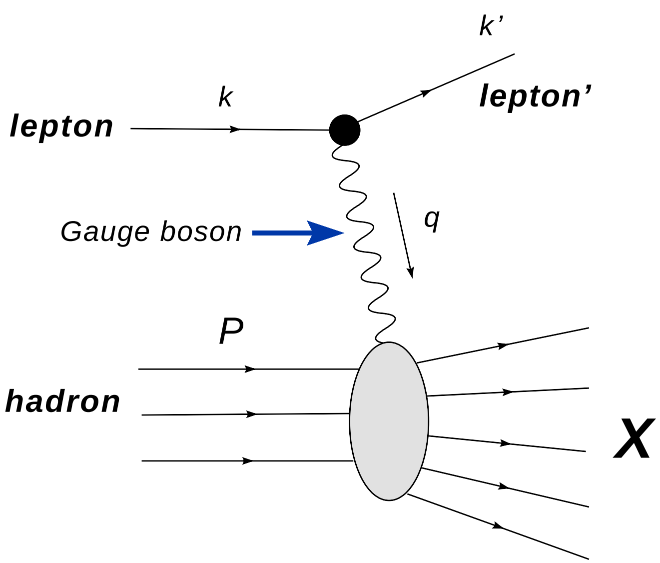
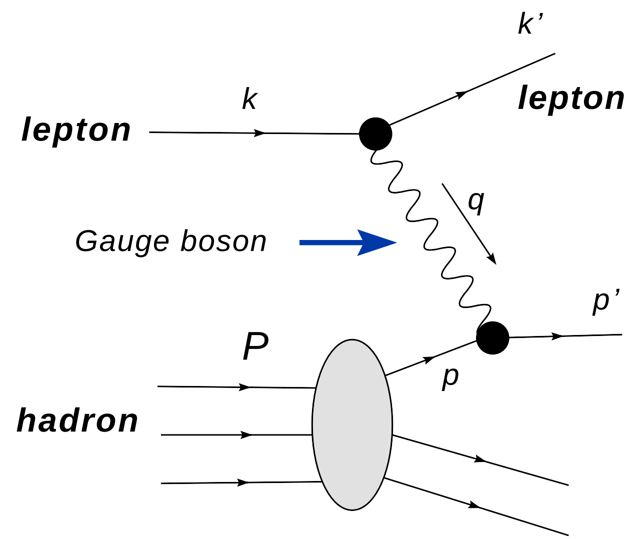


$\Longrightarrow$



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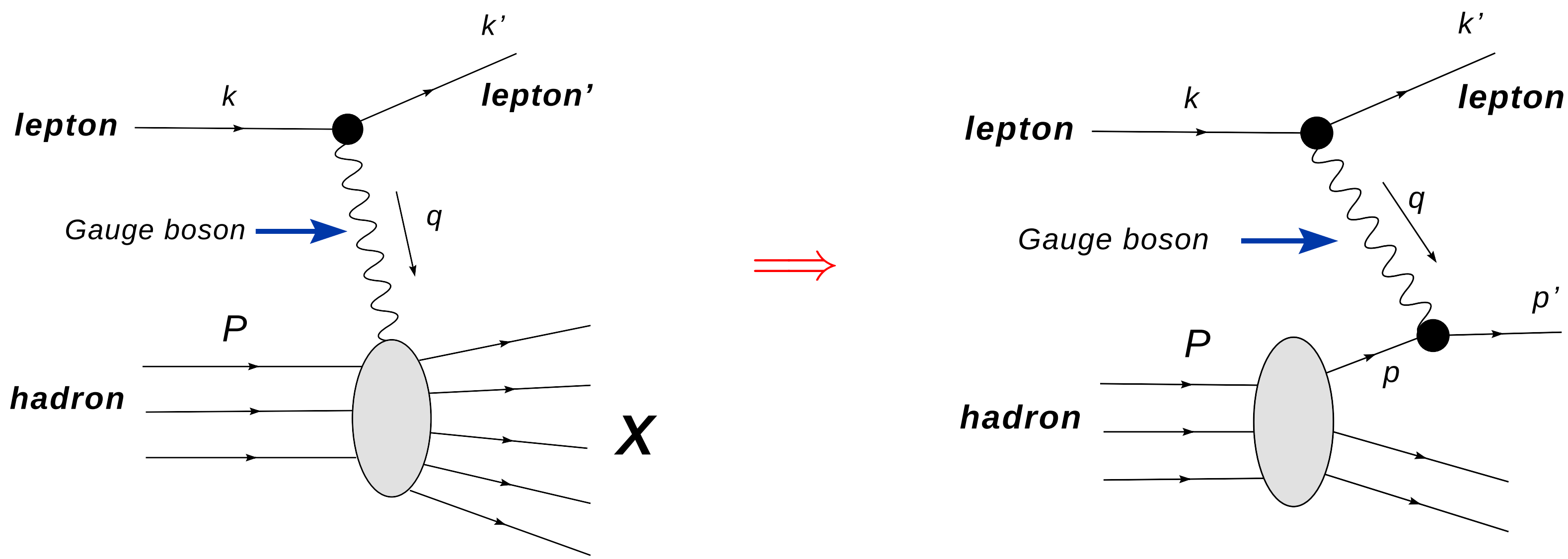
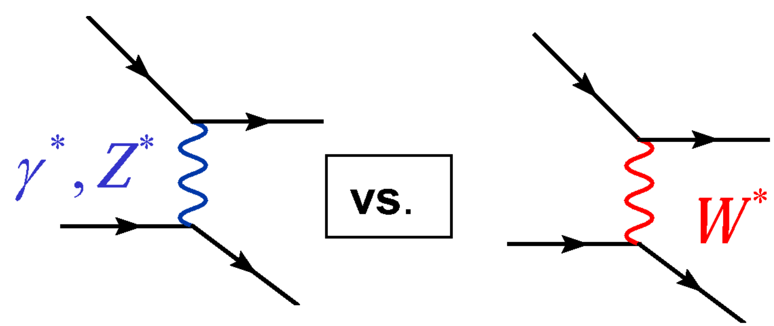


