Matching NLO Calculations with Parton Shower: the POsitive-Weight Hardest Emission Generator

Carlo Oleari
Università di Milano-Bicocca, Milan

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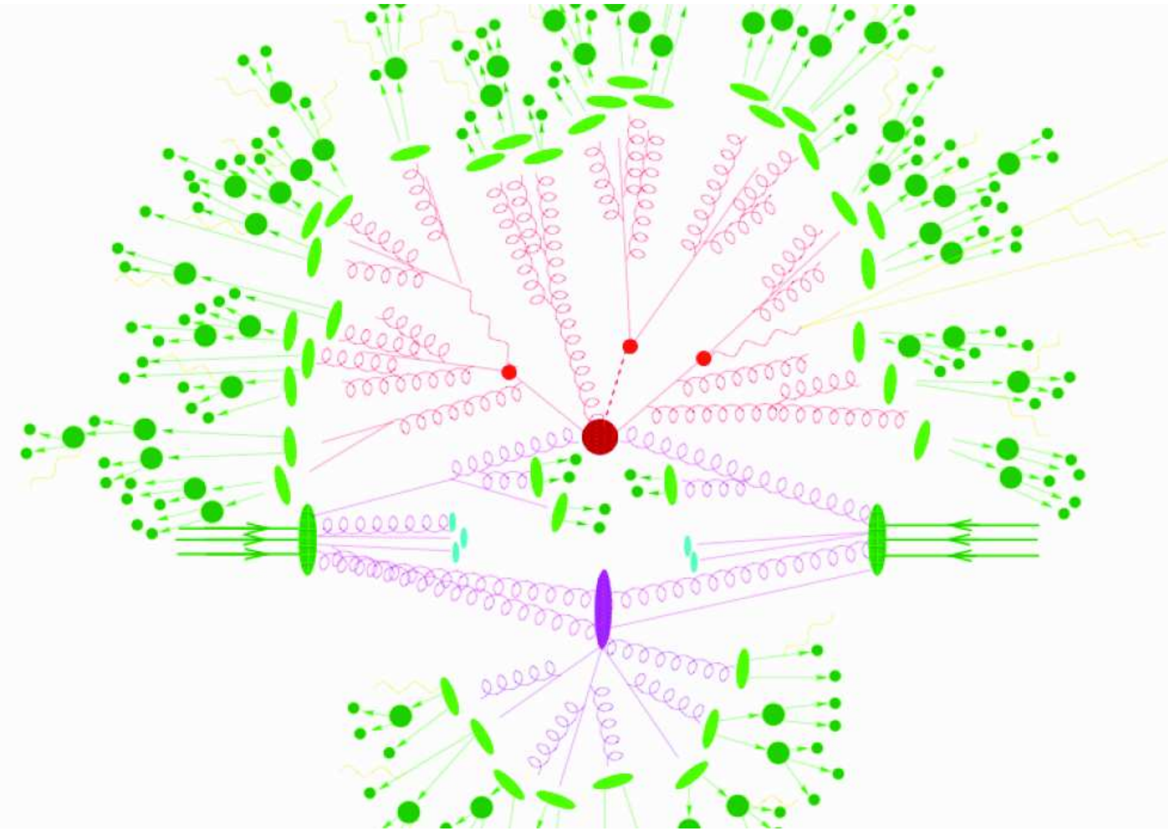
- Basics of shower Monte Carlo programs
- The POWHEG formalism
- First applications
- Conclusions



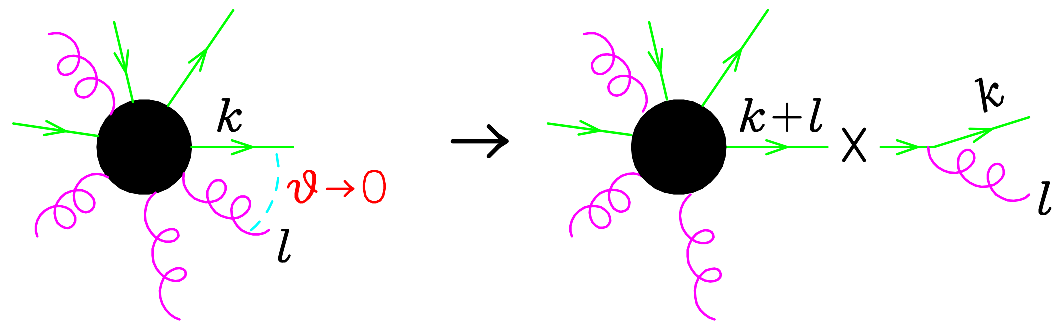
Shower Monte Carlo

- In high-energy collider physics not many questions can be answered without a shower Monte Carlo.
- The name shower comes from the fact that we dress a hard event with QCD radiation.
- After a latency period, many physicists are now looking at shower Monte Carlo models again, under different perspective (Catani, Krauss, Kühn & Webber; Mangano, Moretti, Piccinini, Pittau, Polosa & Treccani; Frixione & Webber; Kramer, Mrenna, Nagy & Soper; Giele, Kosower & Skands; Bauer & Schwartz; Schumann & Krauss; Dinsdale, Ternick & Weinzierl; ...)
- Shower algorithms summarize most of our knowledge in perturbative QCD: infrared cancellations, Altarelli-Parisi equations, soft coherence, Sudakov form factors. All have a simple interpretation in terms of shower algorithms.

Shower basics: collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 \, d\Phi_n \, \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) \, dz \, \frac{d\varphi}{2\pi}$$

$$d\Phi_{n+1} = d\Phi_n \, d\Phi_r \qquad d\Phi_r \div dt \, dz \, d\varphi$$

t : $(k+l)^2,\ p_T^2,\ E^2\theta^2 \ldots$

$z = k^0/(k^0 + l^0)$: energy (or $p_\parallel$ or p^+)fraction of quark

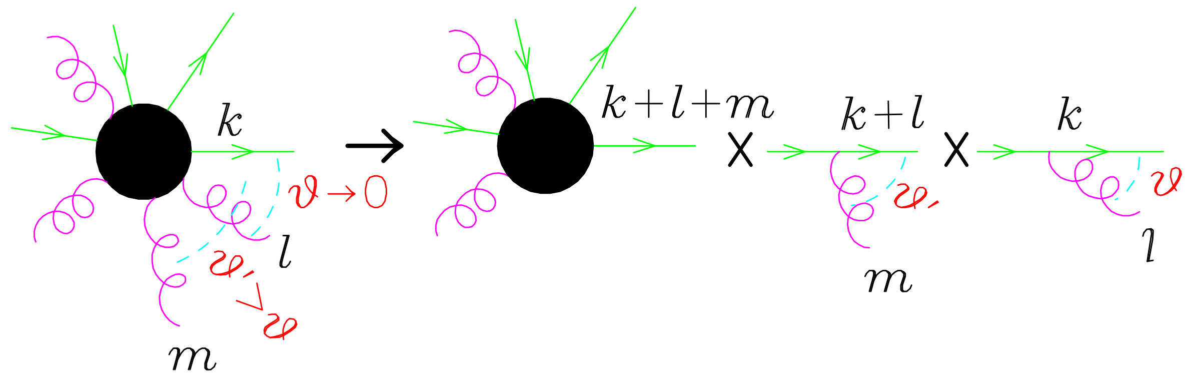
$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli-Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

