α_s^{GUT} *at three loop accuracy*

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Bergische Universität Wuppertal

RADCOR 2007, Florence, Italy

work in collaboration with

D.R.T. Jones, P. Kant, L. Mihaila, M. Steinhauser

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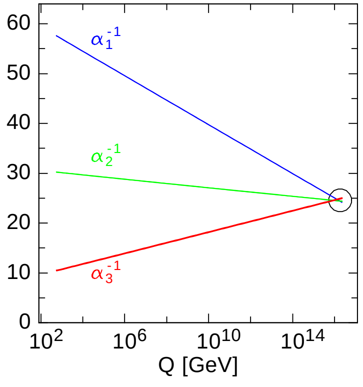

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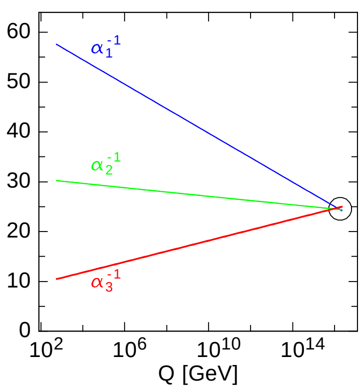
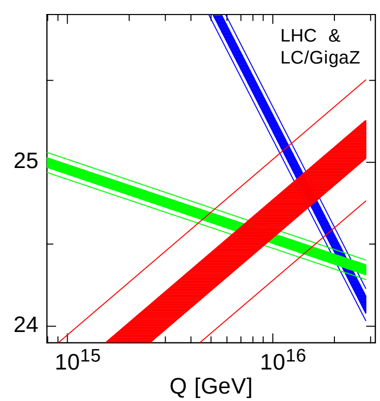


Allanach et al '04



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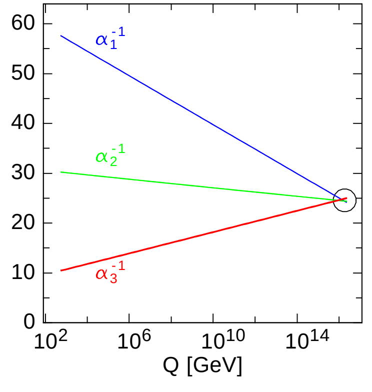

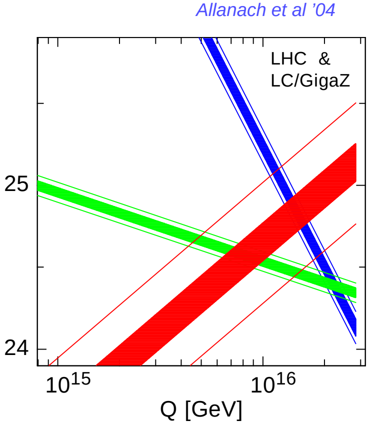

- need methods that preserve the new symmetries

  e.g. Dimensional Regularization breaks SUSY!

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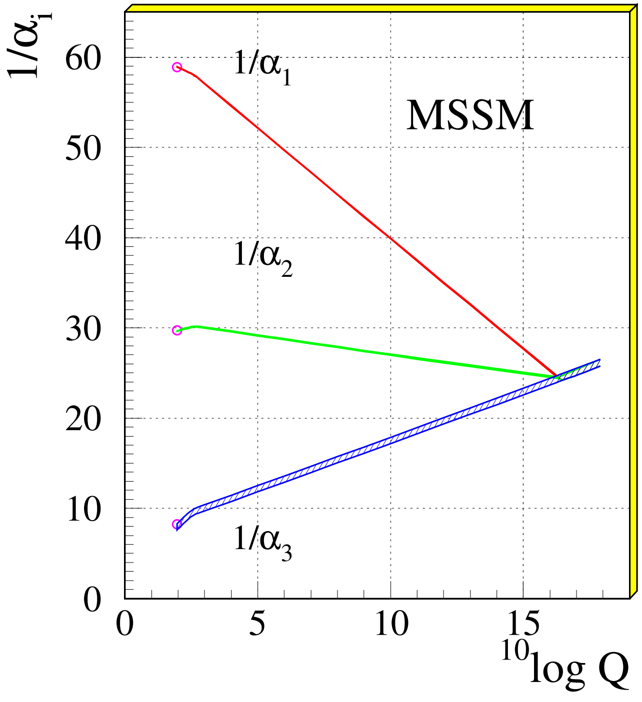

[Amaldi, de Boer, Fürstenau]

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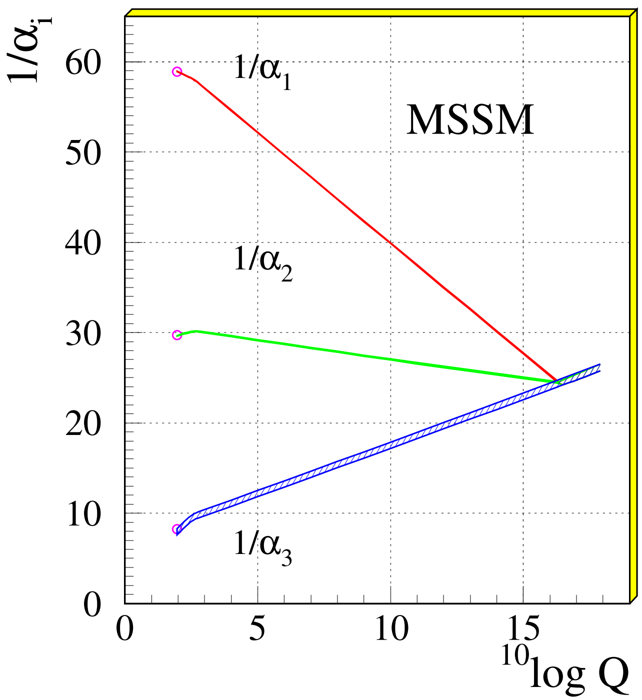

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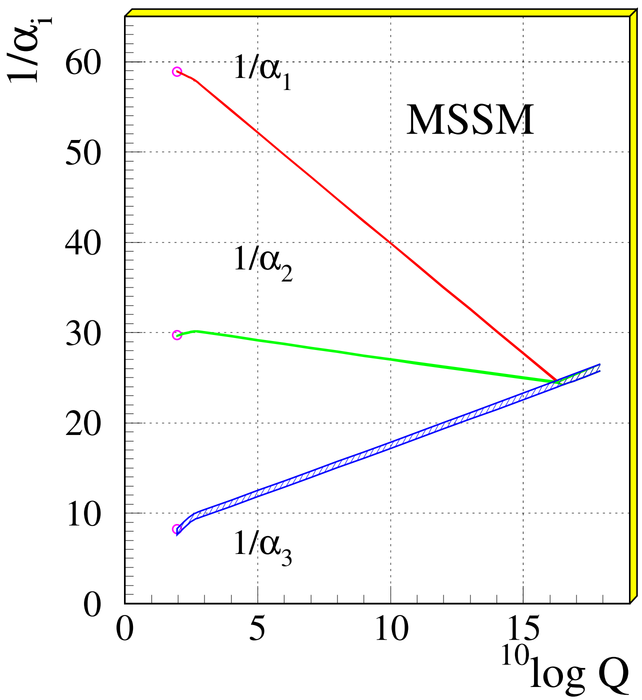

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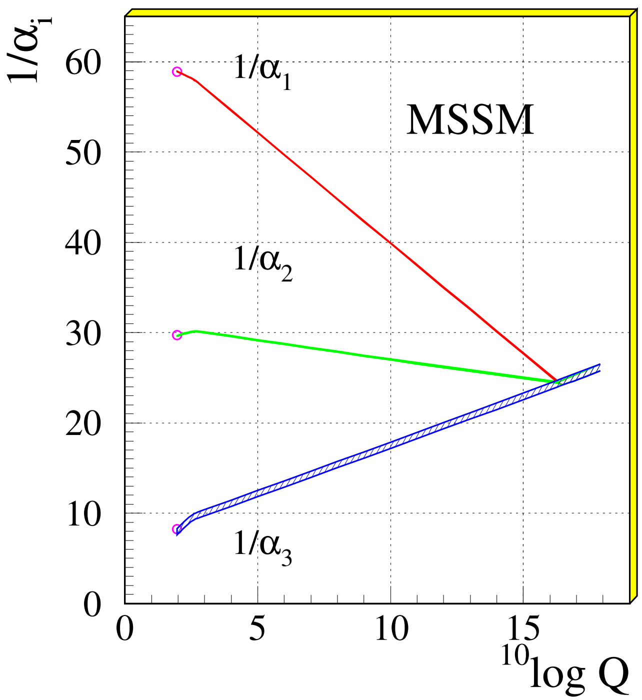