- *Gauge boson*:
  γ, Z^0 - NC
  $W^{\pm}$ - CC

- **Kinematic variables**
  - momentum transfer $Q^2 = -q^2 > 0$
  - Bjorken variable $x = Q^2/(2P \cdot q)$
  - Inelasticity $y = (P \cdot q)/(P \cdot k)$

DIS experiments

- **EW unification at HERA:**
  neutral vs . charged current



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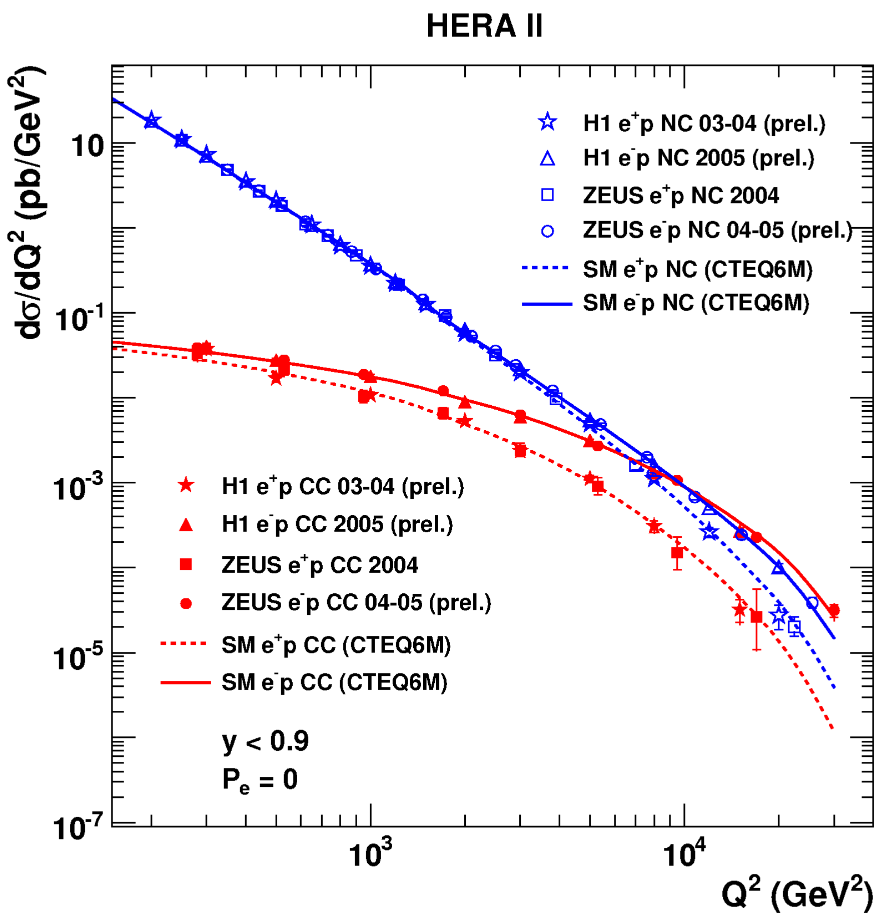
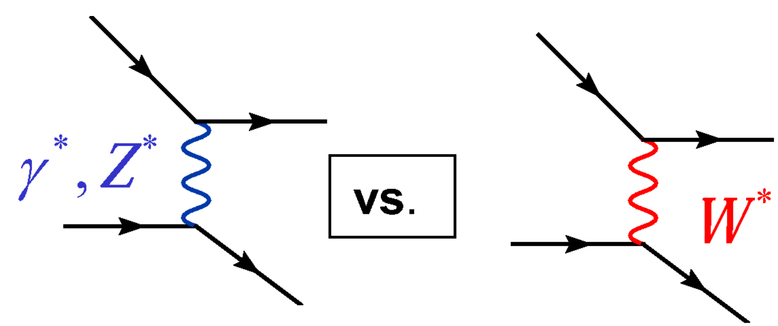


Charged and neutral deep in-elastic scattering cross sections become comparable when Q^2 reaches the electroweak scale

$\Longleftarrow$



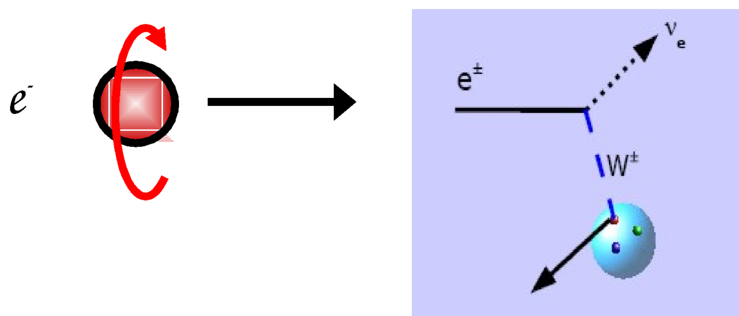
- Polarized charged current DIS at HERA

CC cross section modified by polarization:

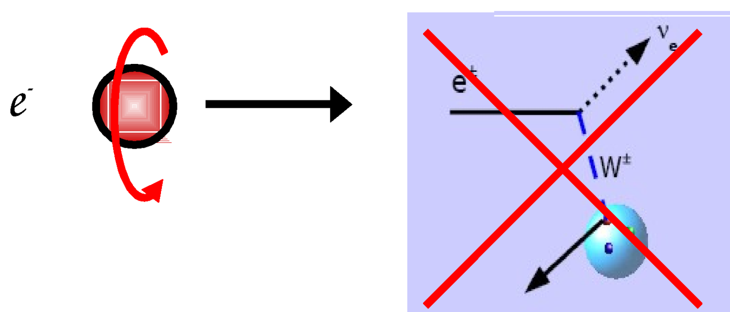
$$\sigma_{CC}^{e^\pm p}(P_e) = (1 \pm P_e) \cdot \sigma_{CC}^{e^\pm p}(P_e = 0) \qquad P_e = \frac{N_R - N_L}{N_R + N_L}$$

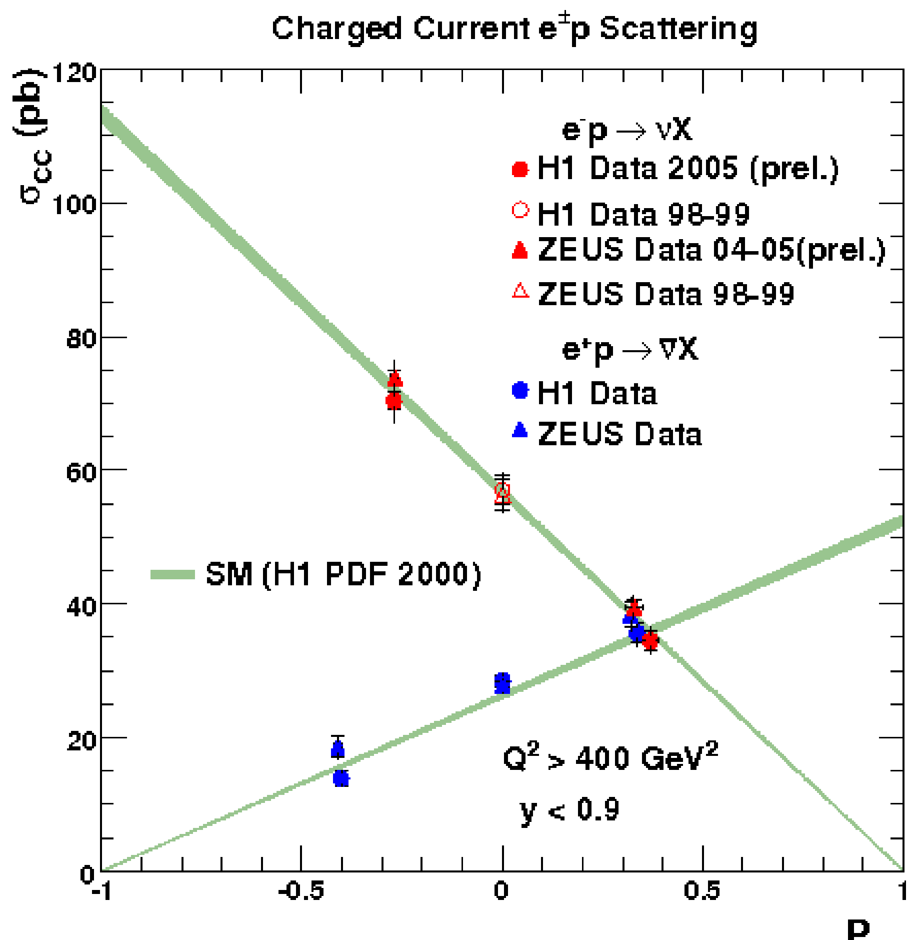
- Cross section is linearly proportional to polarization P_e
- Standard model prediction: vanishing cross section for $P_e = +1(-1)$ in $e^{-(+)}$ scattering

lefthanded electrons interact (CC)



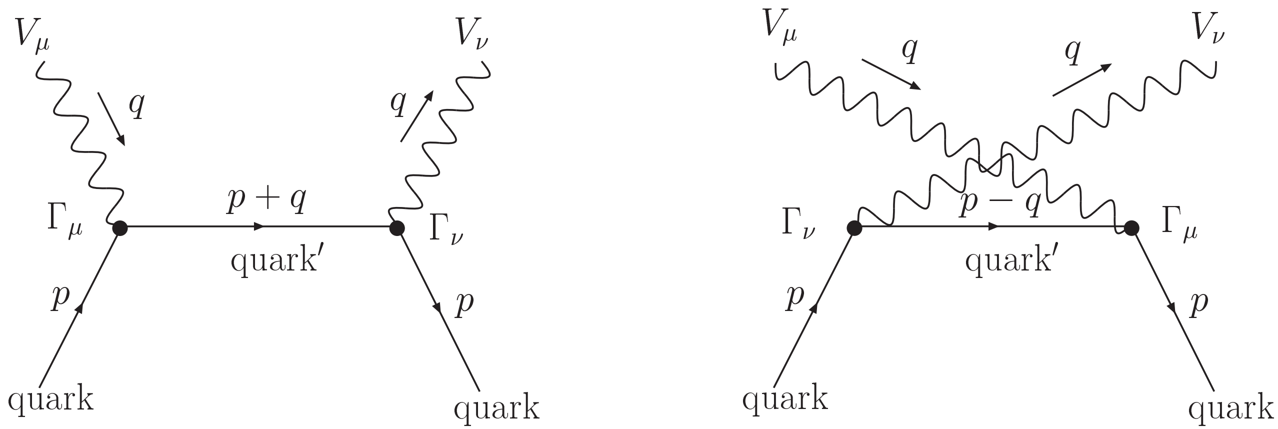
righthanded electrons do not!





