The Higgs sector in the CP-violating MSSM at $\mathcal{O}(\alpha_t\alpha_s)$

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Outline

- Higgs sector in the MSSM with CP-phases
- Mass of the lightest Higgs boson
- Higgs mass calculators

Higgs bosons

At Born level: no CP-violation:

- one phase in the Higgs potential: $V_{\text{Higgs}} = \cdots + \epsilon_{ij} |m_3^2| e^{i\varphi_{m_3^2}} H_1^i H_2^j + \ldots$
  elimination via Peccei-Quinn transformation
- phase difference of Higgs doublets:
  vanishes because of minimum condition

Physical mass eigenstates (at Born level):

- 5 Higgs bosons: 3 neutral H^0, h^0, A^0; 2 charged $H^\pm$

Masses of the Higgs bosons:

- not all independent: here: $H^\pm$-mass $M_{H^\pm}$ (and $\tan\beta$) as free parameter
  $\tan\beta = \frac{v_2}{v_1}$: ratio of the Higgs vac. expect. values
- lightest Higgs boson: h^0

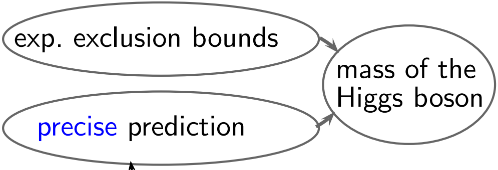
Mass of the lightest Higgs boson

Upper theoretical Born mass bound: $M_{h^0} \leq M_Z = 91$ GeV

with quantum corrections of higher orders: $M_{h^0} \lesssim 135$ GeV

dependent on the MSSM parameters:
particularly on parameter phases

- Before the discovery of the Higgs boson:



$\Rightarrow$ exclusion of parts of the parameter space

i.e. inclusion of higher order corrections
with full phase dependence

Mass of the lightest Higgs boson

Upper theoretical Born mass bound: $M_{h^0} \leq M_Z = 91$ GeV

with quantum corrections of higher orders: $M_{h^0} \lesssim 135$ GeV

dependent on the MSSM parameters:
particularly on parameter phases

- Discovery of the Higgs boson:

accurate measurement

precise prediction

mass of the Higgs boson

$\Rightarrow$ strong bounds on the MSSM parameter, e.g. on $\tan\beta = \frac{v_2}{v_1}$

i.e. inclusion of higher order corrections with full phase dependence

Determination of the Higgs masses

Two-point-function:

$$-i\hat{\Gamma}(p^2) = p^2 - \mathbf{M}(\mathbf{p^2})$$

with the matrix:

$$\mathbf{M}(\mathbf{p^2}) = \begin{pmatrix} M^2_{H^0_{\text{Born}}} - \hat{\Sigma}_{H^0H^0}(p^2) & -\hat{\Sigma}_{H^0h^0}(p^2) & -\hat{\Sigma}_{H^0A^0}(p^2) \\ -\hat{\Sigma}_{H^0h^0}(p^2) & M^2_{h^0_{\text{Born}}} - \hat{\Sigma}_{h^0h^0}(p^2) & -\hat{\Sigma}_{h^0A^0}(p^2) \\ -\hat{\Sigma}_{H^0A^0}(p^2) & -\hat{\Sigma}_{h^0A^0}(p^2) & M^2_{A^0_{\text{Born}}} - \hat{\Sigma}_{A^0A^0}(p^2) \end{pmatrix}$$

Real parameters: $\hat{\Sigma}_{H^0A^0}(p^2) = \hat{\Sigma}_{h^0A^0}(p^2) = 0$

no mixing between CP-even and CP-odd states

Calculate the zeros of the determinant of $\hat{\Gamma}$: $\det[p^2 - \mathbf{M}(\mathbf{p^2})] = 0$

