Parton distributions in the virtual photon up to NNLO in QCD and factorization scheme dependence

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Introduction & Motivation

- Deep-inelastic electron-nucleon scattering

⇨ $F_2(x, Q^2),\ F_L(x, Q^2)$ Nucleon structure functions

$x = \dfrac{Q^2}{2p \cdot q}$:Bjorken variable $0 \leq x \leq 1$

$-Q^2 = q^2 \leq 0$:mass squared of the probe photon

⇨ $q^i(x, Q^2),\ G(x, Q^2)$

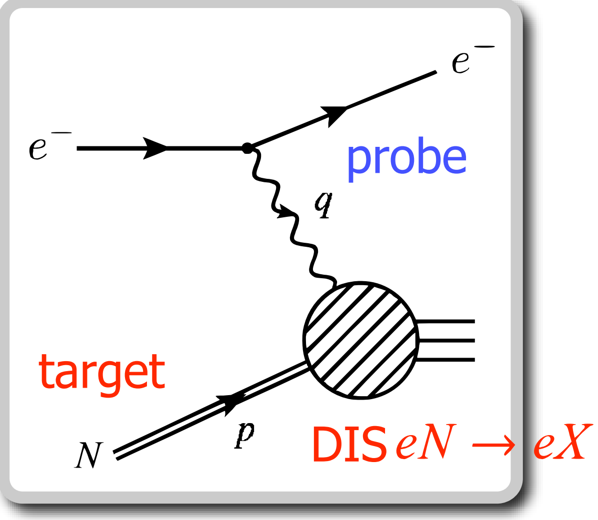
Parton distribution functions (PDFs)
inside a nucleon

- necessary for the analysis of semi-inclusive reactions

- factorization-scheme dependent

Some assumptions made to extract PDFs from data

Rather difficult to see the features of factorization schemes



Introduction & Motivation

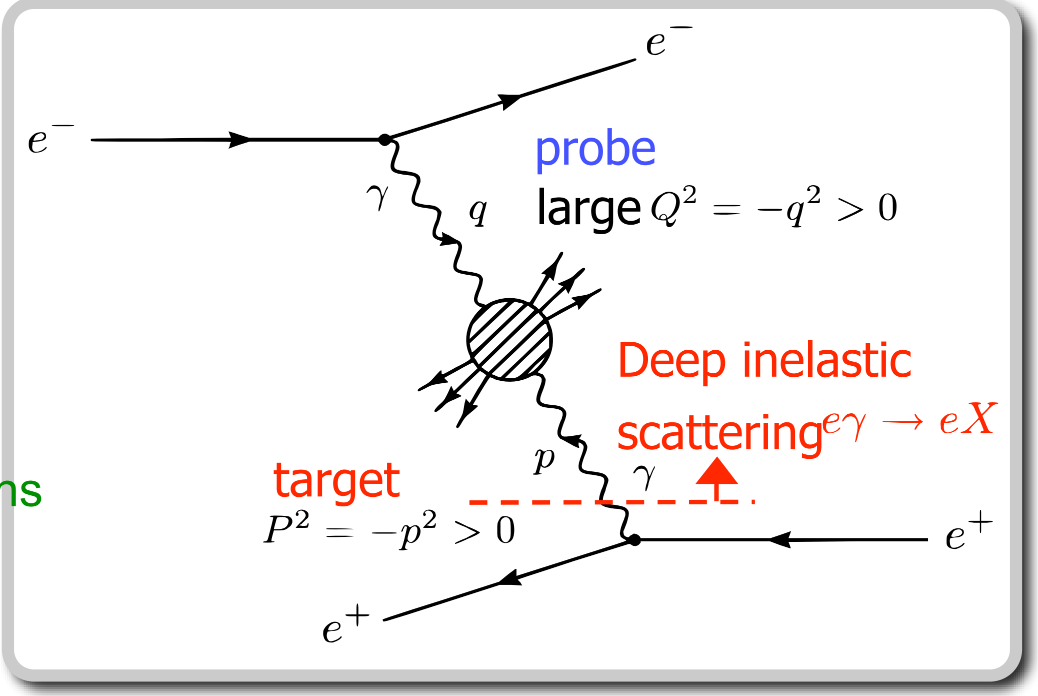
- Future linear collider experiment (e.g. ILC)

Two-photon process $e^+e^- \to (e^+e^-\gamma\gamma) \to e^+e^-X$

Viewed as a deep-inelastic electron-photon scattering

We can study the structures of photon

⇨ $F_2^\gamma(x, Q^2, P^2)$
$F_L^\gamma(x, Q^2, P^2)$



- Highly virtual photon target

$(\Lambda^2 \ll P^2 \ll Q^2)$

pQCD gives definite predictions for

- $F_2^\gamma(x, Q^2, P^2)$ $F_L^\gamma(x, Q^2, P^2)$
- pdf in the virtual photon

$q_S^\gamma(x, Q^2, P^2),\ G^\gamma(x, Q^2, P^2),\ q_{NS}^\gamma(x, Q^2, P^2)$ $-P^2 = p^2 \leq 0$:mass squared of the target photon

- A good playground to see the scheme-dependence of pdf

F_2^γ in Perturbative QCD

- For highly virtual photon target ($\Lambda^2 \ll P^2 \ll Q^2$)

Λ : QCD scale parameter

$$F_2^\gamma(x, Q^2, P^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} \hat{A} + \hat{B} + \alpha_s(Q^2) \hat{C} \right]$$

(LO) (NLO) (NNLO)

(Uematsu' talk)

Hadronic piece can also be dealt with perturbatively

Definite prediction of F_2^γ , its shape and magnitude, is possible

LO, NLO Uematsu-Walsh (1981,1982)

NNLO Ueda-Uematsu-Sasaki (OPE+RG method)

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Motivated by the calculation of 3-loop anomalous dimensions
Vogt-Moch-Vermaseren (2004,2006)

QCD-improved Parton Model Approach

$q^i_\pm(x,Q^2,P^2)$:Quark DF with $\pm$ helicity of the virtual photon ($-P^2$)

$G^\gamma_\pm(x,Q^2,P^2)$: Gluon DF with $\pm$ helicity

$\Gamma^\gamma_\pm(x,Q^2,P^2)$: Photon DF with $\pm$ helicity

- Unpolarized PDFs

$$q^i \equiv \frac{1}{2}[q^i_+ + \overline{q}^i_+ + q^i_- + \overline{q}^i_-] \qquad G^\gamma \equiv \frac{1}{2}[G^\gamma_+ + G^\gamma_-] \qquad \Gamma^\gamma \equiv \frac{1}{2}[\Gamma^\gamma_+ + \Gamma^\gamma_-]$$

- In LO in QED coupling $\alpha = e^2/4\pi$: $\Gamma^\gamma(x,Q^2,P^2) = \delta(1-x)$

Photon DF does not evolve

- Singlet quark DF: $q^\gamma_S \equiv \sum q^i$

- Non-singlet quark DF: $q^\gamma_{NS} \equiv \sum_i e_i^2 \left(q^i - \frac{1}{n_f} q^\gamma_S\right)$

QCD-improved Parton Model Approach

- Parton distribution functions in the virtual photon

$$\overrightarrow{q}^{\gamma} = (\boldsymbol{q}^{\gamma}, \Gamma^{\gamma}), \qquad \boldsymbol{q}^{\gamma} \equiv (q_S^{\gamma}, G^{\gamma}, q_{NS}^{\gamma})$$

- Factorization:

$$F_2^{\gamma} = \overrightarrow{q}^{\gamma} \otimes \overrightarrow{C}_2^{\gamma} \qquad \overrightarrow{C}_2^{\gamma} = (\boldsymbol{C}_2, C_{2,\gamma}), \qquad \boldsymbol{C}_2 \equiv (C_{2,S}, C_{2,G}, C_{2,NS})$$

- DGLAP evolution equation

$$\frac{d\boldsymbol{q}^{\gamma}(x, Q^2, P^2)}{d\ln Q^2} = \boldsymbol{k}(x, Q^2) + \int_x^1 \frac{dy}{y} \boldsymbol{q}^{\gamma}(y, Q^2, P^2) \times P(\frac{x}{y}, Q^2)$$

- $\boldsymbol{k}(x, Q^2)$:Splitting fn. of photon into quark and gluon

- $P(\frac{x}{y}, Q^2)$:Splitting fn. of quark and gluon

$$P(z, Q^2) = \begin{pmatrix} P_{qq}^{S}(z, Q^2) & P_{Gq}(z, Q^2) & 0 \\ P_{qG}(z, Q^2) & P_{GG}(z, Q^2) & 0 \\ 0 & 0 & P_{qq}^{NS}(z, Q^2) \end{pmatrix}$$

