Soft gluon contributions to Drell-Yan and Higgs productions beyond NNLO

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- Introduction
- Scale ambiguity
- Sudakov Resummation of soft gluons at $N^3\text{LO}$
- Drell-Yan and Higgs productions
- Conclusions

Dedicated to

W.L. van Neerven

In collaboration with

W.L. van Neerven, J. Blümlein and J. Smith
Snap shot of the talk
Snap shot of the talk

• Perturbative QCD provides a frame work to compute observables at high energies.
Snap shot of the talk

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- They are "often" sensitive to:
  1) Renormalisation scale
  2) Factorisation scale
  3) Non-perturbative quantities that enter
  4) Missing higher order contributions (stability of perturbation)
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- Higher order QCD corrections reduce these effects.
- Soft gluons dominate in some kinematic regions that are accessible at hadron colliders.
- Sudakov resummation of soft gluons can be used to predict for Higgs and Drell-Yan total cross section and rapidity distribution beyond $NNLO$. 
Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

\[ 2S \, d\sigma_{P_1P_2}^{P_1P_2} (\tau, m_h^2) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab} (x, \mu_F) \, 2\hat{s} \, d\hat{\sigma}_{ab} \left( \frac{\tau}{x}, m_h^2, \mu_F \right) \]
Factorisation Theorem (QCD improved Parton Model)

Collins, Soper, Sterman

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- The perturbatively calculable partonic cross section:

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d\hat{\sigma}^{ab} (z, m_h^2, \mu_F) = \sum_{i=0}^{\infty} \left( \frac{\alpha_s (\mu_R)}{4\pi} \right)^i d\hat{\sigma}^{ab,(i)} (z, m_h^2, \mu_F, \mu_R)
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- The non-perturbative flux:

\[ \Phi_{ab} (x, \mu_F) = \int_{x}^{1} \frac{dz}{z} f_a (z, \mu_F) f_b \left( \frac{x}{z}, \mu_F \right) \]
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Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S \, d\sigma^{P_1P_2}_{12} (\tau, m_{h}^2) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab}(x, \mu_F) \, 2\hat{s} \, d\hat{\sigma}^{ab}_{12} \left( \frac{\tau}{x}, m_{h}^2, \mu_F \right)$$

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- $f^{P_1}_{a}(x, \mu_F)$ are Parton distribution functions with momentum fraction $x$. 
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- \( f_a^{P_1} (x, \mu_F) \) are Parton distribution functions with momentum fraction \( x \).
- \( \mu_R \) is the Renormalisation scale and \( \mu_F \), Factorisation scale
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- \(\mu_R\) is the Renormalisation scale and \(\mu_F\), Factorisation scale

- The Renormalisation group invariance:

\[
\frac{d}{d\mu} \sigma_{P_1P_2}^{P_1P_2}(\tau, m_h^2) = 0, \quad \mu = \mu_F, \mu_R
\]
Higgs production at LHC and Scale dependence

$\sigma(pp\rightarrow H+X)$ [pb]

$\sqrt{s} = 14$ TeV

$M_H$ [GeV]

- LO
- NLO
- NNLO

Harlander
Higgs production at LHC and Scale dependence

Harlander, Kilgore/ Anastasiou, Melnikov/ van Neerven, Smith, VR

$\sigma(pp\to H+X)$ [pb] for $\sqrt{s} = 14$ TeV

- See Hinchcliff,... for LO and see Dawson, Djouadi et.al for NLO (with finite top mass), NNLO is done in the large top limit $N = \frac{\sigma(\mu_R = \mu_F = \mu)}{\sigma(\mu_0)}$. 

![Graph showing Higgs production at LHC and Scale dependence with $\sigma(pp\to H+X)$ and $Z$ as functions of $M_H$ and $\mu/\mu_0$.](image)
Higgs production at LHC and Scale dependence

Harlander, Kilgore/ Anastasiou, Melnikov/ van Neerven, Smith, VR

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- Is it the end?
Soft part of NNLO

Catani et al, Harlander and Kilgore
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\[ 2S \, d\sigma^{P_1 P_2} (\tau, m_h) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab} (x) \, 2\hat{s} \, d\hat{\sigma}^{ab} \left( \frac{\tau}{x}, m_h \right) \quad \tau = \frac{m_h^2}{S} \]

Gluon flux is largest at LHC
Soft part of NNLO

Catani et al, Harlander and Kilgore

\[ 2S \ d\sigma^{P_1P_2}_\tau (\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab} (x) \ 2\hat{s} \ d\hat{\sigma}^{ab} \left( \frac{\tau}{x}, m_h \right) \]

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- \( \Phi_{ab}(x) \) becomes large when \( x \to x_{min} = \tau \)