$\Rightarrow$ Higgs masses $M_{h_1} \leq M_{h_2} \leq M_{h_3}$

Determination of the Higgs masses

Calculate the zeros of the determinant of $\hat{\Gamma}$:

$$\det[p^2 - \mathbf{M}(\mathbf{p^2})] = 0$$

or calculate the eigenvalues $\lambda(p^2)$ of $\mathbf{M}(\mathbf{p^2})$:

$$\det[\lambda(p^2) - \mathbf{M}(\mathbf{p^2})] = 0$$

and solve iteratively:

$$p^2 - \lambda(p^2) = 0$$

Other possibility:

First: Inversion of $\hat{\Gamma}$ $\Rightarrow$ propagator matrix

Next: Expansion about the real part of the complex pole of the diagonal propagator $\Rightarrow$ equation that can be solved iteratively

Higgs masses at higher orders (incl. CP-phases)

Status:

- Higgs masses at higher order without CP-phases $\Rightarrow$ in good shape (up to leading 3-loop [S. Martin 07])

Including CP-phases:

- Eff. potential approach, up to two-loop leading-log contributions (sfermionic/fermionic contributions) [Pilaftsis, Wagner], [Demir], [Choi, Drees, Lee], [Carena, Ellis, Pilaftsis, Wagner]
- Gaugino contributions at one-loop [Ibrahim, Nath]
- Effects of imaginary parts at one-loop [Ellis, Lee, Pilaftsis], [Choi, Kalinowski, Liao, Zerwas], [Bernabeu, Binosi, Papavassiliou]

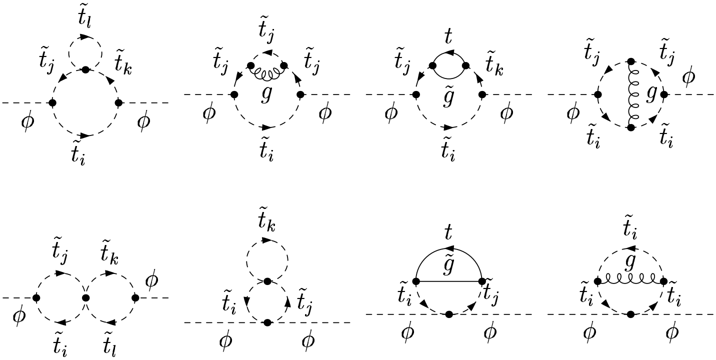
Here: full one-loop + two-loop $\mathcal{O}(\alpha_t\alpha_s)$ $\alpha_t = \frac{\lambda_t^2}{4\pi}$, λ_t = Yukawa coupl.

Renormalized Higgs self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- Terms of order $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters

  contributing self energy diagrams ($\phi = h^0, H^0, A^0$):



Renormalized Higgs self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- Terms of order $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters

  contributing self energy diagrams:

  - generation of diagrams with `FeynArts`

    [Küblbeck, Böhm, Denner],[Hahn]

  - tensor reduction with `TwoCalc` [Weiglein, Scharf, Böhm]
  - extraction of relevant terms:
    - use vanishing external momenta
    - use vanishing electroweak gauge couplings g, g'

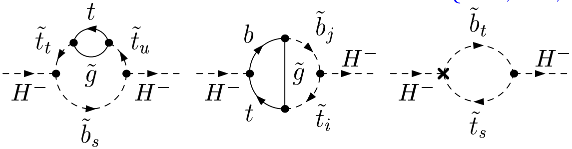
Renormalized Higgs self energies $\hat{\Sigma}$

Parameters of the Higgs sector need to be defined up to $\mathcal{O}(\alpha_t \alpha_s)$:

- define the $H^{\pm}$-, W- as well as the Z-mass as **pole mass**

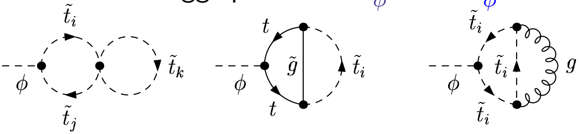
  $\Rightarrow$ directly related to a **physical observable** $\delta M_X^{2\,(i)} = \mathrm{Re}\Sigma_{XX}^{(i)}$,
  $X = \{H^{\pm}, W, Z\}$

calculation of selfenergy diagrams like:



- **no** shift of the minimum of the Higgs potential $\delta t_\phi^{(i)} = -T_\phi^{(i)}$

calculation of tadpole diagrams like ($\phi = h^0, H^0, A^0$):



- $\overline{\mathbf{DR}}$-scheme for field and $\tan\beta$ renormalization

  $\delta \tan^{(i)} \beta = \delta \tan^{(i)} \beta^{\overline{\mathrm{DR}}}$, $\delta Z_{H_i}^{(i)} = \delta Z_{H_i}^{(i)\,\overline{\mathrm{DR}}}$

Renormalized Higgs self energies $\hat{\Sigma}$

Renormalized $h^0 h^0$-self energy at one-loop:

$$\hat{\Sigma}^{(1)}_{h^0 h^0}(p^2) = \Sigma^{(1)}_{h^0 h^0}(p^2) + \delta Z^{(1)}_{h^0 h^0}(p^2 - M^2_{h^0_{\text{Born}}}) - \delta M^{(1)}_{h^0}$$

with the mass counterterm:

$$\begin{aligned}\delta M^{(1)}_{h^0} = {} & c^2_{\alpha-\beta}\left(\delta M^2_{H^\pm}{}^{(1)} - \delta M^2_W{}^{(1)}\right) + s^2_{\alpha+\beta}\delta M^2_Z{}^{(1)} \\ & + \left[\left(M^2_{H^\pm} - M^2_W\right) s_{2(\alpha-\beta)} + M^2_Z s_{2(\alpha+\beta)}\right] c^2_\beta \, \delta \tan^{(1)} \beta \\ & + \frac{e\, s_{\alpha-\beta}}{4 M_W s_W}\left[\left(3 + c_{2(\alpha-\beta)}\right)\delta t^{(1)}_{h^0} + s_{2(\alpha-\beta)}\delta t^{(1)}_{H^0}\right]\end{aligned}$$

Renormalized $h^0 A^0$-mixing at two-loop:

$$\hat{\Sigma}^{(2)}_{h^0 A^0}(p^2) = \Sigma^{(2)}_{h^0 A^0}(p^2) - \delta M^{(1)}_{h^0 A^0} \quad \text{with} \quad \delta M^{(2)}_{h^0 A^0} = \frac{e\, s_{\alpha-\beta}}{2 M_W s_W}\delta t^{(2)}_{A^0}$$

Renormalized Higgs self energies $\hat{\Sigma}$

Parameters of the top (bottom) sector defined at one-loop:

- top quark mass and top squark masses on-shell
- $\tilde{b}_1$-mass ($= \tilde{b}_L$-mass) determined by $SU(2)$-relation ($m_b = 0$):

  $$m_{\tilde{b}_1} = m_{\tilde{b}_1}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})$$

- generalization of the mixing angle condition:

  $$\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\text{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_2}) = 0$$

  $$\Rightarrow \quad (\delta\theta_{\tilde{t}} + i \sin\theta_{\tilde{t}} \cos\theta_{\tilde{t}} \delta\varphi_{\tilde{t}}) e^{i\varphi_{\tilde{t}}} = \frac{\widetilde{\text{Re}}\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\text{Re}}\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_2})}{2(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})}$$

- $A_t = |A_t| e^{i\varphi_{A_t}}$ is then determined (A_t: trilinear coupling):

  $$A_t = A_t(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})$$

Phases in other sectors

- Sfermion sector:
  - phase φ_{A_f} of the trilinear coupling A_f
- Higgsino sector:
  - phase of μ (small), μ: Higgsino mass parameter

