Soft gluon contributions to Drell-Yan and Higgs productions beyond NNLO

V. Ravindran

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- Introduction
- Scale ambiguity
- Sudakov Resummation of soft gluons at N^3LO
- Drell-Yan and Higgs productions
- Conclusions

Dedicated to

W.L. van Neerven

In collaboration with

W.L. van Neerven, J. Blümlein and J. Smith

Snap shot of the talk

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- Perturbative QCD provides a frame work to compute observables at high energies

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2) Factorisation scale

3) Non-perturbative quantities that enter

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- Soft gluons dominate in some kinematic regions that are accessible at hadron colliders.
- Sudakov resummation of soft gluons can be used to predict for Higgs and Drell-Yan total cross section and rapidity distribution beyond $NNLO$.

Factorisation Theorem (QCD improved Parton Model)

Collins, Soper, Sterman

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

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- The Renormalisation group invariance:

$$\frac{d}{d\mu}\sigma^{P_1 P_2}(\tau, m_h^2) = 0, \qquad \mu = \mu_F, \mu_R$$

Higgs production at LHC and Scale dependence

Harlander, Kilgore/ Anastasiou, Melnikov/ van Neerven, Smith, VR

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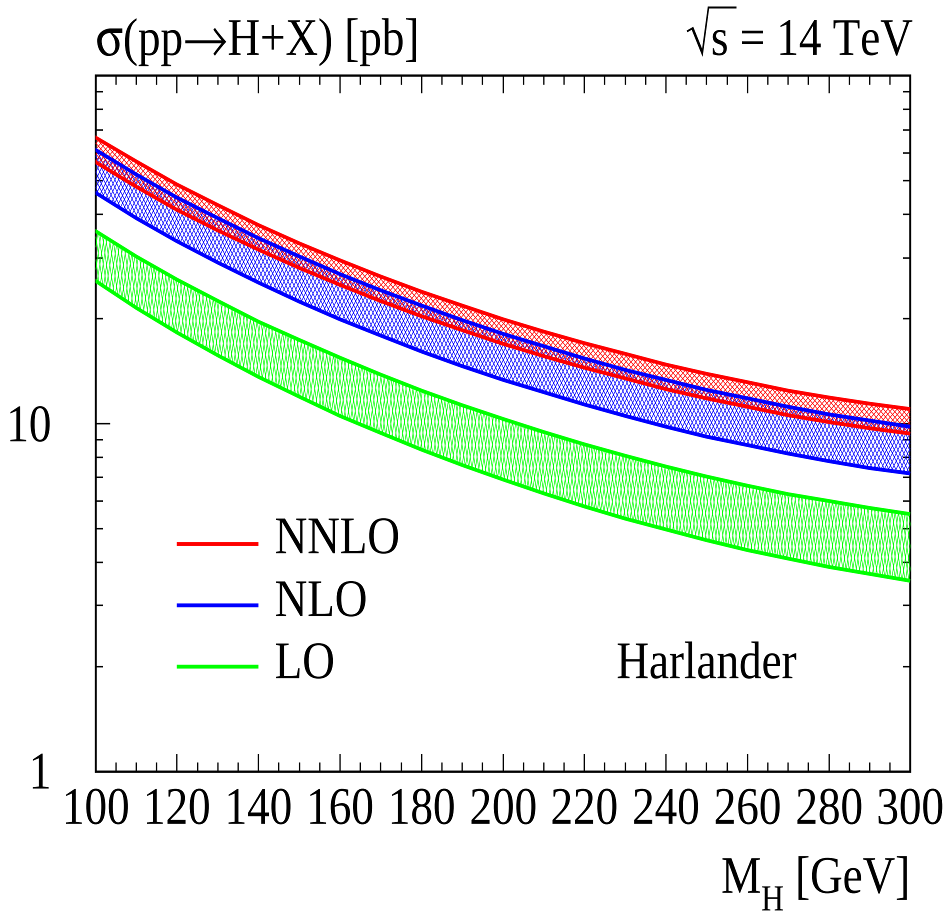

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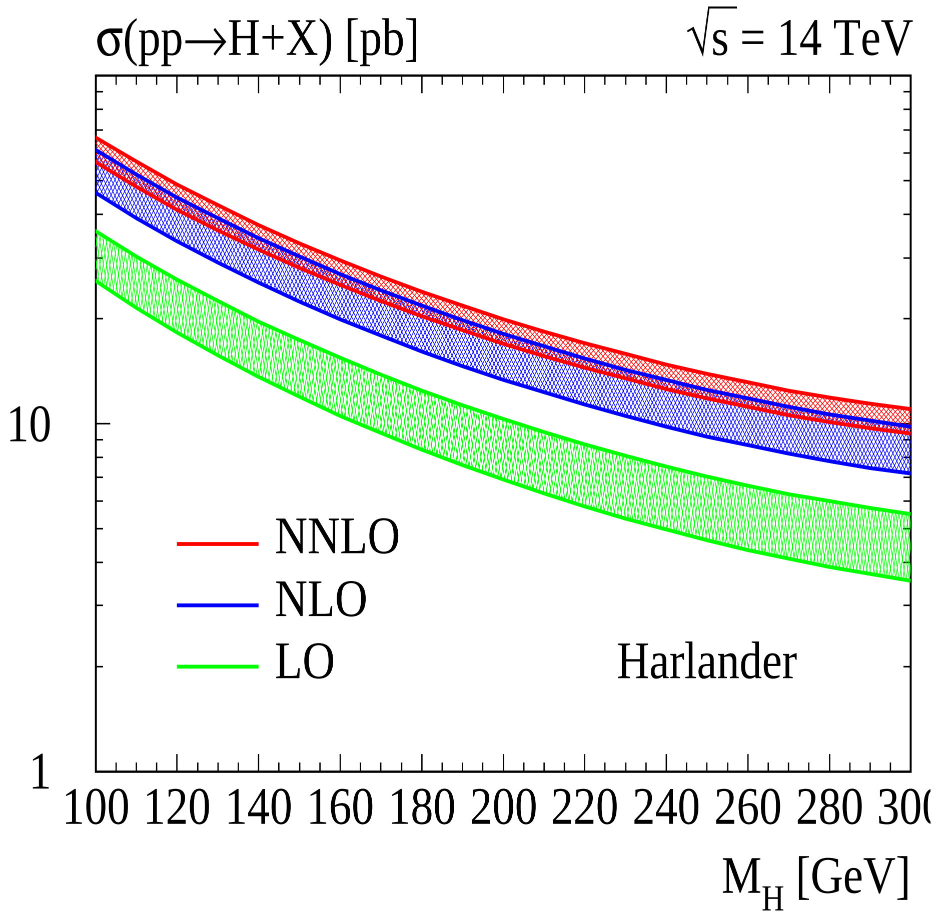

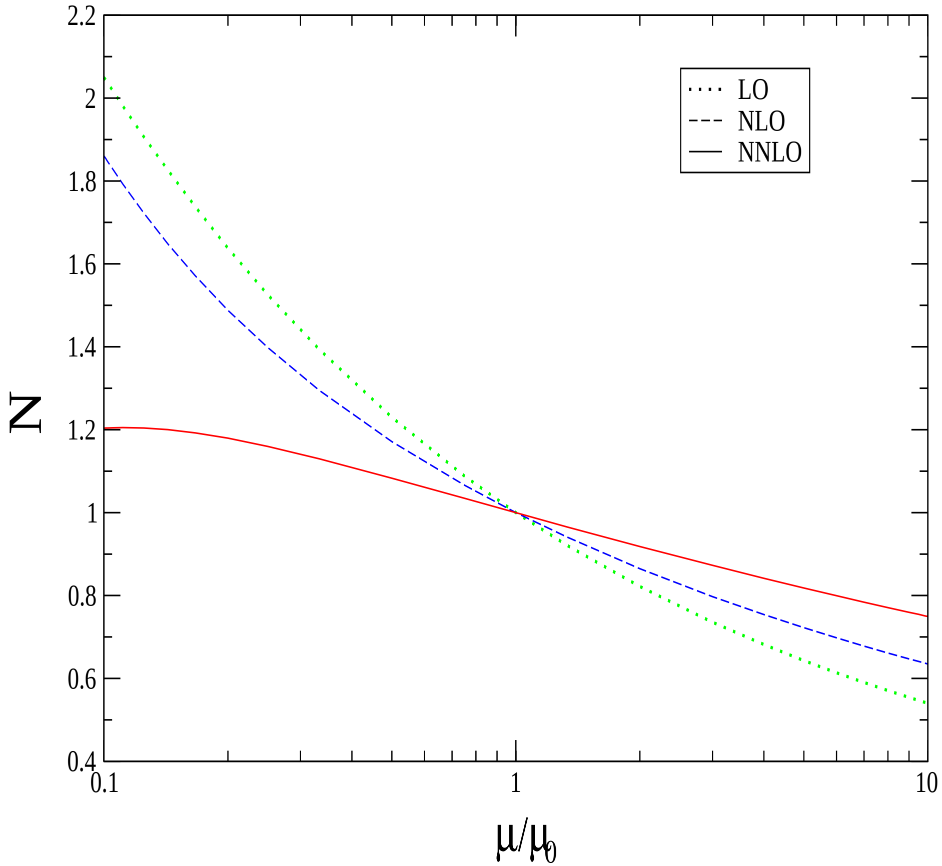

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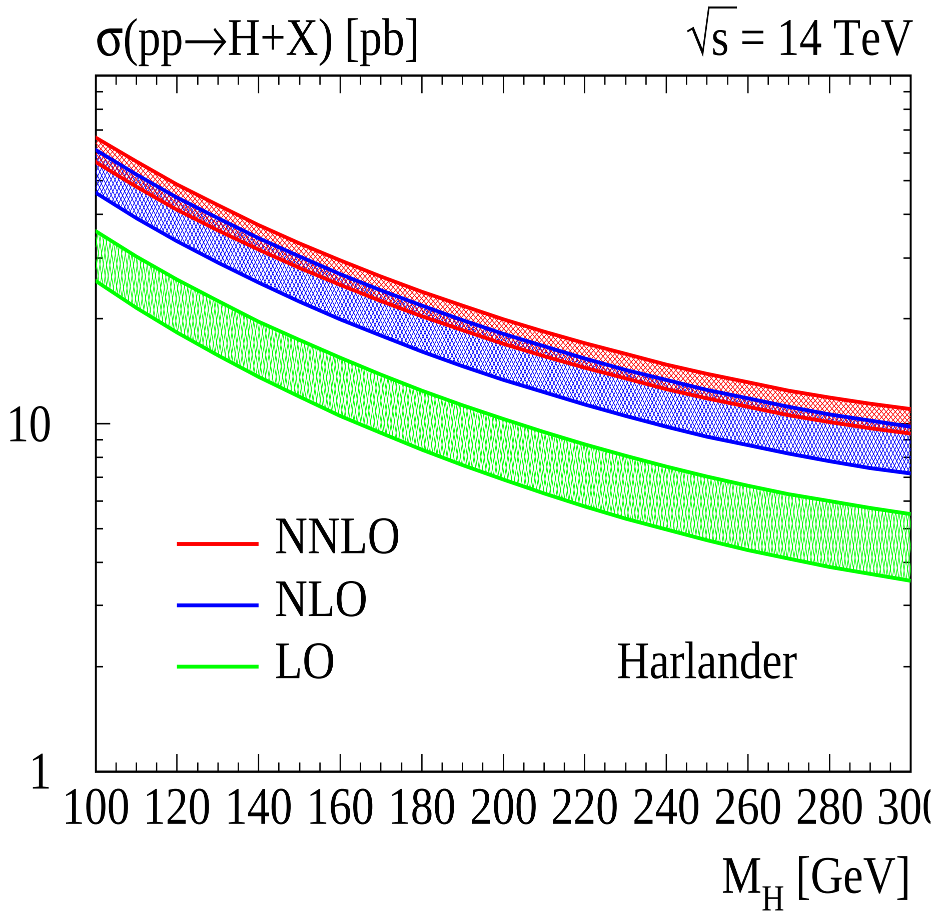

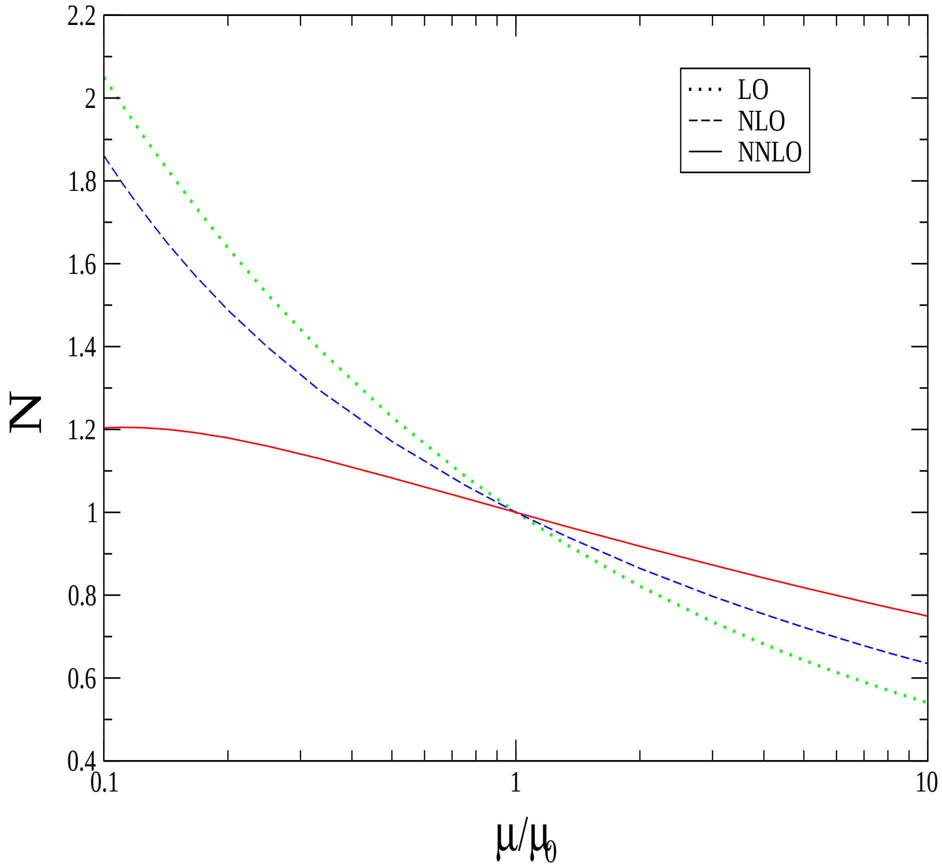

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- Is it the end?

