Double and Triple Higgs boson production at $\mathcal{O}(\alpha_{ew}^3)$ at the ILC within a generic 2HDM.

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Based on: G. F., J. Guasch, D. López-Val, J. Solà, arXiv:0707.3162 [hep-ph]

Our aim is to consider $\mathcal{O}(\alpha_{ew}^3)$ triple and double Higgs boson production at the ILC in the general 2HDM and compare with the MSSM case.

$$e^+e^- \to 2H \qquad (2H \equiv h^0A^0;\ H^0A^0;\ H^+H^-)\,,$$

$$e^+e^- \to 3H \qquad (3H \equiv H^+H^-h;\ h\,h\,A^0;\ h^0H^0A^0)\,, \quad (h = h^0, H^0, A^0)$$

The cross-sections for the 2H final states lie within the same order of magnitude in both the MSSM and 2HDM.

We find that for the 3H states the maximum 2HDM cross-sections, being of order 0.1 pb, are much larger than the MSSM ones which in most cases are of order 10^{-6} pb or less.

This fact originates in the structure of the trilinear Higgs boson couplings.

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Outline

- Double Higgs boson production in a linear collider has been investigated, mainly in the MSSM, in [Djouadi, Haber, Zerwas ('93,'96); Djouadi, Driesen, Hollik, Rosiek ('96); Feng, Moroi ('96)]
- The one-loop calculation of the cross-sections for e^+e^- 2H production, essentially in the MSSM case, has been investigated in [Driesen, Hollik, Rosiek ('96); Coniavitis, Ferrari ('07); Heinemeyer ('06)]
- There are also studies considering radiative corrections to charged Higgs production in e^+e^- collisions within the 2HDM [Guasch, Hollik, Kraft('01)]
- Double and multiple Higgs production at the LHC has been investigated in [Djouadi, Kilian, Muhlleitner, Zerwas ('99); Binoth, Karg, Kauer, Ruckl ('06)],
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Two-Higgs-Doublet Model (2HDM)

- 2HDM extend the SM with a second $SU_L(2)$ doublet with $Y = 1$

$$\Phi_1 = \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \quad (Y = +1)\,, \quad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \quad (Y = +1)$$

- In the SUSY case instead we have

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \equiv \epsilon\, \Phi_1^* = \begin{pmatrix} \Phi_1^{0*} \\ -\Phi_1^- \end{pmatrix} \qquad (Y = -1) \qquad \epsilon = i\,\sigma_2.$$

- In 2HDM the most general structure of the CP-conserving, gauge invariant, renormalizable Higgs potential reads:

$$V(\Phi_1,\Phi_2) = \lambda_1(\Phi_1^\dagger\Phi_1 - v_1^2)^2 + \lambda_2(\Phi_2^\dagger\Phi_2 - v_2^2)^2 + \lambda_3\left[(\Phi_1^\dagger\Phi_1 - v_1^2) + (\Phi_2^\dagger\Phi_2 - v_2^2)\right]^2$$

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λ_i, $(i = 1 \dots 6)$ dimensionless real parameters and $v_{1,2} = \langle \Phi_{1,2}^0 \rangle$.

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- Free parameters in the general 2HDM are usually chosen to be:

$$M_{h^0},\ M_{H^0},\ M_{A^0},\ M_{H^\pm},\ \tan\beta,\ \alpha,\ \lambda_5$$

In order to keep closer to the MSSM structure of the Higgs sector we adopt the SUSY constrain $\lambda_5 = \lambda_6 = 2\sqrt{2}\, G_F\, M_{A^0}^2$

- There are two possible 2HDM scenarios that ensure the absence of tree-level FCNC

1) In the type I 2HDM one Higgs doublet (Φ_2) couples to all of the SM fermions, whereas the other one (Φ_1) does not couple to them at all;

2) In the type II 2HDM the doublet Φ_1 (resp. Φ_2) couples only to down-like (resp. up-like) fermions.

The MSSM Higgs sector is a type II one. We shall not dwell here on the Higgs boson interactions with fermions because they are not involved in any of our tree-level Higgs boson production processes.