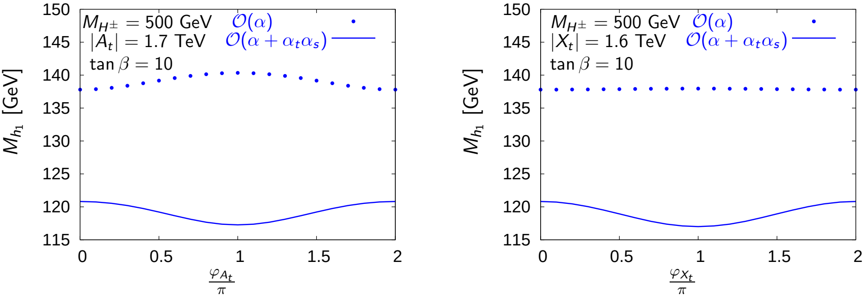
    constraints from measurements of electr. dipole moments
- Gaugino sector:
  - phases of the gaugino mass parameters M_1, M_2, M_3
    - — one phase can be eliminated (R-Transformation), often φ_{M_2}
    - — phase φ_{M_3} is the phase of the gluino mass parameter

      $\Rightarrow$ enters into the Higgs sector at two-loop level

soft breaking parameter

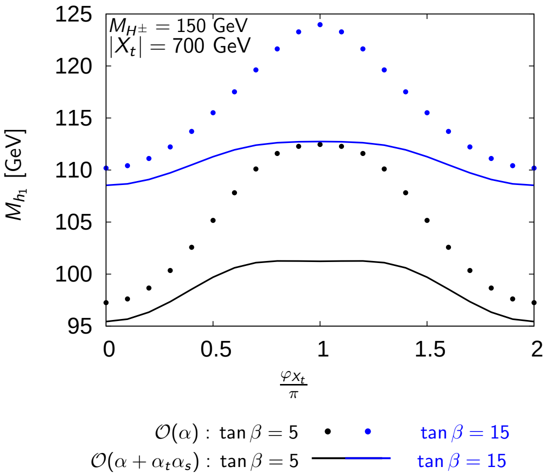
Results: φ_{A_t}- versus φ_{X_t}-dependence (large $M_{H^\pm}$)

size of the squark mixing:
$X_t := A_t - \mu^* \cot\beta$



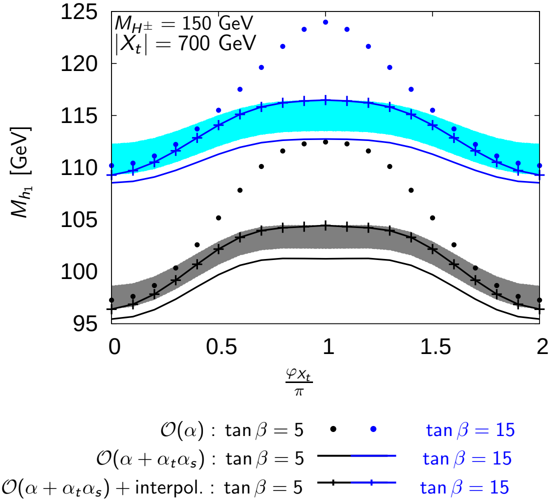
- The qualitative behaviour of M_{h_1} can change with the inclusion of quantum corrections of $\mathcal{O}(\alpha_t\alpha_s)$.
- Quantum corrections tend to be smaller for constant absolute value of of the squark mixing, $|X_t| = \text{const}$.

Results: φ_{X_t}-dependence (small $M_{H^\pm}$)



- Higgs mass M_{h_1} does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- One-loop corrections are more sensitive to φ_{X_t} for small $M_{H^\pm}$.

Results: φ_{X_t}-dependence (small $M_{H^\pm}$)



Bands: Estimate of the size of the corrections of $\mathcal{O}(\alpha_b\alpha_s + \alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ [Slavich et al.], FeynHiggs

Interpolation: Size of above corrections known for the MSSM with **real** parameters: Evaluate for $\varphi_{X_t} = 0$ and $\varphi_{X_t} = \pi$ and interpolate

- Higgs mass M_{h_1} does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- One-loop corrections are more sensitive to φ_{X_t} for small $M_{H^\pm}$.