Soft part of NNLO

Catani et al, Harlander and Kilgore

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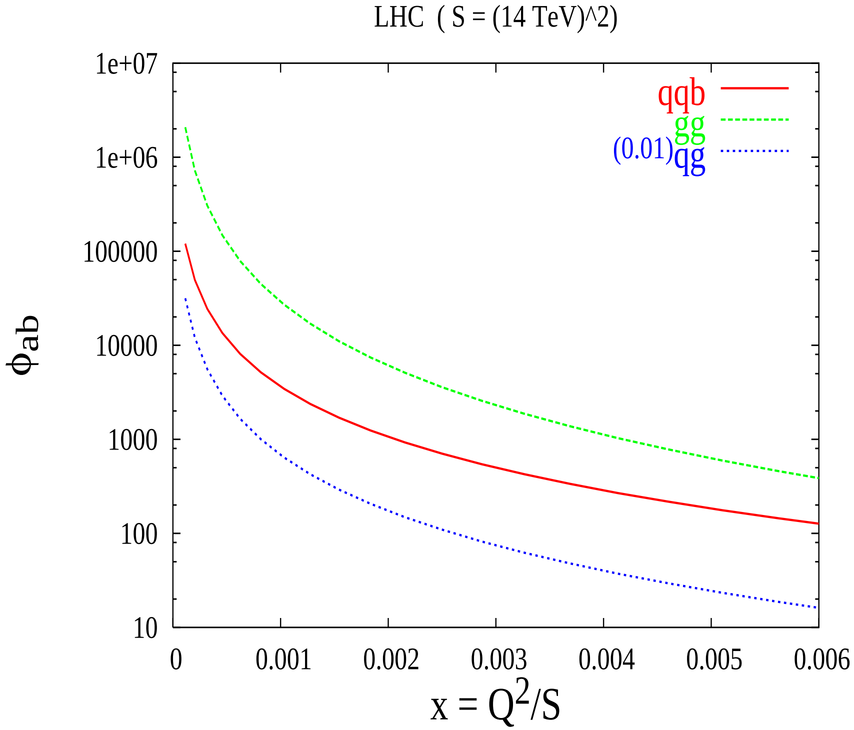
$$2S\, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\, 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$



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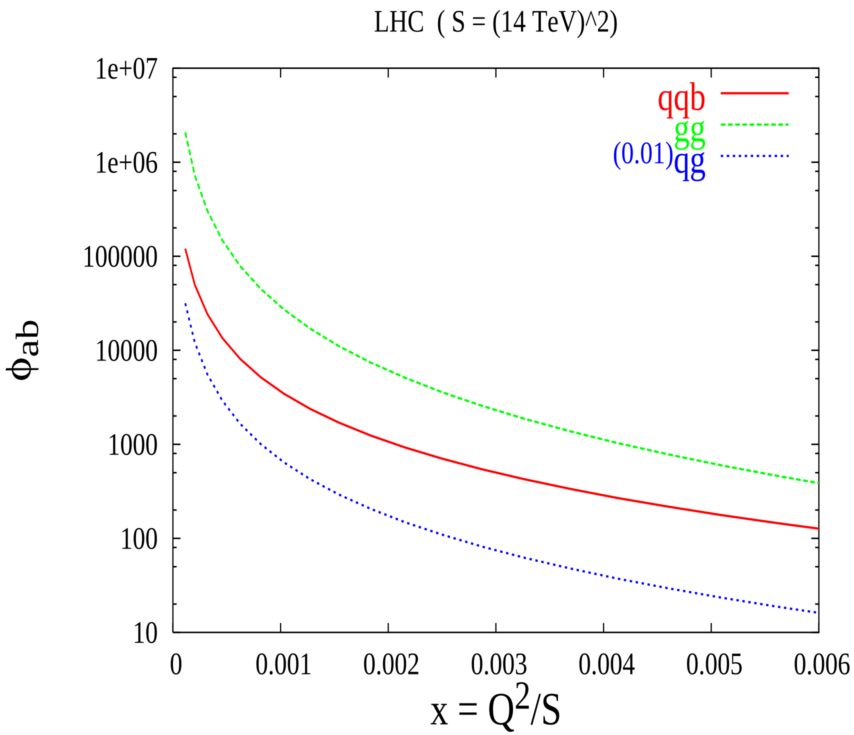

- $\Phi_{ab}(x)$ becomes large when $x \longrightarrow x_{min} = \tau$

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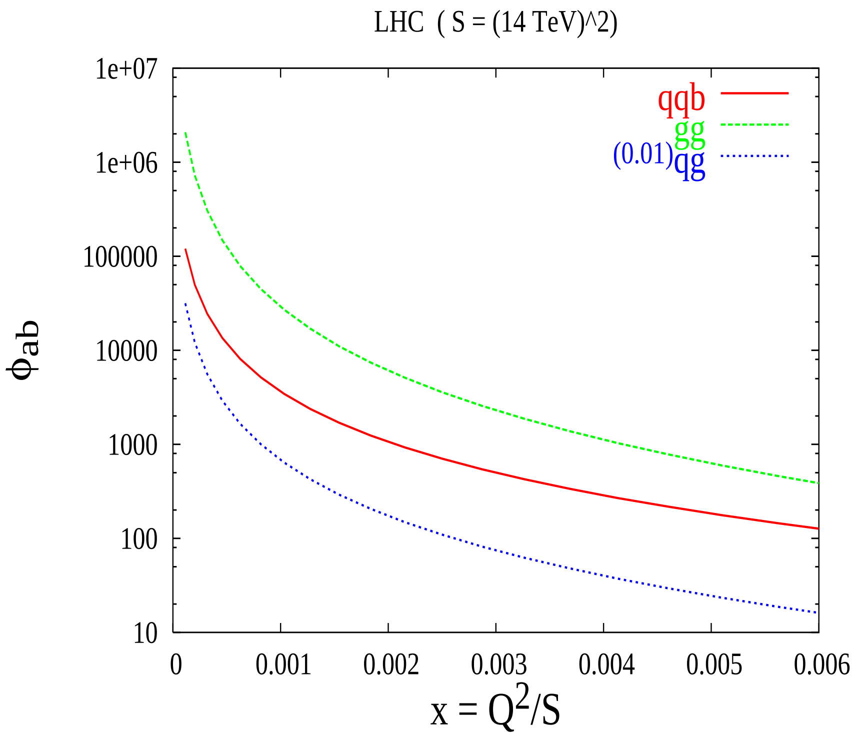

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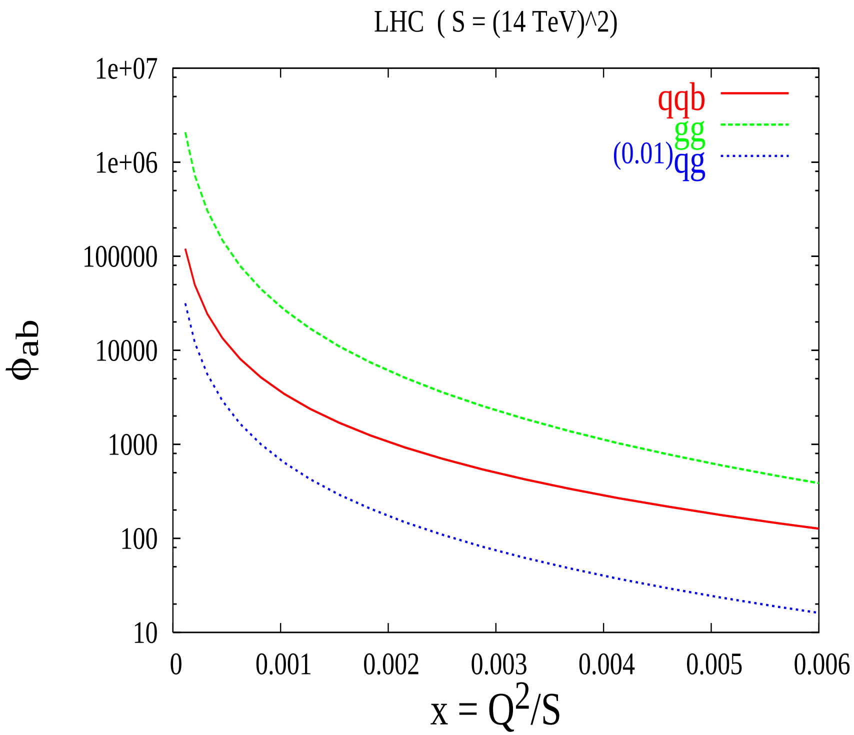

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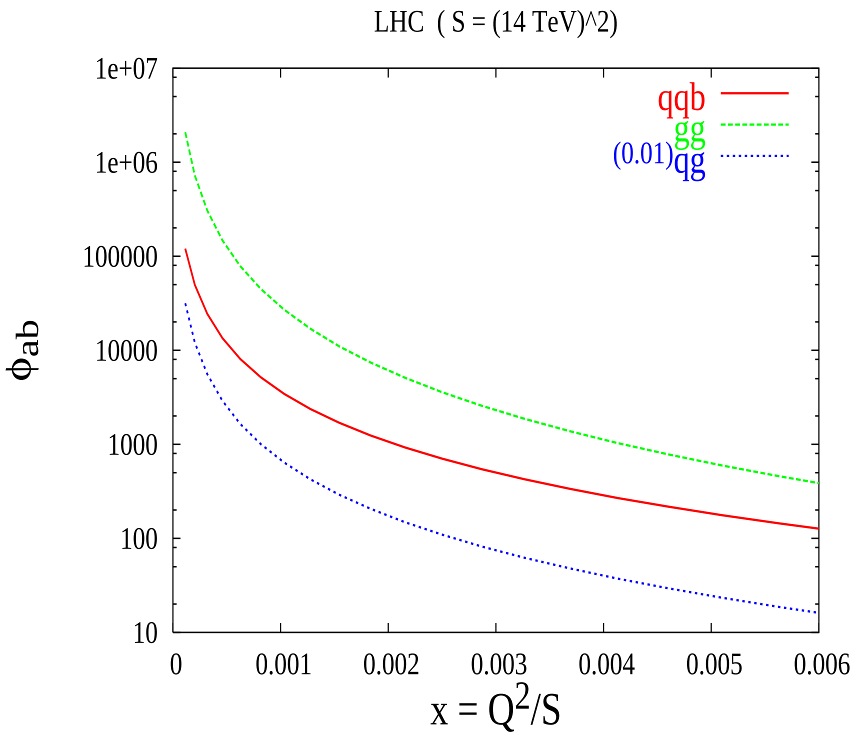

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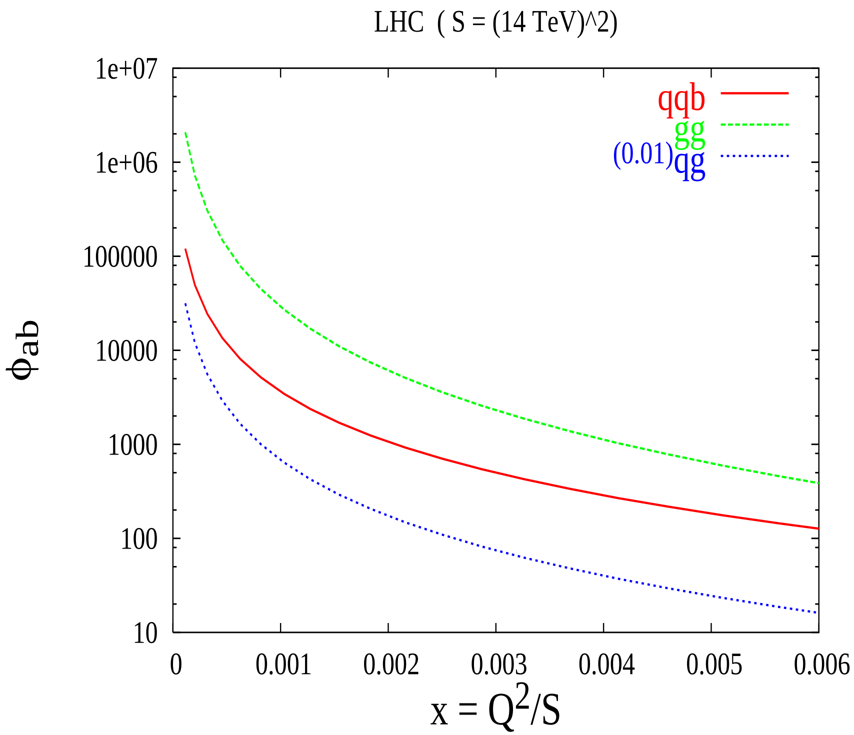

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- Expand the partonic cross section around $x = \tau$.