QCD-improved Parton Model Approach

- Taking moments: $f(n) \equiv \int_0^1 dx x^{n-1} f(x)$

$$\frac{d\ \boldsymbol{q}^\gamma(n,Q^2,P^2)}{d\ \ln Q^2} = \boldsymbol{k}(n,Q^2) + \boldsymbol{q}^\gamma(n,Q^2,P^2)P(n,Q^2)$$

- Expanding in powers of α_s

$$\boldsymbol{k}(Q^2) = \frac{\alpha}{2\pi}\boldsymbol{k}^{(0)} + \frac{\alpha\alpha_s(Q^2)}{(2\pi)^2}\boldsymbol{k}^{(1)} + \frac{\alpha}{2\pi}\left[\frac{\alpha_s(Q^2)}{2\pi}\right]^2\boldsymbol{k}^{(2)}\dots$$

$$P(Q^2) = \frac{\alpha_s(Q^2)}{2\pi}P^{(0)} + \left[\frac{\alpha_s(Q^2)}{2\pi}\right]^2 P^{(1)} + \left[\frac{\alpha_s(Q^2)}{2\pi}\right]^3 P^{(2)}$$

$$\frac{d\alpha_s(Q^2)}{d\mathrm{ln}Q^2} = -\beta_0\frac{\alpha_s(Q^2)^2}{4\pi} - \beta_1\frac{\alpha_s(Q^2)^3}{(4\pi)^2} - \beta_2\frac{\alpha_s(Q^2)^4}{(4\pi)^3} +$$

- Introducing $t \equiv \frac{2}{\beta_0}\ln\frac{\alpha_s(P^2)}{\alpha_s(Q^2)}$

$$\begin{aligned}\frac{d\boldsymbol{q}^\gamma(t)}{dt} = \frac{\alpha}{2\pi}\Bigg\{&\frac{2\pi}{\alpha_s}\boldsymbol{k}^{(0)} + \left[\boldsymbol{k}^{(1)} - \frac{\beta_1}{2\beta_0}\boldsymbol{k}^{(0)}\right]\\ &+\frac{\alpha_s}{2\pi}\left[\boldsymbol{k}^{(2)} - \frac{\beta_1}{2\beta_0}\boldsymbol{k}^{(1)} + \frac{1}{4}\left((\frac{\beta_1}{\beta_0})^2 - \frac{\beta_2}{\beta_0}\right)\boldsymbol{k}^{(0)}\right] + \mathcal{O}(\alpha_s^2)\Bigg\}\\ +\boldsymbol{q}^\gamma(t)\Bigg\{&P^{(0)} + \frac{\alpha_s}{2\pi}\left[P^{(1)} - \frac{\beta_1}{2\beta_0}P^{(0)}\right]\\ &+\frac{\alpha_s^2}{(2\pi)^2}\left[P^{(2)} - \frac{\beta_1}{2\beta_0}P^{(1)} + \frac{1}{4}\left((\frac{\beta_1}{\beta_0})^2 - \frac{\beta_2}{\beta_0}\right)P^{(0)}\right] + \mathcal{O}(\alpha_s^3)\Bigg\}\end{aligned}$$

QCD-improved Parton Model Approach

- Solution to DGLAP evolution eq.

$$\boldsymbol{q}^{\gamma}(t) = \boldsymbol{q}^{\gamma(0)}(t) + \boldsymbol{q}^{\gamma(1)}(t) + \boldsymbol{q}^{\gamma(2)}(t)$$

- Initial conditions (factorization scheme dependent)

$$\boldsymbol{q}^{\gamma(0)}(0) = \boldsymbol{0}\ , \qquad \boldsymbol{q}^{\gamma(1)}(0) = \frac{\alpha}{4\pi}\boldsymbol{A}^{(1)}\ , \qquad \boldsymbol{q}^{\gamma(2)}(0) = \frac{\alpha}{4\pi}\frac{\alpha_s(P^2)}{4\pi}\boldsymbol{A}^{(2)}$$

$$\langle \gamma(p) \mid O_n^i(\mu) \mid \gamma(p) \rangle|_{\mu^2=P^2} = \frac{\alpha}{4\pi}\Big\{A_n^{i(1)} + \frac{\alpha_s(P^2)}{4\pi}A_n^{i(2)}\Big\}\ , \qquad i = S, G, NS$$

finite photon matrix element

- Splitting functions $\longleftrightarrow$ anomalous dimensions

$$P^{(0)} = -\frac{1}{4}\widehat{\gamma}_n^{(0)}\ , \qquad P^{(1)} = -\frac{1}{8}\widehat{\gamma}_n^{(1)}\ , \qquad P^{(2)} = -\frac{1}{16}\widehat{\gamma}_n^{(2)}$$

$$\boldsymbol{k}^{(0)} = \frac{1}{4}\boldsymbol{K}_n^{(0)}, \qquad \boldsymbol{k}^{(1)} = \frac{1}{8}\boldsymbol{K}_n^{(1)}, \qquad \boldsymbol{k}^{(2)} = \frac{1}{16}\boldsymbol{K}_n^{(2)}$$

- When parameters calculated in $\overline{\mathrm{MS}}$ scheme are used, we obtain PDFs in the virtual photon up to NNLO in $\overline{\mathrm{MS}}$ scheme

QCD-improved Parton Model Approach

- Solution

$$
\begin{aligned}
\boldsymbol{q}^{\gamma(0)}(t)/\Big[\frac{\alpha}{8\pi\beta_0}\Big] &= \frac{4\pi}{\alpha_s(Q^2)}\,\boldsymbol{K}_n^{(0)}\sum_i P_i^n\,\frac{1}{1+d_i^n}\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{1+d_i^n}\right\}, \\
\boldsymbol{q}^{\gamma(1)}(t)/\Big[\frac{\alpha}{8\pi\beta_0}\Big] &= \Bigg\{\boldsymbol{K}_n^{(1)}\sum_i P_i^n\,\frac{1}{d_i^n}+\frac{\beta_1}{\beta_0}\boldsymbol{K}_n^{(0)}\sum_i P_i^n\left(1-\frac{1}{d_i^n}\right) \\
&\qquad -\boldsymbol{K}_n^{(0)}\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}\,\frac{1}{d_i^n}-2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\sum_i P_i^n\Bigg\} \\
&\qquad\qquad \times\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{d_i^n}\right\} \\
&\quad+\Bigg\{\boldsymbol{K}_n^{(0)}\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}\,P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{1}{1+d_i^n}-\frac{\beta_1}{\beta_0}\boldsymbol{K}_n^{(0)}\sum_i P_i^n\,\frac{d_i^n}{1+d_i^n}\Bigg\} \\
&\qquad\qquad \times\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{1+d_i^n}\right\} \\
&\quad+\,2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\,,
\end{aligned}
$$