Calculation

- Leading order diagrams at parton level
  - Vector and axial-vector interaction $a\gamma^{\mu} + b\gamma^{\mu}\gamma^{5}$



- Mellin moments with definite symmetry properties
  - process dependent distinction even/odd N (from OPE)

$$F_i(N,Q^2) = \int_0^1 dx\, x^{N-2} F_i(x,Q^2)\,, \quad i = 2, L$$

$$F_3(N,Q^2) = \int_0^1 dx\, x^{N-1} F_3(x,Q^2)$$

Known

- NC (exchange via γ gauge boson) $\longrightarrow F_2^{eP}$
  CC (exchange via $W^\pm$ gauge boson) $\longrightarrow F_2^{\nu p+\bar{\nu}p}$, $F_3^{\nu p+\bar{\nu}p}$

  even N for F_2, odd N for F_3

  - NLO Bardeen, Buras, Duke, Muta '78
  - N^{2}LO Zijlstra, van Neerven '92
  - N^{3}LO Moch, Vermaseren, Vogt '05/'06

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New

- NC $\gamma - Z$ interference at N^3LO still missing
- CC (exchange via $W^\pm$ gauge boson) $\longrightarrow F_2^{\nu p-\bar{\nu}p}$, $F_3^{\nu p-\bar{\nu}p}$

  odd N for F_2, even N for F_3

  - order N^3LO *already known* Moch, M. R. '07
    best use: difference "even-odd" Moch, M. R. and Vogt. '07

The calculation

- Big number of diagrams $\Rightarrow$ need of automatization
  e.g. DIS structure functions $F_{2,L}^{\nu p \pm \bar{\nu} p}$ - 1076 diagrams, $F_3^{\nu p \pm \bar{\nu} p}$ - 1314 diagrams up to 3 loops

In practice

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- latest version of FORM and TFORM (multi-threaded version)
  Vermaseren, FORM version 3.2 (Apr 16 2007); Tentyukov, Vermaseren '07
  TFORM up to 5 times faster with 8 threads!

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- QGRAF $\mapsto$ generation of diagrams for DIS structure functions
  Nogueira '93

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  Nogueira '93

Calculation of diagrams $\mapsto$

- MINCER in FORM Larin, Tkachev, Vermaseren '91

What does MINCER do?

MINCER minces integrals

Results **Nucl. Phys. B 782, 51 (2007)**

$$
\begin{aligned}
C^{\rm ns}_{3,10} \quad = \quad & 1 + a_{\rm s} C_F \frac{1953379}{138600} + a_{\rm s}^2 C_F n_f \left(-\frac{537659500957277}{15975002736000} \right) + a_{\rm s}^2 {C_F}^2 \left(\frac{597399446375524589}{14760902528064000} \right. \\
& \left. + \frac{7202}{105} \zeta_3 \right) + a_{\rm s}^2 C_A C_F \left(\frac{5832602058122267}{29045459520000} - \frac{99886}{1155} \zeta_3 \right) \\
& + a_{\rm s}^3 C_F {n_f}^2 \left(\frac{51339756673194617191}{996360920644320000} + \frac{48220}{18711} \zeta_3 \right) \\
& + a_{\rm s}^3 {C_F}^2 n_f \left(-\frac{125483817946055121351353}{209235793335307200000} - \frac{59829376}{3274425} \zeta_3 + \frac{24110}{693} \zeta_4 \right) \\
& + a_{\rm s}^3 {C_F}^3 \left(-\frac{744474223606695878525401307}{7088908678200207936000000} + \frac{28630985464358}{24960941775} \zeta_3 \right. \\
& \qquad \left. + \frac{151796299}{8004150} \zeta_4 - \frac{53708}{99} \zeta_5 \right) \\
& + a_{\rm s}^3 C_A C_F n_f \left(-\frac{1852213500455074877 53}{226445663782800000} + \frac{8071097}{39690} \zeta_3 - \frac{24110}{693} \zeta_4 \right) \\
& + a_{\rm s}^3 C_A {C_F}^2 \left(\frac{19770078729338607732075449}{8369431733412288000000} - \frac{619383700181}{5546875950} \zeta_3 \right. \\
& \qquad \left. - \frac{151796299}{5336100} \zeta_4 - \frac{37322}{99} \zeta_5 \right) \\
& + a_{\rm s}^3 {C_A}^2 C_F \left(\frac{93798719639056648125143}{36231306205248000000} - \frac{43202630363}{20582100} \zeta_3 \right. \\
& \qquad \left. + \frac{151796299}{16008300} \zeta_4 + \frac{195422}{231} \zeta_5 \right) .
\end{aligned}
$$

Checks

- Known Mellin moments for $F_{2,L}^{\nu P+\bar{\nu}P}$ (even) and $F_3^{\nu P+\bar{\nu}P}$ (odd) recalculated

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- All calculations with gauge parameter ξ for gluon propagator (Up to 10'th MM)

$$\mathrm{i}\frac{-g^{\mu\nu} + (1-\xi)q^\mu q^\nu}{q^2}$$

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$$\mathrm{i}\frac{-g^{\mu\nu} + (1-\xi)q^{\mu}q^{\nu}}{q^2}$$

- Adler sum rule for DIS structure functions $\longrightarrow C_{2,1}^{\rm ns} = 1$

$$\int_0^1 \frac{dx}{x}\left(F_2^{\nu P}(x,Q^2) - F_2^{\nu N}(x,Q^2)\right) = 2$$

  - measures isospin of the nucleon in the quark-parton model
  - neither perturbative nor non-perturbative corrections in QCD

Applications

- Gottfried type sum rule (charged lepton(l)-proton(P) or neutron(N) DIS)

$$\int_0^1 \frac{dx}{x}\left(F_2^{lP}(x,Q^2) - F_2^{lN}(x,Q^2)\right)$$

- Study of difference between subjects corresponding to even and odd Mellin moments

  Broadhurst, Kataev, Maxwell ‘04

  Suppressed by $[C_F - C_A/2] \sim 1/N_c$

- Checked for anomalous dimensions

$$\delta\gamma^{\rm ns} = \gamma^{\rm ns+} - \gamma^{\rm ns-}$$

up to 3 loops.