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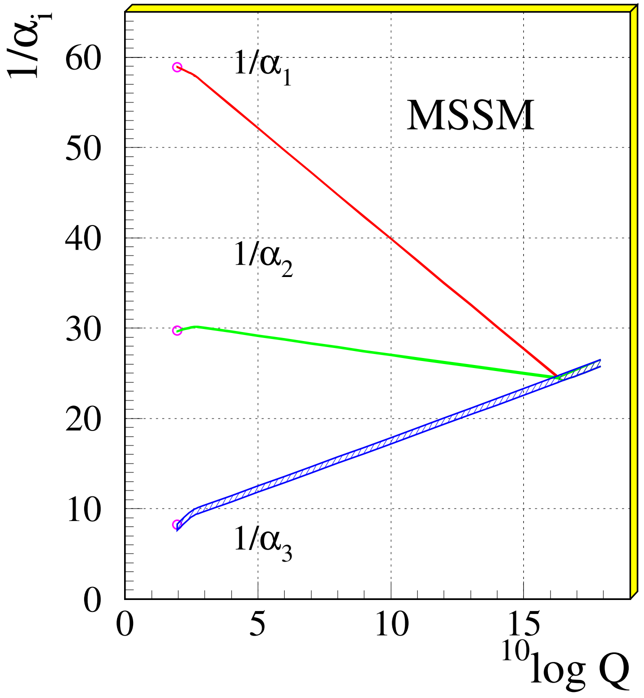

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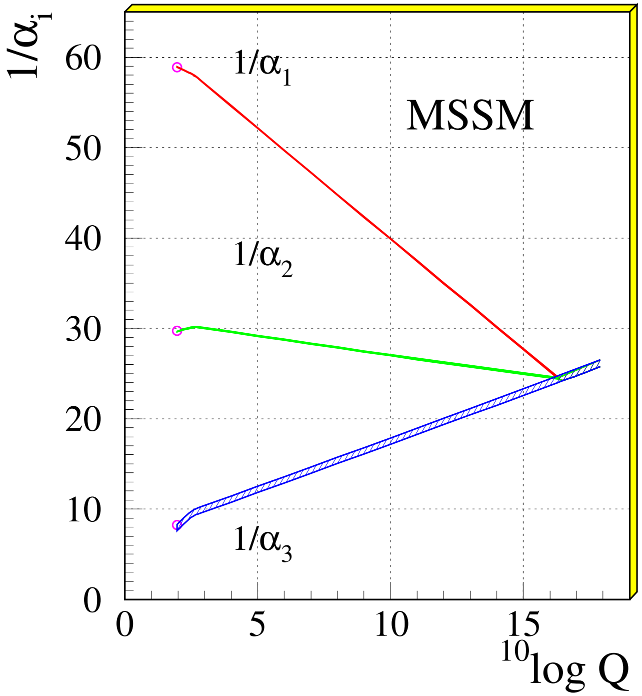

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running:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s(\mu^2) = \beta(\alpha_s)$$

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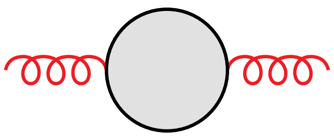
-  — $\beta(\alpha_s)$

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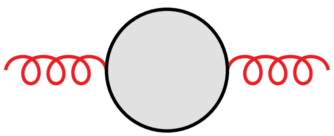
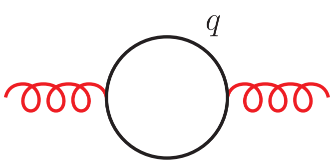
-  — $\beta(\alpha_s)$
- 

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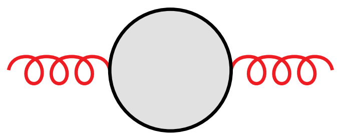
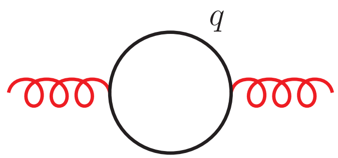
-  — $\beta(\alpha_s)$
- 

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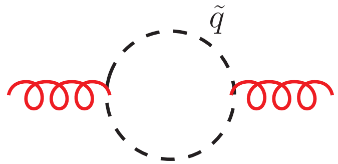
- 

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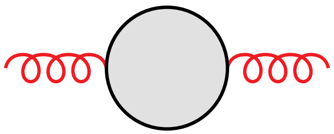
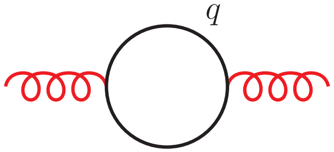
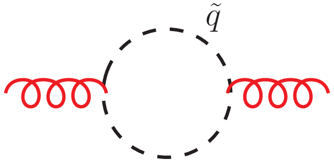
-  — $\beta(\alpha_s)$

-  — $\beta^{\mathrm{SM}}(\alpha_s) = \frac{17}{6}\frac{\alpha_s^2}{\pi} + \ldots \qquad (n_f = 5)$

-  — $\beta^{\mathrm{SUSY}}(\alpha_s) = \frac{3}{4}\frac{\alpha_s^2}{\pi} + \ldots \qquad (n_f = 6)$

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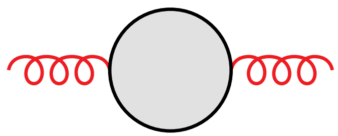

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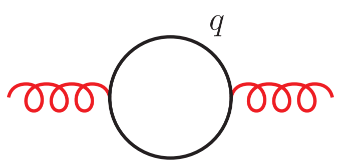

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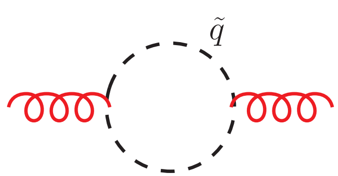

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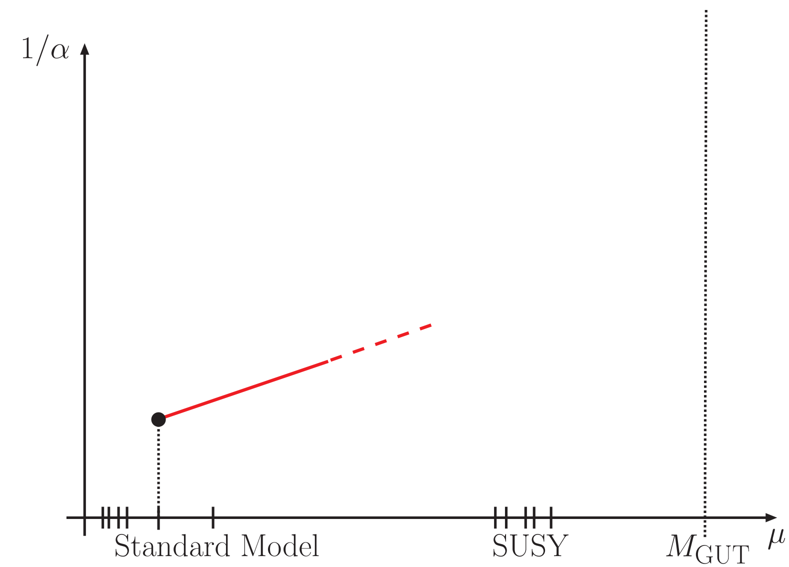

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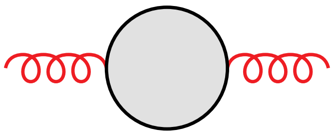
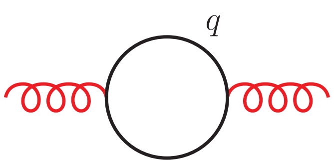
-  — $\beta(\alpha_s)$
- 

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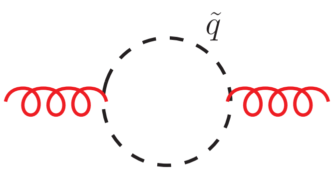
- 