$$q_S^\gamma(n,Q^2,P^2)=(1,1)\ \text{component}$$

$$q_G^\gamma(n,Q^2,P^2)=(1,2)\ \text{component}$$

$$q_{NS}^\gamma(n,Q^2,P^2)=(1,3)\ \text{component}$$

$$
\begin{aligned}
&\boldsymbol{q}^{\gamma(2)}(t)/\Big[\frac{\alpha}{8\pi\beta_0}\Big]\Big[\frac{\alpha_s(Q^2)}{4\pi}\Big] \\
&= \Bigg\{-\boldsymbol{K}_n^{(0)}\Big(\frac{\beta_1}{\beta_0}\Big)^2\sum_i P_i^n\left(1-\frac{d_i^n}{2}\right)+\boldsymbol{K}_n^{(0)}\frac{\beta_2}{\beta_0}\sum_i P_i^n\,\frac{1}{1-d_i^n}\Big(1-\frac{d_i^n}{2}\Big) \\
&\quad -\boldsymbol{K}_n^{(0)}\frac{\beta_1}{\beta_0}\Bigg[\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}\,\frac{1-d_j^n}{1-d_i^n}+\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_i^n}{4\beta_0+\lambda_j^n-\lambda_i^n}\,\frac{1-d_i^n+d_j^n}{1-d_i^n}\Bigg] \\
&\quad +\boldsymbol{K}_n^{(0)}\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(2)}P_i^n}{4\beta_0+\lambda_j^n-\lambda_i^n}\,\frac{1}{1-d_i^n} \\
&\quad -\boldsymbol{K}_n^{(0)}\sum_{j,k,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_k^n\,\widehat{\gamma}_n^{(1)}\,P_i^n}{(2\beta_0-\lambda_i^n+\lambda_k^n)(4\beta_0+\lambda_j^n-\lambda_i^n)}\,\frac{1}{1-d_i^n} \\
&\quad +\boldsymbol{K}_n^{(1)}\frac{\beta_1}{\beta_0}\sum_i P_i^n+\boldsymbol{K}_n^{(1)}\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}\,\frac{1}{1-d_i^n}-\boldsymbol{K}_n^{(2)}\sum_i P_i^n\,\frac{1}{1-d_i^n} \\
&\quad +2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}-2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\frac{\beta_1}{\beta_0}\sum_i P_i^n d_i^n-2\beta_0\widetilde{\boldsymbol{A}}_n^{(2)}\sum_i P_i^n\Bigg\} \\
&\qquad\qquad \times\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{d_i^n-1}\right\} \\
&+\Bigg\{\boldsymbol{K}_n^{(0)}\Big(\frac{\beta_1}{\beta_0}\Big)^2\sum_i P_i^n\,(1-d_i^n)-\boldsymbol{K}_n^{(0)}\frac{\beta_1}{\beta_0}\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}\,P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{1-d_i^n}{d_i^n} \\
&\quad +\boldsymbol{K}_n^{(0)}\frac{\beta_1}{\beta_0}\sum_{j,i}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_i^n}{2\beta_0+\lambda_j^n-\lambda_i^n}-\boldsymbol{K}_n^{(0)}\sum_{j,i,k}\frac{P_j^n\,\widehat{\gamma}_n^{(1)}\,P_i^n\,\widehat{\gamma}_n^{(1)}\,P_k^n}{(2\beta_0+\lambda_i^n-\lambda_k^n)(2\beta_0+\lambda_j^n-\lambda_i^n)}\,\frac{1}{d_i^n} \\
&\quad -\boldsymbol{K}_n^{(1)}\frac{\beta_1}{\beta_0}\sum_i P_i^n+\boldsymbol{K}_n^{(1)}\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{1}{d_i^n} \\
&\quad -2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}+2\beta_0\widetilde{\boldsymbol{A}}_n^{(1)}\frac{\beta_1}{\beta_0}\sum_i P_i^n d_i^n\Bigg\}\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{d_i^n}\right\} \\
&+\Bigg\{\boldsymbol{K}_n^{(0)}\Big(\frac{\beta_1}{\beta_0}\Big)^2\sum_i P_i^n\,\frac{d_i^n}{2}-\boldsymbol{K}_n^{(0)}\frac{\beta_2}{\beta_0}\sum_i P_i^n\,\frac{d_i^n}{2(1+d_i^n)} \\
&\quad -\boldsymbol{K}_n^{(0)}\frac{\beta_1}{\beta_0}\Bigg[\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}\,P_j^n}{2\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{d_j^n}{1+d_i^n}+\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}\,P_j^n}{4\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{1+d_i^n-d_j^n}{1+d_i^n}\Bigg] \\
&\quad +\boldsymbol{K}_n^{(0)}\sum_{i,j}\frac{P_i^n\,\widehat{\gamma}_n^{(2)}P_j^n}{4\beta_0+\lambda_i^n-\lambda_j^n}\,\frac{1}{1+d_i^n} \\
&\quad +\boldsymbol{K}_n^{(0)}\sum_{i,j,k}\frac{P_i^n\,\widehat{\gamma}_n^{(1)}\,P_j^n\,\widehat{\gamma}_n^{(1)}\,P_k^n}{(2\beta_0+\lambda_i^n-\lambda_j^n)(4\beta_0+\lambda_i^n-\lambda_k^n)}\,\frac{1}{1+d_i^n}\Bigg\}\left\{1-\left[\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right]^{d_i^n+1}\right\} \\
&+2\beta_0\widetilde{\boldsymbol{A}}_n^{(2)}\,,
\end{aligned}
$$

Moment Sum Rule for F_2^γ

- Since $F_2^\gamma = \overrightarrow{q}^\gamma \otimes \overrightarrow{C}_2^\gamma$

$$\int_0^1 dx\, x^{n-2} F_2^\gamma(x, Q^2, P^2)$$

LO ($\alpha\alpha_s^{-1}$)

for even n

$$= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{ \underbrace{\frac{4\pi}{\alpha_s(Q^2)}\sum_i \mathcal{L}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right]}_{} \right.$$

NLO (α)

$$\underbrace{+\sum_i \mathcal{A}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right] + \sum_i \mathcal{B}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] + \mathcal{C}^n}_{}$$

$$\underbrace{+\frac{\alpha_s(Q^2)}{4\pi}\left(\sum_i \mathcal{D}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n-1}\right] + \sum_i \mathcal{E}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right]\right.}_{}$$

$$\left.\left.\underbrace{+\sum_i \mathcal{F}_i^n\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] + \mathcal{G}^n}_{}\right) + \mathcal{O}(\alpha_s^2)\right\}$$

NNLO ($\alpha\alpha_s$)

Moment Sum Rule for F_2^γ

LO :

$$\mathcal{L}_i^n = \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{1}{d_i^n + 1}$$

$$d_i^n = \frac{\lambda_i^n}{2\beta_0} \qquad i = +, -, NS$$

NLO :

$$\mathcal{A}_i^n = -\boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{d_i^n} - \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{1 - d_i^n}{d_i^n}$$
$$+\boldsymbol{K}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{1}{d_i^n} - 2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)}$$

$$\mathcal{B}_i^n = \boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1 + d_i^n}$$
$$+ \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(1)} \frac{1}{1 + d_i^n} - \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{d_i^n}{1 + d_i^n}$$

$$\mathcal{C}^n = 2\beta_0 (C_{2,n}^{\gamma(1)} + \widetilde{\boldsymbol{A}}_n^{(1)} \cdot \boldsymbol{C}_{2,n}^{(0)})$$

Moment Sum Rule for F_2^γ

NNLO :

$$
\begin{aligned}
\mathcal{D}_i^n \;=\; & -\boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \frac{1}{1-d_i^n} \right) \left(1 - \frac{d_i^n}{2} \right) \\
& -\boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{1-d_j^n}{1-d_i^n} \\
& -\boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 4\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \left(\frac{1 - d_i^n + d_j^n}{1-d_i^n} \right) \\
& +\boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(2)} P_i^n}{\lambda_j^n - \lambda_i^n + 4\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1-d_i^n} \\
& -\boldsymbol{K}_n^{(0)} \sum_{j,k} \frac{P_k^n \hat{\gamma}_n^{(1)} P_j^n \hat{\gamma}_n^{(1)} P_i^n}{(\lambda_j^n - \lambda_i^n + 2\beta_0)(\lambda_k^n - \lambda_i^n + 4\beta_0)} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1-d_i^n} \\
& +\boldsymbol{K}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} + \boldsymbol{K}_n^{(1)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1-d_i^n} \\
& -\boldsymbol{K}_n^{(2)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1-d_i^n} + 2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \\
& -2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} d_i^n - 2\beta_0 \widetilde{\boldsymbol{A}}_n^{(2)} P_i^n \boldsymbol{C}_{2,n}^{(0)} ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}_i^n \;=\; & -\boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(1)} \frac{\beta_1}{\beta_0} \frac{1-d_i^n}{d_i^n} - \boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(1)} \frac{1}{d_i^n} \\
& +\boldsymbol{K}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(1)} \frac{1}{d_i^n} + \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1^2}{\beta_0^2} (1 - d_i^n) \\
& -\boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{1-d_i^n}{d_i^n} \\
& +\boldsymbol{K}_n^{(0)} \sum_j \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n}{\lambda_j^n - \lambda_i^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \\
& -\boldsymbol{K}_n^{(0)} \sum_{j,k} \frac{P_j^n \hat{\gamma}_n^{(1)} P_i^n \hat{\gamma}_n^{(1)} P_k^n}{(\lambda_i^n - \lambda_k^n + 2\beta_0)(\lambda_j^n - \lambda_i^n + 2\beta_0)} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{d_i^n} \\
& -\boldsymbol{K}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} + \boldsymbol{K}_n^{(1)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{d_i^n} \\
& -2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} + 2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} d_i^n \\
& -2\beta_0 \widetilde{\boldsymbol{A}}_n^{(1)} P_i^n \boldsymbol{C}_{2,n}^{(1)} ,
\end{aligned}
$$