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LHC (\( S = (14 \text{ TeV})^2 \))
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- Dominant contribution to Higgs production comes from the region when \( x \to \tau \)

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$2S \frac{d\sigma^{P_1P_2}}{d\tau,m_h} = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab}(x) 2\hat{s} \frac{d\hat{\sigma}^{ab}}{d\tau,m_h}$

$\tau = \frac{m_h^2}{S}$

- $\Phi_{ab}(x)$ becomes large when $x \rightarrow x_{min} = \tau$
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- It is sufficient if we know the partonic cross section when $x \rightarrow \tau$

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\[ 2S \, d\sigma_{P_1 P_2}^{P_1 P_2} (\tau, m_h) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab} (x) \, 2\hat{s} \, d\hat{\sigma}^{ab} \left( \frac{\tau}{x}, m_h \right) \quad \tau = \frac{m_h^2}{S} \]

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- \( x \rightarrow \tau \) is called soft limit.

Gluon flux is largest at LHC
Soft part of NNLO

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\[ 2S \, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_0^1 \frac{dx}{x} \Phi_{ab}(x) \; 2\hat{s} \; d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \quad \tau = \frac{m_h^2}{S} \]

- $\Phi_{ab}(x)$ becomes large when $x \rightarrow x_{\text{min}} = \tau$
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- $x \rightarrow \tau$ is called \textit{soft limit}.
- Expand the partonic cross section around $x = \tau$.

Gluon flux is largest at LHC

\[ \phi_{ab}(x) \text{ vs. } x = Q^2/S \]

LHC ( $S = (14 \text{ TeV})^2$)
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Catani et al, Harlander and Kilgore
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C^{(0)} = C_0^{(0)} \delta(1 - z) + \sum_{k=0}^{\infty} C_0^{(k)} \left( \frac{\log^k(1 - z)}{(1 - z)} \right)
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Soft part

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- \( \mathcal{C}_0^{(i)} \) will be pure constants such as \( \zeta(2), \zeta(3) \).
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OR

Extract from "Form factors and DGLAP kernels" using

1) Factorisation theorem
2) Renormalisation Group Invariance
3) Drell-Yan NNLO results
Soft plus Virtual at $N^3LO$ and beyond

VR
Soft plus Virtual at $N^3LO$ and beyond

Using "factorisation" of Virtual, Soft and Collinear:

$$\Delta_{I,P}^{sv}(z, q^2, \mu_R^2, \mu_F^2) = C \exp \left( \Psi_I^P (z, q^2, \mu_R^2, \mu_F^2, \varepsilon) \right) \bigg|_{\varepsilon=0}$$

$I = q, g$  \hspace{0.5cm} $n = 4 + \varepsilon$
Soft plus Virtual at $N^3 LO$ and beyond

Using "factorisation" of Virtual, Soft and Collinear:

$$\Delta_{I,P}^{sv}(z, q^2, \mu_R^2, \mu_F^2) = C \exp \left( \Psi_P^I (z, q^2, \mu_R^2, \mu_F^2, \epsilon) \right) \bigg|_{\epsilon=0}$$

$$\Psi_P^I (z, q^2, \mu_R^2, \mu_F^2, \epsilon) = \left( \ln \left( Z^I(\hat{a}_s, \mu_R^2, \mu_F^2, \epsilon) \right)^2 + \ln |\hat{F}_I(\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \right) \delta(1 - z)$$

$$+ 2 \Phi_P^I (\hat{a}_s, q^2, \mu^2, z, \epsilon) - 2 m C \ln \Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon)$$
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- $Z^I(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon)$ is operator renormalisation constant with $\mu$ is mass parameter in $n = 4 + \varepsilon$ dimensional regularisation $\rightarrow N^3 LO$

- $\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon)$ is the Form factor with $Q^2 = -q^2 \rightarrow N^3 LO$

- $\Phi_P^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon)$ is the soft distribution function $\rightarrow NNLO$ level

- $\Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon)$ is mass factorisation kernel $\rightarrow N^3 LO$
Soft plus Virtual at $N^3LO$ and beyond

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$$\hat{\alpha}_s = \frac{g_s^2}{16\pi^2} \quad m = \frac{1}{2} \quad \text{for DIS} \quad m = 1 \quad \text{for DY, Higgs}$$
Sudakov Resummation for Form factors

Vogt, Vermaseren, Moch, VR
Sudakov Resummation for Form factors

\[ Q^2 \frac{d}{dQ^2} \ln \hat{F}^I (\hat{a}_s, Q^2, \mu^2, \varepsilon) = \frac{1}{2} \left[ K^I \left( \hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon \right) + G^I \left( \hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu^2}{\mu_R^2}, \varepsilon \right) \right] \]