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  OR

  Extract from "Form factors and DGLAP kernels" using

  1) Factorisation theorem 2) Renormalisation Group Invariance

  3) Drell-Yan NNLO results

Soft plus Virtual at N^3LO and beyond

VR

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VR

Using "factorisation" of Virtual, Soft and Collinear:

$$\Delta^{sv}_{I,P}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi^I_P(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)\right)\Big|_{\varepsilon=0} \qquad I=q,g \qquad n=4+\varepsilon$$

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$$\hat{a}_s = \frac{\hat{g}_s^2}{16\pi^2} \qquad m=\frac{1}{2} \quad \text{for} \quad \text{DIS}, \quad m=1 \qquad \text{for} \quad \text{DY, Higgs}$$

Sudakov Resummation for Form factors

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$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I \left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) \quad = \quad \frac{1}{2}\left[K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

Solution : $\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \hat{\mathcal{L}}_F^{I,(i)}(\varepsilon)$

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Formal solution upto 4 loops:

$$\hat{\mathcal{L}}_F^{I,(1)} = \frac{1}{\varepsilon^2}\Big(-2A_1^I\Big) + \frac{1}{\varepsilon}\Big(G_1^I(\varepsilon)\Big)$$

$$\hat{\mathcal{L}}_F^{I,(2)} = \frac{1}{\varepsilon^3}\Big(\beta_0 A_1^I\Big) + \frac{1}{\varepsilon^2}\Big(-\frac{1}{2}A_2^I - \beta_0 G_1^I(\varepsilon)\Big) + \frac{1}{2\varepsilon}G_2^I(\varepsilon)$$

$$\ldots. \quad \ldots. \quad \ldots. \quad \ldots.$$

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Sudakov Resummation for Form factors

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$$Q^2 \frac{d}{dQ^2} \ln \hat{F}^I \left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) = \frac{1}{2}\left[K^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right]$$

$$\textbf{Solution :} \quad \ln \hat{F}^I (\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\mathcal{L}}_F^{I,(i)}(\varepsilon)$$

Formal solution upto 4 loops:

$$\hat{\mathcal{L}}_F^{I,(1)} = \frac{1}{\varepsilon^2}\left(-2A_1^I\right) + \frac{1}{\varepsilon}\left(G_1^I(\varepsilon)\right)$$

$$\hat{\mathcal{L}}_F^{I,(2)} = \frac{1}{\varepsilon^3}\left(\beta_0 A_1^I\right) + \frac{1}{\varepsilon^2}\left(-\frac{1}{2}A_2^I - \beta_0 G_1^I(\varepsilon)\right) + \frac{1}{2\varepsilon} G_2^I(\varepsilon)$$

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- A^I $\quad are \quad$ **maximally non − abelian** $\quad A_i^g = \frac{C_A}{C_F} A_i^q \qquad i = 1, 2, 3.$

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- A^I *are* **maximally non − abelian** $A_i^g = \frac{C_A}{C_F} A_i^q$ $i = 1, 2, 3.$

- Every order in $\hat{a}_s$, all the poles except the lowest one can be predicted from the previous order results using A and β function.

New observation for single pole in ε

$VR, Smith, van\ Neerven$

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$$G_1^I(\varepsilon) = 2(B_1^I - \gamma_1^I) + f_1^I + \sum_{k=1}^{\infty} \varepsilon^k g_1^{I,k}$$

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B_i^I are $\delta(1-z)$ part of P_{II} splitting functions. The new constants "f_1^I and f_2^I" satisfy

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This completes the understanding of all the poles of the form factors.

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DGLAP kernels satisfy Renormalisation Group Equations:

$$\mu_F^2 \frac{d}{d\mu_F^2}\Gamma(z, \mu_F^2, \varepsilon) = \frac{1}{2} P\left(z, \mu_F^2\right) \otimes \Gamma\left(z, \mu_F^2, \varepsilon\right) \; .$$

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The diagonal terms of the splitting functions $P^{(i)}(z)$ have the following structure

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$$\mathcal{D}_0 = \left(\frac{1}{1-z}\right)_+ \; , \qquad P_{reg,II}^{(i)} \text{are} \quad \text{regular} \quad \text{when} \quad z \longrightarrow 1.$$

We will be left with only maximally non-abelian constants A_i^I and f_i^I

Finiteness of the Cross section

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Observable $\boldsymbol{\Delta^I(\alpha_s, Q^2)}$ are finite:

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$$\frac{\mathbf{1}}{\boldsymbol{\varepsilon^{i+1}}} \quad \mathbf{at} \quad \mathbf{i^{th}loop}$$

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- The structure of soft part should be "similar" to the Form Factors.
- Hence using gauge invariance and RG invariance, we can propose

$$q^2 \frac{d}{dq^2} \Phi^I\left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) \quad = \quad \frac{1}{2}\left[\overline{K}^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) + \overline{G}^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right)\right]$$

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RG invariance of Φ^I implies:

$$\mu_R^2 \frac{d}{d\mu_R^2}\overline{K}^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) = -\mu_R^2 \frac{d}{d\mu_R^2}\overline{G}^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) = -\overline{A}^I(a_s(\mu_R^2))\delta(1-z)$$

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$$\Phi^I\left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\Phi}^{I,(i)}(z, \varepsilon)$$

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Most general solution:

$$\begin{aligned}
\Phi^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) &= \Phi^I(\hat{a}_s, q^2(1-z)^{2m}, \mu^2, \varepsilon) \\
&= \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^{2m}}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\, m\, \varepsilon}{2(1-z)}\right) \hat{\phi}^{\,I,(i)}(\varepsilon)
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All the poles in ε are predictable.

Universal Soft part

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Single pole in ε:

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Higgs productions from Drell-Yan beyond $NNLO$

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$$\sigma(g + g \longrightarrow Higgs)$$

without explicitly calculating the soft part of Higgs production.

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- Soft plus Virtual part Higgs production through bottom quark fusion

$$\sigma(b + \overline{b} \longrightarrow Higgs)$$

can be predicted upto N^3LO(without $\delta(1-z)$).

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- The scalar form factor $\mathcal{F}^S = \langle P|\overline{\psi}\psi|P\rangle$ can be predicted at three loop from the known three loop A_i, B_i, f_i and γ_i^m.
- Soft plus Virtual part Higgs production through bottom quark fusion

$$\sigma(b + \overline{b} \longrightarrow Higgs)$$

can be predicted upto N^3LO(without $\delta(1-z)$).

- Our $NNLO$ predictions agrees with the results of by Harlander and Kilgore.

Hadro production in e^+e^- annihilation from DIS

Blümlein and VR

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Threshold Resummation

- Alternate derivation for the threshold resummation formula in z space for both DY and DIS:

VR

$$
\begin{aligned}
\Phi_P^I(\hat{a}_s, q^2, \mu^2, z, \varepsilon) = & \left(\frac{m}{1-z} \left\{ \int_{\mu_R^2}^{q^2(1-z)^{2m}\delta_P} \frac{d\lambda^2}{\lambda^2} A_I\left(a_s(\lambda^2)\right) \right.\right. \\
& \left.\left. + \overline{G}_P^I\left(a_s\left(q^2(1-z)^{2m}\delta_P\right), \varepsilon\right) \right\} \right)_+ \\
& + \delta(1-z) \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2\delta_P}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i\, \hat{\phi}_P^{I,(i)}(\varepsilon) \\
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- $\overline{G}_P^I(\varepsilon = 0)$ upto three loop gives D_i^I and B_i^I for $i = 1, 2, 3$
- Expansion of $\mathcal{C}e^{\left(2\Phi_P^I\right)}$ leads to soft part of the cross section.
- Fixed order N^3LO soft plus virtual cross sections can be computed(except $\delta(1-z)$)

Soft plus Virtual part at N^3LO for Higgs Production

Moch, Vogt and VR

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$$2S\, d\sigma^{P_1P_2}(\tau, m_h) = \sum_{ab}\int_\tau^1 \frac{dx}{x}\Phi_{ab}(x)\, 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$

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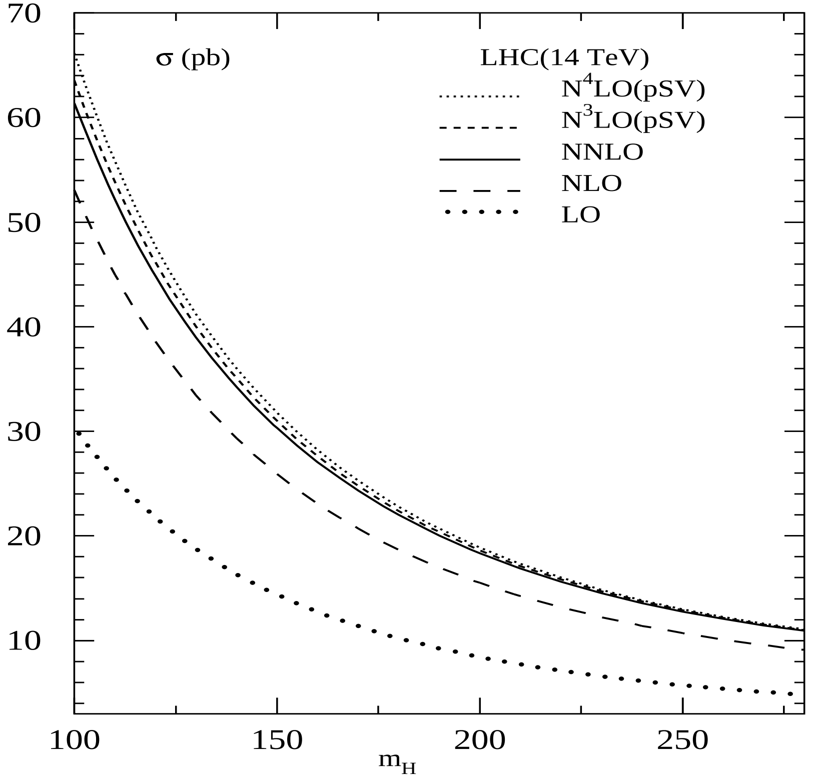

Gluon flux is largest at LHC

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- They contribute bulk of the cross section

Scale variation at N^3LO for Higgs production

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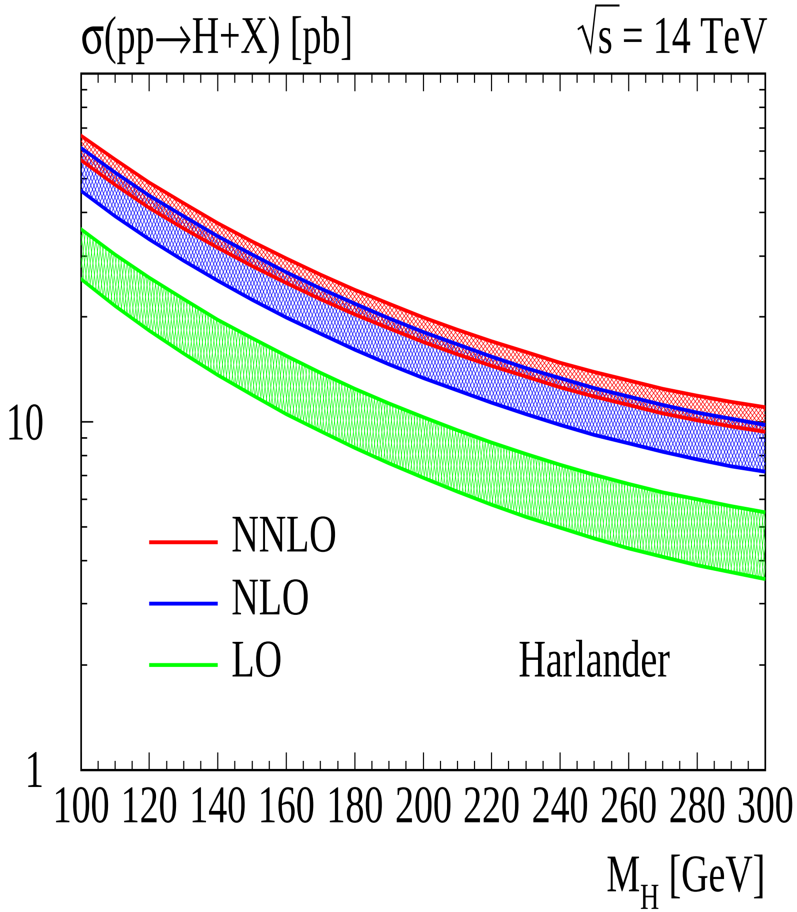