Shower basics: collinear factorization

If another gluon becomes collinear, iterate the previous formula

$\theta', \theta \to 0$ with $\theta' > \theta$

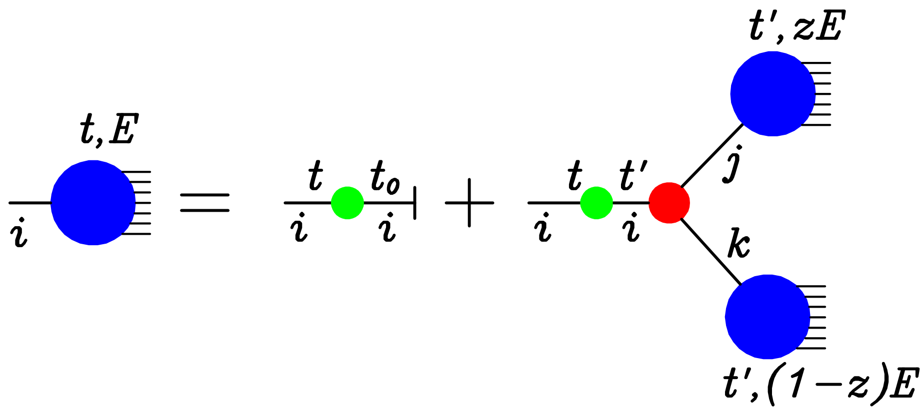


$$|M_{n+1}|^2 d\Phi_{n+1} \quad \Longrightarrow \quad |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\varphi'}{2\pi} \times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\varphi}{2\pi} \theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions

$$\int_{t_{\min}}^{t^{\max}} \frac{dt_1}{t_1} \int_{t_{\min}}^{t_1} \frac{dt_2}{t_2} \cdots \int_{t_{\min}}^{t_{m-1}} \frac{dt_m}{t_m} \propto \frac{\log^m \frac{t_{\max}}{t_{\min}}}{m!} \approx \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}, \qquad \Lambda \approx \Lambda_{\mathrm{QCD}}$$

Final recipe I



$$\mathcal{S}_i(t,E) = \Delta_i(t,t_0)\langle\mathbb{I}| + \sum_{(jk)}\int_{t_0}^{t}\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\int dz\int\frac{d\varphi}{2\pi}\,\Delta_i(t,t')\,P_{i,jk}(z)\,\mathcal{S}_j(t',zE)\;\mathcal{S}_k(t',(1-z)E)$$

- consider all tree graphs.
- assign values to the radiation variables Φ_r (t, z and φ) to each vertex.
- at each vertex, $i \to jk$, include a factor

$$\frac{dt}{t}\,dz\,\frac{\alpha_s(t)}{2\pi}\,P_{i,jk}(z)\,\frac{d\varphi}{2\pi}$$

Simple probabilistic interpretation of “not-resolved” corrections

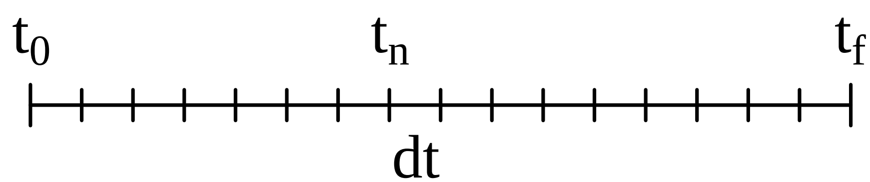
- probability of emission in the interval dt

$$dP_{\rm emis}(t+dt,t) = \frac{dt}{t}\,\frac{\alpha_s(t)}{2\pi}\int dz\,P_{i,jk}(z)$$

- probability of no emission in the interval dt

$$dP_{\rm no\ emis}(t+dt,t) = 1 - dP_{\rm emis}(t+dt,t) = 1 - \frac{dt}{t}\,\frac{\alpha_s(t)}{2\pi}\int dz\,P_{i,jk}(z)$$

- divide a finite interval $[t_0, t_f]$ in N small intervals $dt = (t_f - t_0)/N$.

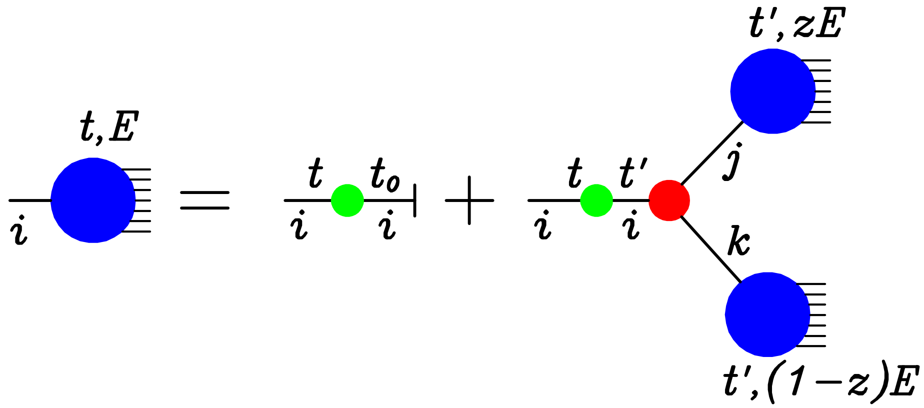


The probability of not emitting radiation between the two virtuality scales t_f and t_0 is given by the product

$$P_{\rm no\ emis}(t_f,t_0) = \lim_{N\to\infty}\prod_{n=1}^{N}\left[1 - \frac{dt}{t_n}\,\frac{\alpha_s(t_n)}{2\pi}\int dz\,P_{i,jk}(z)\right] = \exp\left\{-\int_{t_0}^{t_f}\frac{dt}{t}\,\frac{\alpha_s(t)}{2\pi}\int dz\,P_{i,jk}(z)\right\}$$

$$\equiv \Delta(t_f,t_0)$$

Final recipe II



- include a factor $\Delta_i(t_1, t_2)$ to each internal parton i, from hardness t_1 to hardness t_2.

$$\Delta_i(t_1, t_2) = \exp\left[- \sum_{(jk)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz\, P_{i,jk}(z) \int \frac{d\varphi}{2\pi} \right]$$

The weights $\Delta_i(t_1, t_2)$ are called Sudakov form factors. They resum all the dominant virtual corrections to the tree graph (in the collinear approximation).
Notice that, when $t_2 \ll t_1$, $\Delta \to 0$, i.e. the probability that a hard parton turns into a narrow jet, or that it does not radiate at all, is small (it is Sudakov suppressed)