Solution: \[ \ln \hat{F}^I (\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{Q^2}{\mu^2} \right)^{i \frac{\varepsilon}{2}} S_{\varepsilon}^i \hat{L}_{F}^{I,(i)} (\varepsilon) \]
Sudakov Resummation for Form factors

\[ Q^2 \frac{d}{dQ^2} \ln \hat{F}^I (\hat{a}_s, Q^2, \mu^2, \varepsilon) = \frac{1}{2} \left[ K^I \left( \hat{a}_s, \frac{\mu^2_R}{\mu^2}, \varepsilon \right) + G^I \left( \hat{a}_s, \frac{Q^2}{\mu^2_R}, \mu^2, \varepsilon \right) \right] \]

Solution:

\[ \ln \hat{F}^I (\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{Q^2}{\mu^2} \right)^i \frac{\varepsilon^i}{2} S_\varepsilon \hat{L}_F^{I,(i)} (\varepsilon) \]

Formal solution upto 4 loops:

\[ \hat{L}_F^{I,(1)} = \frac{1}{\varepsilon^2} \left( -2A_1^I \right) + \frac{1}{\varepsilon} \left( G_1^I (\varepsilon) \right) \]

\[ \hat{L}_F^{I,(2)} = \frac{1}{\varepsilon^3} \left( \beta_0 A_1^I \right) + \frac{1}{\varepsilon^2} \left( -\frac{1}{2} A_2^I - \beta_0 G_1^I (\varepsilon) \right) + \frac{1}{2\varepsilon} G_2^I (\varepsilon) \]

\[ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \]
Sudakov Resummation for Form factors

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\end{align*}
\]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \]

\[ \bullet A^I \text{ are maximally non-abelian} \quad A^g_i = \frac{C_A}{C_F} A^q_i \quad i = 1, 2, 3. \]
Sudakov Resummation for Form factors

Vogt, Vermaseren, Moch, VR

\[ Q^2 \frac{d}{dQ^2} \ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \frac{1}{2} \left[ K^I(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon) + G^I(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon) \right] \]

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\[ \hat{L}_F^{I,(2)} = \frac{1}{\varepsilon^3} \left( \beta_0 A_1^I \right) + \frac{1}{\varepsilon^2} \left( -\frac{1}{2} A_2^I - \beta_0 G_1^I(\varepsilon) \right) + \frac{1}{2\varepsilon} G_2^I(\varepsilon) \]

\[ \ldots \ldots \ldots \ldots \ldots \]

- \( A^I \) are maximally non-abelian \( A_i^q = \frac{C_A}{C_F} A_i^q \) \( i = 1, 2, 3. \)

- Every order in \( \hat{a}_s \), all the poles except the lowest one can be predicted from the previous order results using \( A \) and \( \beta \) function.
New observation for single pole in $\varepsilon$
New observation for single pole in $\varepsilon$

Two loop results for $\hat{F}^q$ and $\hat{F}^g$ in $SU(N)$ solves the single pole problem:
New observation for single pole in $\varepsilon$

Two loop results for $\hat{F}^q$ and $\hat{F}^g$ in $SU(N)$ solves the single pole problem: $G^I$'s have interesting structure:

$$G^I_1(\varepsilon) = 2(B^I_1 - \gamma^I_1) + f^I_1 + \sum_{k=1}^{\infty} \varepsilon^k g^I_{1,k}$$

$$G^I_2(\varepsilon) = 2(B^I_2 - \gamma^I_2) + f^I_2 - 2\beta_0 g^I_{1,1} + \sum_{k=1}^{\infty} \varepsilon^k g^I_{2,k}$$
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$B^I_i$ are $\delta(1 - z)$ part of $P_{II}$ splitting functions. The new constants "$f^I_1$ and $f^I_2$" satisfy

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$$

$$
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Even the single pole can be predicted: $G_i^I = 2(B_i^I - \gamma_i^I) + f_i^I + \cdots$
New observation for single pole in $\varepsilon$

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Recent three loop result by Moch, Vermaseren, Vogt confirms our prediction:

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Recent three loop result by Moch, Vermaseren, Vogt confirms our prediction:

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f^g_3 = \frac{C_A}{C_F} f^q_3
\]

This completes the understanding of all the poles of the form factors.
Mass factorisation using DGLAP kernel

VR
Mass factorisation using DGLAP kernel

Due to the massless partons, collinear singularities appear in
- the phase space of the real emission processes
- loop integrals of the virtual corrections
Mass factorisation using DGLAP kernel

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They are removed by Mass Factorisation by adding:

$$- \ln \Gamma(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon)$$
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DGLAP kernels satisfy Renormalisation Group Equations:

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\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z, \mu_F^2, \epsilon) = \frac{1}{2} P(z, \mu_F^2) \otimes \Gamma(z, \mu_F^2, \epsilon).
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The diagonal terms of the splitting functions \(P^{(i)}(z)\) have the following structure

\[P^{(i)}_{II}(z) = 2 \left[ B^I_{i+1} \delta(1 - z) + A^I_{i+1} D_0 \right] + P^{(i)}_{reg,II}(z) ,\]
Mass factorisation using DGLAP kernel

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\]

The diagonal terms of the splitting functions \(P^{(i)}(z)\) have the following structure

\[
P^{(i)}_{II}(z) = 2 \left[ B_{i+1}^I \delta(1-z) + A_{i+1}^I D_0 \right] + P^{(i)}_{reg,II}(z),
\]

\[
D_0 = \left( \frac{1}{1-z} \right)_+, \quad P^{(i)}_{reg,II} \text{ are regular when } z \to 1.
\]

We will be left with only maximally non-abelian constants \(A^I_i\) and \(f^I_i\).
Finiteness of the Cross section

VR
Finiteness of the Cross section

Observable $\Delta^I(\alpha_s, Q^2)$ are finite:

$Infra – red safe$
Finiteness of the Cross section

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$\textit{Infra – red safe}$

The remaining poles after UV Operator Renormalisation ($Z_{\alpha_s}$ and $Z^I$) and Mass factorisation:

$$\frac{1}{\varepsilon^{i+1}} \text{ at } i^{\text{th}} \text{ loop}$$

Highest poles are not removed by renormalisation and factorisation
Finiteness of the Cross section

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- The structure of soft part should be "similar" to the Form Factors.
- Hence using gauge invariance and RG invariance, we can propose

$$q^2 \frac{d}{dq^2} \Phi^I (\hat{\alpha}_s, q^2, \mu^2, z, \varepsilon) = \frac{1}{2} \left[ K^I \left( \hat{\alpha}_s, \frac{\mu^2}{\mu^2_R}, z, \varepsilon \right) + G^I \left( \hat{\alpha}_s, \frac{q^2}{\mu^2_R}, \frac{\mu^2}{\mu^2}, z, \varepsilon \right) \right]$$
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Observable $\Delta^I(\alpha_s, Q^2)$ are finite:

\[ V_R \]

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q^2 \frac{d}{dq^2} \Phi^I(\hat{\alpha}_s, q^2, \mu^2, z, \varepsilon) = \frac{1}{2} \left[ K^I \left( \hat{\alpha}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon \right) + G^I \left( \hat{\alpha}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon \right) \right]
\]

RG invariance of $\Phi^I$ implies:

\[
\mu_R^2 \frac{d}{d\mu_R^2} K^I \left( \hat{\alpha}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon \right) = -\mu_R^2 \frac{d}{d\mu_R^2} G^I \left( \hat{\alpha}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon \right) = -A^I(\alpha_s(\mu_R^2)) \delta(1-z)
\]
Solution to (Soft)Sudakov Equation

VR
Infra-red safeness of the cross section implies

\[ \overline{A}^I = -A^I \]
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Solution to (soft) Sudakov equation:

\[
\Phi^I (\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{q^2}{\mu^2} \right)^{i \varepsilon / 2} S^i_{\varepsilon} \hat{\Phi}^I, (i) (z, \varepsilon)
\]
Infra-red safeness of the cross section implies

$$\overline{A^I} = -A^I$$

Solution to (soft) Sudakov equation:

$$\Phi^I (\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{q^2}{\mu^2} \right)^{i \frac{\varepsilon}{2}} S_{\varepsilon}^i \hat{\Phi}^{I,(i)}(z, \varepsilon)$$

where

$$\hat{\Phi}^{I,(i)}(z, \varepsilon) = \hat{L}_F^{I,(i)}(\varepsilon) \begin{pmatrix} A^I \rightarrow -\delta(1-z) A^I, & G^I(\varepsilon) \rightarrow \overline{G}^I(z, \varepsilon) \end{pmatrix}$$
In red safeness of the cross section implies
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Solution to (soft) Sudakov equation:
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where
\[ \hat{\Phi}^{I,(i)}(z, \varepsilon) = \mathcal{L}^{I,(i)}_{F}(\varepsilon) \left( A^I \rightarrow -\delta(1-z) A^I, \ G^I(\varepsilon) \rightarrow \overline{G}^I(z, \varepsilon) \right) \]