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Trilinear Higgs couplings

- In the case of the MSSM, and due to the SUSY invariance, the Higgs self-couplings are of pure gauge nature. This is the reason for the tiny triple-Higgs boson production rates obtained within the framework of the MSSM [Djouadi ('99)].
- In contrast, the general 2HDM accommodates trilinear Higgs couplings with great potential enhancement. For instance we have

$$C(H^{\pm}H^{\mp}H^0) = \frac{-ie\cos(\beta-\alpha)}{M_W\sin\theta_W\sin2\beta}\left[(M^2_{H^{\pm}}-M^2_{A^0}+\frac{M^2_{H^0}}{2})\sin2\beta-(M^2_{H^0}-M^2_{A^0})\tan(\beta-\alpha)\right]$$

$$C_{MSSM}(H^{\pm}H^{\mp}H^0) = \frac{-ieM_W}{\sin\theta_W}\left[\cos(\beta-\alpha)-\frac{\cos2\beta\,\cos(\alpha+\beta)}{2\cos^2\theta_W}\right]$$

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Constrains

- Keep the theory within a perturbative regime: $0.1 \lesssim \tan\beta \lesssim 60$
- Maintain the (approximate) $SU(2)$ custodial symmetry: $\rho = \rho_0 + \delta\rho$ where $\rho_0 = M_W^2/M_Z^2 \cos^2\theta_W = 1$. $|\delta\rho_{2HDM}| \leq 10^{-3}$ from experimental constrain.

  Since for $M_{A^0} \to M_{H^\pm}$ then $\delta\rho_{2HDM} \to 0$, if $M_{A^0} \sim M_{H^\pm}$ $\delta\rho_{2HDM}$ can be kept within bounds.
- Restrictions coming from FCNC radiative B decays $\mathcal{B}(b \to s\gamma) \simeq 3 \times 10^{-4}$, implies a bound of $M_{H^\pm} > 350\ GeV$ for $\tan\beta \geq 1$. (This bound does not apply to type-I models.)
- Unitarity bounds: we bound the size of the trilinear Higgs boson couplings by the value of their single SM counterpart at the scale of $1\ TeV$.

$$|C(HHH)| \leq \left|\lambda_{HHH}^{(SM)}(M_H = 1\ TeV)\right| = \left.\frac{3\, e\, M_H^2}{2\sin\theta_W M_W}\right|_{M_H = 1\ TeV}.$$

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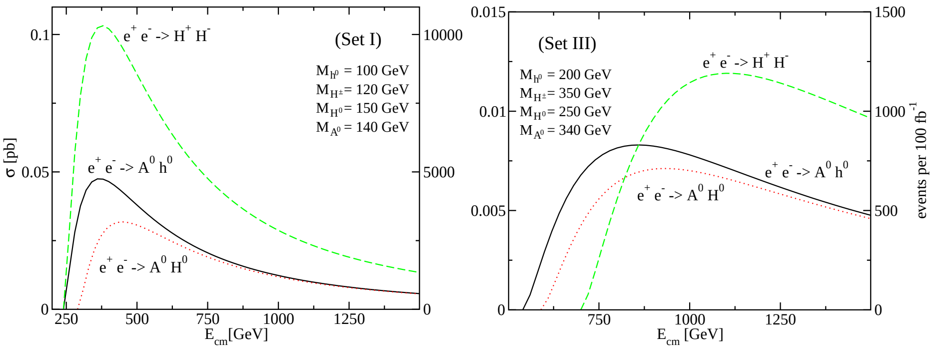
Numerical Analysis: 2HDM 2H production



(a) light masses (b) heavy masses

Figure: Total cross section σ (in *pb*) and number of events per 100 fb^{-1} as a function of $E_{cm} = \sqrt{s}$ for the 2HDM tree-level Higgs boson pair production channels: a) *light* Higgs bosons and b) *heavy* Higgs bosons.

MSSM 2H production

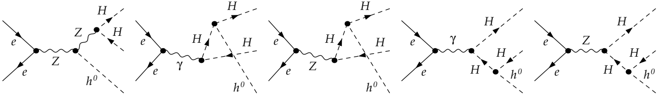
	σ_{max} ($\sqrt{s} = 1$ TeV)	M_{A^0} (GeV)	$\tan\beta$
$e^+e^- \to A^0h^0$	0.013	100	60
$e^+e^- \to A^0H^0$	0.012	130	60
$e^+e^- \to H^+H^-$	0.028	100	5.5

Table: Maximum cross sections (in pb) for the 2H production channels within the MSSM at $\sqrt{s} = 1$ TeV.