FeynHiggs and CPsuperH

Two programs: input: MSSM parameters
output: Higgs masses

Features (not complete):

FeynHiggs:	CPsuperH:
full one-loop corr.	gaugino/higgsino leading log one-loop contr.
two-loop corr.:	fermionic/sfermionic contr.:
$\mathcal{O}(\alpha_t \alpha_s)$ complex	up to two-loop leading log
$\mathcal{O}(\alpha_b \alpha_s + \alpha^2_{\{t,b\}})$ real	$\mathcal{O}(\alpha_{\{t,b\}} \alpha_s + \alpha^2_{\{t,b\}})$ complex

Input:

on-shell squark parameters	$\overline{\text{DR}}$ squark parameters

FeynHiggs and CPsuperH: Comparison

Input:

on-shell squark parameters	$\overline{\text{DR}}$ squark parameters
FeynHiggs	CPsuperH

Transformation from one scheme to another necessary:

Use relation:

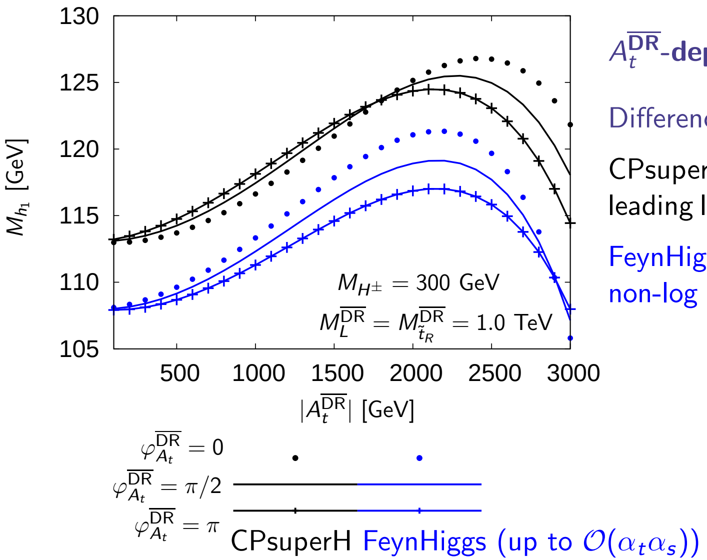
$$X^{\overline{\text{DR}}} + \delta X^{\overline{\text{DR}}} = X^{\text{OS}} + \delta X^{\text{OS}}$$

with $X = \{A_t, M_L, M_{\tilde{t}_R}\}$: squark soft breaking parameter

δX^{OS} is then determined by the on-shell counterterms:

$$\delta X^{\text{OS}} = \delta X^{\text{OS}}(\delta m^{\text{OS}}_{\tilde{t}_1}, \delta m^{\text{OS}}_{\tilde{t}_2}, \delta m^{\text{OS}}_t, \delta\theta^{\text{OS}}_{\tilde{t}}, \delta\varphi^{\text{OS}}_{\tilde{t}})$$

FeynHiggs and CPsuperH: First Results:



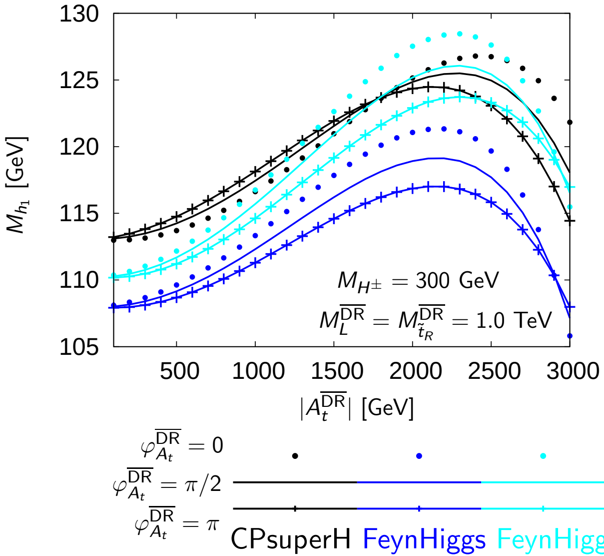
$A_t^{\overline{\rm DR}}$**-dependence:**