Most general solution:
\[ \Phi^I (\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \Phi^I (\hat{a}_s, q^2 (1-z)^{2m}, \mu^2, \varepsilon) \]
\[ = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{q^2 (1-z)^{2m}}{\mu^2} \right)^{i \frac{\varepsilon}{2}} S^i_{\varepsilon} \left( \frac{i m \varepsilon}{2(1-z)} \right) \hat{\phi}^{I,(i)}(\varepsilon) \]
Infra-red safeness of the cross section implies

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Solution to (soft) Sudakov equation:

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\]

All the poles in \( \varepsilon \) are predictable.
Universal Soft part

VR
Universal Soft part

Single pole in $\varepsilon$:

$$
\overline{G}_1^I(\varepsilon) = -f_1^I + \sum_{k=1}^{\infty} \varepsilon^k \overline{G}_1^I,(k)
$$

$$
\overline{G}_2^I(\varepsilon) = -f_2^I - 2\beta_0 \overline{G}_1^I,(1) + \sum_{k=1}^{\infty} \varepsilon^k \overline{G}_2^I,(k)
$$

\[\vdots \qquad \vdots \qquad \vdots \]

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Maximally non-abelian:

$$\overline{g}_i^g(\varepsilon) = \frac{C_A}{C_F} \overline{g}_i^g(\varepsilon) \quad i = 1, 2, 3$$
Universal Soft part

Single pole in $\varepsilon$:

$$\mathcal{G}^I_1(\varepsilon) = -f^I_1 + \sum_{k=1}^{\infty} \varepsilon^k \mathcal{G}^{I,(k)}_1$$

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Soft part of the any cross section are independent of spin, colour, flavour or other quantum numbers.
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$$\Phi^q (\hat{a}_s, q^2, z, \mu^2, \varepsilon) = \frac{C_F}{C_A} \Phi^g (\hat{a}_s, q^2, z, \mu^2, \varepsilon)$$
Higgs productions from Drell-Yan beyond $\text{NNLO}$

Universal soft function: $VR$
Higgs productions from Drell-Yan beyond $NNLO$

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$$\Phi^g (\hat{a}_s, q^2, z, \mu^2, \varepsilon) = \frac{C_A}{C_F} \Phi^q (\hat{a}_s, q^2, z, \mu^2, \varepsilon)$$

- From Drell-Yan $\Phi^q (\hat{a}_s, q^2, z, \varepsilon)$, Gluon form factor $\mathcal{F}^g$, and operator renormalisation constant $Z_g$ and DGLAP kernel $\Gamma_{gg}$, we can compute soft plus virtual part of

$$\sigma(g + g \rightarrow Higgs)$$

without explicitly calculating the soft part of Higgs production.
Higgs productions from Drell-Yan beyond **NNLO**

Universal soft function: \( VR \)

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\Phi^g (\hat{a}_s, q^2, z, \mu^2, \varepsilon) = \frac{C_A}{C_F} \Phi^q (\hat{a}_s, q^2, z, \mu^2, \varepsilon)
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- From Drell-Yan \( \Phi^q (\hat{a}_s, q^2, z, \varepsilon) \), Gluon form factor \( F^g \) and operator renormalisation constant \( Z_g \) and DGLAP kernel \( \Gamma_{gg} \), we can compute soft plus virtual part of

\[
\sigma (g + g \rightarrow \text{Higgs})
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- Our **NNLO** predictions agrees with the results by Catani et al, Harlander and Kilgore. No need for explicit computation of soft contributions for the Higgs production.
Higgs productions from Drell-Yan beyond $NNLO$

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- The scalar form factor $\mathcal{F}^S = \langle P \bar{\psi} \psi | P \rangle$ can be predicted at three loop from the known three loop $A_i, B_i, f_i$ and $\gamma_i^m$. 
Higgs productions from Drell-Yan beyond $NNLO$

Universal soft function:

$$\Phi^g (\hat{a}_s, q^2, z, \mu^2, \epsilon) = \frac{C_A}{C_F} \Phi^q (\hat{a}_s, q^2, z, \mu^2, \epsilon)$$

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- Soft plus Virtual part Higgs production through bottom quark fusion

$$\sigma(b + \overline{b} \rightarrow \text{Higgs})$$

can be predicted upto $N^3LO$(without $\delta(1 - z)$).
Higgs productions from Drell-Yan beyond $NNLO$

Universal soft function:

$$\Phi^g (\hat{a}_s, q^2, z, \mu^2, \epsilon) = \frac{C_A}{C_F} \Phi^q (\hat{a}_s, q^2, z, \mu^2, \epsilon)$$

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• Soft plus Virtual part Higgs production through bottom quark fusion

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can be predicted upto $N^3LO$(without $\delta(1 - z))$.