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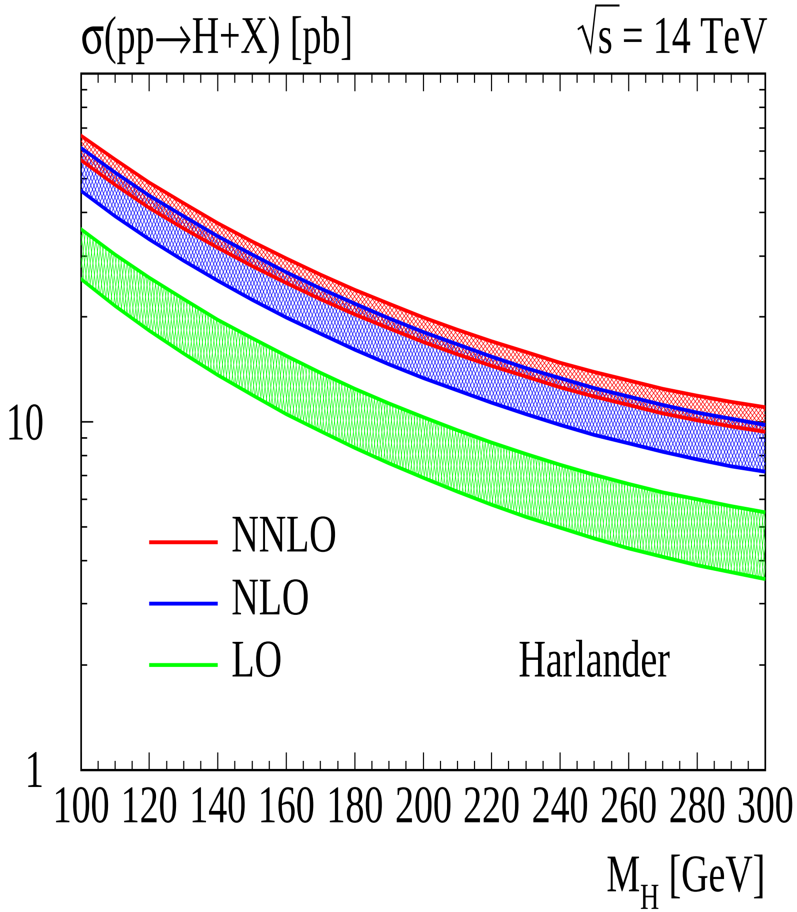

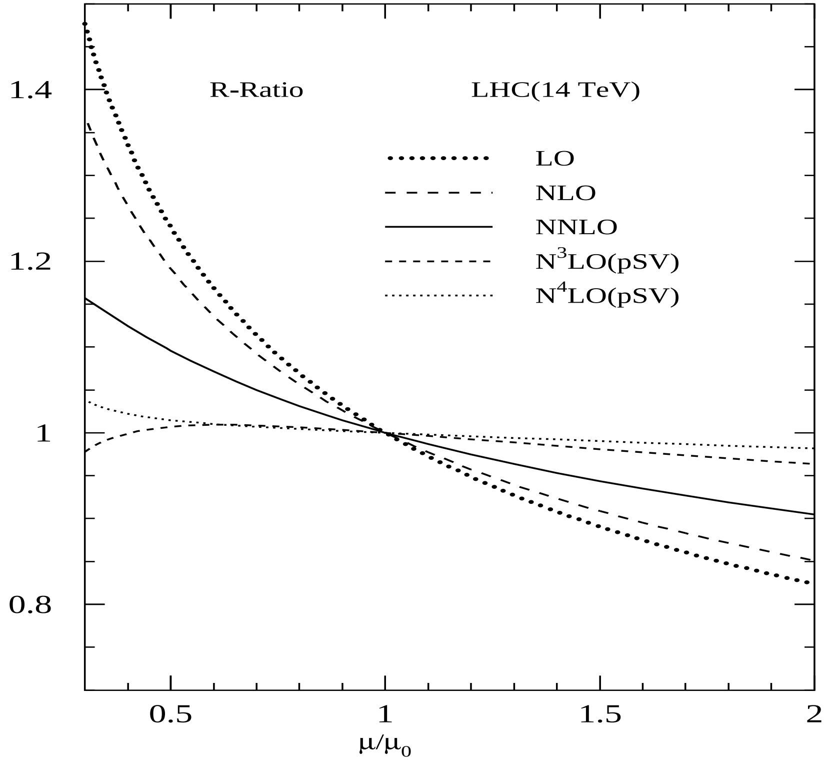

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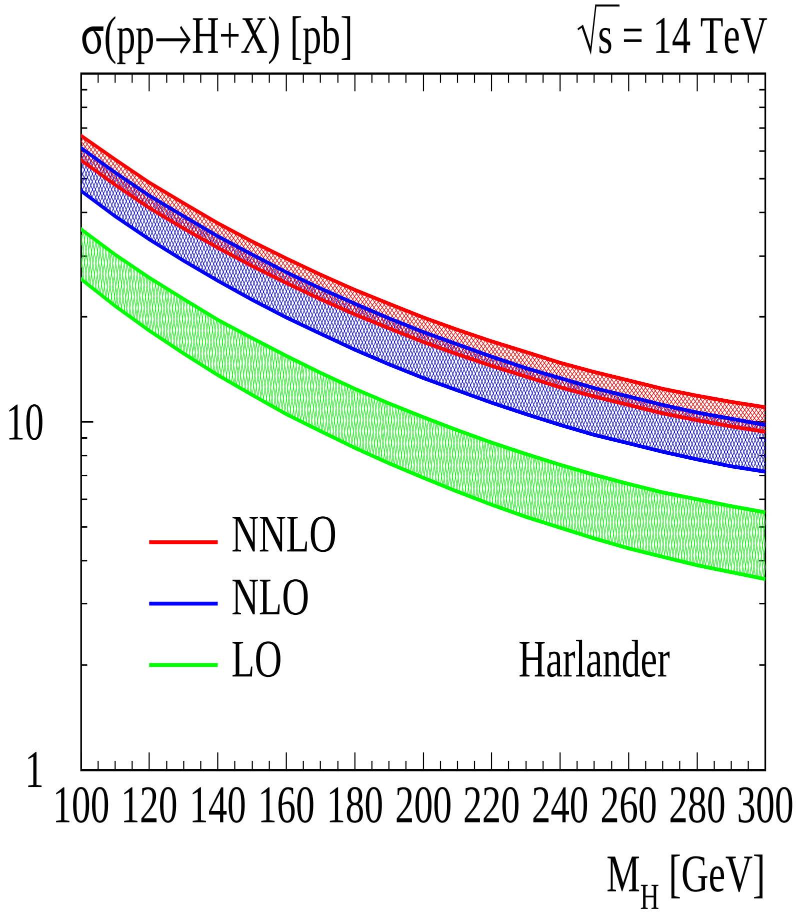

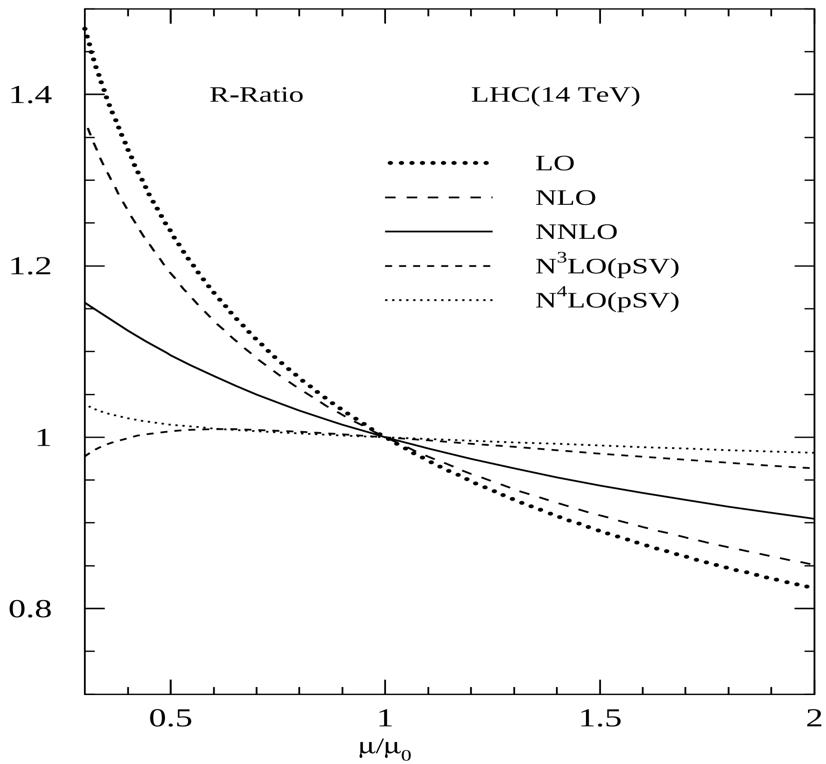

- Scale uncertainity improves a lot
- Perturbative QCD works at LHC

Soft distribution for rapidity

VR, Smith and van Neerven

Using RGE and Factorisation:

Soft distribution for rapidity

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$$\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \varepsilon) = \Phi^I_{d,finite} + \Phi^I_{d,singular}$$

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where

$$\Phi^{I}_{d,finite} = \frac{1}{2}\delta(1-z_2)\left(\frac{1}{1-z_1}\left\{\int_{\mu_R^2}^{q^2(1-z_1)}\frac{d\lambda^2}{\lambda^2}A_I\left(a_s(\lambda^2)\right)\right.\right.$$

$$\left.\left.+\overline{G}^{I}_{d}\left(a_s\left(q^2(1-z_1)\right),\varepsilon\right)\right\}\right)_+$$

$$+q^2\frac{d}{dq^2}\left[\left(\frac{1}{4(1-z_1)(1-z_2)}\left\{\int_{\mu_R^2}^{q^2(1-z_1)(1-z_2)}\frac{d\lambda^2}{\lambda^2}A^I\left(a_s(\lambda^2)\right)\right.\right.\right.$$

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$$+z_1 \leftrightarrow z_2$$

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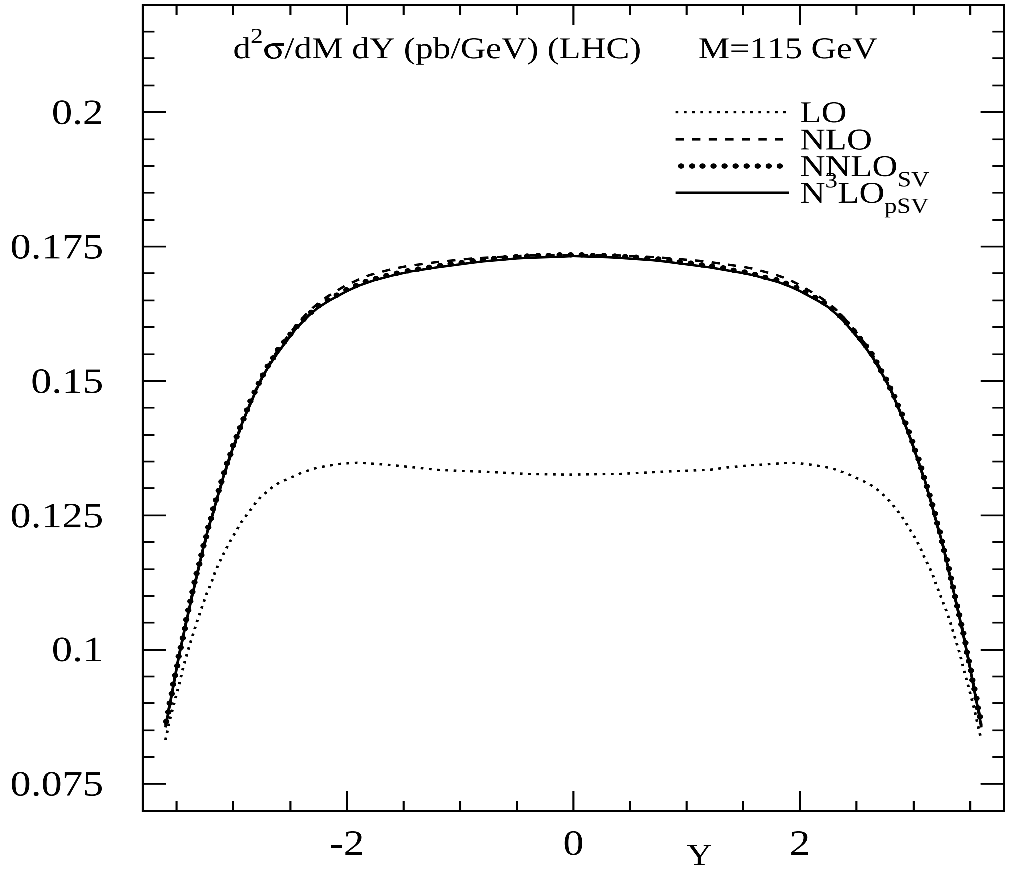