Differences:

CPsuperH:
leading log $\mathcal{O}(\alpha_t^2)$ terms

FeynHiggs:
non-log $\mathcal{O}(\alpha_t\alpha_s)$ terms

FeynHiggs and CPsuperH: First Results:



$A_t^{\overline{\rm DR}}$-dependence:

Differences:

CPsuperH:
leading log $\mathcal{O}(\alpha_t^2)$ terms

FeynHiggs:
non-log $\mathcal{O}(\alpha_t\alpha_s)$ terms

FeynHiggs:
non-log $\mathcal{O}(\alpha_t\alpha_s)$ terms
$+$ interpolation of
$\mathcal{O}(\alpha_t^2)$ terms

Summary

- At **Born** level: **no** CP-violation in the Higgs sector
- **Quantum corrections**:
  - can **induce** CP-violation.
  - have to be taken into account for the **prediction of Higgs masses**.
- The dominant two-loop contributions $\mathcal{O}(\alpha_t \alpha_s)$ with complete phase dependence are **included** into FeynHiggs.
- First results of the comparison between FeynHiggs and CPsuperH

The Higgs potential in the MSSM

Higgs potential:

gauge couplings

$$V_{\text{Higgs}} = \frac{g^2 + g'^2}{8}(H_1^+ H_1 - H_2^+ H_2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2$$

$$+ |\mu|^2 (H_1^+ H_1 + H_2^+ H_2) \qquad \mu\text{: coupl. betw. Higgs superfields}$$

$$+ (m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2) \qquad \text{soft breaking terms}$$

$$+ (\epsilon_{ij} |m_3^2| e^{i\varphi_{m_3^2}} H_1^i H_2^j + h.c.)$$

Two Higgs doublets (v_i: Higgs vac. exp. value):

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - i\zeta_1^0) \\ -\phi_1^- \end{pmatrix}, \qquad H_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\zeta_2^0) \end{pmatrix}$$

$\Longrightarrow$ 2 phases

Determination of the Higgs masses

Propagator matrix:

$$\Delta(p^2) = -\left[\hat{\Gamma}(p^2)\right]^{-1}$$

Inversion of $\hat{\Gamma} \Rightarrow$ diagonal propagators ($i = H^0,\ h^0,\ A^0$):

$$\Delta_{ii}(p^2) = \frac{i}{p^2 - M^2_{i_{\text{Born}}} + \hat{\Sigma}^{\text{eff}}_{ii}(p^2)}$$

with the self energy term:

$$\hat{\Sigma}^{\text{eff}}_{ii}(p^2) = \hat{\Sigma}_{ii}(p^2) - i\,\text{funct}(\hat{\Gamma}_{jk}(p^2)) \qquad i,\ j,\ k\ = 1,\ 2,\ 3.$$

Determination of the Higgs masses

Poles $\mathcal{M}_i^2$ of the diag. propagators:

$$\mathcal{M}_i^2 - M_{i_{\text{Born}}}^2 + \hat{\Sigma}_{ii}^{\text{eff}}(\mathcal{M}_i^2) = 0$$

Complex pole with phys. mass M and width Γ :

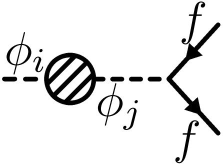
$$\mathcal{M}^2 = M^2 - i M \Gamma$$

Expansion around M^2 up to first order in Γ:

$$M_i^2 - M_{i\text{Born}}^2 + \text{Re}\hat{\Sigma}_{ii}^{\text{eff}}(M_i^2) + \frac{\text{Im}\hat{\Sigma}_{ii}^{\text{eff}}(M_i^2)\left(\text{Im}\hat{\Sigma}_{ii}^{\text{eff}}\right)'(M_i^2)}{1 + \left(\text{Re}\hat{\Sigma}_{ii}^{\text{eff}}\right)'(M_i^2)} = 0$$