• Our $NNLO$ predictions agrees with the results of by Harlander and Kilgore.
Hadro production in $e^+e^-$ annihilation from DIS

Blümlein and VR
Hadro production in $e^+e^-$ annihilation from DIS

- The scaling variable in DIS is

$$x_{Bj} = -\frac{q^2}{2p \cdot q} \quad -q^2 > 0$$
Hadro production in $e^+ e^-$ annihilation from DIS

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$$x_{ee} = \frac{2p \cdot q}{q^2} \quad q^2 > 0$$
Hadro production in $e^+e^-$ annihilation from DIS

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- Drell-Levy-Yan showed that these two processes are related by crossing relation.
Hadro production in $e^+ e^- \text{ annihilation from DIS}$

**Blümlein and VR**

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- Drell-Levy-Yan showed that these two processes are related by crossing relation.
- Gribov-Lipatov relation in the soft limit:

\[ \Phi_{DIS}(\hat{a}_s, Q^2, \mu^2, x_{Bj}, \varepsilon) = \Phi_{ee}(\hat{a}_s, q^2, \mu^2, x_{ee}, \varepsilon) \]

\[ P_{II}(x_{Bj}) = \tilde{P}_{II}(x_{ee}) \quad \text{Distributions} \]
Hadro production in $e^+e^-$ annihilation from DIS

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$$\Phi_{DIS}(\hat{a}_s, Q^2, \mu^2, x_{Bj}, \varepsilon) = \Phi_{ee}(\hat{a}_s, q^2, \mu^2, x_{ee}, \varepsilon)$$

$$P_{II}(x_{Bj}) = \tilde{P}_{II}(x_{ee}) \quad \text{Distributions}$$

- From DIS results, we can predict soft plus virtual part of the coefficient functions for hadro production in $e^+e^-$ annihilation up to three loop level.

$$C^{(3),sv}_{ee}(\alpha_s, z) \quad \text{New result}$$
**Hadro production in $e^+e^-$ annihilation from DIS**

- The scaling variable in DIS is

\[ x_{Bj} = -\frac{q^2}{2p \cdot q} \quad -q^2 > 0 \]

- The scaling variable in hadro production is

\[ x_{ee} = \frac{2p \cdot q}{q^2} \quad q^2 > 0 \]

- Drell-Levy-Yan showed that these two processes are related by crossing relation.

- Gribov-Lipatov relation in the soft limit:

\[ \Phi_{DIS}(\hat{a}_s, Q^2, \mu^2, x_{Bj}, \varepsilon) = \Phi_{ee}(\hat{a}_s, q^2, \mu^2, x_{ee}, \varepsilon) \]

\[ P_{II}(x_{Bj}) = \tilde{P}_{II}(x_{ee}) \quad \text{Distributions} \]

- From DIS results, we can predict soft plus virtual part of the coefficient functions for hadro production in $e^+e^-$ annihilation up to three loop level.

\[ C_{ee}^{(3), sv}(\alpha_s, z) \quad \text{New result} \]
Threshold Resummation

\[ \Phi^I_P(\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \left( \frac{m}{1 - z} \left\{ \int_{\mu_R^2}^{q^2(1-z)^{2m} \delta_P} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) \right\} + \right. \\
+ \bar{G}^I_P(a_s(q^2(1-z)^{2m} \delta_P), \varepsilon) \right) + \\
+ \delta(1 - z) \sum_{i=1}^{\infty} \hat{a}^i_s \left( \frac{q^2 \delta_P}{\mu^2} \right)^i S^i_\varepsilon \hat{\Phi}^I_{P,(i)}(\varepsilon) \\
+ \left( \frac{m}{1 - z} + \sum_{i=1}^{\infty} \hat{a}^i_s \left( \frac{\mu_R^2}{\mu^2} \right)^i \right) S^i_\varepsilon K^{I,(i)}(\varepsilon) \]
Threshold Resummation

- Alternate derivation for the threshold resummation formula in $z$ space for both DY and DIS:

$$\Phi^I_P(\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \left( \frac{m}{1 - z} \right) \left\{ \int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) \right. + \left[ G^I_P(a_s(q^2(1-z)^{2m}\delta_P), \varepsilon) \right] \right\} +$$

$$+ \delta(1 - z) \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{q^2\delta_P}{\mu^2} \right)^{i \frac{\varepsilon}{2}} S_{\varepsilon}^i \hat{\Phi}^I_P(\varepsilon)$$

$$+ \left( \frac{m}{1 - z} \right) + \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{\mu_R^2}{\mu^2} \right)^{i \frac{\varepsilon}{2}} S_{\varepsilon}^i \overline{K}^I(\varepsilon)$$

- The threshold exponents $D^I_i$ for DY and $B^I_i$ for DIS are related to $\overline{G}^I_P(\varepsilon = 0)$.
- $\overline{G}^I_P(\varepsilon = 0)$ up to three loop gives $D^I_i$ and $B^I_i$ for $i = 1, 2, 3$.
Threshold Resummation

- Alternate derivation for the threshold resummation formula in $z$ space for both DY and DIS:

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\Phi_P^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) = \left( \frac{m}{1 - z} \left\{ \int_{\mu_R^2}^{q^2(1-z)^2m\delta_P} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) \right. \right.