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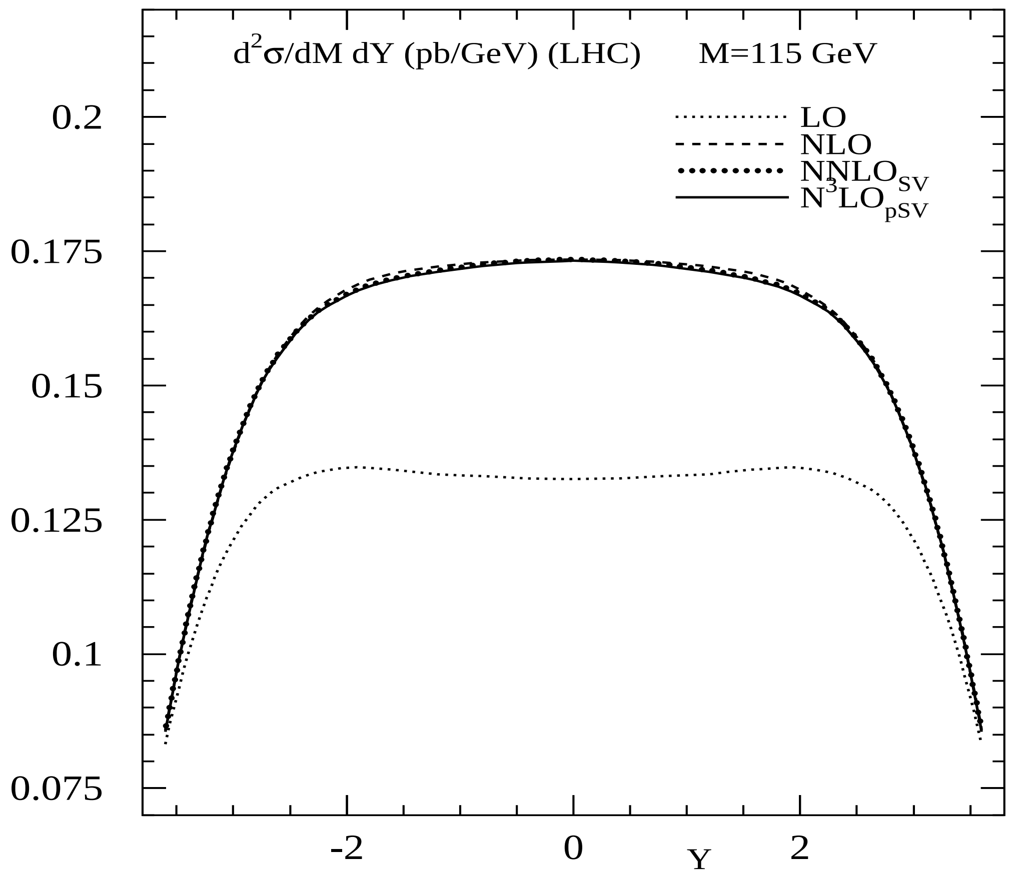

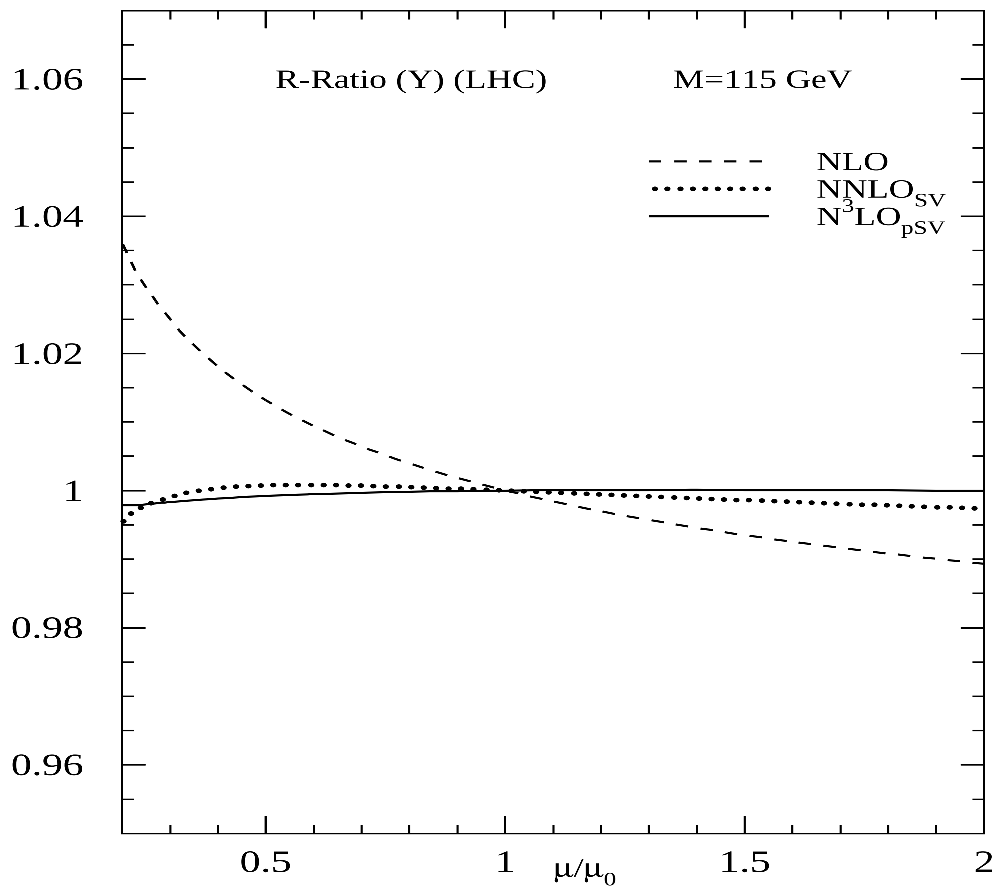

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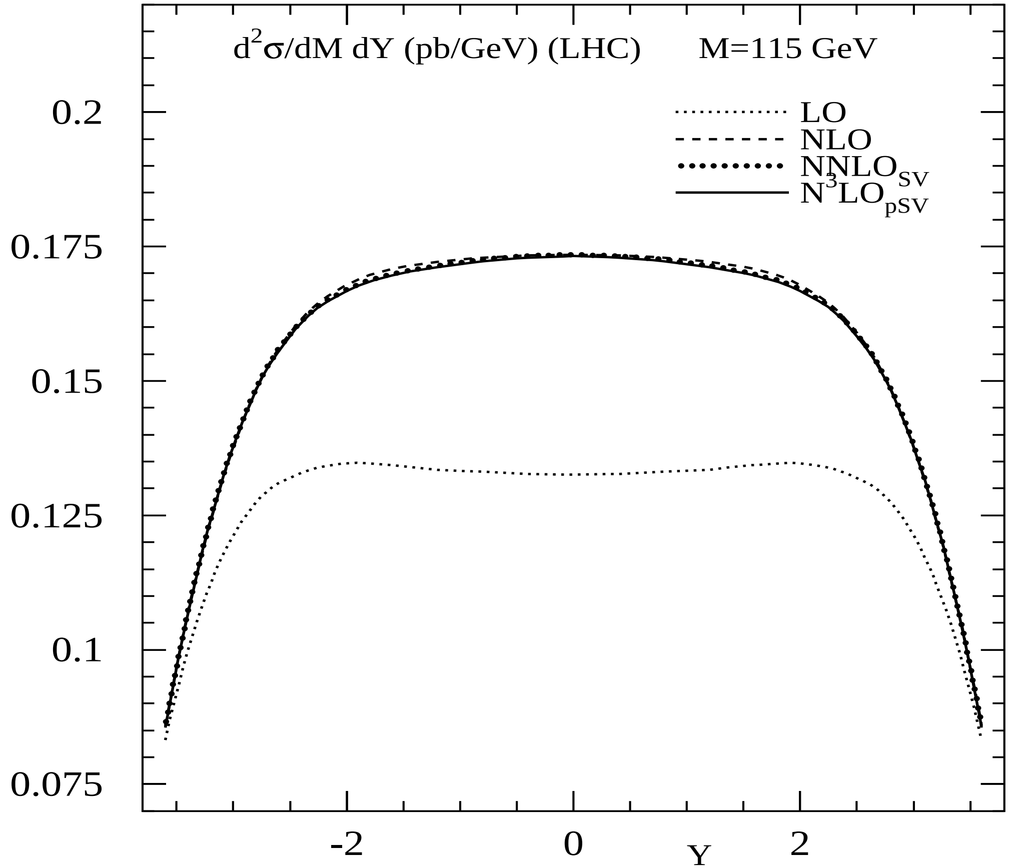

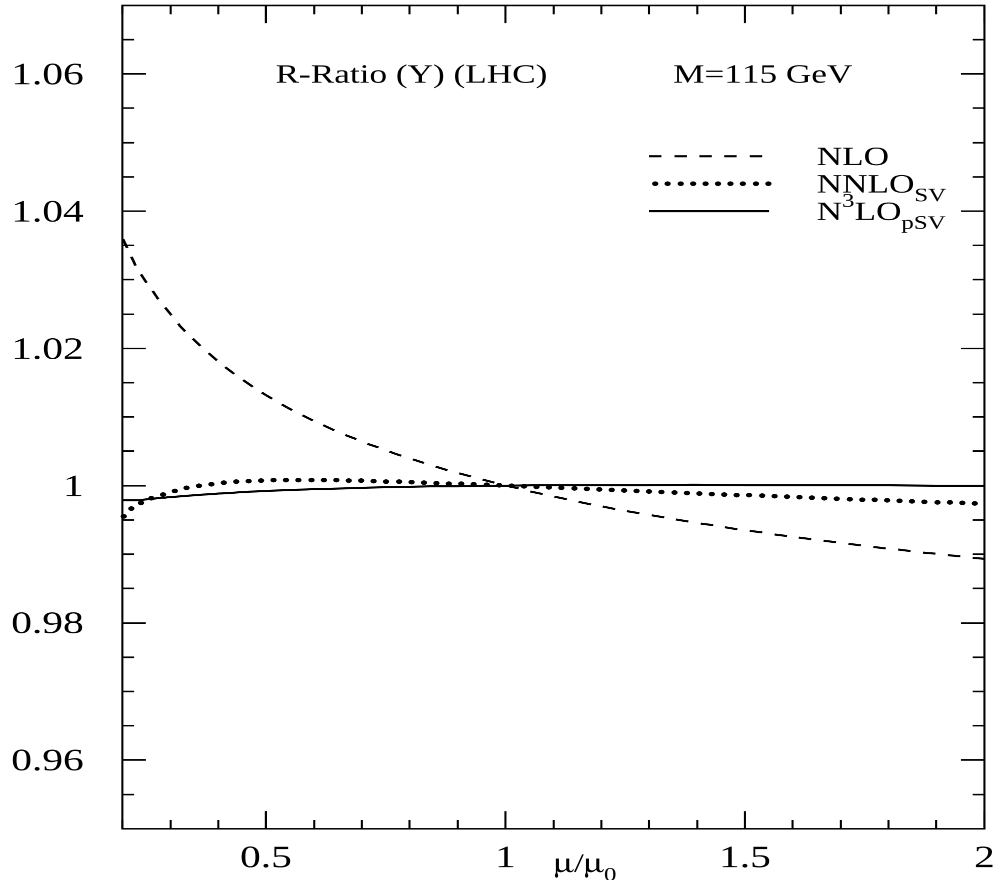

- Compared against Dixon, Anastasiou, Melnikov, Petriello NNLO results for Drell-Yan, $\boldsymbol{Higgs, Z, W^{\pm}}$ productions.