Iterative solution $\Rightarrow$ Higgs masses M_i^2

Amplitudes with external Higgs bosons



Mixing between the Higgs bosons:

($\overline{\text{DR}}$/on-shell scheme) $\phi_{\{i,j\}} = H^0,\ h^0,\ A^0$

Finite wave function normalization factors needed:

$$\sqrt{\hat{Z}_i}(\Gamma_i + \hat{Z}_{ij}\Gamma_j + \hat{Z}_{ik}\Gamma_k)$$

$\hat{Z}_i$ ensures that residuum is set to 1:

$$\hat{Z}_i = \frac{1}{1 + \left(\text{Re}\hat{\Sigma}^{\text{eff}}_{ii}\right)'(M_i^2)}$$

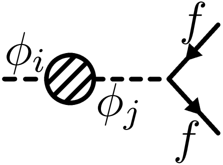
$\hat{Z}_{ij}$ describes transition $i \to j$:

$$\hat{Z}_{ij} = \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)}\bigg|_{p^2=M_i^2}$$

Amplitudes with external Higgs bosons

Mixing between the Higgs bosons:

($\overline{\mathrm{DR}}$/on-shell scheme) $\phi_{\{i,j\}} = H^0,\, h^0,\, A^0$



Finite wave function normalization factors needed:

$$\sqrt{\hat{Z}_i}(\Gamma_i \,+\, \hat{Z}_{ij}\Gamma_j \,+\, \hat{Z}_{ik}\Gamma_k)$$

Mixing matrix ($\hat{Z}_{ii} = 1$):

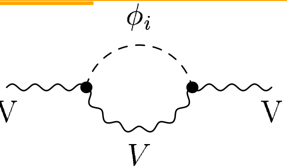
$$\tilde{\mathbf{Z}}_{ij} = \sqrt{\hat{Z}_i}\hat{Z}_{ij}$$

Vertex with external Higgs boson:

$$\tilde{\mathbf{Z}}_{ii}\Gamma_i \,+\, \tilde{\mathbf{Z}}_{ij}\Gamma_j \,+\, \tilde{\mathbf{Z}}_{ik}\Gamma_k$$

Amplitudes with internal Higgs bosons

Diagrams with internal Higgs bosons enter precision observables (W-mass, ...):



- Calculation with Born states $\phi_i = H^0,\ h^0,\ A^0$: no problem
- Calculation with $\phi_i = h_1,\ h_2,\ h_3 \Rightarrow$ Inclusion of higher order effects:

One possibility: Use of effective couplings:

Consider $\tilde{\mathbf{Z}}_{ij}$ as mixing matrix:

Problem: $\tilde{\mathbf{Z}}_{ij}$ is a non-unitary matrix (no rotation matrix)

Further approximations:

effective potential approach: $\tilde{\mathbf{Z}}(\hat{\Sigma}(p^2)) \rightarrow \tilde{\mathbf{Z}}(\hat{\Sigma}(0)) = \mathcal{R}$

on-shell approximation: $\tilde{\mathbf{Z}}(\hat{\Sigma}(p^2)) \rightarrow \tilde{\mathbf{Z}}(\mathrm{Re}\hat{\Sigma}(p^2_{\mathrm{OS}})) = \mathcal{U}$ $\hat{\Sigma}_{ii}(p^2_{\mathrm{os}} = M^2_{i_{\mathrm{Born}}})$, $\hat{\Sigma}_{ij}(p^2_{\mathrm{os}} = (M^2_{i_{\mathrm{Born}}} + M^2_{j_{\mathrm{Born}}})/2)$