- include a factor $\Delta_i(t, t_0)$ on final lines (t_0 = IR cutoff)

First branching

The probability of the first branching is independent of subsequent branchings because of KLN cancellation. It is given by

$$dP_{\rm first} = \Delta_i(t,t')\,\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\,P_{i,jk}(z)\,dz\,\frac{d\varphi}{2\pi}$$

Integrating in $dz, d\varphi$, summing over jk, the t' distribution is

$$dP_{\rm first} = \Delta_i(t,t')\,\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\int\sum_{(jk)} P_{i,jk}(z)\,dz\frac{d\varphi}{2\pi} = d\Delta_i(t,t')$$

i.e. the distribution is uniform in the Sudakov form factor.
Notice that

$$\int_{\infty}^{t_{\rm min}} dP_{\rm first} = \int_{\infty}^{t_{\rm min}} d\Delta_i(t_{\rm min},t) = \Delta_i(t_{\rm min},t_{\rm min}) - \Delta_i(t_{\rm min},\infty) = 1$$

as it should be for a correct probabilistic interpretation.

Shower algorithm

We start from a given value of the virtuality variable t. We want to generate the value t' for the next emission, according to the probability

$$dP = \Delta_i(t,t') \frac{\alpha_S(t')}{2\pi} \frac{1}{t'} \sum_{(jk)} P_{i,jk}(z) \frac{1}{2\pi} dt'\, dz\, d\varphi = d\Delta_i(t,t')$$

The algorithm works as follow:

- generate a uniform random number $0 < r < 1$
- solve the equation $\Delta_i(t,t') = r$ for t'
- if $t' < t_0$ stop here (final state line)
- generate z, jk with probability $P_{i,jk}(z)\, dz$, and $0 < \varphi < 2\pi$ uniformly
- restart from each of the two branches, with hardness parameter t'.

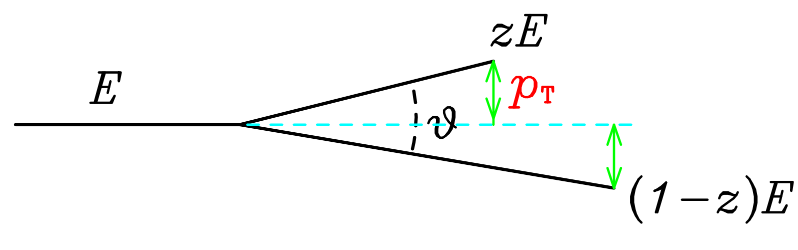
$$dP = f(X)\, dX = dF(X) = 1\, dR \implies \int_{x_{\min}}^{x} f(X)\, dX = F(x) = \int_0^r 1\, dR = r \quad \implies \quad x = F^{-1}(r)$$

Accuracy: soft divergences and double-log regions

$z \to 1$ ($z \to 0$) region problematic. In fact, for $z \to 1$, $P_{qq}, P_{gg} \div 1/(1-z)$

The choice of the ordering variable t makes a difference

$$
\begin{aligned}
\text{virtuality:} \quad & t \equiv E^2 z(1-z) \overbrace{\theta^2}^{2(1-\cos\theta)} \\
p_T^2: \quad & t \equiv E^2 z^2 (1-z)^2 \theta^2 \\
\text{angle:} \quad & t \equiv E^2 \theta^2
\end{aligned}
$$



$$\text{virtuality}: z(1-z) > t/E^2 \implies \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{4}$$

$$p_T^2 : z^2(1-z)^2 > t/E \implies \int \frac{dt}{t} \int_{t/E}^{1-t/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{2}$$

$$\text{angle}: \implies \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \log \Lambda$$

Sizable difference in double-log structure!

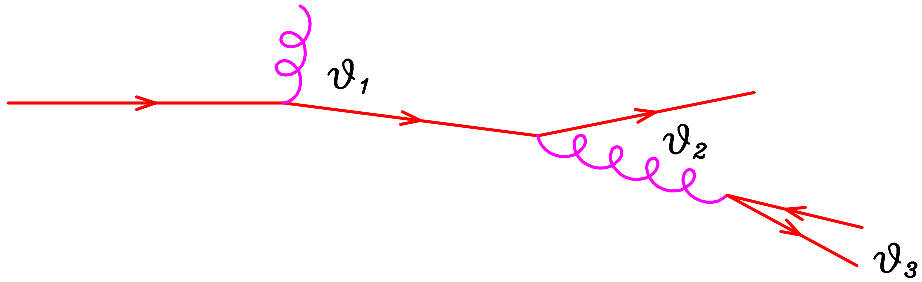
Angular ordering

Mueller (1981) showed that angular ordering is the correct choice

$$\frac{d\theta}{\theta}\,\frac{\alpha_s\left(p_T^2\right)}{2\pi}\,P(z)\,dz$$

$$\theta_1 > \theta_2 > \theta_3 \ldots$$

$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region (p_T^2 equals to the maximum virtuality of the gluon line).

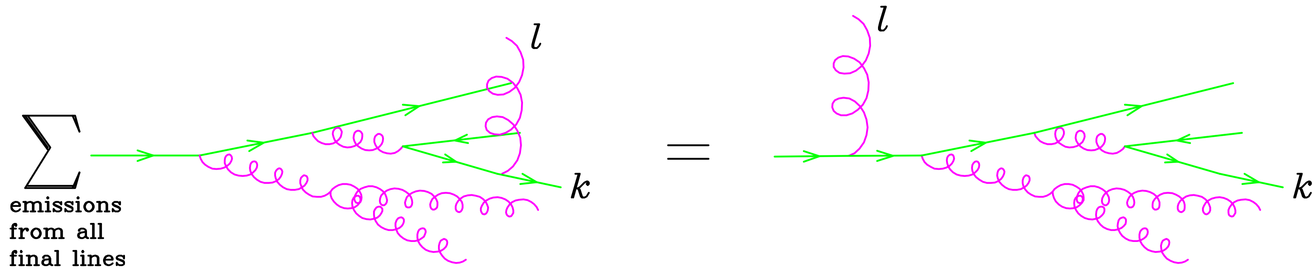
$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t}\frac{dt}{t}\int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz\,\frac{\alpha_s(p_T^2)}{2\pi}\sum_{(jk)} P_{i,jk}(z)\right]$$

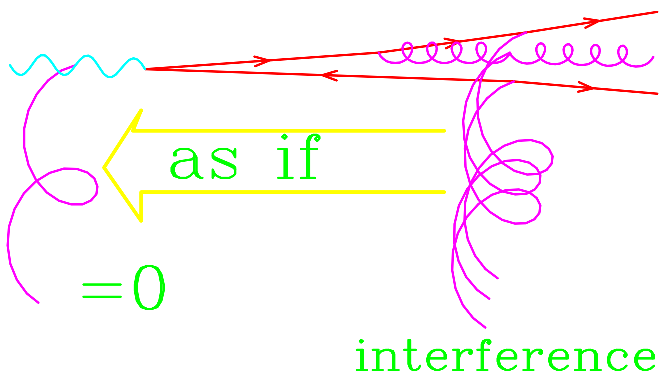
$$\approx \exp\left\{-\frac{c_i}{4\pi b_0}\left[\log\frac{t}{\Lambda^2}\log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0}\right]_{t'}^{t}\right\} \qquad (c_q = C_F, c_g = 2C_A)$$

Sudakov dumping stronger than any power of t.