+ \mathcal{G}_P^I(a_s(q^2(1-z)^2m\delta_P), \varepsilon) \left. \right\} \right) +

+ \delta(1 - z) \sum_{i=1}^{\infty} \hat{\alpha}_s^i \left( \frac{q^2\delta_P}{\mu^2} \right)^{i\frac{\varepsilon}{2}} S^i_\varepsilon \hat{\Phi}_P^{I,(i)}(\varepsilon)

+ \left( \frac{m}{1 - z} \right) + \sum_{i=1}^{\infty} \hat{\alpha}_s^i \left( \frac{\mu_R^2}{\mu^2} \right)^{i\frac{\varepsilon}{2}} S^i_\varepsilon \overline{K}^{I,(i)}(\varepsilon)

- The threshold exponents $D_i^I$ for DY and $B_i^I$ for DIS are related to $\mathcal{G}_P^I(\varepsilon = 0)$.
- $\mathcal{G}_P^I(\varepsilon = 0)$ up to three loop gives $D_i^I$ and $B_i^I$ for $i = 1, 2, 3$
- Expansion of $C e^{(2\Phi_P^I)}$ leads to soft part of the cross section.
- Fixed order $N^3 LO$ soft plus virtual cross sections can be computed (except $\delta(1 - z)$)
Soft plus Virtual part at $N^3LO$ for Higgs Production

Moch, Vogt and VR
Soft plus Virtual part at $N^3LO$ for Higgs Production

\[ 2S \, d\sigma_{P_1P_2}^{P_1P_2}(\tau, m_h) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi_{ab}(x) \, 2\hat{s} \, d\hat{\sigma}^{ab}_{\tau}(\frac{\tau}{x}, m_h) \quad \tau = \frac{m_h^2}{S} \]
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- Finite terms in $F^I$ and $\Phi^I$ at 3-loop are still missing
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\[ \mathcal{D}_j \quad j = 5, 4, 3, 2, 1, 0 \]

- At 4-loop, we can predict only

\[ \mathcal{D}_j \quad j = 7, 6, 5, 4, 3, 2 \]
Soft plus Virtual part at $N^3LO$ for Higgs Production

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2S \ d\sigma^{P_1P_2} (\tau, m_h) = \sum_{ab} \int_{\tau}^{1} \frac{dx}{x} \Phi^{ab} (x) \ 2\hat{s} \ \hat{d}\sigma^{ab} \left( \frac{\tau}{x}, m_h \right) \quad \tau = \frac{m_h^2}{S}
\]

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- We can not predict $\delta (1 - z)$ part at 3-loop.

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- At 4-loop, we can predict only $\mathcal{D}_j \quad j = 7, 6, 5, 4, 3, 2$

- They contribute bulk of the cross section

Gluon flux is largest at LHC
Scale variation at $N^3LO$ for Higgs production

\[ N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)} \]
Scale variation at $N^3LO$ for Higgs production

$$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$

![Graph showing scale variation at $N^3LO$ for Higgs production](image-url)

- $\sigma(pp\rightarrow H+X)$ [pb]
- $\sqrt{s} = 14$ TeV
- $M_H$ [GeV]
- Harlander

Options: LO, NLO, NNLO

- Red: NNLO
- Blue: NLO
- Green: LO
Scale variation at $N^3LO$ for Higgs production

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![Graph showing scale variation]
Scale variation at $N^3LO$ for Higgs production

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- Scale uncertainty improves a lot
- Perturbative QCD works at LHC
Soft distribution for rapidity

Using RGE and Factorisation:
Soft distribution for rapidity

Using RGE and Factorisation:

\[
\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) = \Phi_d^{I, finite} + \Phi_d^{I, singular}
\]
Soft distribution for rapidity

Using RGE and Factorisation:

\[ \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = \Phi_{d,\text{finite}}^I + \Phi_{d,\text{singular}}^I \]

where

\[
\Phi_{d,\text{finite}}^I = \frac{1}{2} \delta(1 - z_2) \left( \frac{1}{1 - z_1} \int_{\mu_R^2}^{q^2(1-z_1)} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) \right.
\]
\[
+ \overline{G}_d^I(a_s(q^2(1-z_1), \epsilon) \bigg) \bigg) +
\]
\[
+ q^2 \frac{d}{dq^2} \left[ \left( \frac{1}{4(1 - z_1)(1 - z_2)} \int_{\mu_R^2}^{q^2(1-z_1)(1-z_2)} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) \right. \right.
\]
\[
+ \overline{G}_d^I(a_s(q^2(1-z_1)(1 - z_2)), \epsilon) \bigg) \bigg) \bigg) +
\]
\[
+ z_1 \leftrightarrow z_2
\]