Couplings

One example:

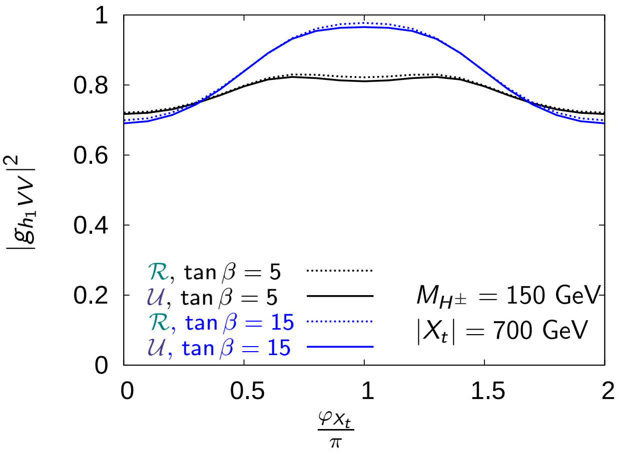
Coupling of two gauge bosons ($V = W,\ Z$) and one Higgs boson:

$$g_{h_i VV} = [U_{i1}\cos(\beta-\alpha) + U_{i2}\sin(\beta-\alpha)]\, g_{H_{\mathrm{SM}} VV}$$

standard model coupling ↗

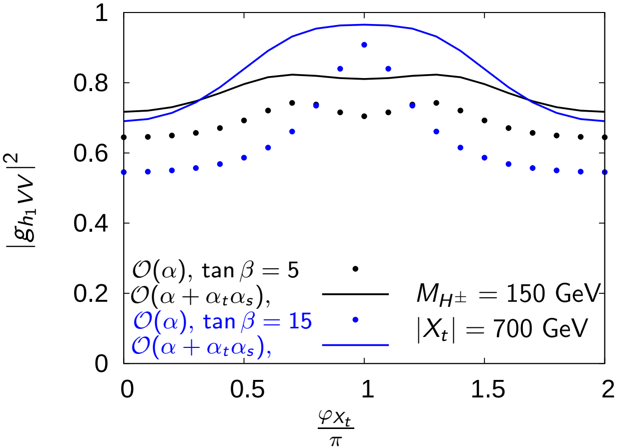
- only CP-even components of the Higgs bosons couple to V
- all three Higgs bosons can have a CP-even component

Results: φ_{X_t}-dependence of couplings



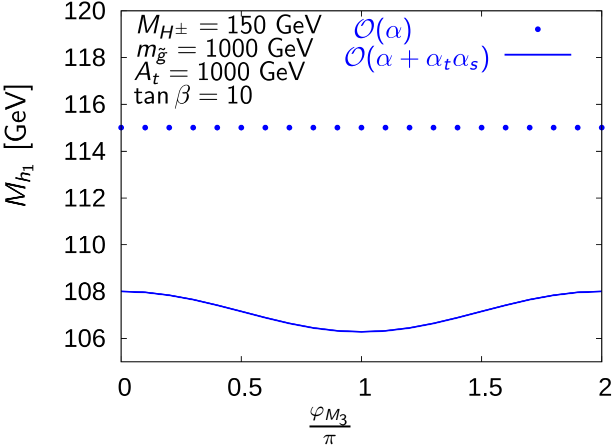
- Here: g_{h_1VV} is normalized to the standard model coupling.
- $|g_{h_1VV}|^2$ does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- $\mathcal{R}_{p^2=0}$ and $\mathcal{U}_{p_{OS}}$ give similar results with only tiny differences.

Results: φ_{X_t}-dependence of couplings



- Here: $g_{h_1 VV}$ is normalized to the standard model coupling.
- $|g_{h_1 VV}|^2$ does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.

Results: φ_{M_3}-dependence



- Quantum corrections of $\mathcal{O}(\alpha_s\alpha_t)$ cause a dependence of M_{h_1} on φ_{M_3}.