Color coherence

Soft gluons emitted at large angles from final-state partons add coherently





- angular ordering accounts for soft gluon interference.
- intensity for photon jets $= 0$
- intensity for gluon jets $= C_A$ instead of $2C_F + C_A$

In angular ordered shower Monte Carlo, large-angle soft emission is generated first. Hardest emission, i.e. highest $p_T = E\, z(1-z)\,\theta$, in general, happens later.

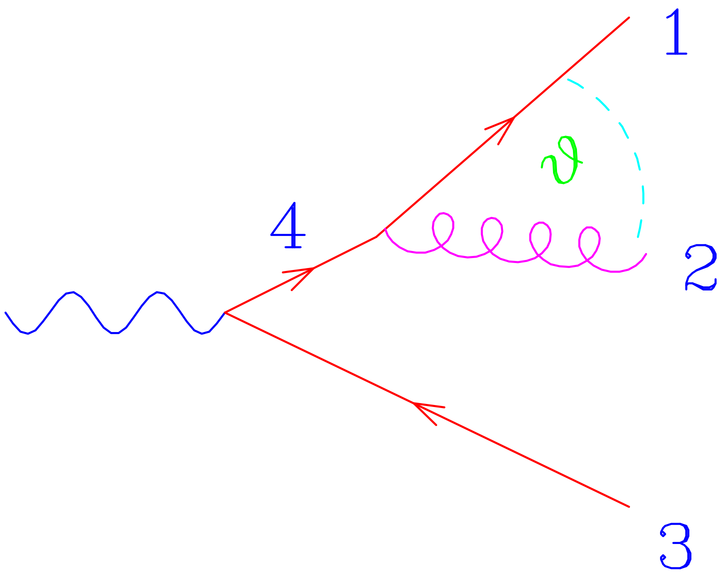
POWHEG

The POWHEG (POsitive-Weight Hardest Emission Generator) method [Nason, hep-ph/0409146] deals with two main issues (no technical details!):

1. transform an angular-ordered shower into a shower where the hardest emission happens first
   - generate first event with hardest emission
   - generate all subsequent emissions with a p_T veto equal to the hardest emission p_T
   - pair up the partons that are nearest in p_T
   - generate an angular-ordered shower associated with the paired parton, stopping at the angle of the paired partons (truncated shower)
   - generate all subsequent vetoed showers

2. include exact NLO cross section

Example of truncated shower: e^+e^-



- nearby partons: 1 and 2
- truncated shower: 1 and 2 pair, from θ up to a maximum angle. The truncated shower reintroduces coherent soft radiation from 1 and 2 at angles larger than θ (angular-ordered shower Monte Carlo programs generate those earlier).
- 1 and 2 shower from θ to cutoff
- 3 showers from maximum to cutoff

Truncated showers not yet implemented.

No evidence of effects from their absence in ZZ and e^+e^- production. Might be some effects in heavy-quark production.

Reaching NLO accuracy

$$\Phi_{n+1} = \Phi_{n+1}(\Phi_n, \Phi_r) \qquad d\Phi_{n+1} = d\Phi_n \, d\Phi_r \qquad d\Phi_r = dt\, dz \, \frac{d\varphi}{2\pi}$$

$$d\sigma^{\rm LO} = B(\Phi_n)\, d\Phi_n \left[\Delta^{\rm LO}(t_{\rm min}) + \int d\Phi_r \, \Delta^{\rm LO}(t) \, \frac{\alpha_s}{2\pi} \, P(z) \, \frac{1}{t}\right]$$

$$d\sigma^{\rm NLO} = \bar{B}(\Phi_n)\, d\Phi_n \left[\Delta^{\rm NLO}_{t_{\rm min}} + \int d\Phi_r \, \Delta^{\rm NLO}_t \, \frac{R(\Phi_{n+1})}{B(\Phi_n)}\right]$$

$$\bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{\overbrace{V(\Phi_n)}^{\text{infinite}} + \overbrace{\int d\Phi_r \, R(\Phi_{n+1})}^{\text{infinite}}}_{\text{FINITE!}}$$

$$\Delta^{\rm LO}(t) = \exp\left[-\int_t d\Phi_r' \, \frac{\alpha_s}{2\pi} \, P(z') \, \frac{1}{t'}\right] \qquad \Delta^{\rm NLO}_t = \exp\left[-\underbrace{\int d\Phi_r' \, \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)} \, \theta(t'-t)}_{\text{FINITE because of } \Theta \text{ function}}\right]$$

with $t' = p_T(\Phi_n, \Phi_r')$ = transverse momentum of the emitted parton.

If we expand the shower expression in α_s, we obtain a result that is accurate at the NLO level.

In addition, the differential cross section is **POSITIVE** if $\bar{B}$ is positive.

Mathematical tricks

- ✓ To generate the underlying Born variables (Φ_n), distributed according to $\bar{B}(\Phi_n)$, one uses programs like BASES/SPRING, that, after a single integration, can generate points distributed according to the integrand function.

- ✓ Use the veto technique and the highest-p_T bid procedure, to generate the radiation variables, distributed according to $d\Delta_i(t, t')$.

These tricks are well known to Monte Carlo experts.

We have collected a few of them in the appendixes of our paper
[Frixione, Nason and C.O., arXiv:0709.2092 [hep-ph]].

POsitive-Weight Hardest Emission Generator

POWHEG is a method, NOT (only) a program

✓ it is independent from parton-shower programs. POWHEG can be interfaced with both PYTHIA and HERWIG, or with your favorite showering program, if the vetoed shower is implemented, according to the Les Houches Interface.

✓ it can use existing NLO results

✓ it generates events with positive weights

✓ NLO accuracy for integrated quantities

✓ leading-log-collinear + double-log (soft-collinear) + large-N_c-soft-log accurate on first radiation (for 3 external colored partons, it is exact at next-to-leading-log accuracy).