- Conjecture for coefficient functions

$$\delta C_{i,n}^{\rm ns} = C_{i,n}^{\nu P + \bar{\nu} P} - C_{i,n}^{\nu P - \bar{\nu} P}$$

with color coefficient $[C_F - C_A/2]$

Results

arXiv:0708.3731v1 [hep-ph]

$$
\begin{aligned}
\delta C_{2,3}^{\rm ns} \quad = \quad & +a_s^2 C_F [C_F - C_A/2] \Bigg(-\frac{4285}{96} - 122\zeta_3 + \frac{671}{9}\zeta_2 + \frac{128}{5}{\zeta_2}^2 \Bigg) \\
& + a_s^3 C_F [C_F - C_A/2]^2 \Bigg(\frac{1805677051}{466560} - \frac{2648}{9}\zeta_5 + \frac{10093427}{810}\zeta_3 - \frac{1472}{3}{\zeta_3}^2 \\
& \qquad - \frac{7787113}{1944}\zeta_2 + \frac{55336}{9}\zeta_2\zeta_3 - \frac{378838}{45}{\zeta_2}^2 - \frac{8992}{63}{\zeta_2}^3 \Bigg) \\
& + a_s^3 {C_F}^2 [C_F - C_A/2] \Bigg(-\frac{5165481803}{1399680} + \frac{40648}{9}\zeta_5 - \frac{9321697}{810}\zeta_3 + \frac{1456}{3}{\zeta_3}^2 \\
& \qquad + \frac{8046059}{1944}\zeta_2 - 4984\zeta_2\zeta_3 + \frac{798328}{135}{\zeta_2}^2 - \frac{56432}{315}{\zeta_2}^3 \Bigg) \\
& + a_s^3 n_f C_F [C_F - C_A/2] \Bigg(\frac{20396669}{116640} - \frac{1792}{9}\zeta_5 + \frac{405586}{405}\zeta_3 - \frac{139573}{486}\zeta_2 \\
& \qquad + \frac{1408}{9}\zeta_2\zeta_3 - \frac{50392}{135}{\zeta_2}^2 \Bigg) .
\end{aligned}
$$

▲ Remarkable: appearance of values of weight 6.

OPE based moments $C_{2,L}^{\nu p - \bar{\nu} p} \; - \; 1, 3, 5, \cdots \; ; \; C_3^{\nu p - \bar{\nu} p} \; - \; 2, 4, 6, \cdots \quad \Rightarrow$
weight w of zeta functions up to $2l - 1$ (l - number of loops)

“Unnatural“ moments $C_{2,L}^{\nu p + \bar{\nu} p} \; - \; 1, 3, 5, \cdots \; ; \; C_3^{\nu p + \bar{\nu} p} \; - \; 2, 4, 6, \cdots \quad \Rightarrow$
weight up to $2l$

Results in x-Bjorken space

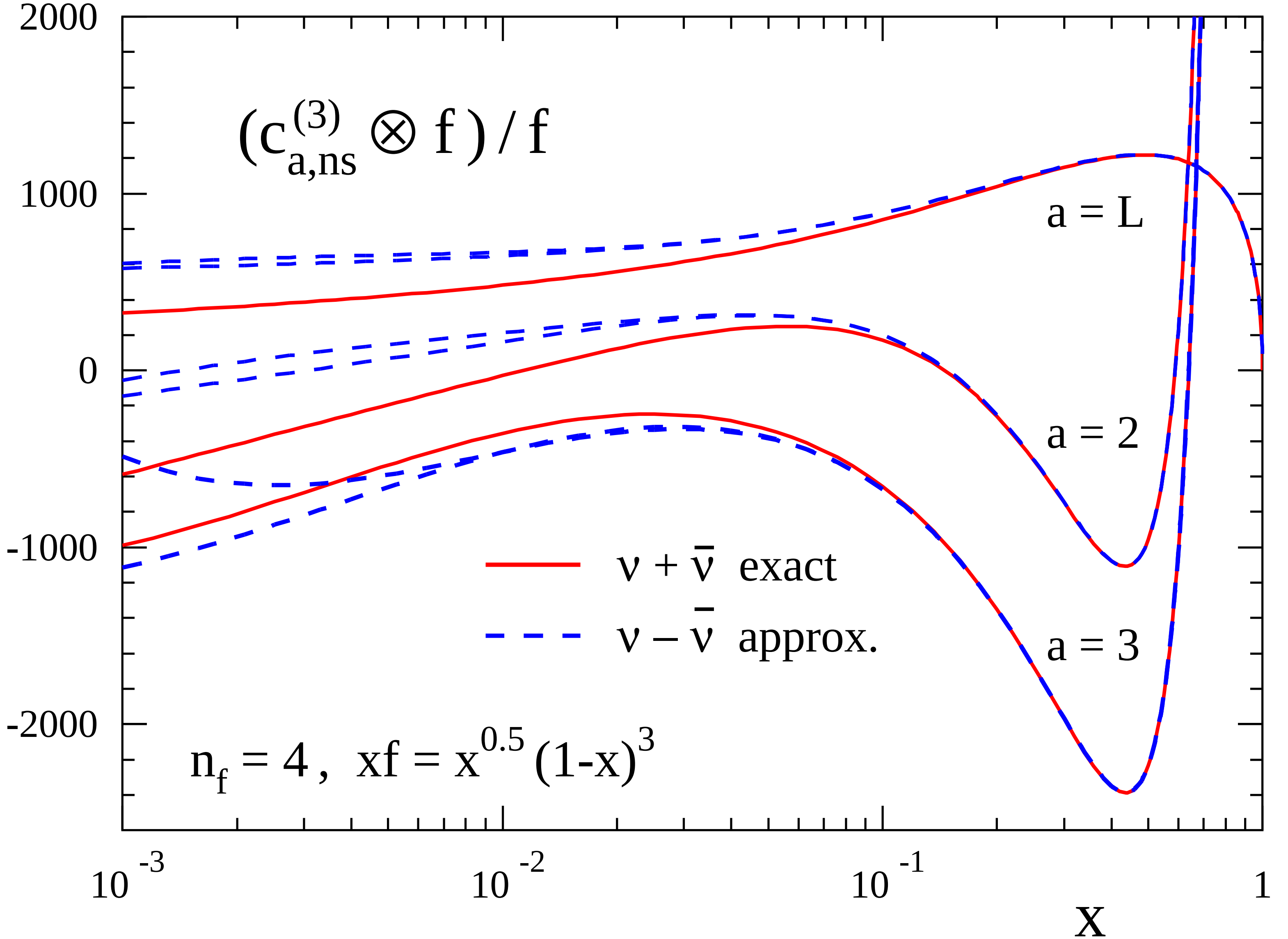
- Easy to use parameterization, ready for phenomenology
- Known 5 Mellin moments, fit functional form (Ansatz)
- Two extremum curves A, B chosen out of about 50. It indicates the width of the uncertainty band