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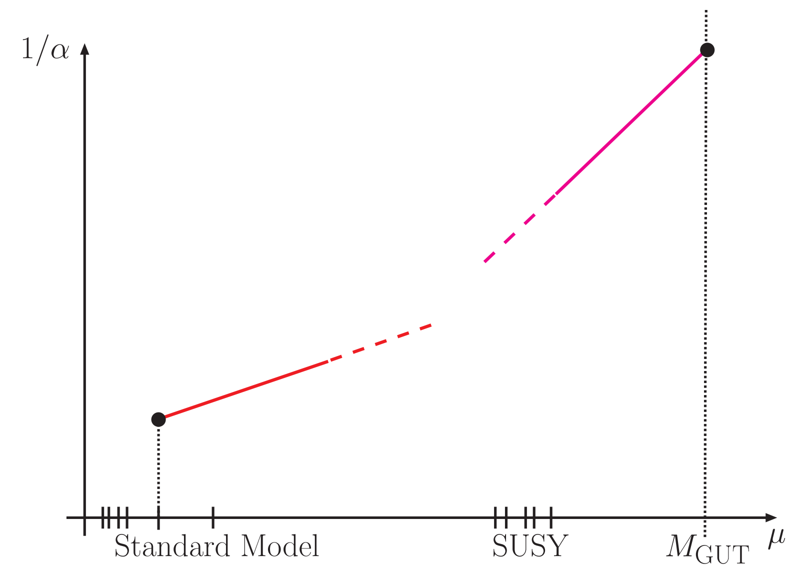

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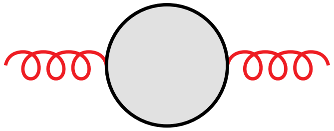
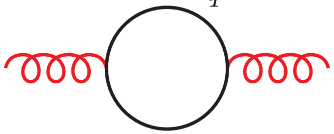
-  — $\beta(\alpha_s)$
- 

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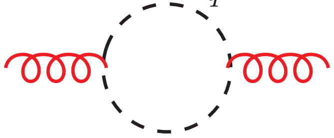
- 

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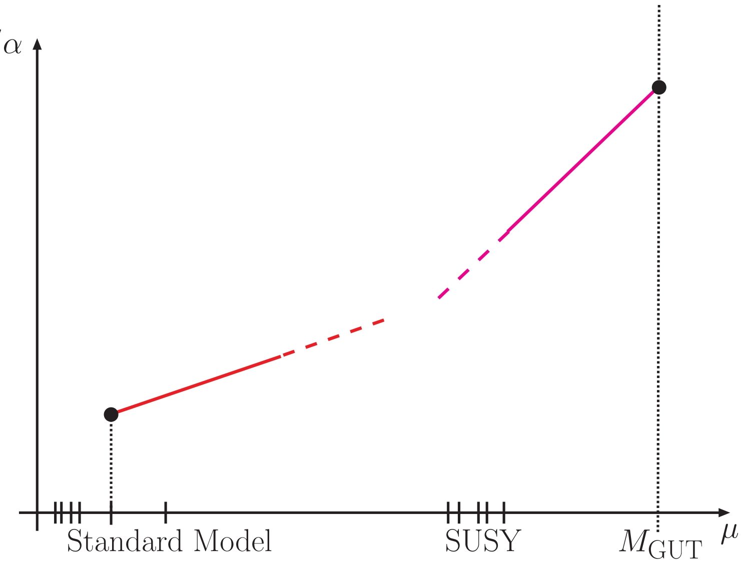

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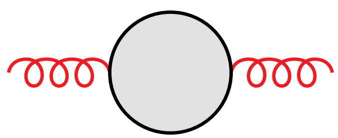
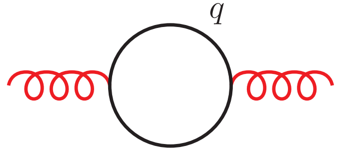
-  — $\beta(\alpha_s)$
- 

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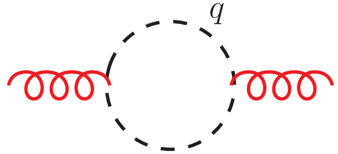
- 

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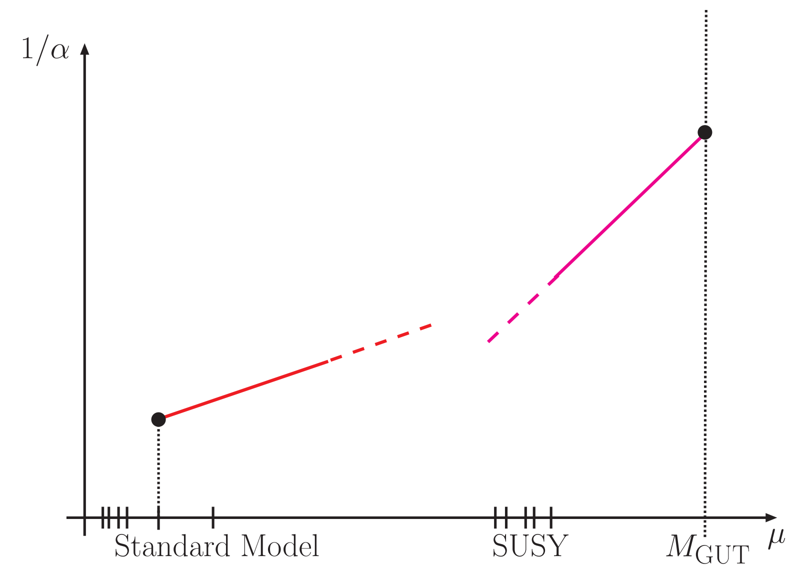

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Decoupling coefficients

$$\mathcal{L}_{\rm SQCD}(\alpha_s^{\rm (full)}, m_q^{\rm (full)}, m_t, M_{\rm SUSY}, A^{\rm (full)}, \tilde{A}, \ldots)$$

$$\updownarrow$$

$$\mathcal{L}_{\rm QCD}(\alpha_s^{(5)}, m_q^{(5)}, A^{(5)}, \ldots)$$

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determined from: $\Gamma^{\rm SQCD}_{\rm light\ fields} = \Gamma^{\rm QCD}_{\rm light\ fields} + \mathcal{O}(\frac{1}{M_{\rm SUSY}}, \frac{1}{m_t})$

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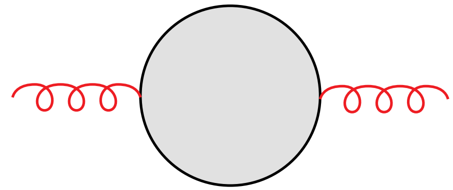

$$\int \mathrm{d}^4x\,\mathrm{e}^{ipx}\langle \mathrm{T} A'(x) A'(0)\rangle_{\mathrm{eff}} = \zeta_A^2 \int \mathrm{d}^4x\,\mathrm{e}^{ipx}\langle \mathrm{T} A(x) A(0)\rangle_{\mathrm{full}} + \mathcal{O}(\frac{p^2}{M})$$

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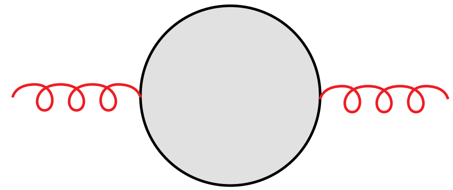

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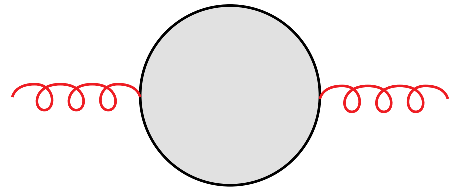

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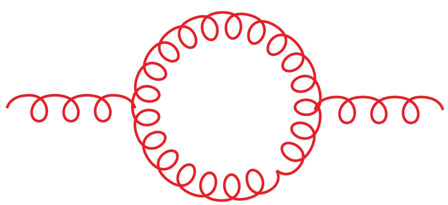

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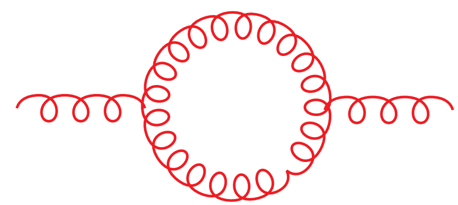

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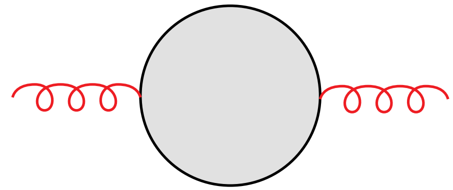