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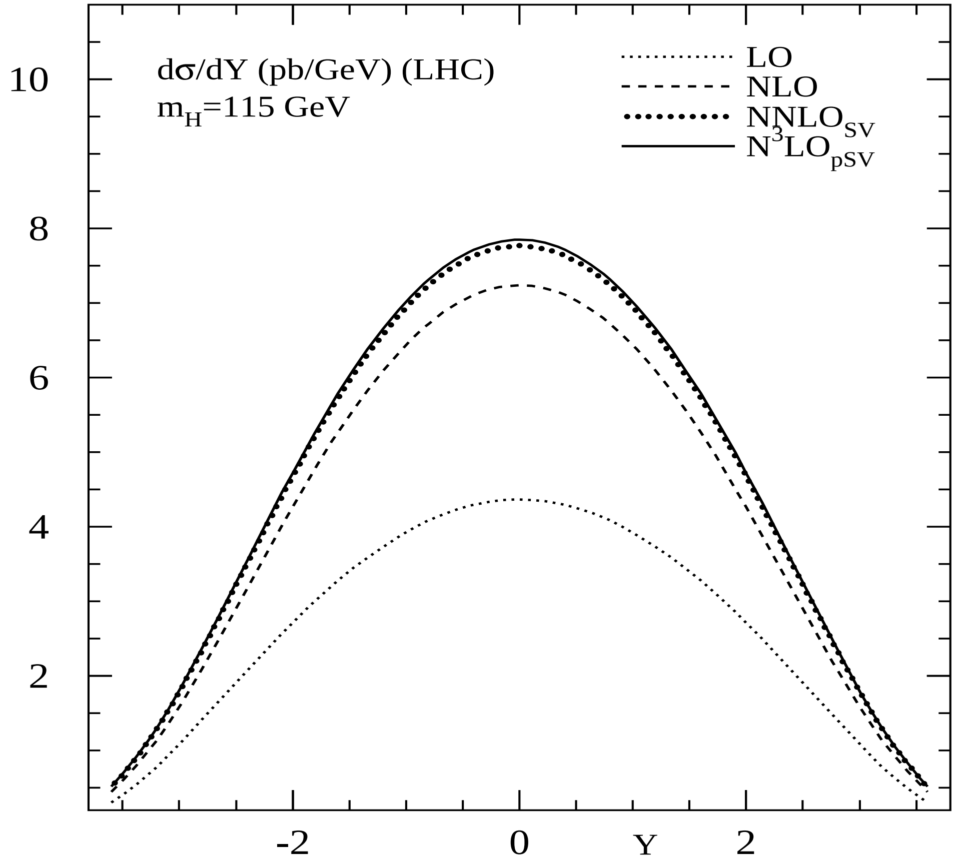

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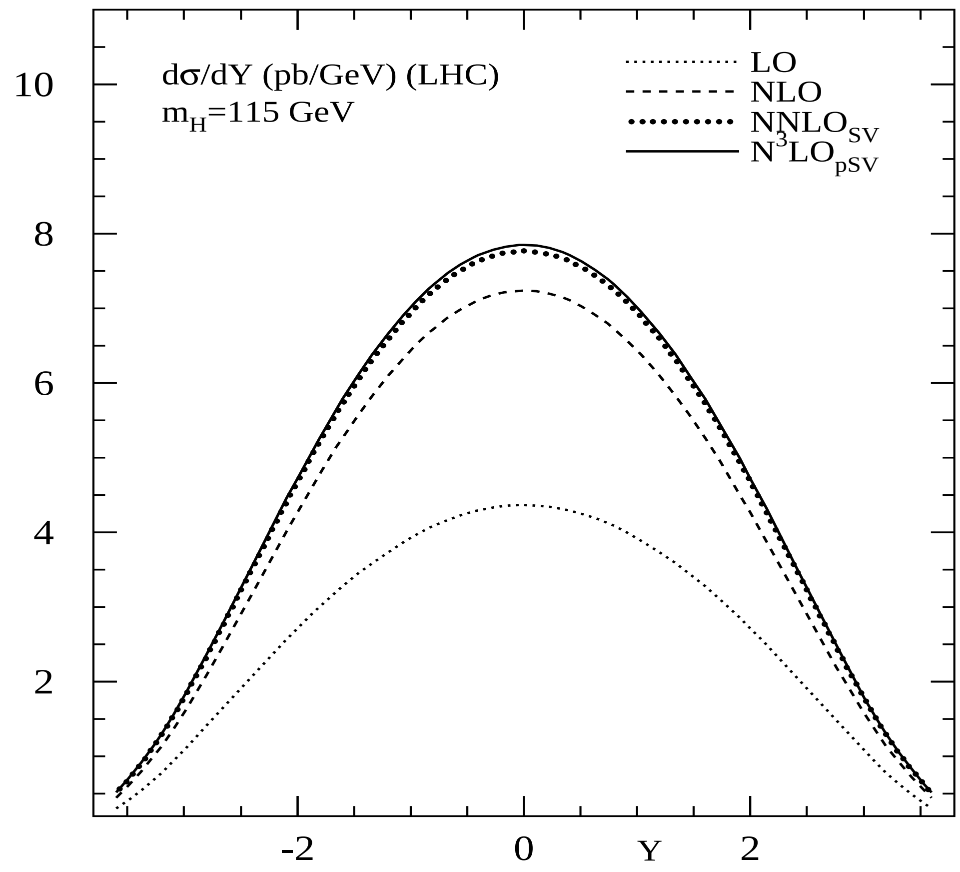

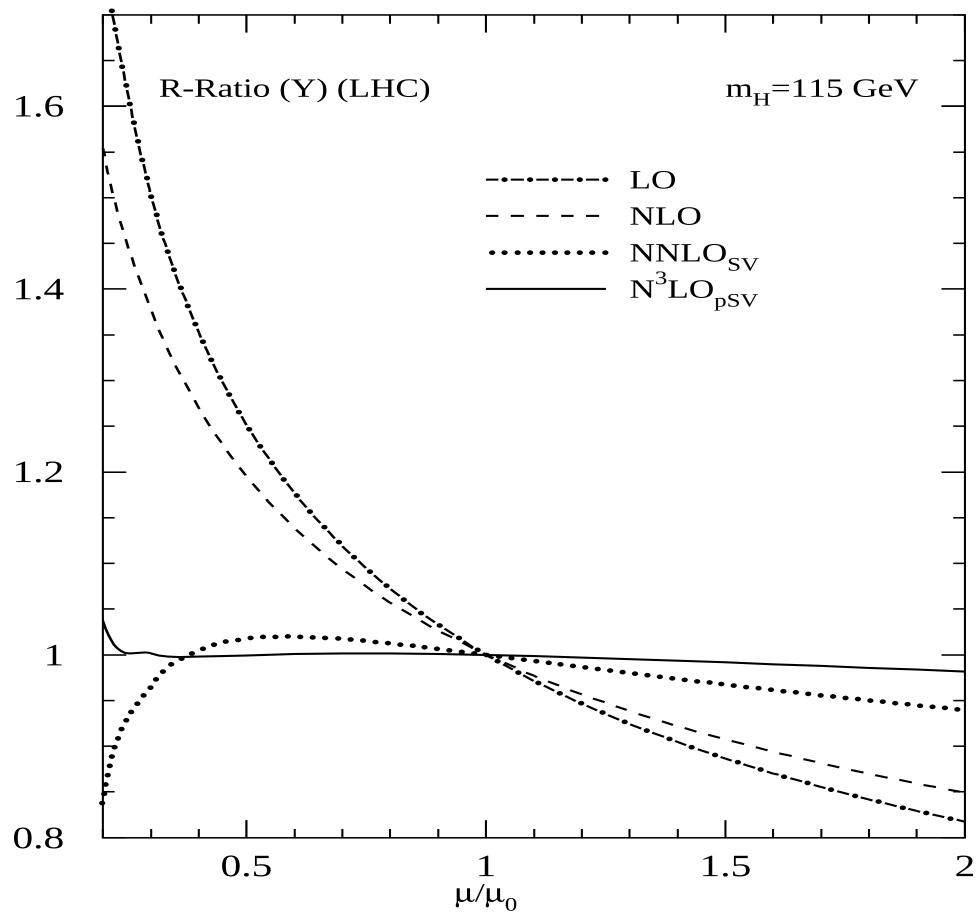

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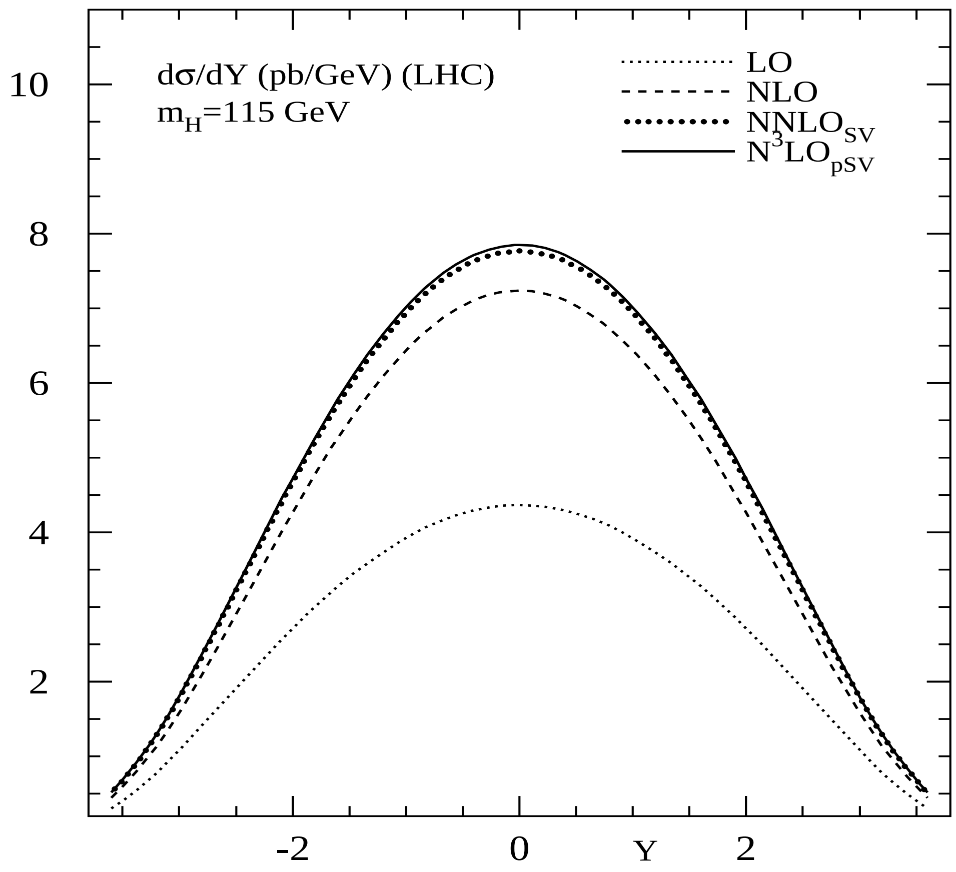