Moment Sum Rule for F_2^γ

NNLO :

$$
\begin{aligned}
\mathcal{F}_i^n \;=\;& \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(2)} \frac{1}{1+d_i^n} - \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(1)} \frac{\beta_1}{\beta_0} \frac{d_i^n}{1+d_i^n} \\
&+ \boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(1)} \frac{1}{1+d_i^n} \\
&+ \boldsymbol{K}_n^{(0)} P_i^n \boldsymbol{C}_{2,n}^{(0)} \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \frac{1}{1+d_i^n} \right) \frac{d_i^n}{2} \\
&- \boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 2\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{d_j^n}{1+d_i^n} \\
&- \boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n}{\lambda_i^n - \lambda_j^n + 4\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{\beta_1}{\beta_0} \frac{1+d_i^n - d_j^n}{1+d_i^n} \\
&+ \boldsymbol{K}_n^{(0)} \sum_j \frac{P_i^n \hat{\gamma}_n^{(2)} P_j^n}{\lambda_i^n - \lambda_j^n + 4\beta_0} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1+d_i^n} \\
&+ \boldsymbol{K}_n^{(0)} \sum_{j,k} \frac{P_i^n \hat{\gamma}_n^{(1)} P_j^n \hat{\gamma}_n^{(1)} P_k^n}{(\lambda_i^n - \lambda_j^n + 2\beta_0)(\lambda_i^n - \lambda_k^n + 4\beta_0)} \boldsymbol{C}_{2,n}^{(0)} \frac{1}{1+d_i^n}
\end{aligned}
$$

$$
\mathcal{G}^n \;=\; 2\beta_0 \left(C_{2,n}^{\gamma(2)} + \widetilde{\boldsymbol{A}}_n^{(1)} \cdot \boldsymbol{C}_{2,n}^{(1)} + \widetilde{\boldsymbol{A}}_n^{(2)} \cdot \boldsymbol{C}_{2,n}^{(0)} \right)
$$

Factorization-scheme independent combinations

- Since is the moments of F_2^γ are physically measurable quantities, each term, $\mathcal{A}_i^n, \cdots, \mathcal{G}_i^n$, is factorization-scheme (FS) independent

- In the NS sector, the following combinations are FS independent

$$\text{Comb.I}: \quad K_{NS}^{(1),n} + C_{2,n}^{NS(1)} K_{NS}^{(0),n} + C_{2,n}^{\gamma,NS(1)} \gamma_{NS}^{(0),n} ,$$

$$\text{Comb.II}: \quad C_{2,n}^{NS(1)} + \frac{\gamma_{NS}^{(1),n}}{2\beta_0} ,$$

⇐ Bardeen-Buras (1979)

$$\text{Comb.III}: \quad C_{2,n}^{\gamma,NS(1)} + \widetilde{A}_n^{NS(1)} ,$$

$$\begin{aligned} \text{Comb.IV}: \quad & K_{NS}^{(2),n} + C_{2,n}^{NS(1)} K_{NS}^{(1),n} + C_{2,n}^{NS(2)} K_{NS}^{(0),n} + C_{2,n}^{\gamma,NS(2)} \gamma_{NS}^{(0),n} \\ & + C_{2,n}^{\gamma,NS(1)} \gamma_{NS}^{(1),n} + 2\beta_0 \left[C_{2,n}^{NS(1)} C_{2,n}^{\gamma,NS(1)} - C_{2,n}^{\gamma,NS(2)} \right] , \end{aligned}$$

$$\text{Comb.V}: \quad C_{2,n}^{NS(2)} + \frac{\beta_1}{2\beta_0} C_{2,n}^{NS(1)} - \frac{1}{2} \left[C_{2,n}^{NS(1)} \right]^2 + \frac{\gamma_{NS}^{(2),n}}{4\beta_0} ,$$

⇐ Nucleon structure func

$$\text{Comb.VI}: \quad C_{2,n}^{\gamma,NS(2)} + \widetilde{A}_n^{NS(1)} C_{2,n}^{NS(1)} + \widetilde{A}_n^{NS(2)} ,$$

PDFs in $\overline{\mathrm{MS}}$ scheme

$$
\begin{aligned}
F_2^\gamma &= \overrightarrow{q}^{\gamma}|_{\overline{MS}} \otimes \overrightarrow{C}_2^{\gamma}|_{\overline{MS}} = \boldsymbol{q}^\gamma|_{\overline{MS}} \otimes \boldsymbol{C}_2|_{\overline{MS}} + C_2^\gamma|_{\overline{MS}} \\
&= \left\{\boldsymbol{q}^{\gamma(0)} \otimes \boldsymbol{C}_2^{(0)}\right\} && \Longrightarrow \boldsymbol{q}^{\gamma(0)} \\
&\quad + \left\{\boldsymbol{q}^{\gamma(1)}|_{\overline{MS}} \otimes \boldsymbol{C}_2^{(0)} + \boldsymbol{q}^{\gamma(0)} \otimes \boldsymbol{C}_2^{(1)}|_{\overline{MS}}\right\} && \Longrightarrow \boldsymbol{q}^{\gamma(1)}|_{\overline{MS}} \\
&\quad + \left\{\boldsymbol{q}^{\gamma(2)}|_{\overline{MS}} \otimes \boldsymbol{C}_2^{(0)} + \boldsymbol{q}^{\gamma(1)}|_{\overline{MS}} \otimes \boldsymbol{C}_2^{(1)}|_{\overline{MS}} + \boldsymbol{q}^{\gamma(0)} \otimes \boldsymbol{C}_2^{(2)}|_{\overline{MS}}\right\} && \Longrightarrow \boldsymbol{q}^{\gamma(2)}|_{\overline{MS}} \\
&\quad + C_2^\gamma|_{\overline{MS}}
\end{aligned}
$$

In the expression of F_2^γ, put $\boldsymbol{C}_2^{(1)}|_{\overline{MS}} = \boldsymbol{C}_2^{(2)}|_{\overline{MS}} = \boldsymbol{0}$ and $C_2^\gamma|_{\overline{MS}} = 0$

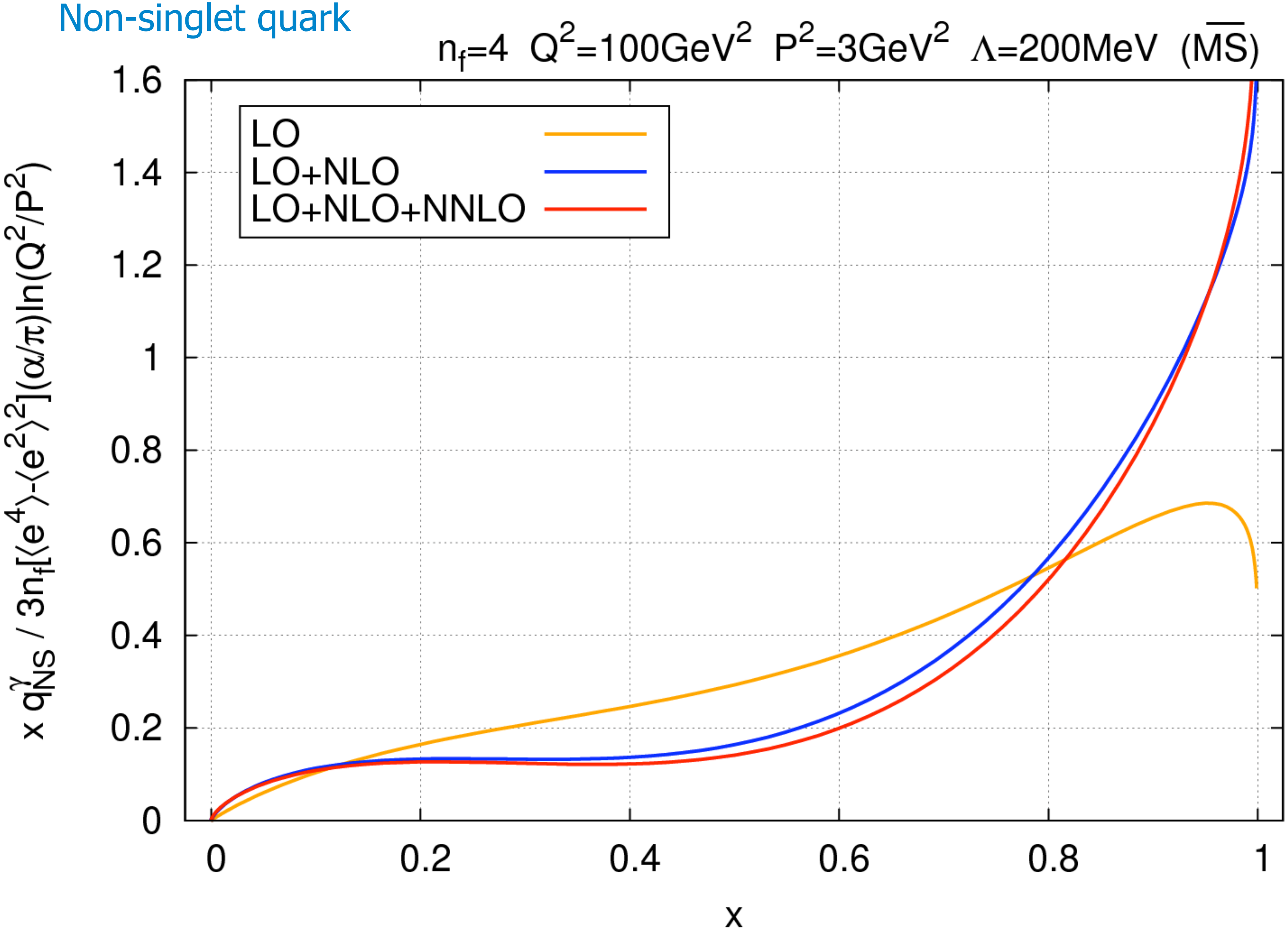
$$\boldsymbol{C}_2^{(0)} = (1,0,0) \Longrightarrow q^\gamma_{S\overline{MS}}$$