VR, Smith and van Neerven
$N^3 L_{PSV}$ results for Drell-Yan rapidity

\[ N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)} \]

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![Graph showing the dependence of differential cross-section on rapidity, with various order approximations.]
$N^3LO_{pSV}$ results for Drell-Yan rapidity

VR, Smith and van Neerven

$$N = \frac{\sigma_{N^3LO}(\mu)}{\sigma_{N^3LO}(\mu_0)}$$

![Graph of d^3σ/dM dY (pb/GeV) (LHC) M=115 GeV](image1)

![Graph of R-Ratio (Y) (LHC) M=115 GeV](image2)
$N^2LO_{pSV}$ results for Drell-Yan rapidity

$VR, Smith and van Neerven$

$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$

- Compared against Dixon, Anastasiou, Melnikov, Petriello NNLO results for Drell-Yan, Higgs, Z, W$^\pm$ productions.
$N^3 LO_{pSV}$ results for Higgs rapidity

$V R, S m i t h ~ a n d ~ v a n ~ N e e r v e n$

$$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$
$N^3 LO_{pSV}$ results for Higgs rapidity

VR, Smith and van Neerven

$$N = \frac{\sigma_{N^3 LO}(\mu)}{\sigma_{N^3 LO}(\mu_0)}$$

\[
\frac{d\sigma}{dY} \text{ (pb/GeV) (LHC)}
\]

$m_H = 115$ GeV
$N^3LO_{pSV}$ results for Higgs rapidity

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\[ N = \frac{\sigma_{N^iLO}^{\mu}}{\sigma_{N^iLO}^{\mu_0}} \]

d$\sigma$/dY (pb/GeV) (LHC) 

$m_H=115$ GeV

R-Ratio (Y) (LHC) 

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$N^3LO_{pSV}$ results for Higgs rapidity

$N = \frac{\sigma_{N_iLO}(\mu)}{\sigma_{N_iLO}(\mu_0)}$

- Scale uncertainty improves a lot
$N^3 L O_{PSV}$ results for rapidity of $Z$

$$N = \frac{\sigma_{N^i LO}^{VR, Smith}(\mu)}{\sigma_{N^i LO}(\mu_0)}$$
$N^3LO_{pSV}$ results for rapidity of $Z$

$$N = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}^{VR, Smith}$$

![Graph showing rapidity distribution of Z boson at various order of perturbative QCD.](image-url)
$N^3LO_{pSV}$ results for rapidity of $Z$ 

$$N = \frac{\sigma_{N^3LO}(\mu)}{\sigma_{N^3LO}(\mu_0)}$$

$VR, Smith$
\( N^3 \text{LO}_{pSV} \) results for rapidity of \( Z \)

\[
N = \frac{\sigma^V_{N^i\text{LO}}(\mu)}{\sigma^V_{N^i\text{LO}}(\mu_0)}
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$N^3LO_{pSV}$ results for rapidity of $W^+$

$$N = \frac{\sigma_{N^3LO}^{VR, Smith}(\mu)}{\sigma_{N^3LO}(\mu_0)}$$

\[d^2\sigma/dydy \text{ pb/GeV (LHC)}\]

- $q=\mu_F=M_W$

- LO

- NLO

- $N^3LO_{SV}$

- $N^3LO_{pSV}$
$N^3 LO_{pSV}$ results for rapidity of $W^+$

$$N = \frac{\sigma_{N^3LO}(\mu)}{\sigma_{N^3LO}(\mu_0)}$$

VR, Smith

Below are plots showing the differential cross sections $d^2\sigma/dqdy$ in pb/GeV for different orders of perturbation theory at the LHC. The plots compare $N^3LO_{pSV}$ and $N^2LO_{pSV}$ to $N^3LO$ and $N^2LO$ at LO and NLO respectively, with $q=\mu_F=M_W$. The plots illustrate the evolution of cross sections with rapidity for varying $q$. The data points and lines represent different theoretical predictions and experimental measurements.
$N^3LO_{pSV}$ results for rapidity of $W^+$

$$
N = \frac{\sigma_{N^3LO}(\mu)}{\sigma_{N^3LO}(\mu_0)}
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Can INDIA be venue for next to next to… RADCOR’07?
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Thank You