$$
\begin{aligned}
\delta c_{3,A}^{(3)}(x) &= (3.216\, L_1^2 + 44.50\, L_1 - 34.588)\, x_1 + 98.719\, L_0^2 + 2.6208\, L_0^5 \\
&\quad - n_f \{(0.186\, L_1 + 61.102\, (1+x))\, x_1 + 122.51\, x L_0 - 10.914\, L_0^2 \\
&\quad - 2.748\, L_0^3\} \;, \\
\delta c_{3,B}^{(3)}(x) &= -(46.72\, L_1^2 + 267.26\, L_1 + 719.49\, x)\, x_1 - 171.98\, L_0 + 9.470\, L_0^3 \\
&\quad + n_f \{(0.8489\, L_1 + 67.928\, (1 + \tfrac{x}{2}))\, x_1 + 97.922\, x L_0 - 17.070\, L_0^2 \\
&\quad - 3.132\, L_0^3\} \;,
\end{aligned}
$$

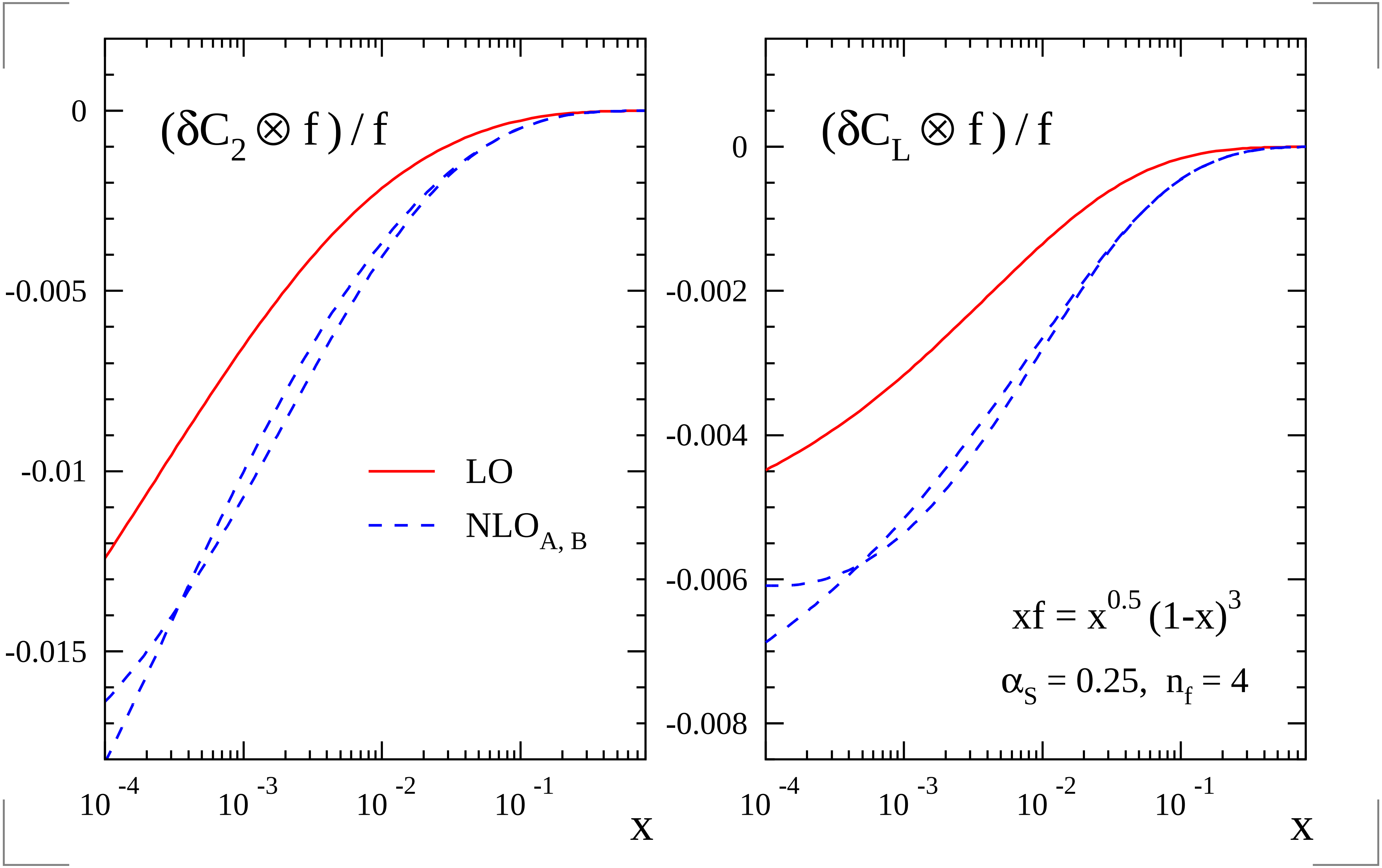
where

$$L_0 = \ln(x),\; x_1 = (1-x),\; L_1 = \ln(x1)\,.$$

Convolution of the α_s^3 order CC coefficient functions



LO (α_s^2) and NLO (α_s^3) of the differences for F_2 and F_L in CC DIS



NuTeV experiment - Paschos-Wolfenstein relation

- Exact relation for massless quarks and isospin zero target in EW
  Paschos,Wolfenstein'73, Llewelin Smith'83

$$R^{-} = \frac{\sigma(\nu_\mu N \to \nu_\mu X) - \sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \to \mu^- X) - \sigma(\bar{\nu}_\mu N \to \mu^+ X)} = \frac{1}{2} - \sin^2\theta_W$$

- measurement of $\sin^2\theta_W$ NuTeV '01 :

Large deviations from Standard model expectations

QCD corrections to Paschos-Wolfenstein relation

- Expansion in α_s and in isoscalar combination $u^- + d^-$,
  Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; Moch, McFarland '03 ,

  $q^- = \int dx\, x(q - \bar{q})$ - second Mellin moments of valence PDFs

$$R^- = \frac{1}{2} - \sin^2\theta_W$$
$$+ \left[1 - \frac{7}{3}\sin^2\theta_W + \frac{8\alpha_s}{9\pi}\{1 + \alpha_s 1.689 + \alpha_s^2(3.661 \pm 0.002)\}\left(\frac{1}{2} - \sin^2\theta_W\right)\right] \times$$