$$\boldsymbol{C}_2^{(0)} = (0,1,0) \Longrightarrow G^\gamma{}_{\overline{MS}}$$

$$\boldsymbol{C}_2^{(0)} = (0,0,1) \Longrightarrow q^\gamma_{NS\overline{MS}}$$

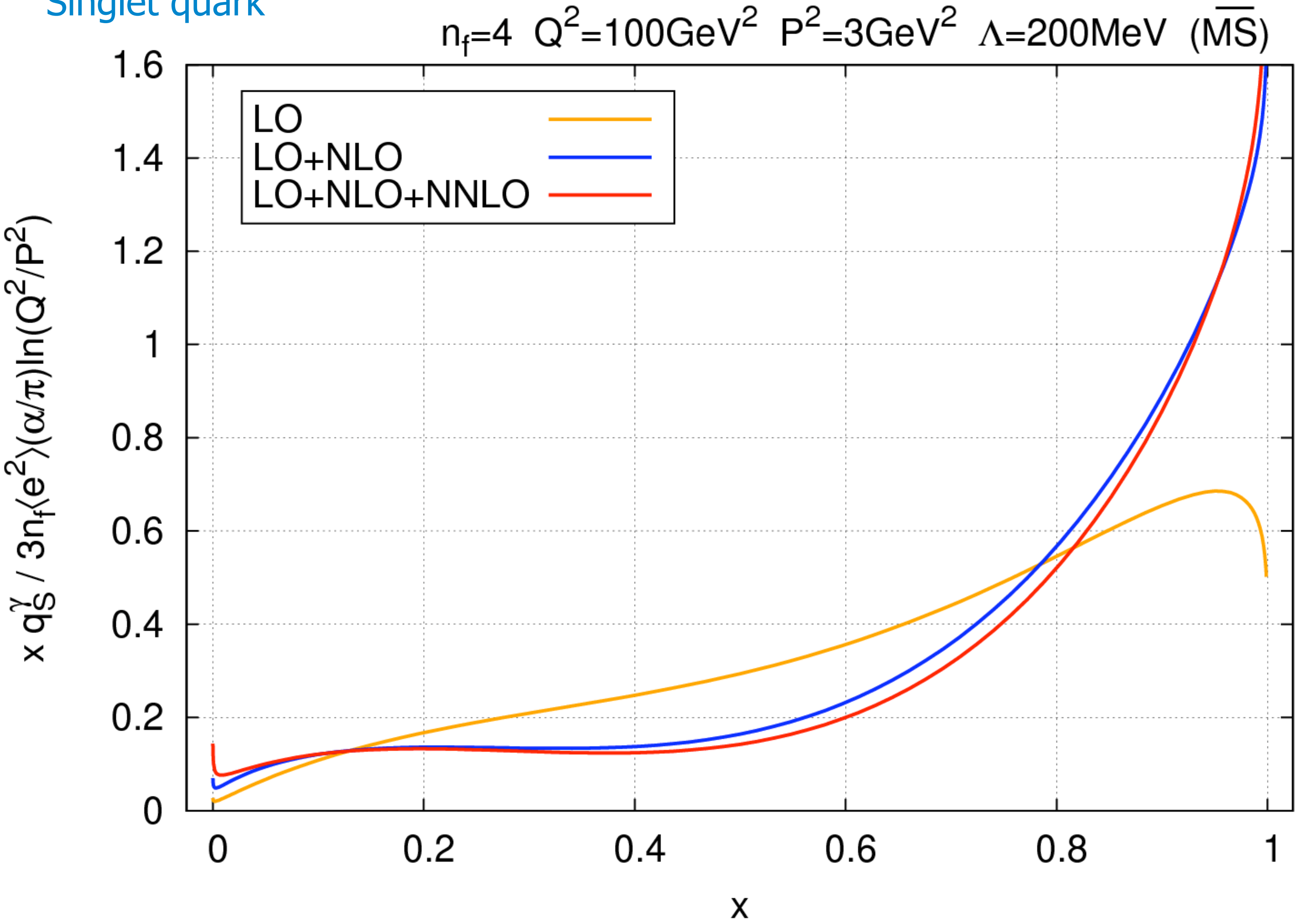
PDFs in $\overline{\mathrm{MS}}$ scheme

Non-singlet quark

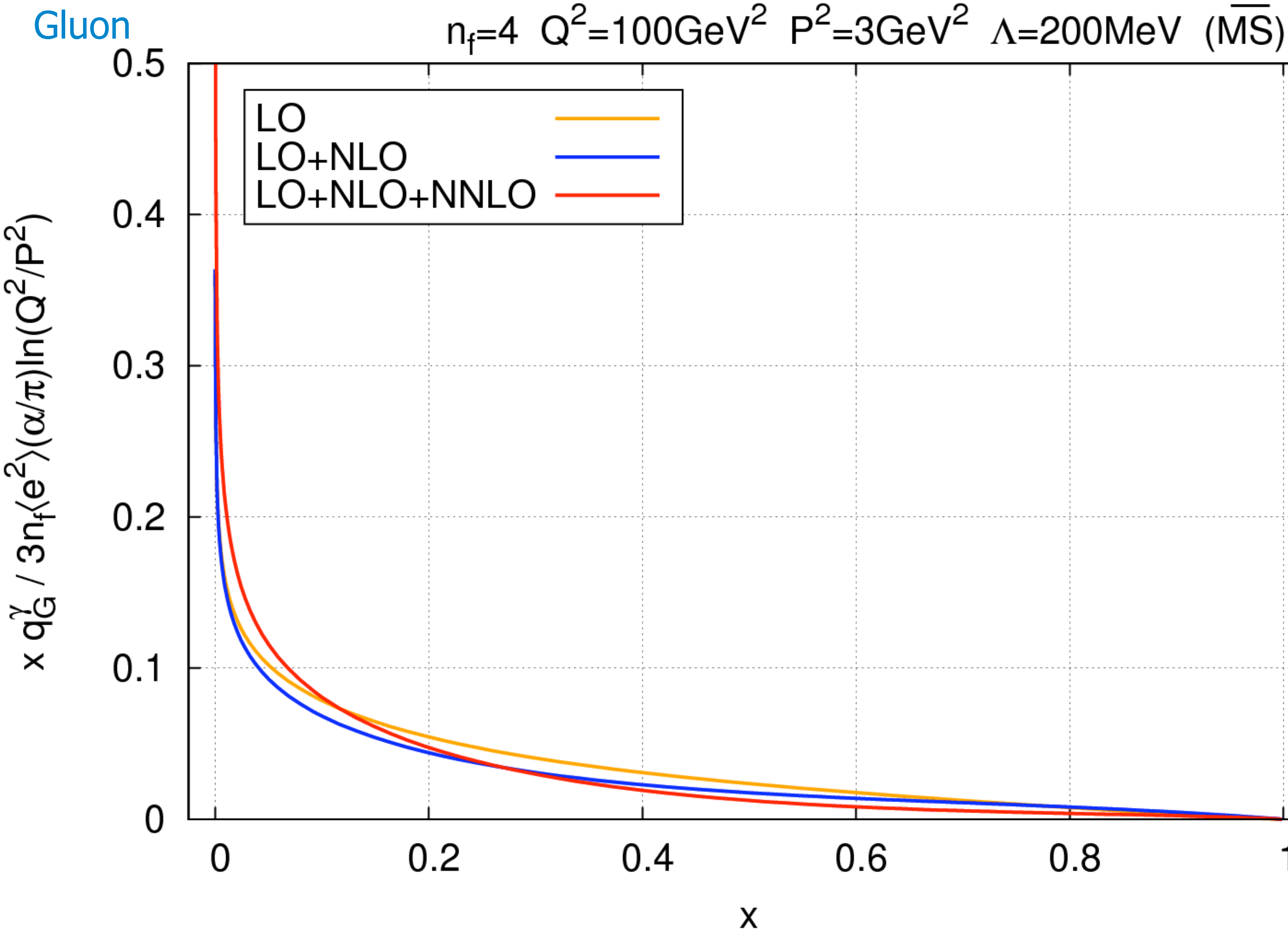


PDFs in $\overline{\mathrm{MS}}$ scheme

Singlet quark



PDFs in $\overline{\mathrm{MS}}$ scheme



Factorization scheme dependence

- Scheme-dependent PDFs $\overrightarrow{q}^{\gamma}|_a$

  $\overline{\mathrm{MS}}$ ⇨ scheme a

$$F_2^{\gamma} = \overrightarrow{q}^{\gamma} \otimes \overrightarrow{C}_2^{\gamma} = \underbrace{\overrightarrow{q}^{\gamma} Z_a}_{\overrightarrow{q}^{\gamma}|_a} \otimes \underbrace{Z_a^{-1} \overrightarrow{C}^{\gamma}}_{\overrightarrow{C}^{\gamma}|_a}$$

scheme
independent

- $a =$ DIS scheme Altarelli-Ellis-Martinelli(1978)