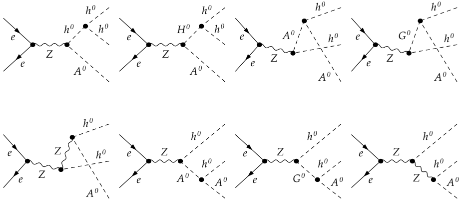
$M_{SUSY}(GeV)$	1000
$\mu(GeV)$	200
$A_t(GeV)$	1000
$A_b(GeV)$	1000
$A_\tau(GeV)$	1000

Table: Choice of parameters used for the computation of 2H and 3H production in the MSSM.

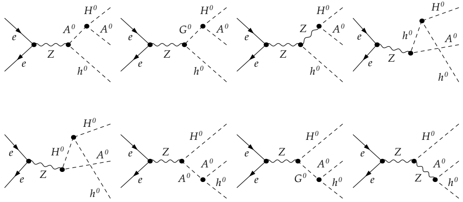
Triple Higgs production



$$e^+e^- \to H^+H^-h^0$$



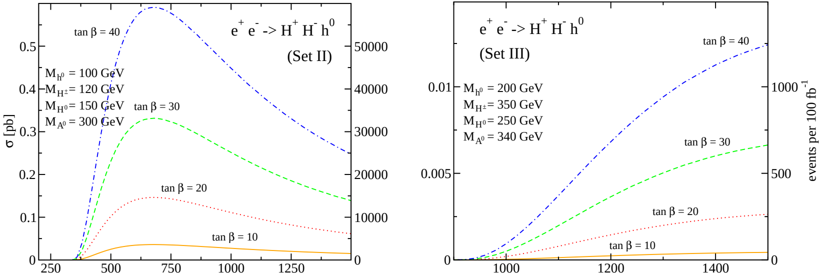
$$e^+e^- \to h^0h^0A^0$$



$$e^+e^- \to H^0A^0h^0$$

Figure: Tree-level Feynman diagrams corresponding to three of the triple Higgs boson production processes. The other four processes proceed through similar collections of diagrams.

2HDM 3H production: $e^+e^- \to H^+H^-h^0$



(a) light masses (b) heavy masses

Figure: Total cross section σ(pb) and number of events per 100 fb^{-1} for the triple Higgs boson production processes $e^+e^- \to H^+H^-h^0$ in the general 2HDM as a function of $\sqrt{s}$ and for different values of $\tan\beta$.

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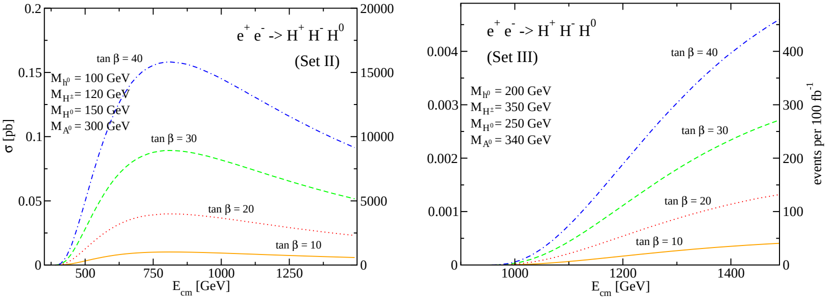

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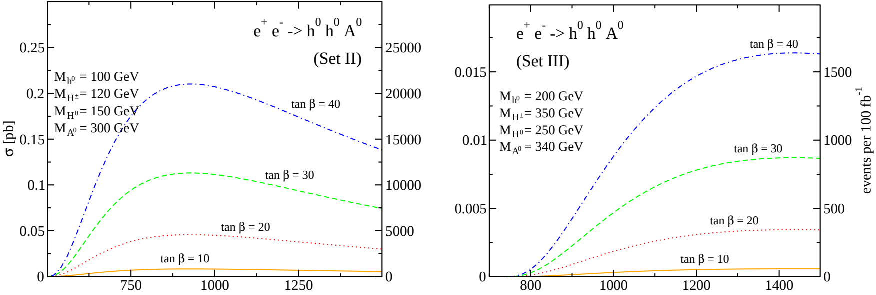