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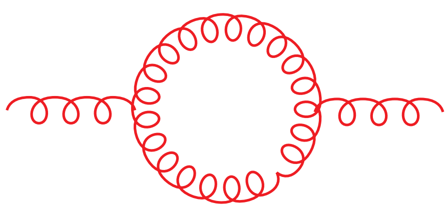
$+$ 

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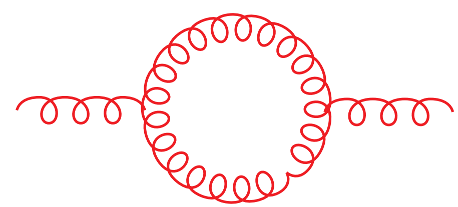
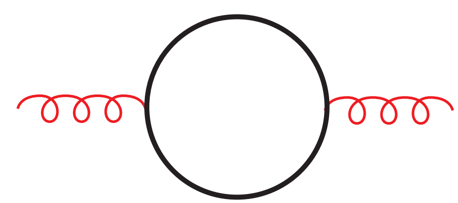
$+$ 

$+$  $)$

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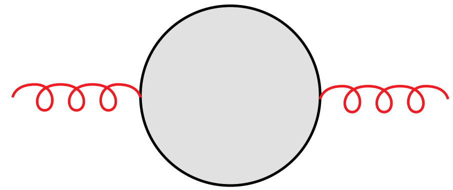

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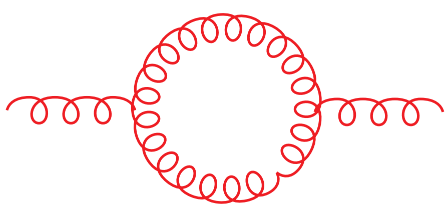
$+$ 

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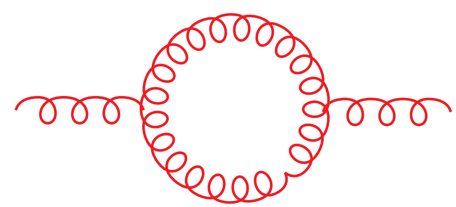
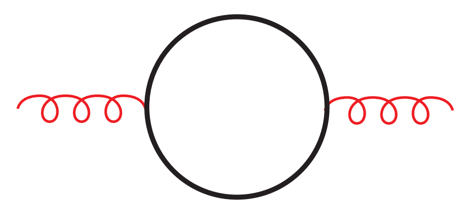
$+$ 

$+$  $)$

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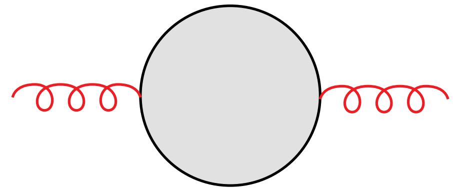

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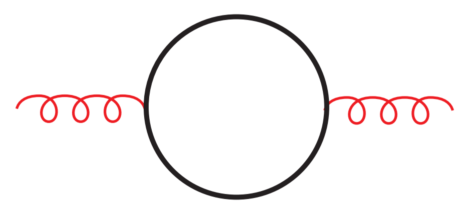
$+$  $)$

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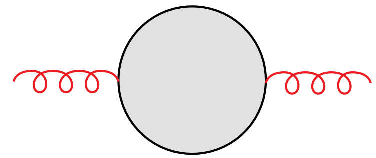

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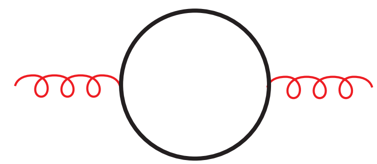

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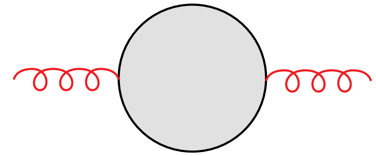

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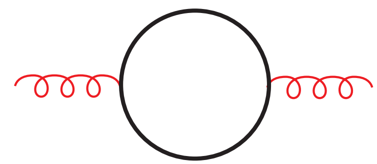


$$\Big|_{p^2=0}$$

calculate using

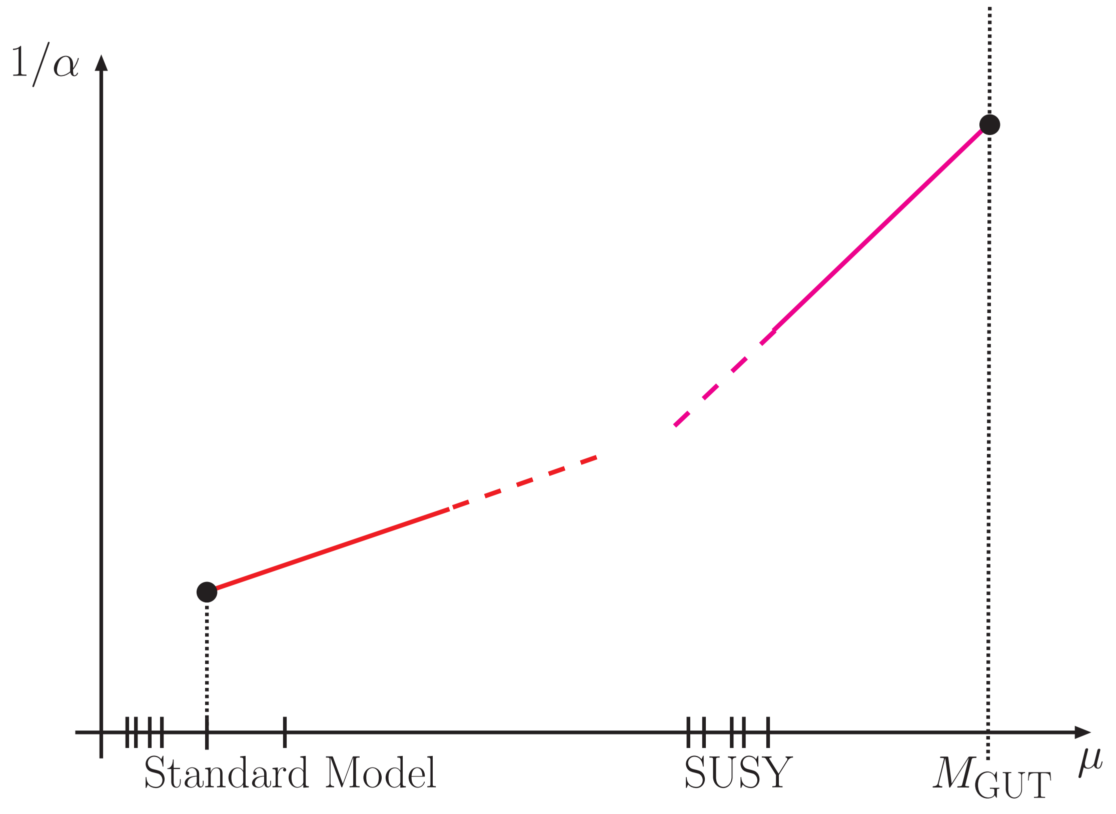
QGRAF, FORM, MINCER, MATAD, EXP, . . .