  F_2 is given by the naïve parton model expression to all orders
  (hadronic coefficient functions are the same as the tree level)

- $a = \mathrm{DIS}_{\gamma}$ scheme Gluck-Reya-Vogt (1992)

  Photonic coefficient function in NLO becomes negative and divergent for $x \to 1$. It is included into quark PDFs.

PDFs in DIS scheme

- In DIS scheme, the coefficient fns are those at the tree level

$$\boldsymbol{C}_2|_{\rm DIS} \;=\; \boldsymbol{C}_2^{(0)} = \big(\langle e^2\rangle, 0, 1\big)$$

- $$\begin{aligned} F_2^\gamma &= \boldsymbol{q}^\gamma|_{\rm DIS} \otimes \boldsymbol{C}_2|_{\rm DIS} + \; C_2^\gamma|_{\overline{MS}} \\ &= q_S^\gamma|_{\rm DIS}\,\langle e^2\rangle + q_{NS}^\gamma|_{\rm DIS} + \; C_2^\gamma|_{\overline{MS}} \end{aligned}$$

- In the expression of F_2^γ, put $C_2^\gamma|_{\overline{MS}} = 0$ and

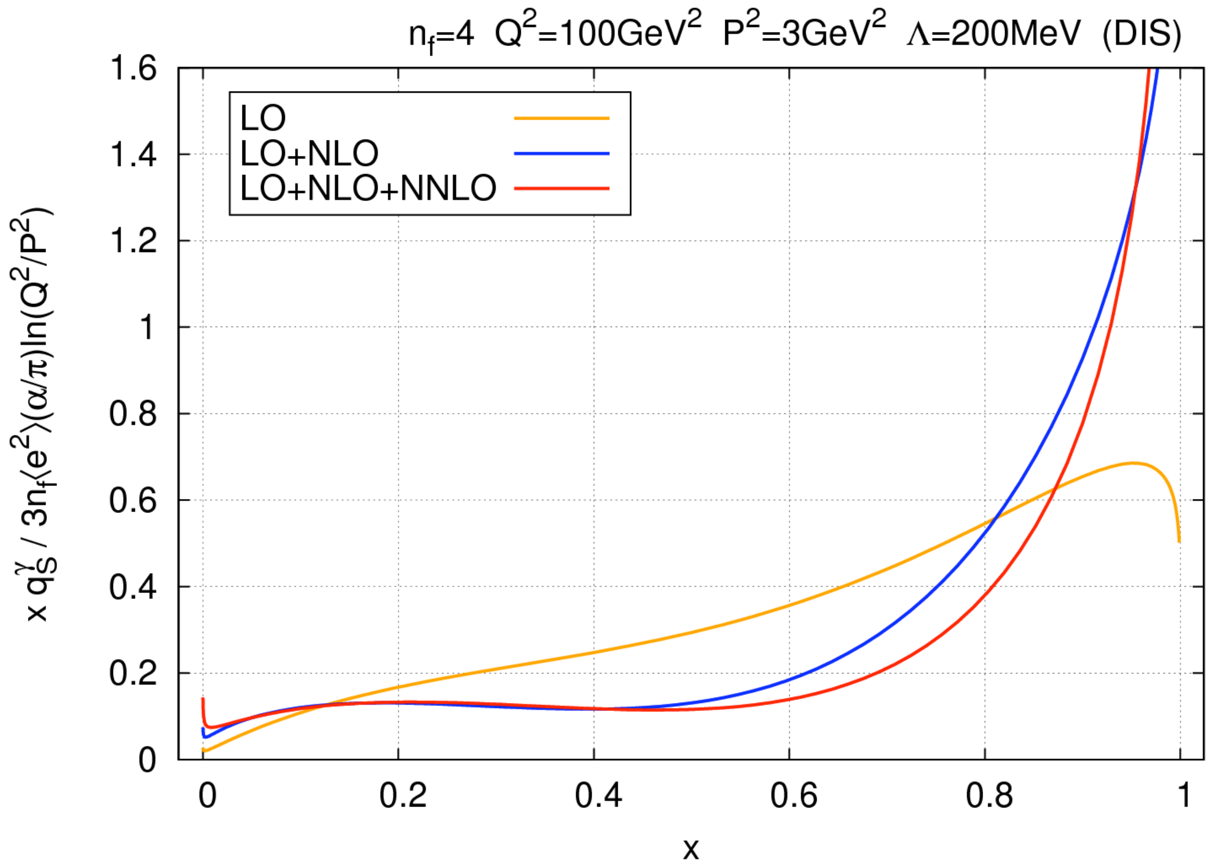
$$\boldsymbol{C}_2^{(0)} = (1,0,0) \qquad \boldsymbol{C}_2^{(1)} \to \frac{1}{\langle e^2\rangle}(C^{(1)}_{2S\,\overline{\rm MS}}, C^{(1)}_{2G\,\overline{\rm MS}}, 0) \qquad \boldsymbol{C}_2^{(2)} \to \frac{1}{\langle e^2\rangle}(C^{(2)}_{2S\,\overline{\rm MS}}, C^{(2)}_{2G\,\overline{\rm MS}}, 0)$$

$\Rightarrow$ $q_S^\gamma|_{DIS}$

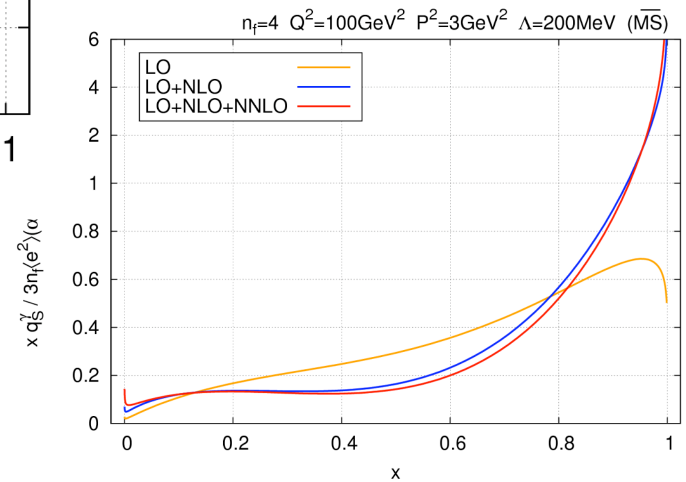
$$\boldsymbol{C}_2^{(0)} = (0,0,1) \qquad \boldsymbol{C}_2^{(1)} \to (0,0,C^{(1)}_{2NS\overline{\rm MS}}) \qquad \boldsymbol{C}_2^{(2)} \to (0,0,C^{(2)}_{2NS\overline{\rm MS}})$$

$\Rightarrow$ $q_{NS}^\gamma|_{DIS}$

PDFs in DIS scheme



Singlet quark



$$\boldsymbol{q}^{\gamma} = \frac{\alpha}{4\pi}\left[\frac{4\pi}{\alpha_s(Q^2)}\boldsymbol{q}_n^{\gamma(0)} + \boldsymbol{q}_n^{\gamma(1)} + \frac{\alpha_s(Q^2)}{4\pi}\boldsymbol{q}_n^{\gamma(2)}\right]$$

(LO) (NLO) (NNLO)

PDFs in DIS_γ scheme

- In DIS_γ scheme, photon coefficient fn is absorbed into quark distribution fns, so that $C_2^\gamma|_{\mathrm{DIS}_\gamma} = 0$