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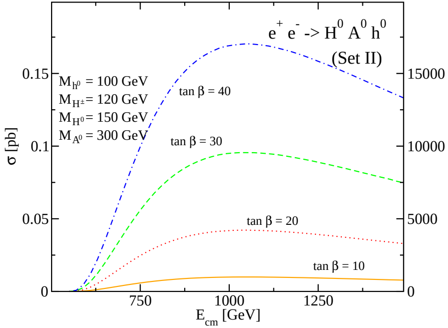

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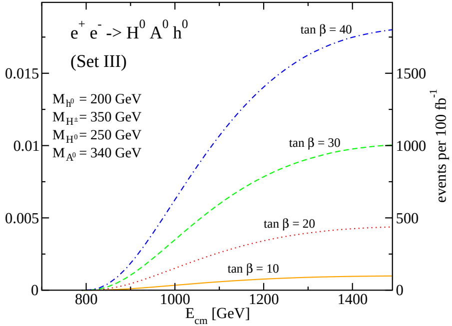

(b) heavy masses

Figure: Total cross section σ(pb) and number of events per 100 fb^{-1} for the triple Higgs boson production processes $e^+e^- \to H^0 A^0 h^0$ in the general 2HDM as a function of $\sqrt{s}$ and for different values of $\tan\beta$.

MSSM Triple Higgs production

	$\sigma_{max}\,(1\ TeV)$	$\sigma_{max}\,(1.4\ TeV)$	$M_{A^0}\ (GeV)$	$\tan\beta$
$e^+e^- \to H^+H^-h^0$	5.6×10^{-6}	3.6×10^{-6}	135	3
$e^+e^- \to H^+H^-H^0$	1.5×10^{-6}	9.1×10^{-7}	100	30
$e^+e^- \to h^0h^0A^0$	1.2×10^{-3}	7.3×10^{-4}	200	2.5
$e^+e^- \to H^0A^0h^0$	2.0×10^{-6}	1.4×10^{-6}	100	5.5

Table: Maximum cross-sections (in pb) for the leading 3H processes within the MSSM at two values of the center of mass energy, $\sqrt{s} = 1$ TeV and 1.4 TeV. The maximizing values of M_{A^0} and $\tan\beta$ are also indicated and are (approximately) the same at the two energies. The 3H processes non-included are even more suppressed. Let us notice that the channel $e^+e^- \to h^0h^0A^0$ has an larger cross-section than the others since it can pick up the resonant decay $H^0 \to h^0\,h^0$ whose branching ratio is non-negligible.

Signatures for triple Higgs production

- We have found that the regions of parameter space with the largest possible values of $\tan\beta$ and relatively small α turn out to maximize the 3H cross-sections.
- For type II models (heavier spectrum of Higgs boson masses) this means typical decays into heavy quarks. The alternate Higgs boson decays into gauge bosons are not dominant.
- Therefore these events should manifest mainly in the form of 6 heavy-quark jet final states.

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Conclusions

- We have computed the cross-sections of double and triple Higgs boson final states produced in a linear e^+e^- collider in the general 2HDM and compared with the MSSM.
- Within the 2HDM type I model the 3H cross-sections may reach 0.1 pb ($\tan\beta \gtrsim 20$ or $\tan\beta < 0.1$) and, in certain regions of parameter space they can be pushed up to 1 pb ($e^+e^- \to H^+H^-h^0$).
- For 2HDM type II models, due to the charged Higgs boson mass bound ($M_{H^\pm} \gtrsim 350\ GeV$) the maximum 3H cross-section is 10 times smaller (~ 0.01 pb) (anyway it means 10^3 events per 100 fb^{-1} of $\int \mathcal{L}$).
- In all cases 2HDM cross-sections can be far larger than in the MSSM. For istance the $\sigma(e^+e^- \to H^+H^-h^0)$ in the MSSM is at most of order 10^{-6}pb, i.e. around 10^4 times smaller than in type II Higgs boson models.
- Finally in 2HDM models maximum cross-sections for the 3H processes are comparable or even larger than the maximum cross-sections for the 2H processes, and can be perfectly competitive, if not the dominant, Higgs boson production mechanism at the ILC.

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