[Nogueira; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker; R.H.; . . .]

method: see [Chetyrkin, Kniehl, Steinhauser '97]

Matching

$$\alpha_s^{(5)}(\mu) = \alpha_s^{(\text{full})}(\mu) \cdot \zeta_s(\mu, M_{\text{SUSY}}, m_t)$$



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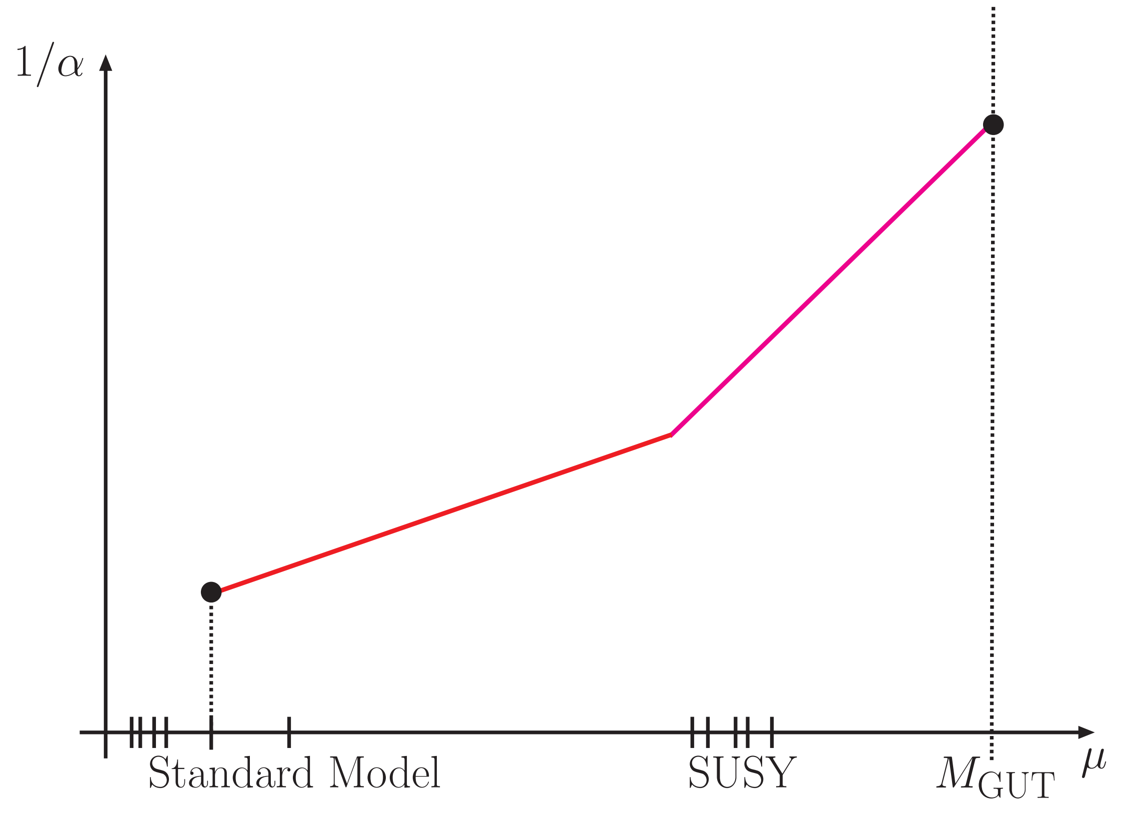

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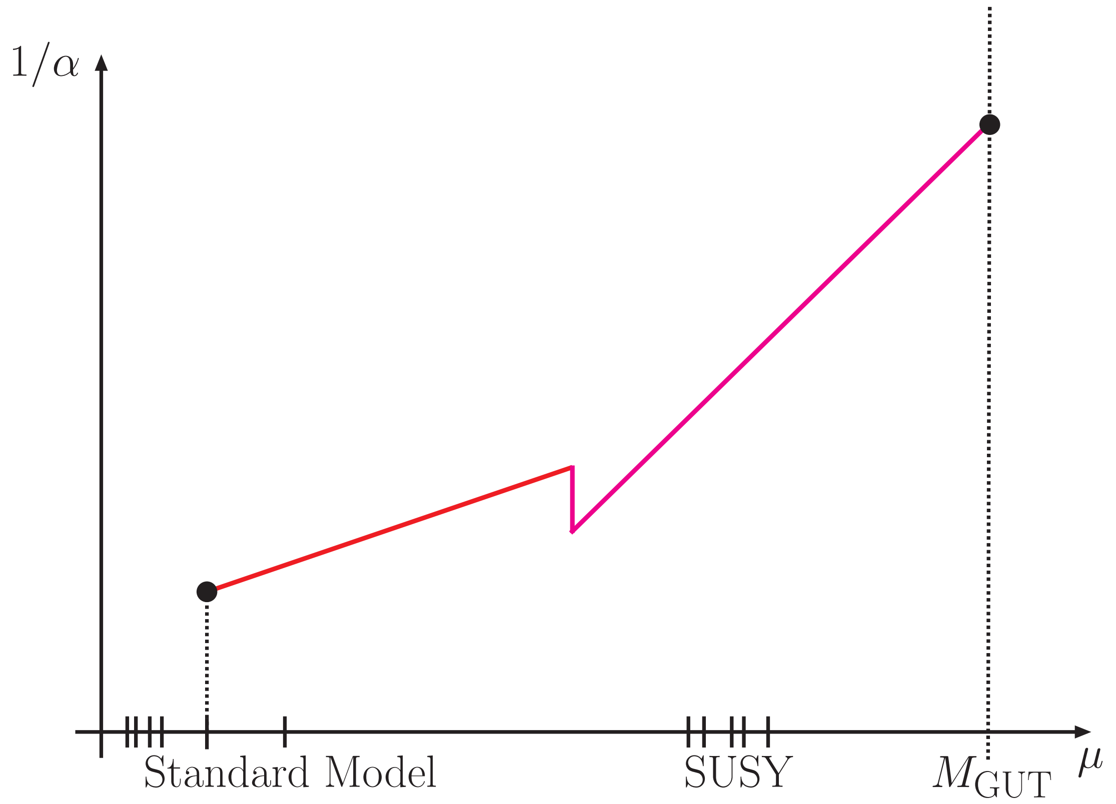

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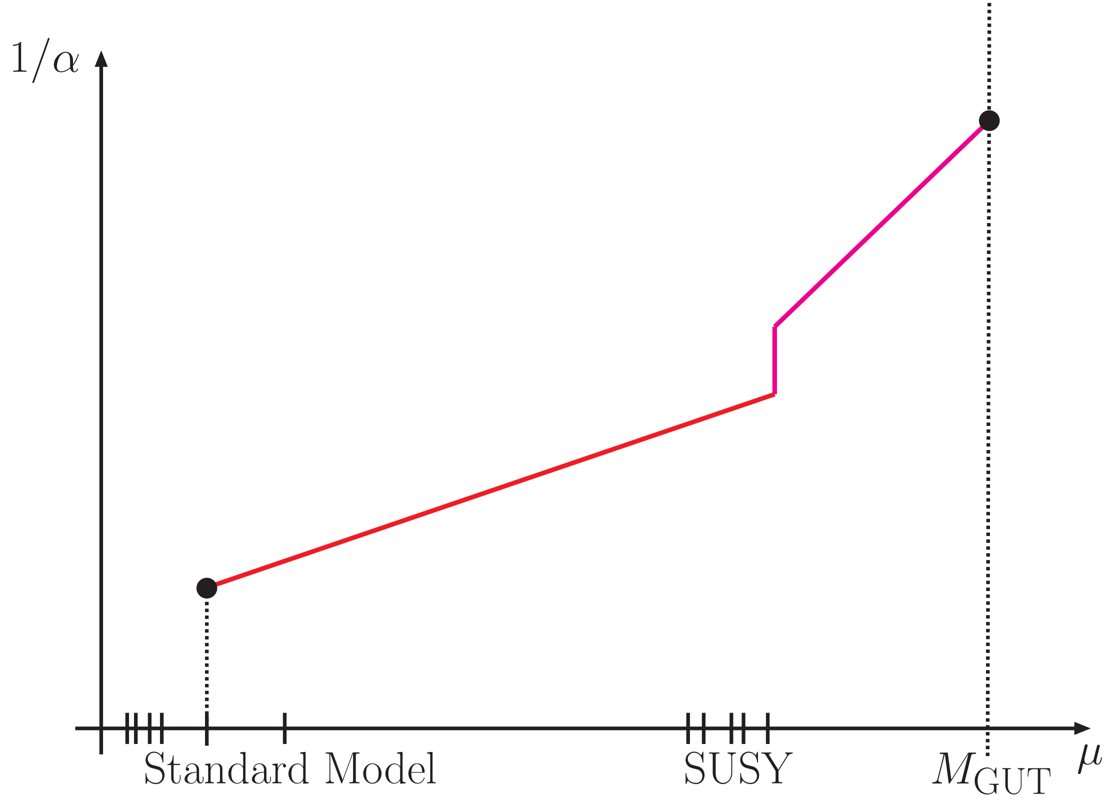