- Expressing $\boldsymbol{q}^{\gamma(1)}|_{\mathrm{DIS}_\gamma}$ and $\boldsymbol{q}^{\gamma(2)}|_{\mathrm{DIS}_\gamma}$ as

$$\begin{aligned}\boldsymbol{q}^{\gamma(1)}|_{\mathrm{DIS}_\gamma} &= \boldsymbol{q}^{\gamma(1)}|_{\overline{MS}} + \delta\boldsymbol{q}^{\gamma(1)}|_{\mathrm{DIS}_\gamma}\\ \boldsymbol{q}^{\gamma(2)}|_{\mathrm{DIS}_\gamma} &= \boldsymbol{q}^{\gamma(2)}|_{\overline{MS}} + \delta\boldsymbol{q}^{\gamma(2)}|_{\mathrm{DIS}_\gamma}\end{aligned}$$

Then we get

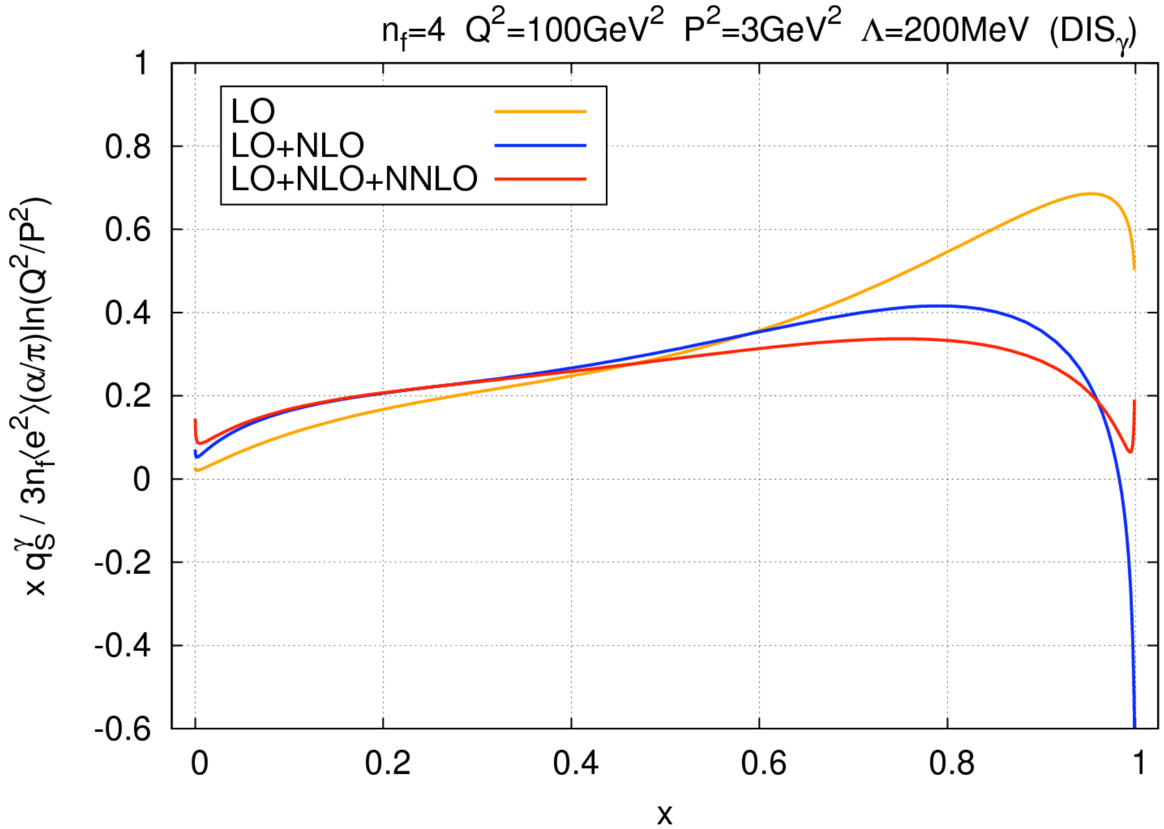
$$\begin{aligned}\frac{\alpha}{4\pi}C_2^{\gamma(1)}|_{\overline{MS}} &= \delta\boldsymbol{q}^{\gamma(1)}|_{\mathrm{DIS}_\gamma} \otimes \boldsymbol{C}_2^{(0)}\\ \frac{\alpha\alpha_s}{(4\pi)^2}C_2^{\gamma(2)}|_{\overline{MS}} &= \delta\boldsymbol{q}^{\gamma(2)}|_{\mathrm{DIS}_\gamma} \otimes \boldsymbol{C}_2^{(0)} + \delta\boldsymbol{q}^{\gamma(1)}|_{\mathrm{DIS}_\gamma} \otimes \boldsymbol{C}_2^{(1)}|_{\overline{MS}}\end{aligned}$$

$$\boldsymbol{C}_2^{(0)} = (1,0,0), \quad \boldsymbol{C}_2^{(1)} = \boldsymbol{0}, \quad C_2^{\gamma,(1)} \longrightarrow C_2^{\gamma,(1)} \times \frac{\langle e^2\rangle}{\langle e^4\rangle} \quad \Rightarrow \quad q_S^{\gamma,(1)}|_{\mathrm{DIS}_\gamma}$$

$$\boldsymbol{C}_2^{(0)} = (1,0,0), \quad \boldsymbol{C}_2^{(1)} = \boldsymbol{C}_2^{(2)} = \boldsymbol{0}, \quad C_2^{\gamma,(2)} \longrightarrow \left(C_2^{\gamma,(2)} - C_2^{\gamma,(1)}\frac{C_2^{S,(1)}}{\langle e^2\rangle}\right) \times \frac{\langle e^2\rangle}{\langle e^4\rangle}$$