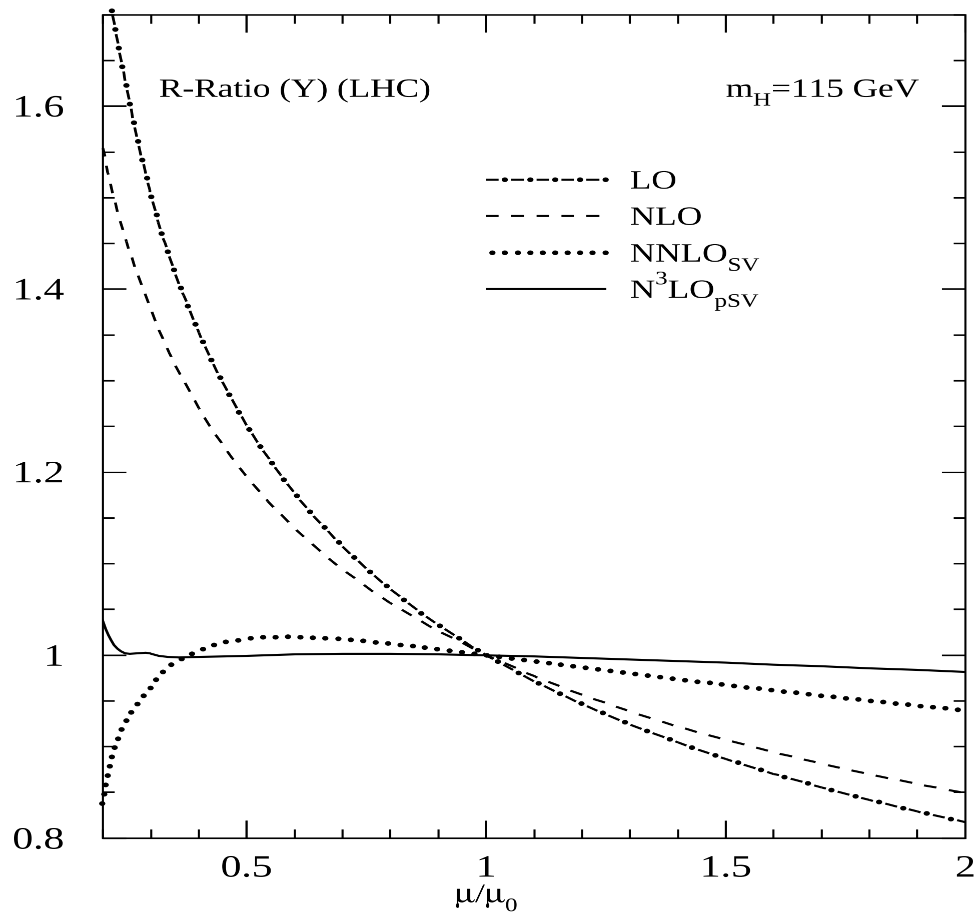

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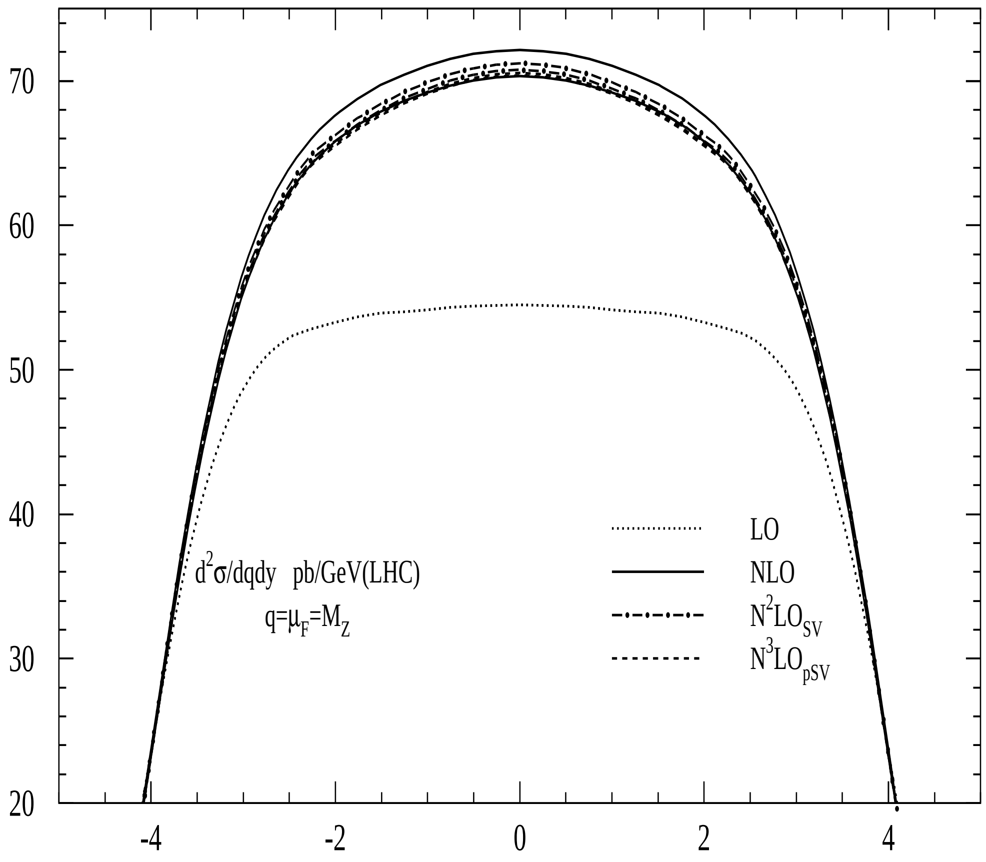

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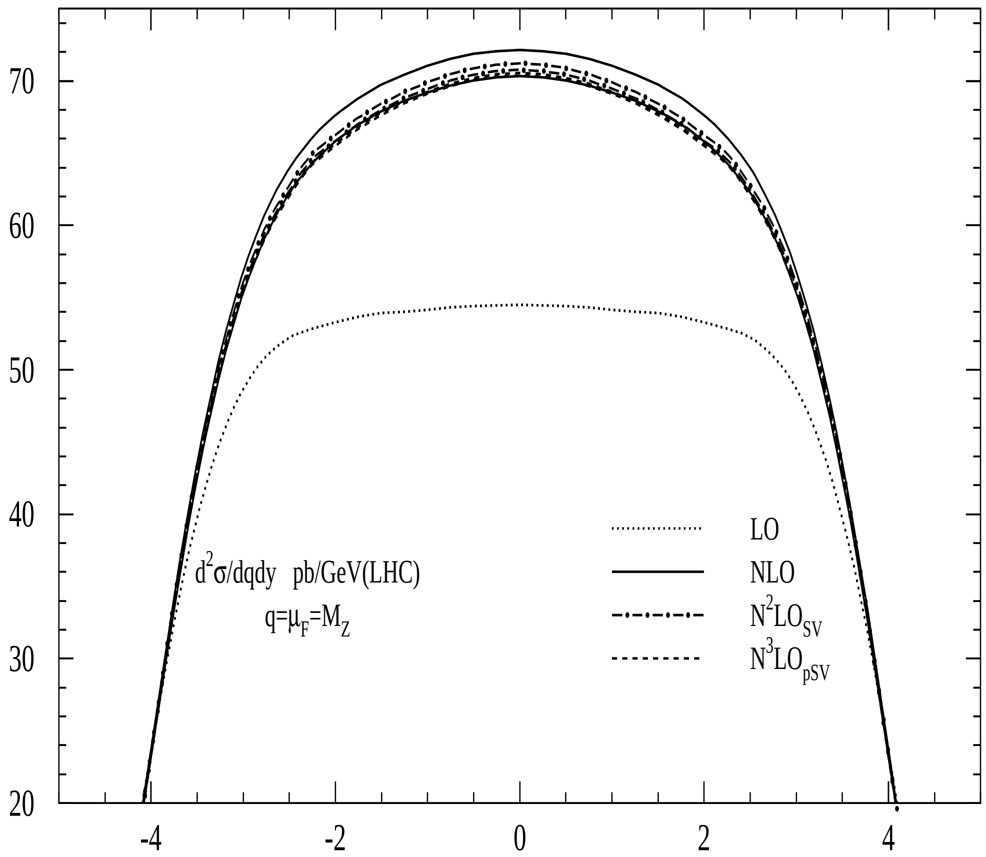

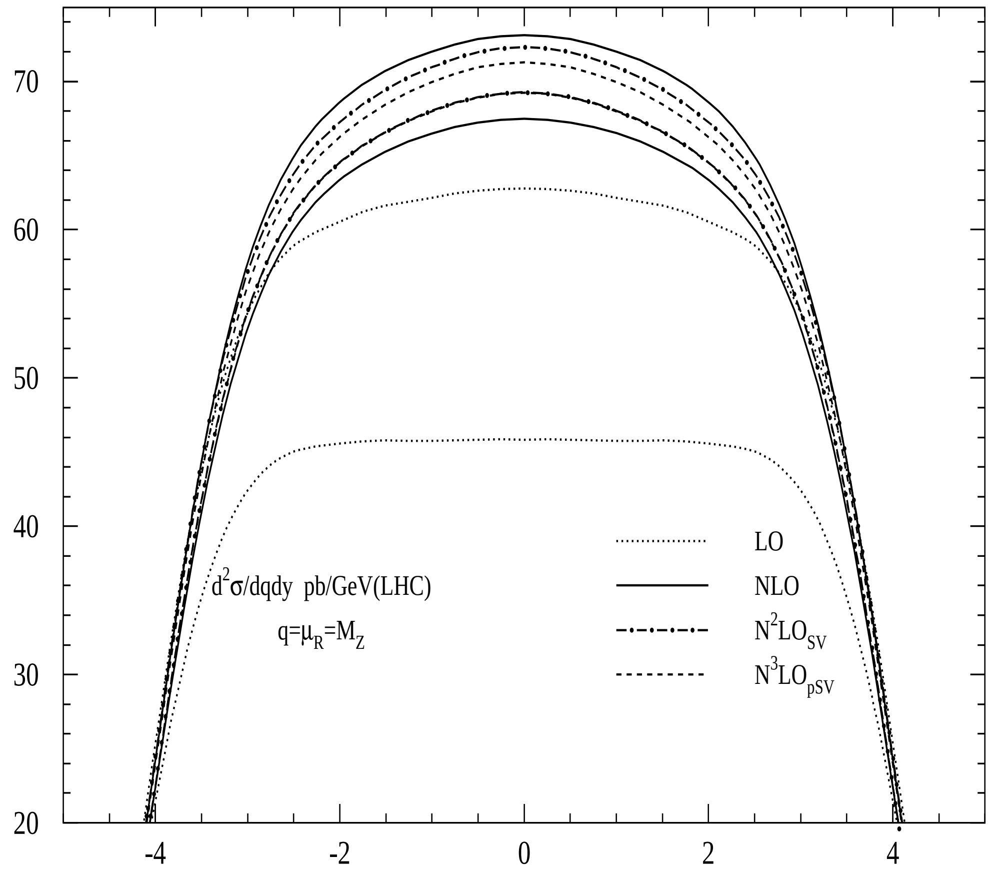

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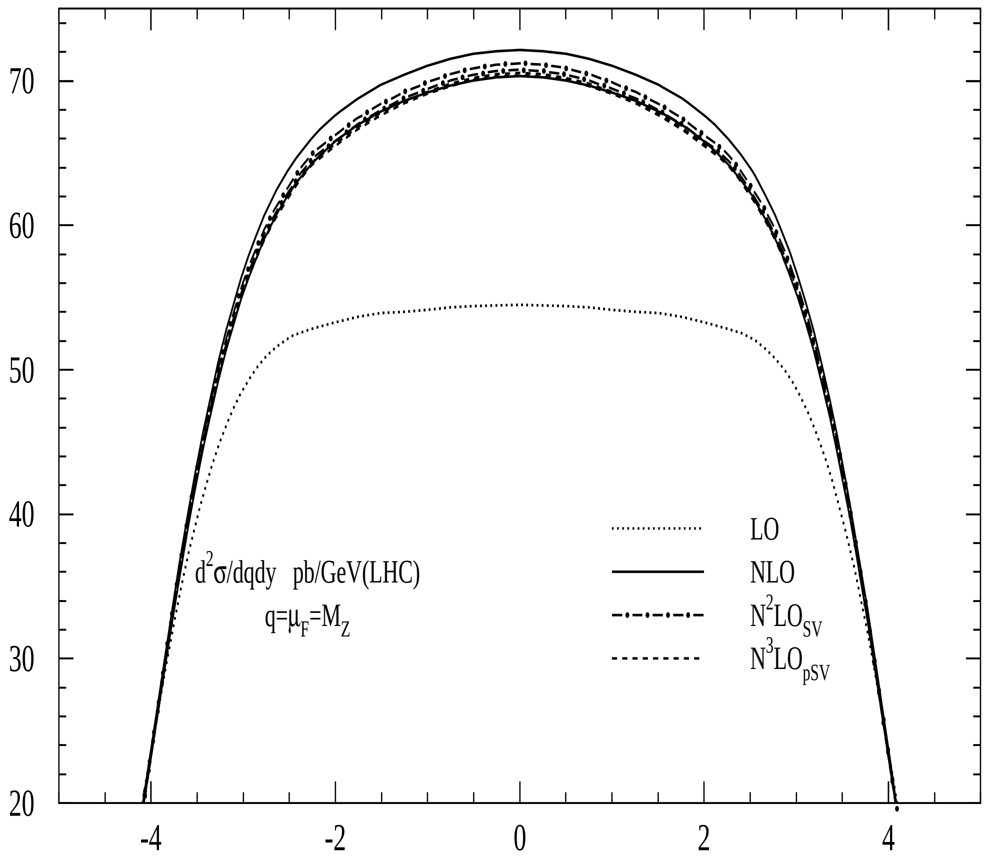