✗ no truncated shower implemented up to now

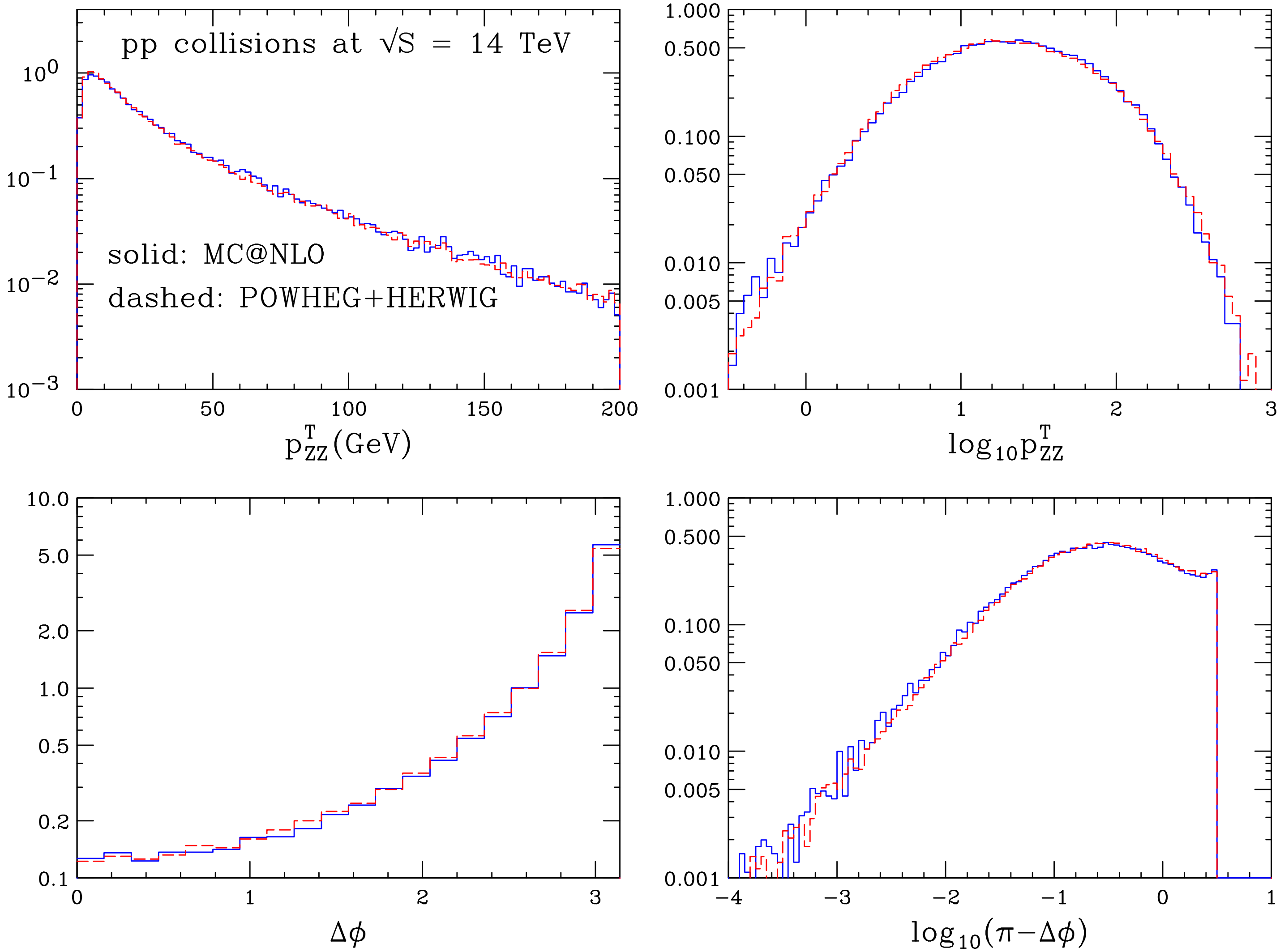
Existing implementations

The POWHEG method has already been successfully used in

- ZZ production [Nason and Ridolfi, `hep-ph/0606275`]
- e^+e^- to hadrons [Latunde-Dada, Gieseke and Webber, `hep-ph/0612281`]
- heavy-quark $Q\overline{Q}$ production ($c\bar{c}$, $b\bar{b}$, $t\bar{t}$) with spin correlations [Frixione, Nason and Ridolfi, arXiv:0707.3088 [hep-ph]].

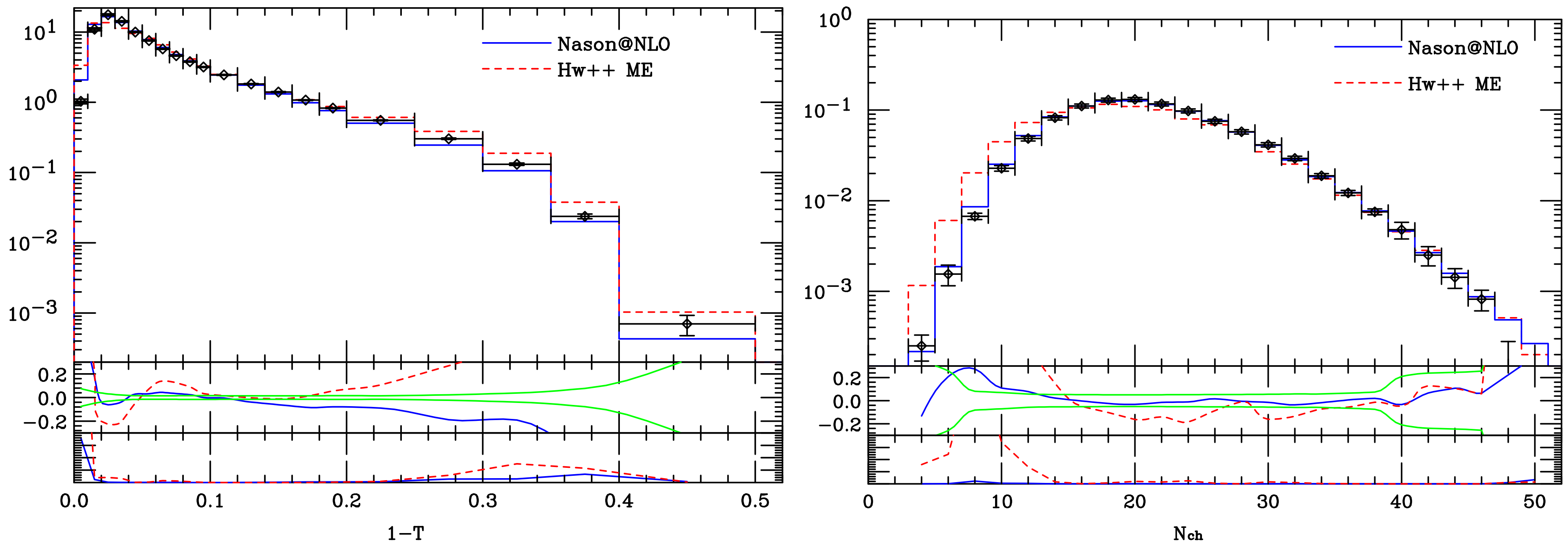
The POWHEG programs for ZZ and $Q\overline{Q}$ production have been interfaced to both PYTHIA and HERWIG.