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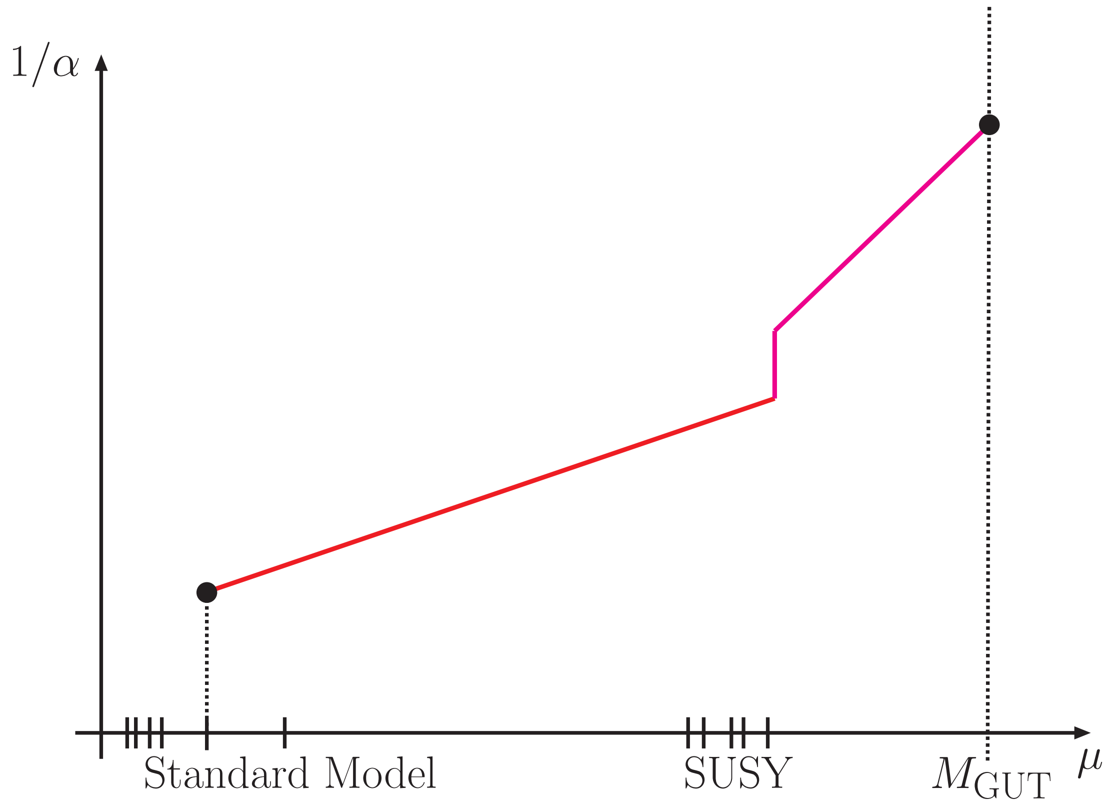

$\zeta_s(\mu, M_{\text{SUSY}}, m_t)$ known to 2 loops [R.H., Mihaila, Steinhauser '06]

$\alpha_s(M_Z) \to \alpha_s(M_{\rm GUT})$

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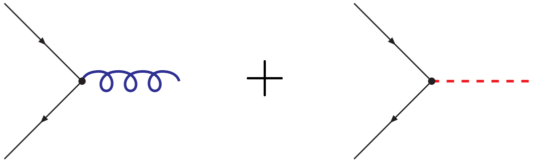

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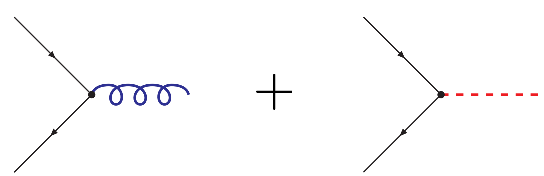

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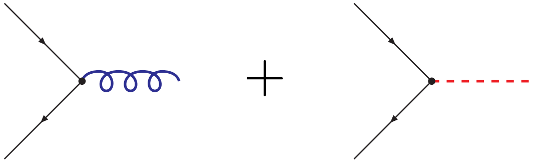

$\rightarrow$ additional Feynman rules for epsilon scalars

$$\{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 2\hat{g}^{\mu\nu}\,, \qquad \{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2\tilde{g}^{\mu\nu}\,, \qquad \{\hat{\gamma}^\mu, \tilde{\gamma}^\nu\} = 0$$

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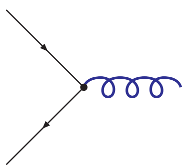
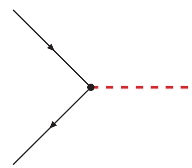
 $\to \alpha_s$  $\to \alpha_e$ “evanescent coupling”

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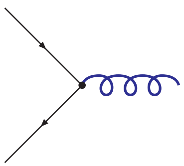
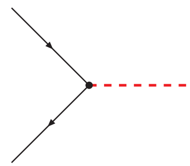
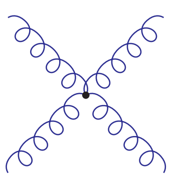
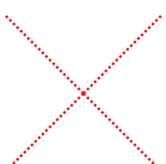
 $\rightarrow \alpha_s$  $\rightarrow \alpha_e$ "evanescent coupling"

even worse:

 $\rightarrow \alpha_s \, f^{abe} f^{cde}$

 $\rightarrow \lambda_1 \, f^{abe} f^{cde}, \quad \lambda_2 \, \delta^{ab}\delta^{cd}, \quad \lambda_3 \, d^{abe} d^{cde}$

$\overline{\text{MS}} - \overline{\text{DR}}$ *conversion*

- value of α_s in *physical scheme* independent of regularization:

$$\alpha_s^{\text{ph}} \equiv z_{\text{ph}}^{\overline{\text{MS}}}\,\alpha_s^{\overline{\text{MS}}} \equiv z_{\text{ph}}^{\overline{\text{DR}}}\,\alpha_s^{\overline{\text{DR}}}$$

- $z_{\text{ph}}^{\overline{\text{MS}}}$ and $z_{\text{ph}}^{\overline{\text{DR}}}$ depend on renormalization point p^2
- momentum dependence drops out in ratio:

$$\Rightarrow \quad \alpha_s^{\overline{\text{DR}}} = \frac{z_{\text{ph}}^{\overline{\text{MS}}}}{z_{\text{ph}}^{\overline{\text{DR}}}}\,\alpha_s^{\overline{\text{MS}}}\,.$$

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$$\alpha_s^{(5),\overline{\text{MS}}} = \alpha_s^{(5),\overline{\text{DR}}} \left[1 - \frac{\alpha_s^{(5),\overline{\text{DR}}}}{4\pi} - \frac{5}{4} \left(\frac{\alpha_s^{(5),\overline{\text{DR}}}}{\pi} \right)^2 + \frac{5}{12} \frac{\alpha_s^{(5),\overline{\text{DR}}}}{\pi} \frac{\alpha_e^{(5)}}{\pi} + \ldots \right]$$

[R.H., Kant, Mihaila, Steinhauser ’06]

even 3-loop: [R.H., Jones, Kant, Mihaila, Steinhauser 06]

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what is $\alpha_e^{(5)}$?

$\overline{\text{MS}} - \overline{\text{DR}}$ *conversion*

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[R.H., Kant, Mihaila, Steinhauser ’06]

even 3-loop: [R.H., Jones, Kant, Mihaila, Steinhauser 06]

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decoupling for α_e:

$$\alpha_e^{(5)} = \zeta_e\, \alpha_e^{(\mathrm{full})} = \zeta_e\, \alpha_s^{(\mathrm{full}),\overline{\mathrm{DR}}}$$

[R.H., Mihaila, Steinhauser '07]

$\alpha_s(M_Z) \to \alpha_s(M_{\rm GUT})$

- Procedure:

$\alpha_s^{(5),\overline{\rm MS}}(M_Z)$

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approximate 2-loop formula [R.H., Mihaila, Steinhauser ’07]:

$$\alpha_s^{({\rm full}),\overline{\rm DR}} \quad \longleftrightarrow \quad \alpha_s^{(5),\overline{\rm MS}}$$

Ready for 3-loop running...