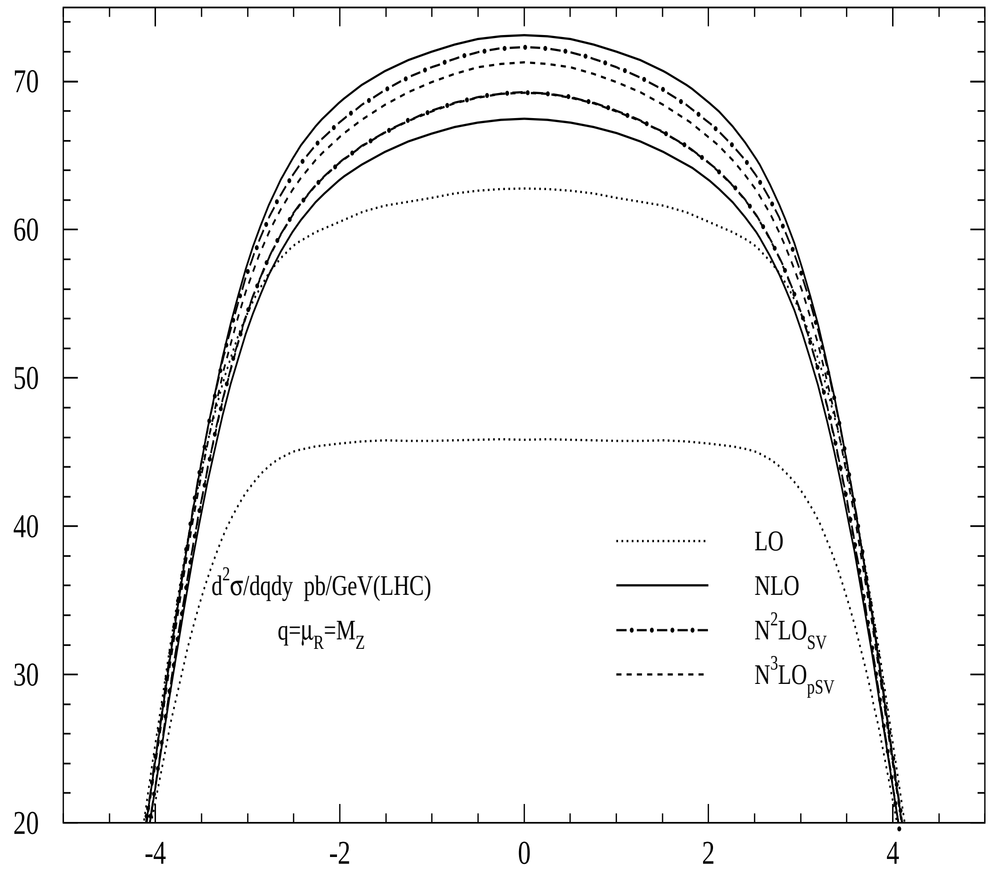

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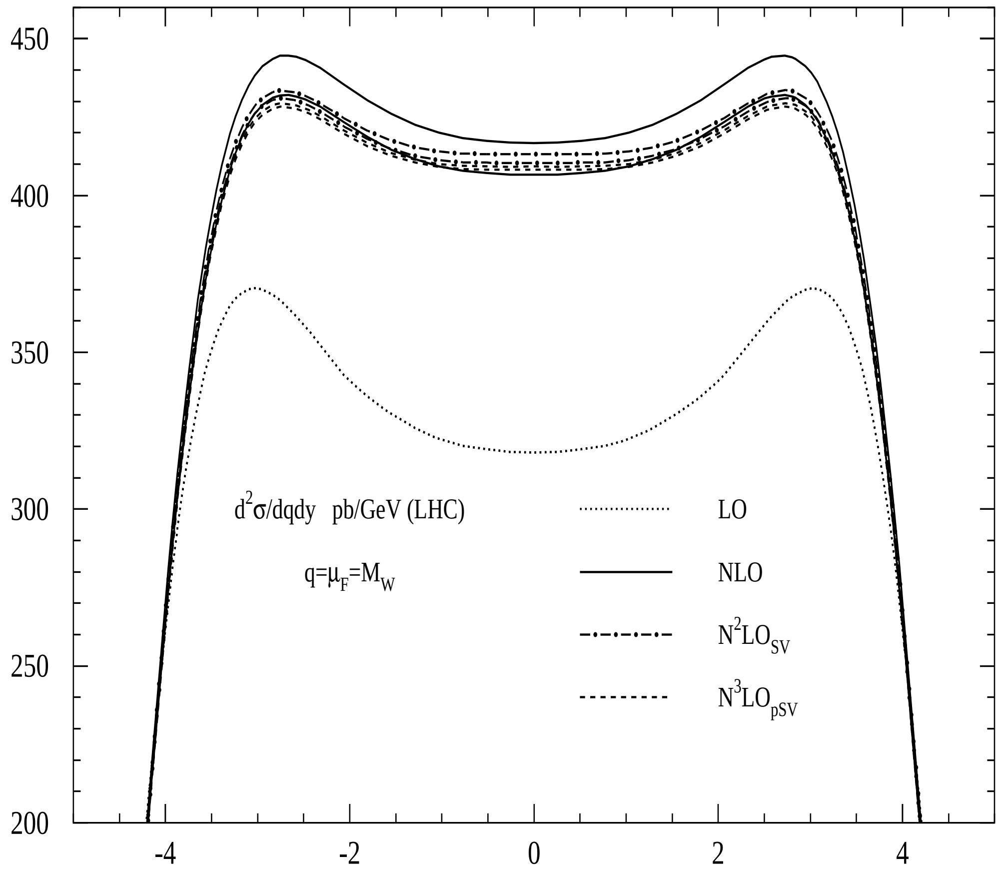

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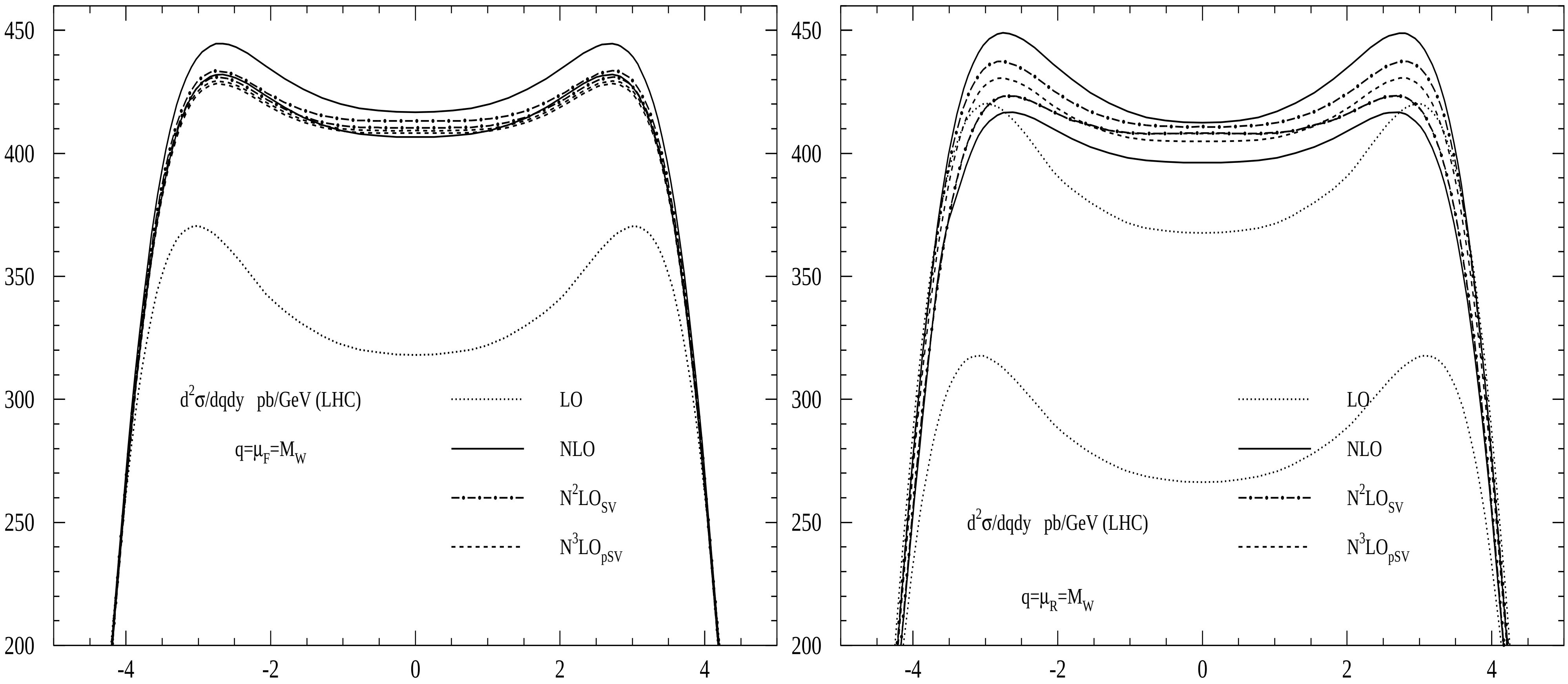

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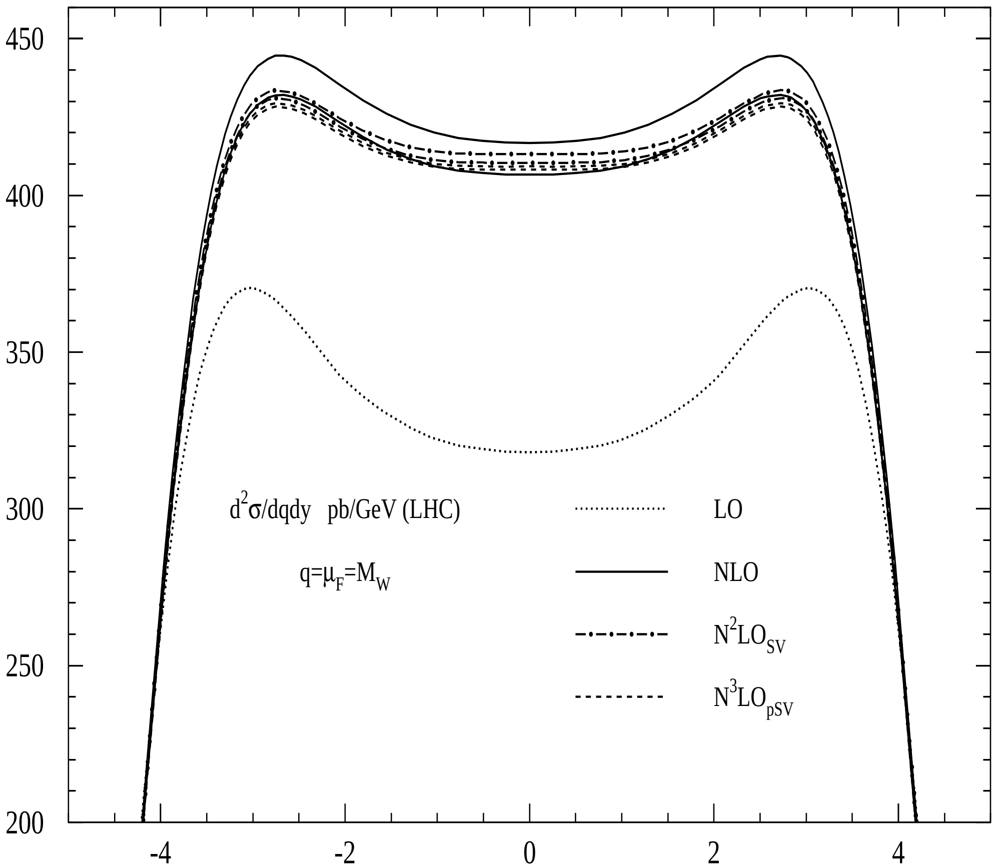

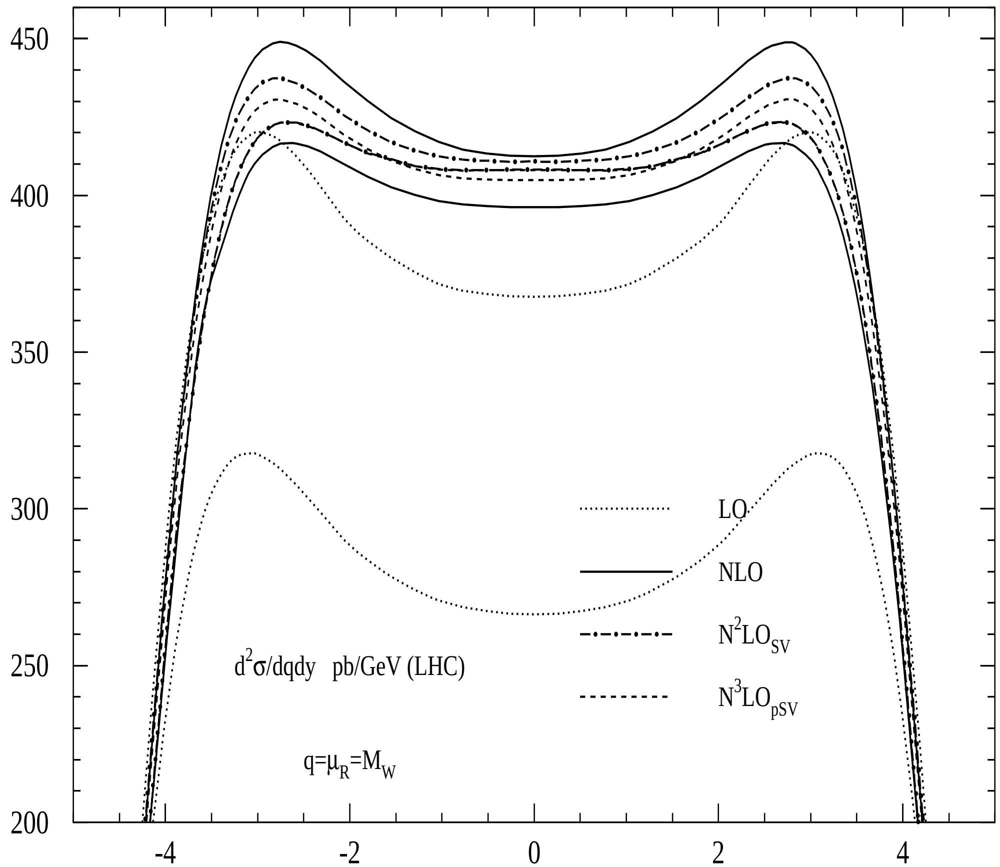

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Thank You