$$\left(\frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-}\right)$$

- QCD corrections in $\{\cdots\}$ with $\delta c_{2,L}^{(3)}(x)$. Under control, relevant:
  Moch, M. R., Vogt '07
  $\{\cdots\} = \{1 + 0.42 + 0.23\}$ for $\alpha_s = 0.25$
- Main uncertainties in s^-
  - either global fit
    Martin, Roberts, Stirling, Thorne '04; Lai, Nadolsky, Pumplin, Stump, Tung, Yuan '07
  - or generated by perturbative evolution
    Catani, de Florian, Rodrigo, Vogelsang '04

Summary

- New results for fixed N Mellin moments at order α_s^3

  $C_{2,L}^{\nu p-\bar{\nu}p}$ (odd) and $C_3^{\nu p-\bar{\nu}p}$ (even)

- and differences "even-odd" in Mellin N-space

- practical approximations in x-space for "even-odd" differences available
  $\Rightarrow$ sufficient for HERA-CC, ν - DIS (e.g. Alekhin makes use of it)

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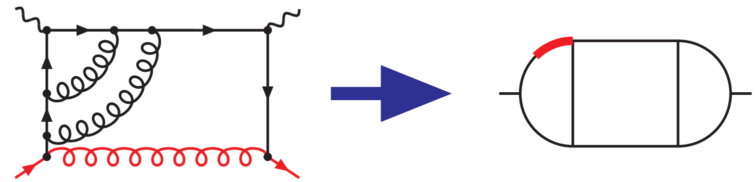
- Stability of QCD α_s expansion for Paschos-Wolfenstein relation

Backup slides

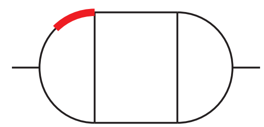
Feynman diag's into MINCER

Method of projection in pictures

- Identify scalar topologies

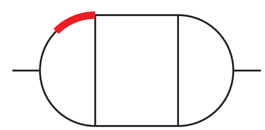


- Scalar diagram with external momenta P and Q



$$= \int \prod_{n}^{3} d^D l_n \, \frac{1}{(P-l_1)^2} \, \frac{1}{l_1^2 \ldots l_8^2}$$

- N-th moment: $\longrightarrow$ coefficient of $(2P \cdot Q)^N$



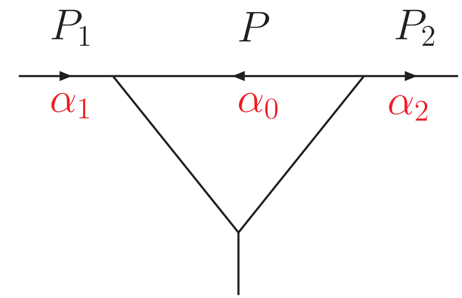
$$= \frac{(2\,P \cdot Q)^N}{(Q^2)^{N+\alpha}} \, C_N$$

- Taylor expansion $$\frac{1}{(P-l_1)^2} = \sum_i \frac{(2P \cdot l_1)^i}{(l_1^2)^{i+1}} \quad \longrightarrow \quad \frac{(2P \cdot l_1)^N}{(l_1^2)^N}$$