ZZ production: POWHEG + HERWIG vs MC@NLO



No significant difference with MCatNLO [Nason and Ridolfi, hep-ph/0606275]

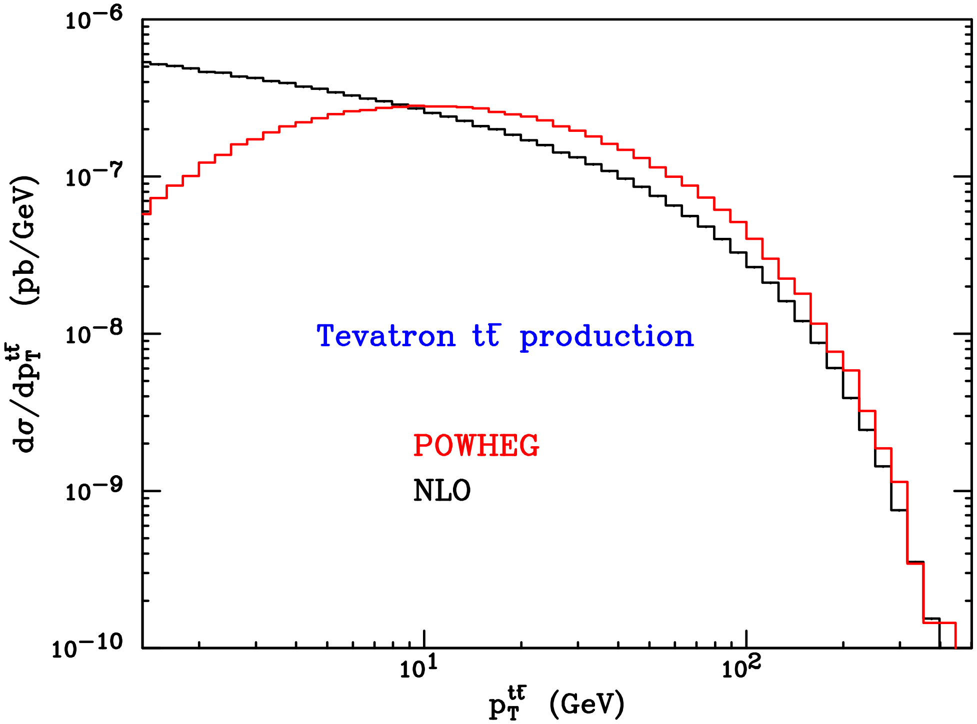
$e^+e^- \longrightarrow$ hadrons

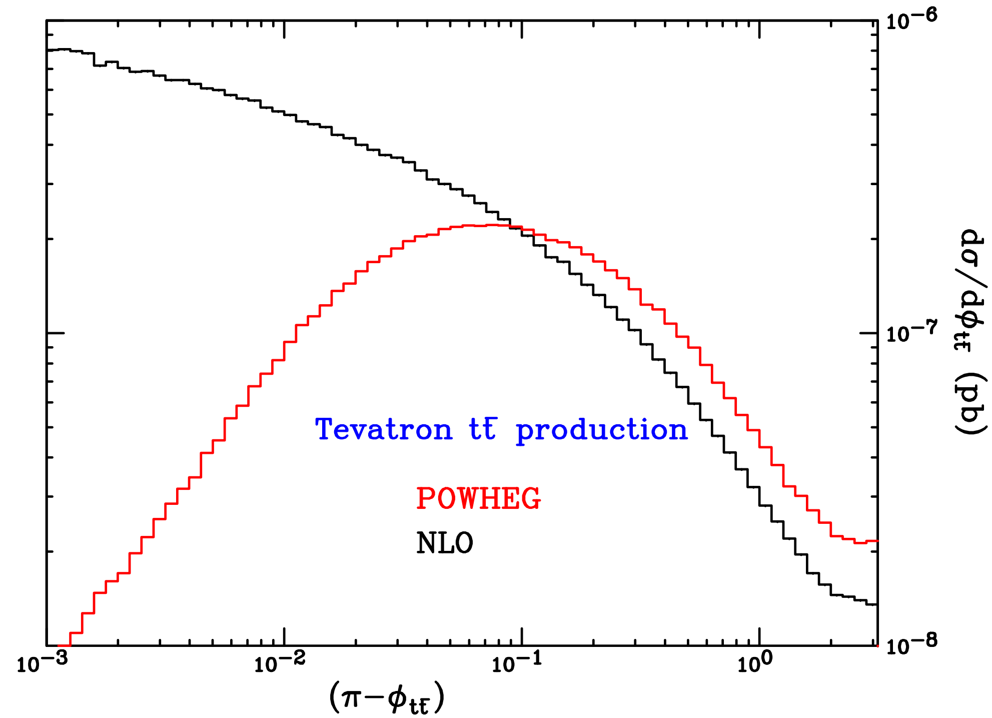


[Latunde-Dada, Gieseke and Webber, hep-ph/0612281]

Fit to e^+e^- data: better agreement than in the standard matrix-element correction approach.