$$\Rightarrow \quad q_S^{\gamma,(2)}|_{\mathrm{DIS}_\gamma}$$

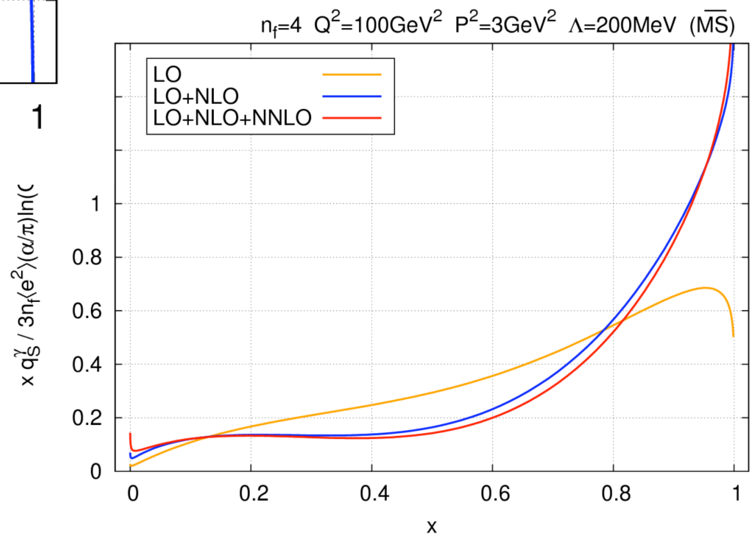
PDFs in DIS_γ scheme



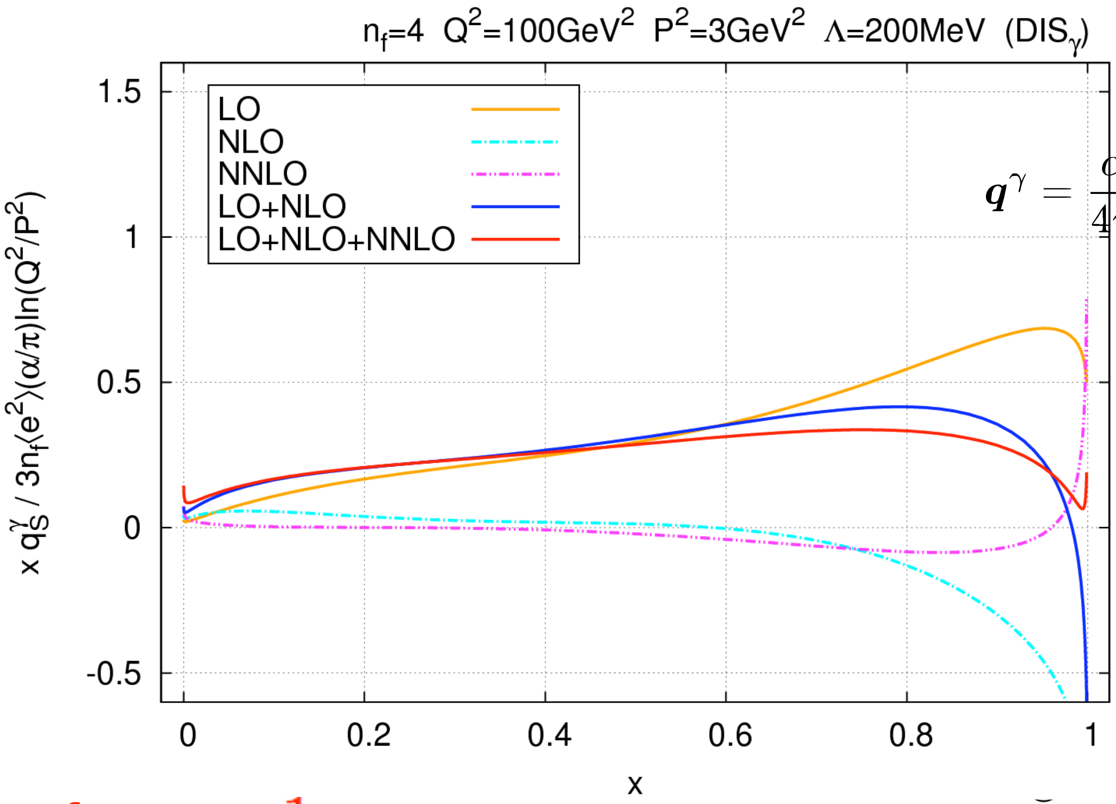
Singlet quark

$$\boldsymbol{q}^\gamma = \frac{\alpha}{4\pi}\left[\frac{4\pi}{\alpha_s(Q^2)}\boldsymbol{q}_n^{\gamma(0)} + \boldsymbol{q}_n^{\gamma(1)} + \frac{\alpha_s(Q^2)}{4\pi}\boldsymbol{q}_n^{\gamma(2)}\right]$$

(LO) (NLO) (NNLO)



PDFs in DIS_γ scheme



Singlet quark

$$\boldsymbol{q}^\gamma = \frac{\alpha}{4\pi}\left[\frac{4\pi}{\alpha_s(Q^2)}\boldsymbol{q}_n^{\gamma(0)} + \boldsymbol{q}_n^{\gamma(1)} + \frac{\alpha_s(Q^2)}{4\pi}\boldsymbol{q}_n^{\gamma(2)}\right]$$

(LO) (NLO) (NNLO)

for $x \to 1$

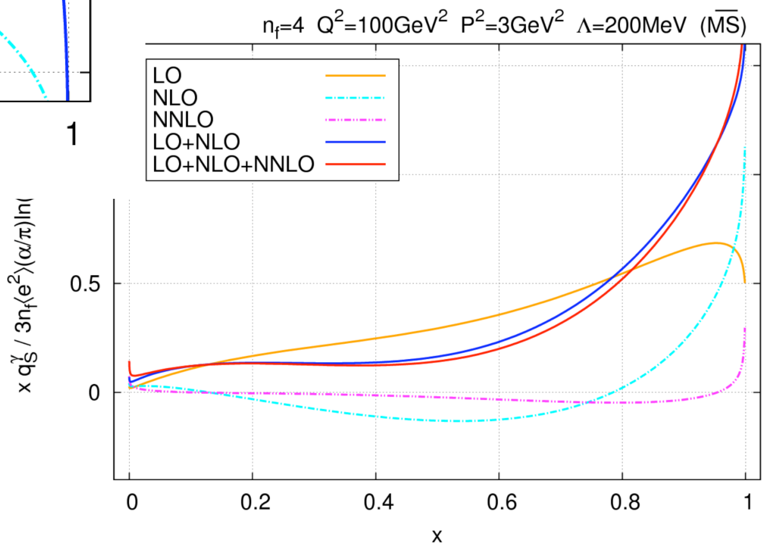
$$q_S^{\gamma(0)} \propto \frac{3}{4}\frac{-1}{\ln(1-x)}$$

$$q_S^{\gamma(2)}{}_{\overline{\mathrm{MS}}} \propto \frac{8}{9}[-\ln^3(1-x)]$$

$$q_S^{\gamma(2)}{}_{\mathrm{DIS}_\gamma} \propto \frac{8}{3}[-\ln^3(1-x)]$$

$$q_S^{\gamma(1)}{}_{\overline{\mathrm{MS}}} \propto 2[-\ln(1-x)]$$

$$q_S^{\gamma(1)}{}_{\mathrm{DIS}_\gamma} \propto -2[-\ln(1-x)]$$



PDFs in DIS_γ scheme

Large n behavior $\Longleftrightarrow$ Behavior at $x \to 1$

$$K_{NS}^{(0),n} \sim \frac{1}{n}$$

$$K_{NS}^{(1),n} \sim \frac{ln^2 n}{n}$$

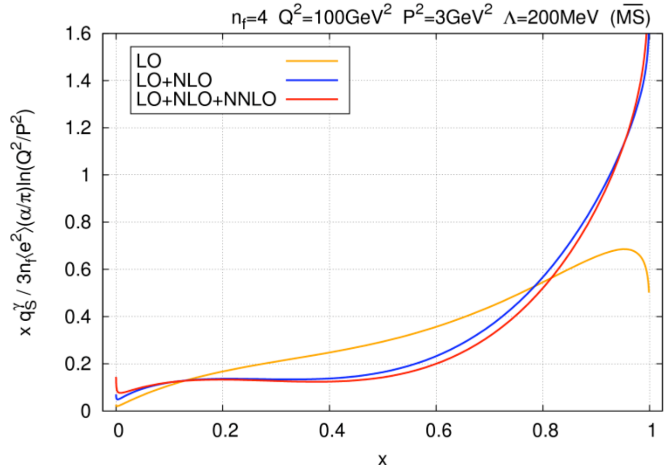
$$K_{NS}^{(0),n} \sim \frac{ln^4 n}{n}$$

$$C_{2,n}^{\gamma(1)} \sim -\frac{ln\ n}{n} \to ln(1-x)$$

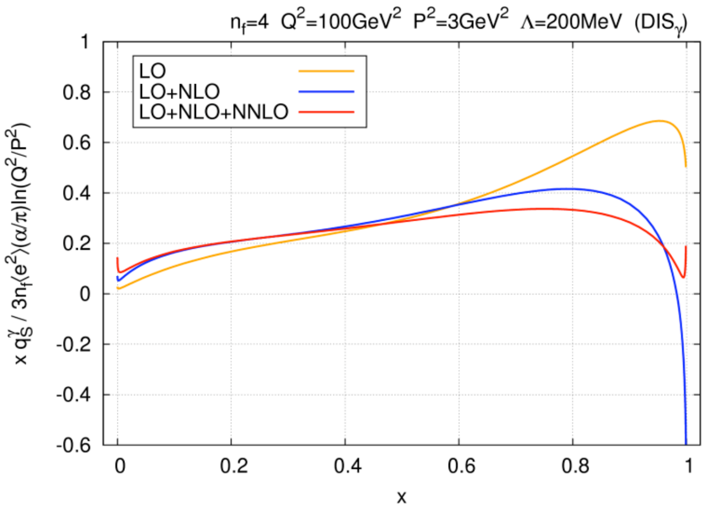
$$C_{2,n}^{\gamma(2)} \sim -\frac{ln^3\ n}{n} \to ln^3(1-x)$$

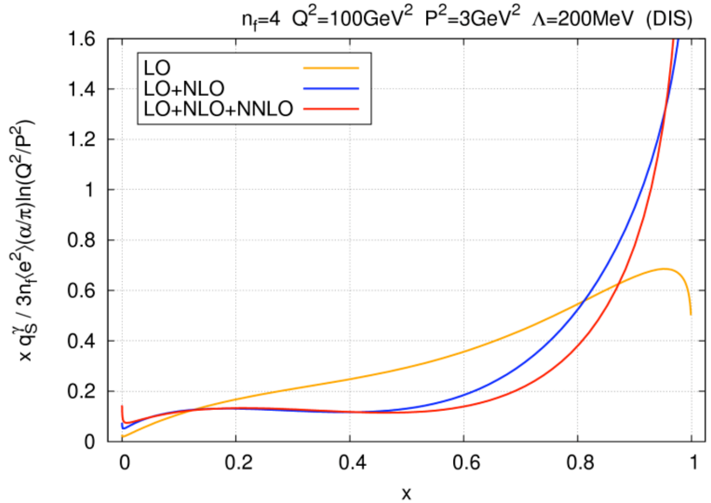
Summary

- PDFs (quark and gluon) in the virtual photon were investigated up to NNLO $(\alpha\alpha_s)$ in the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$
- PDFs were studied in $\overline{\rm MS}$, DIS and $\rm DIS_\gamma$ schemes
- Scheme dependences appear at large x



Singlet quark





Summary

- Gluon distribution