- Feed scalar two-point functions in MINCER

Mincer

- $\int dP \frac{\partial}{\partial P^\mu}\left[(P-l_j)^\mu \times I(l_1,..,P,...)\right] = 0$ - integration by part identities
  t'Hooft, Veltman'72; Chetyrkin , Tkachov '81
  Leibniz, Newton :-)



Triangle rule

Define

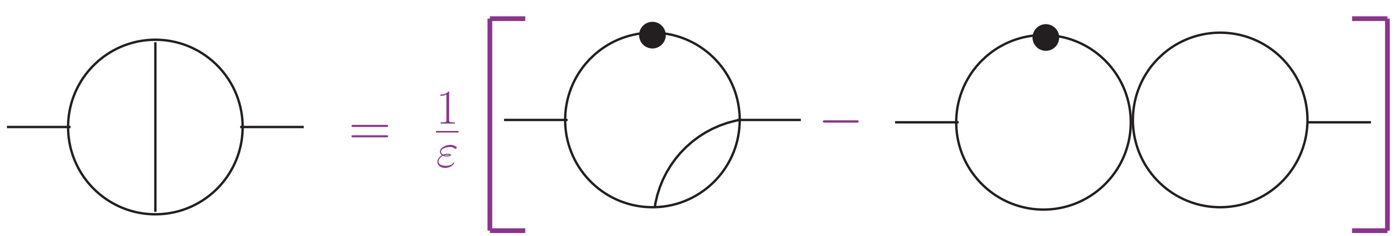
$$I(\alpha_0,\beta_1,\beta_2,\alpha_1,\alpha_2) = \int d^D P \frac{1}{(P^2)^{\alpha_0}((P+P_1)^2)^{\beta_1}(P_1^2)^{\alpha_1}((P+P_2)^2)^{\beta_2}(P_2^2)^{\alpha_2}}$$

and act the integrand with $\frac{\partial}{\partial P_\mu} P_\mu = D + P_\mu \frac{\partial}{\partial P_\mu}$. Result $\Rightarrow$

Recursion relation:

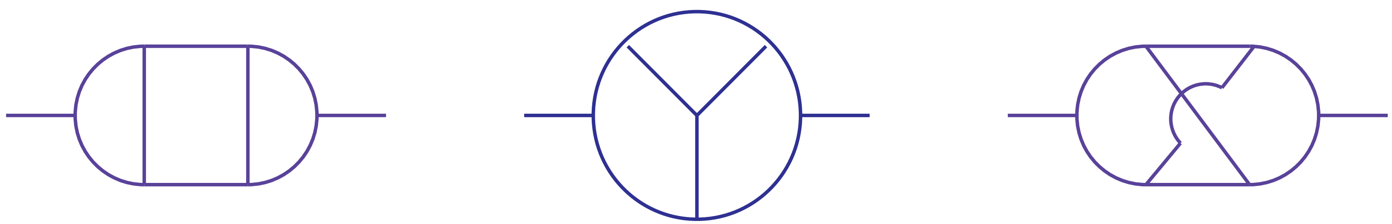
$$I(\alpha_0,\beta_1,\beta_2,\alpha_1,\alpha_2) \times (D - 2\alpha_0 - \beta_1 - \beta_2) =$$
$$\beta_1(I(\alpha_0-1,\beta_1+1,\beta_2,\alpha_1,\alpha_2) - I(\alpha_0,\beta_1+1,\beta_2,\alpha_1-1,\alpha_2))$$
$$\beta_2(I(\alpha_0-1,\beta_1,\beta_2+1,\alpha_1,\alpha_2) - I(\alpha_0,\beta_1,\beta_2+1,\alpha_1,\alpha_2-1))$$

In pictures

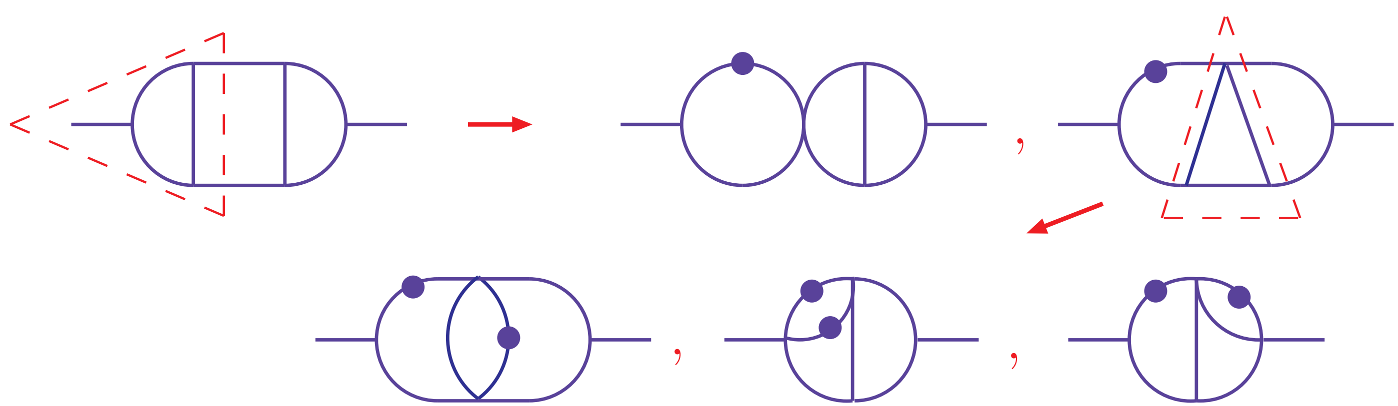


Classification of loop integrals

- Classify according to topology of underlying two-point function
  - top-level topology types ladder, benz, non-planar $\Rightarrow$



- Using **IBP** identities more complicated topologies are reduced to simpler topologies



Strange asymetry