- remark: SPA prescription: [hep-ph/0511344]
  - 1-loop running
  - 1-loop decoupling at M_Z (resummed)
  - 1-loop $\overline{\mathrm{MS}}$ — $\overline{\mathrm{DR}}$ conversion at M_Z (resummed)

resummed: $$\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})} = \frac{\alpha_s^{\overline{\mathrm{MS}},(5)}}{1 - \Delta\alpha_s}$$

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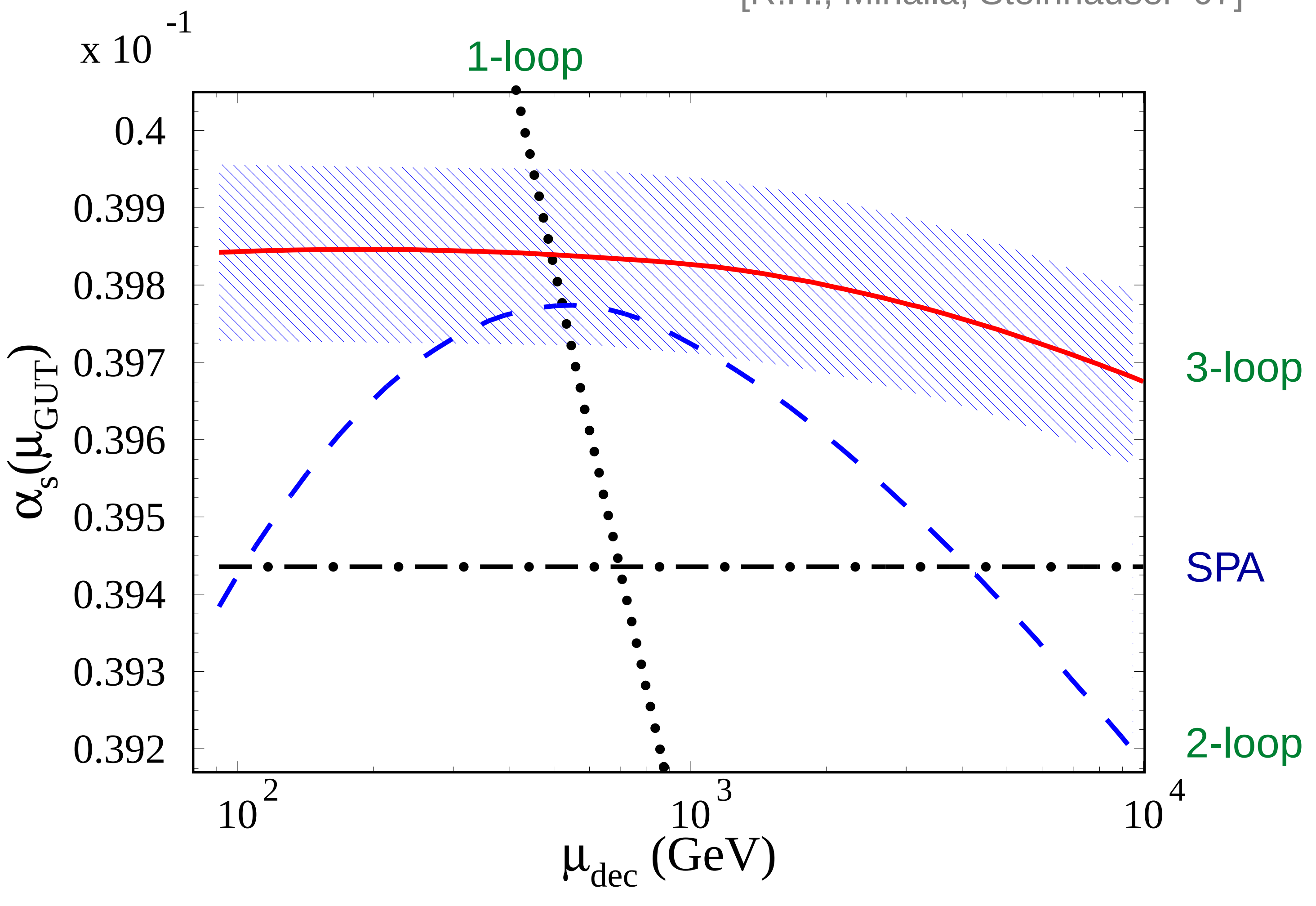
- here:
  - 3-loop running
  - 2-loop matching at $\mu_{\mathrm{dec}} \sim M_{\mathrm{SUSY}}$
  - 2-loop $\overline{\mathrm{MS}}$ — $\overline{\mathrm{DR}}$ conversion at $\mu_{\mathrm{dec}} \sim M_{\mathrm{SUSY}}$

$\alpha_s(M_{\mathrm{GUT}})$ *from* $\alpha_s(M_Z)$

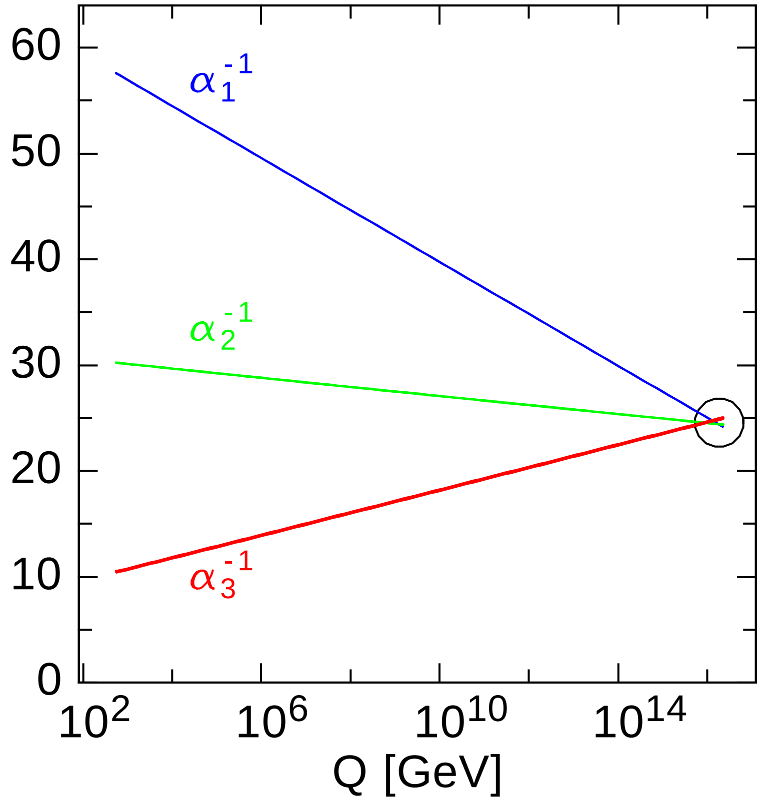
[R.H., Mihaila, Steinhauser '07]