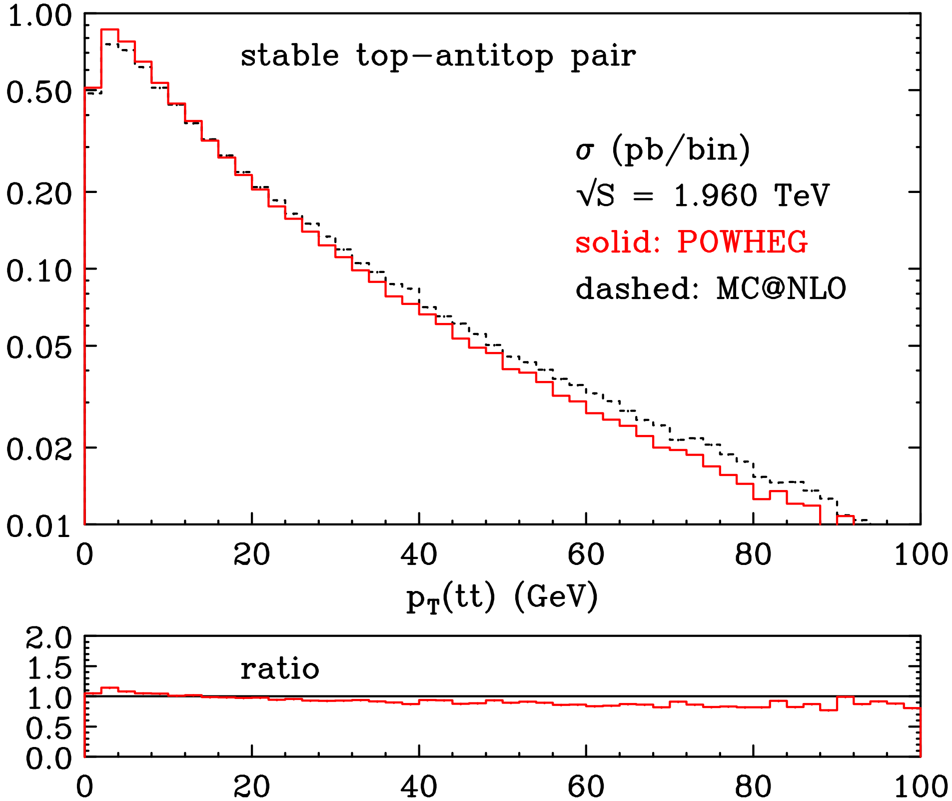
$t\bar{t}$ production: POWHEG vs. NLO

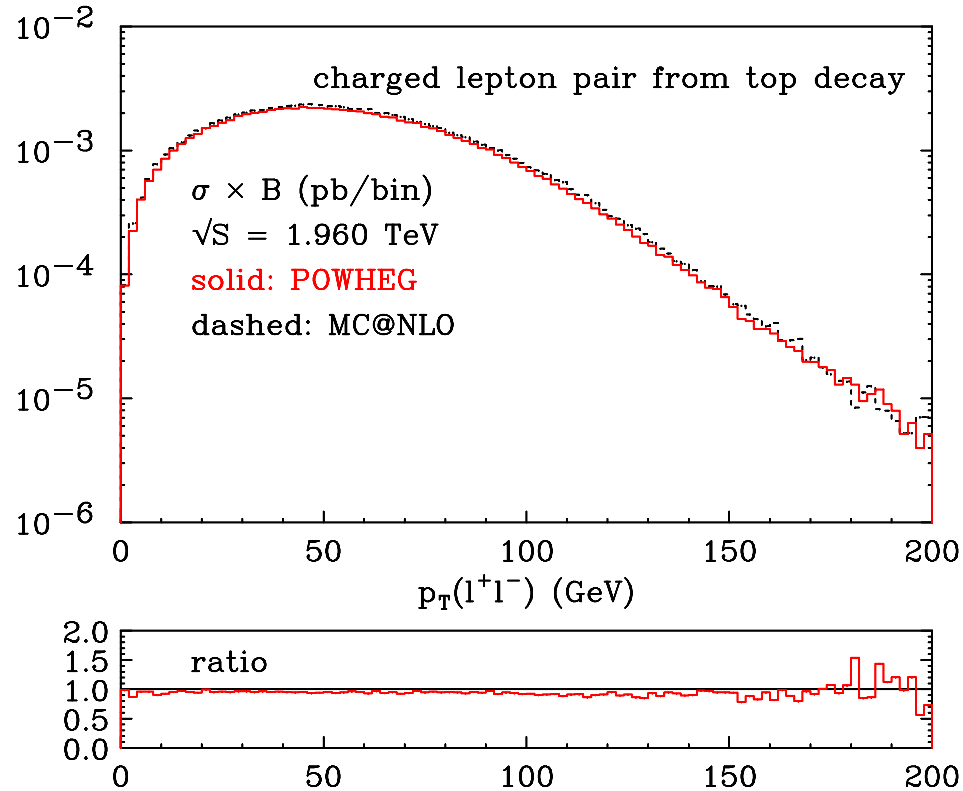




- when $p_T^{t\bar{t}} \to 0$, POWHEG treats correctly the resummation of soft/collinear radiation
- when $p_T^{t\bar{t}}$ becomes large, POWHEG approaches the NLO result
- when $\Phi_{t\bar{t}} \to 0$, the emitted radiation becomes hard and POWHEG goes to the NLO result.

$t\bar{t}$ production





Good agreement for all observables considered. There are sizable differences that can be ascribed to different treatment of higher terms. But more investigation needed (different scale choices, no truncated shower, different hard/soft radiation emission,. . .).

ALPGEN vs MCatNLO: $t\bar{t}$ + 1 jet

[Mangano, Moretti, Piccinini & Treccani, hep-ph/0611129]

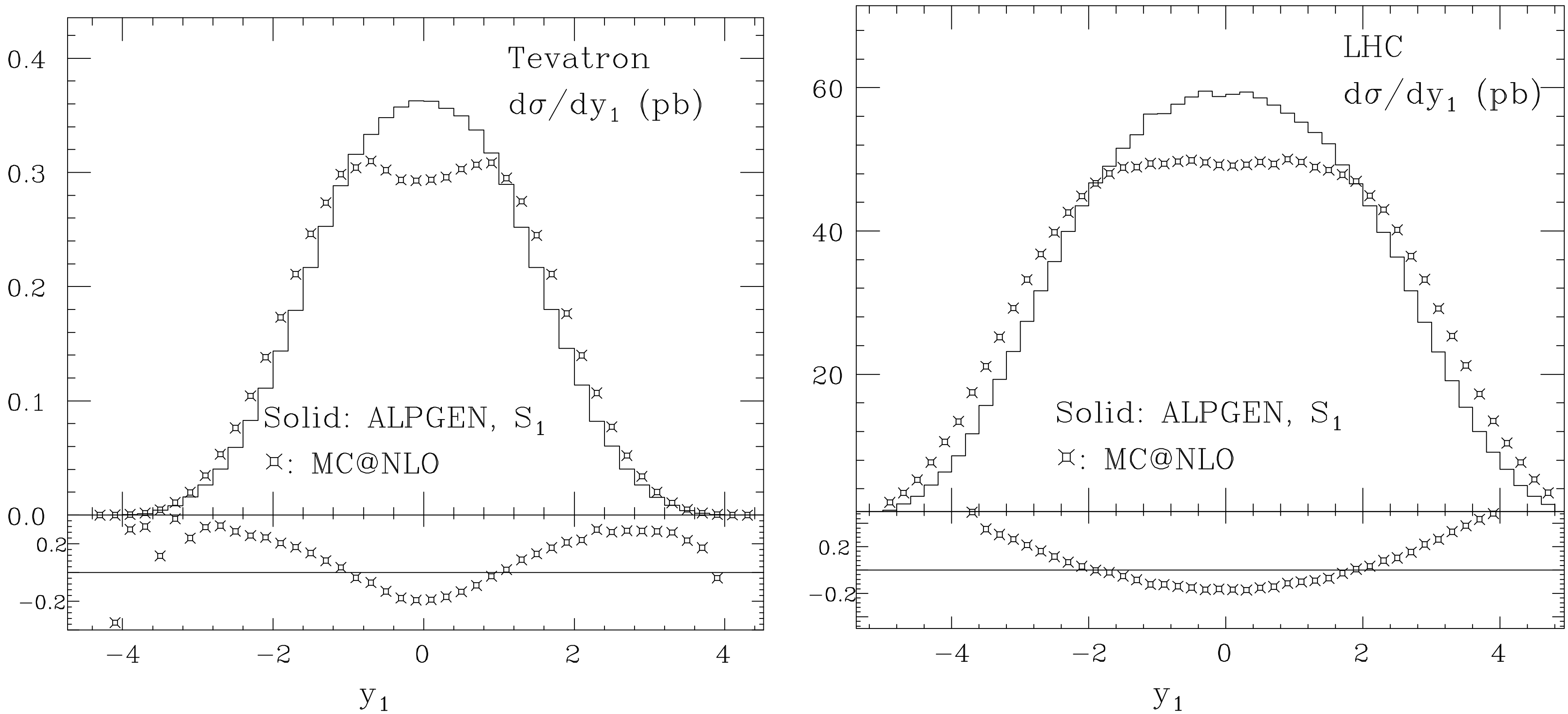
ALPGEN

- Generation: $P^T_{\rm min} = 30$ GeV, $\Delta R = 0.7$
- Matching: $E^T_{\rm min} = 30$ GeV, $\Delta R = 0.7$