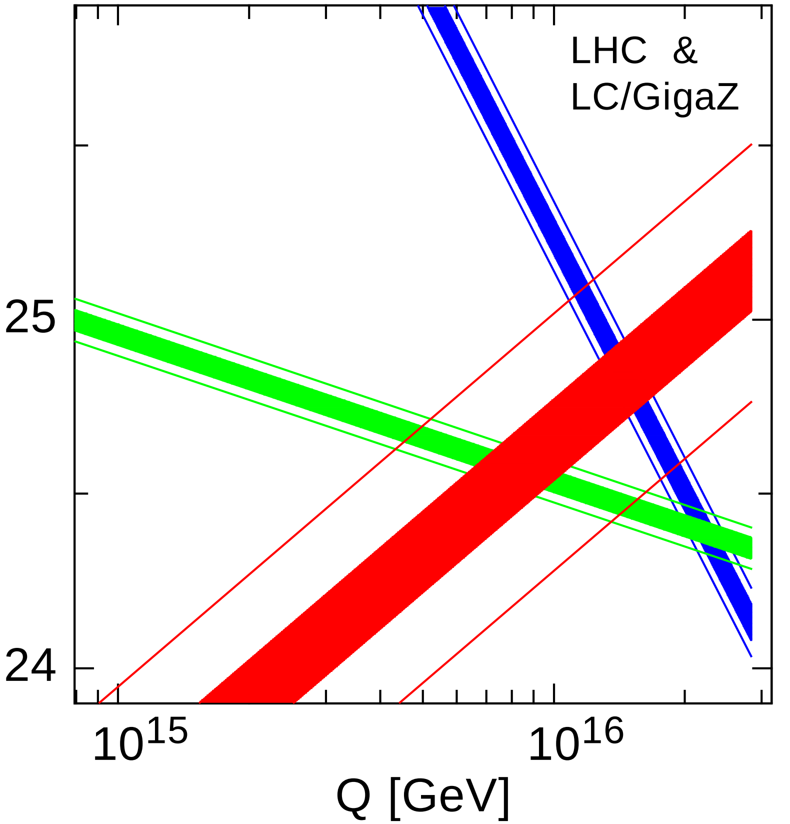


Unification

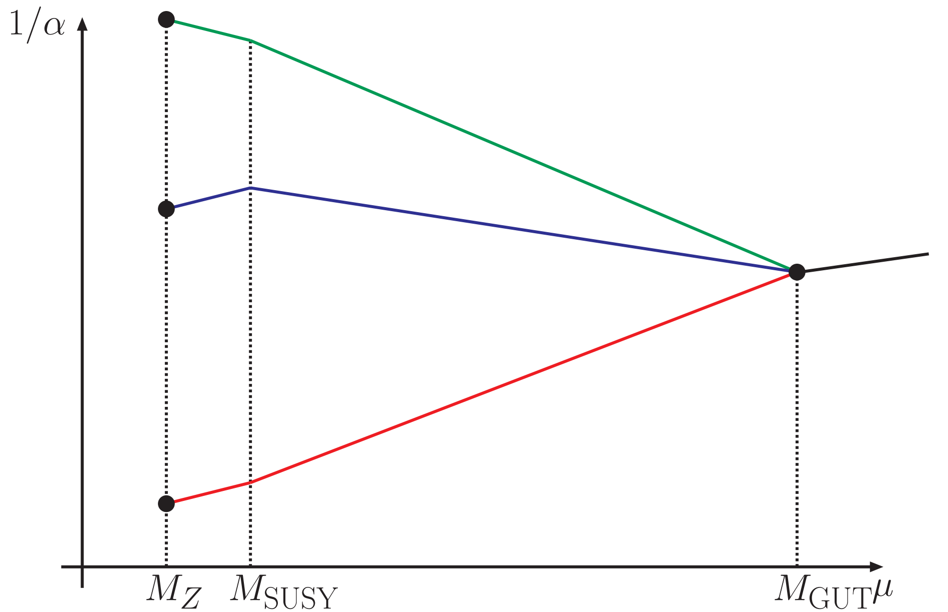
Relate $\alpha_s(M_Z)$ to $\alpha_s(M_{\rm SUSY})$

Allanach et al '04





Decoupling



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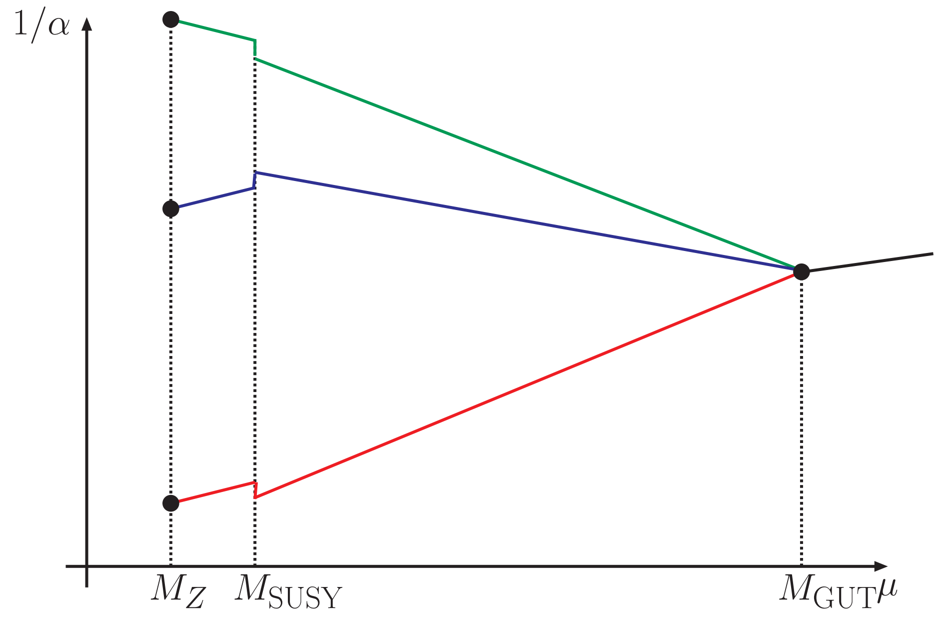

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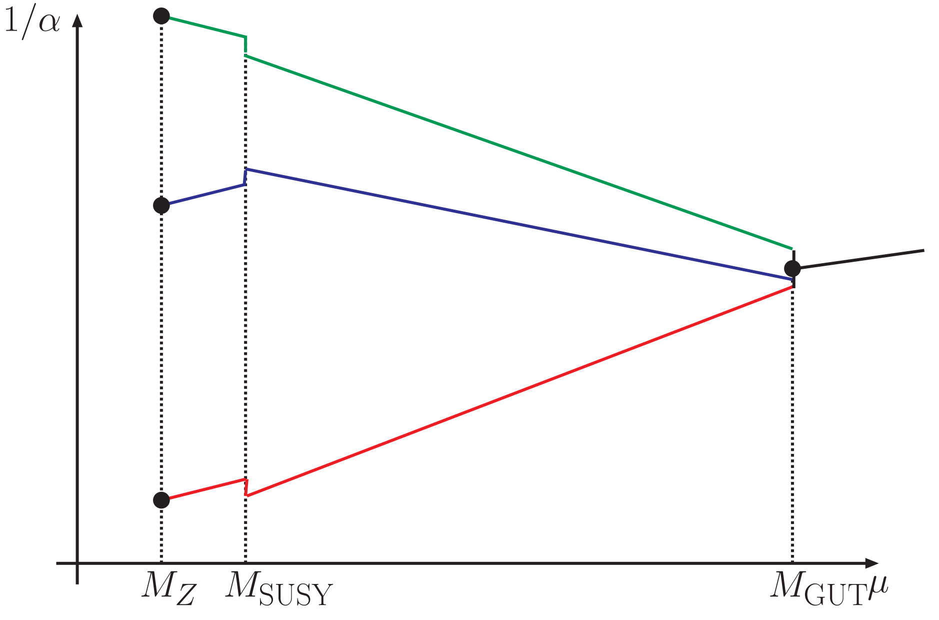

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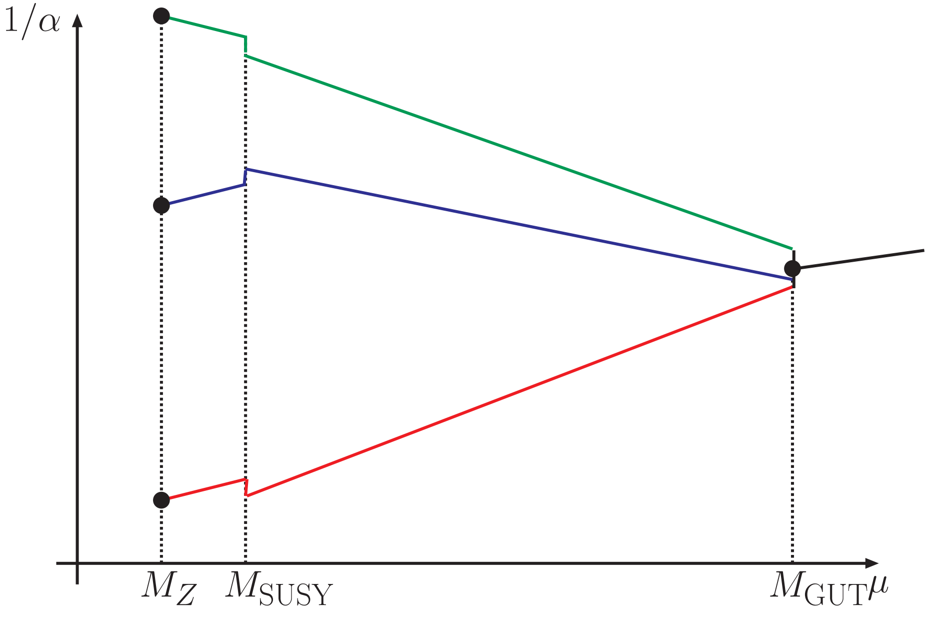

$\rightarrow$ information about GUT theory from measurements at low energies!

[Pierce, Bagger, Matchev, Zhang '97]

DRED in standard QCD

- coupled differential equations:

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s^{\overline{\mathrm{DR}}} = \beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e)\,,$$

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  as opposed to litarature!

Conclusions and Outlook

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- ToDo:

  - quantify validity range of DRED in SUSY
  - combine running with electro-weak couplings