Jet definitions

- Tevatron: $E^T_{\rm min} = 15$ GeV, $\Delta R = 0.4$, K factor = 1.45
- LHC: $E^T_{\rm min} = 20$ GeV, $\Delta R = 0.5$, K factor = 1.57

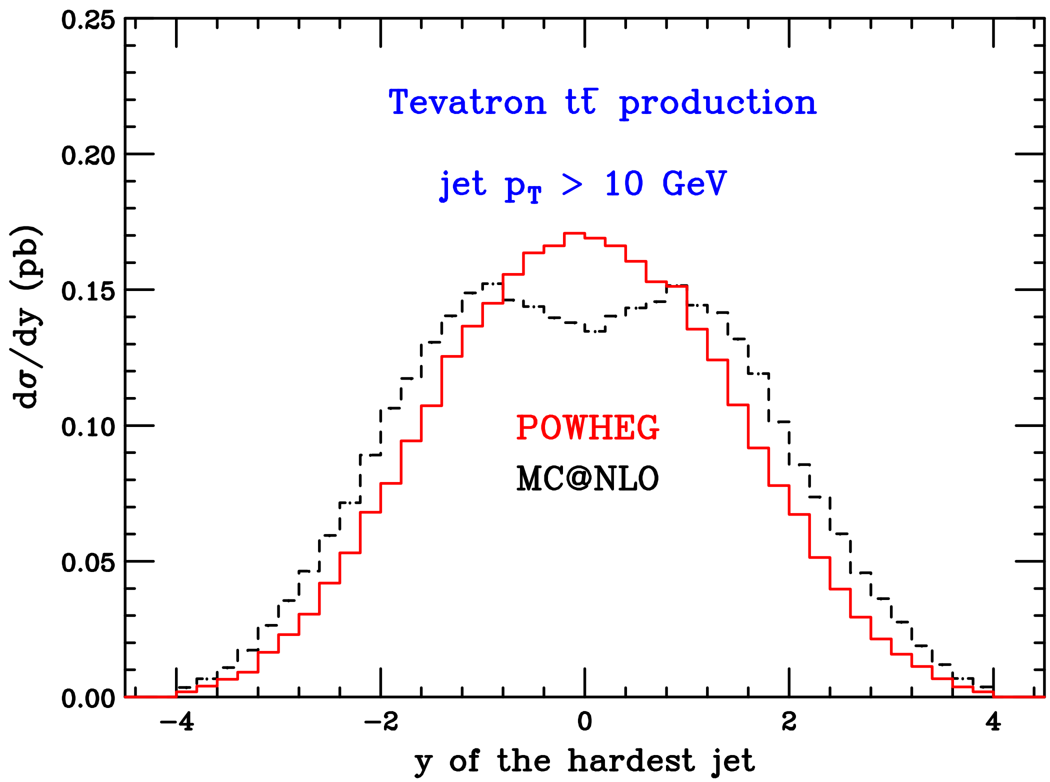
ALPGEN vs MC@NLO: $t\bar{t}$ + 1 jet

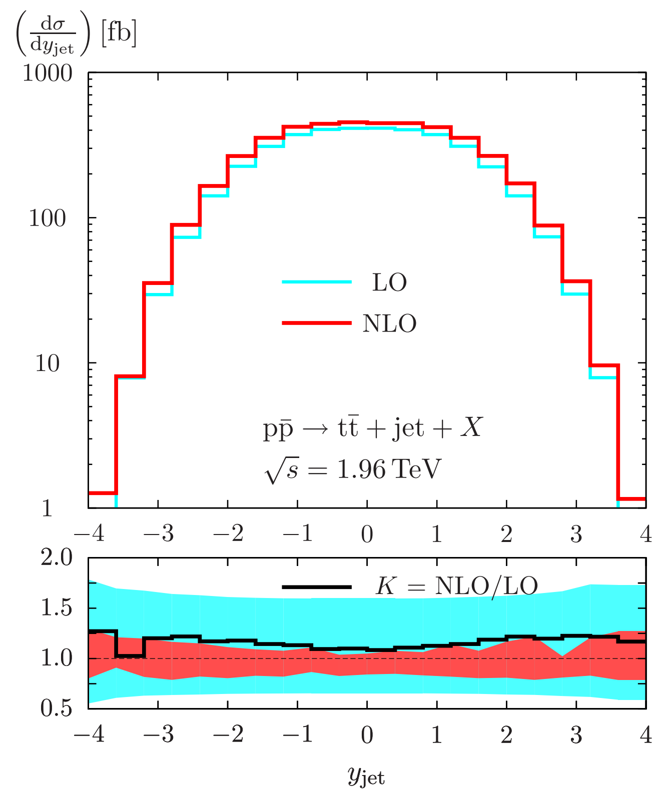


Rapidity y_1 of the leading jet (highest p_T).

Different shapes both at Tevatron and at the LHC

POWHEG: rapidity of the leading jet





POWHEG's distribution as in ALPGEN: no dip present. The size of discrepancy can be attributed to different treatment of higher order terms. Is this "feature" really there?

The new $pp \to t\bar{t}$ + jet at NLO [Dittmaier, Uwer, Weinzierl, hep-ph/0703120] shows no dip too (preliminary result).

From NLO to POWHEG

POWHEG is fully general and can be applied to any NLO subtraction framework.

We have provided any user with all the formulae and ingredients to implement an existing NLO calculation in the POWHEG formalism [Frixione, Nason and C.O., arXiv:0709.2092 [hep-ph]].

We have looked in detail at POWHEG in two subtraction schemes:

- the Frixione, Kunszt and Signer scheme
- the Catani and Seymour scheme.

We have discussed, in a pedagogical way, two examples:

- $e^+e^- \to q\bar{q}$
- $q\bar{q} \to V$

The `fortran` implementation of the POWHEG code for these two processes can be found at:

`http://moby.mib.infn.it/~nason/POWHEG/FNOpaper/`

Strategy and conclusions

✓ Shower Monte Carlo programs to do the final shower already exist

✓ Most of them implement a p_T veto

✓ Most of them comply with a standard interface to hard processes, the so called Les Houches Interface (LHI)

SO. . .

- construct a POWHEG for a NLO process. Output on LHI
- if needed, construct a generator capable to add truncated showers to events from the LHI. Output again on LHI
- use standard Shower Monte Carlo to perform the p_T-vetoed final shower from the event on LHI.