$e^+e^- \longrightarrow 3$ jets at NNLO

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RADCOR 2007

$e^+e^- \to 3$ jets and event shapes

Classical QCD observable

- testing ground for QCD: perturbation theory, power corrections and logarithmic resummation
- precision measurement of strong coupling constant α_s
- current error on α_s from jet observables dominated by theoretical uncertainty: S. Bethke, 2006

$$\alpha_s(M_Z) \quad = \quad 0.121 \pm 0.001(\text{experiment}) \pm 0.005(\text{theory})$$

- theoretical uncertainty largely from missing higher orders
- current status: NLO plus NLL resummation

Theoretical description

- easier than at hadron colliders, since coloured partons only in final state: no initial state emission, no parton distributions
- new calculational methods first developed for e^+e^-, then extended to hadronic processes

$e^+e^- \to 3$ jets and event shapes

Event shape variables

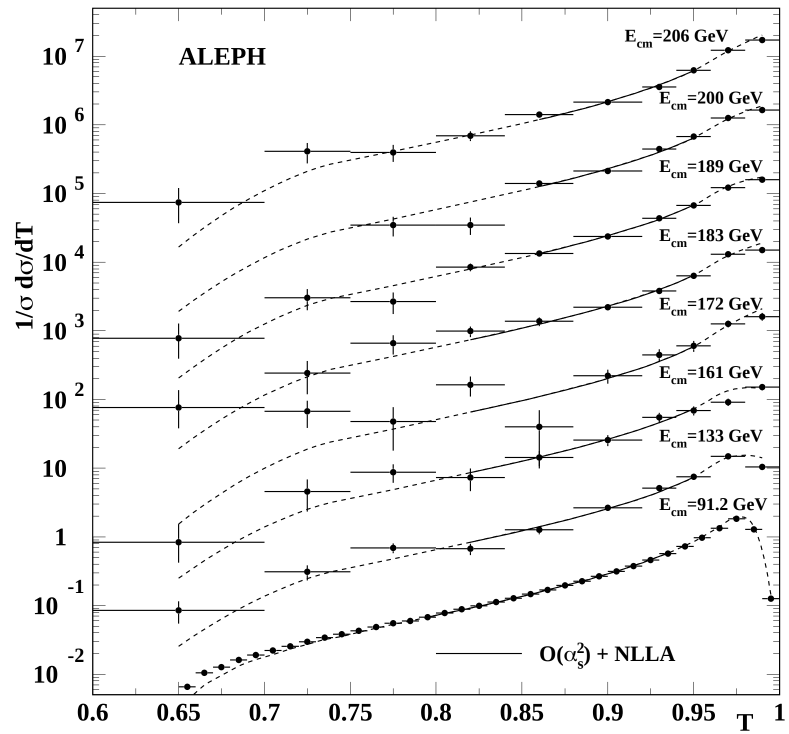
assign a number x to a set of final state momenta: $\{p\}_i \to x$

e.g. Thrust in e^+e^-

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}|}{\sum_{i=1}^{n} |\vec{p_i}|}$$

limiting values:

- back-to-back (two-jet) limit: $T = 1$
- spherical limit: $T = 1/2$



$e^+e^- \to 3$ jets and event shapes

Standard Set of LEP

- Thrust (E. Farhi)

$$T = \max_{\vec{n}} \left(\sum_{i=1}^{n} |\vec{p_i} \cdot \vec{n}| \right) / \left(\sum_{i=1}^{n} |\vec{p_i}| \right)$$

- Heavy jet mass (L. Clavelli, D. Wyler)

$$M_i^2/s = \frac{1}{E_{\rm vis}^2} \left(\sum_{k \in H_i} |\vec{p_k}| \right)^2$$

- C-parameter: eigenvalues of the tensor (G. Parisi)

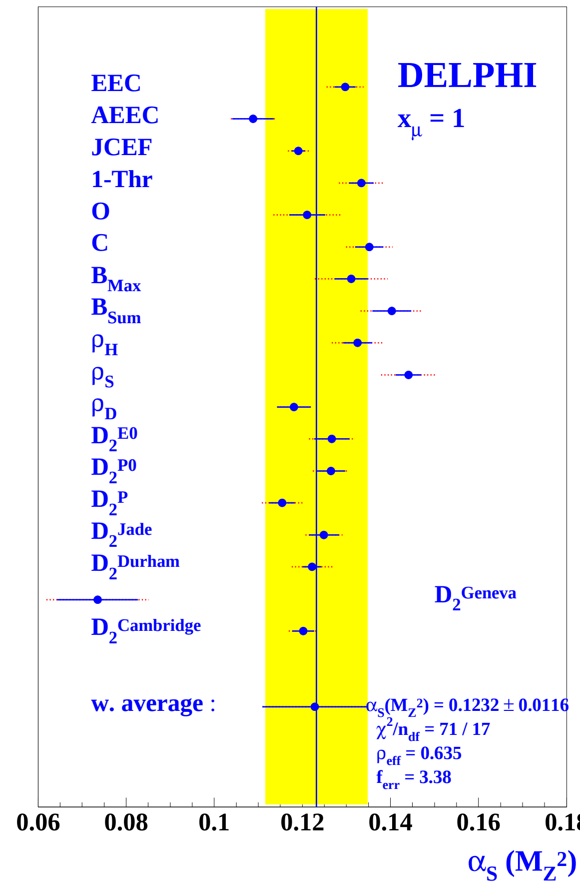
$$\Theta^{\alpha\beta} = \frac{1}{\sum_k |\vec{p_k}|} \frac{\sum_k p_k^\alpha p_k^\beta}{\sum_k |\vec{p_k}|}$$

- Jet broadenings (S. Catani, G. Turnock, B. Webber)

$$B_i = \left(\sum_{k \in H_i} |\vec{p_k} \times \vec{n}_T| \right) / \left(2 \sum_k |\vec{p_k}| \right)$$

$$B_W = \max(B_1, B_2) \qquad B_T = B_1 + B_2$$

- $3j \to 2j$ transition parameter in Durham algorithm y_{23}^D
  S.Catani, Y.L.Dokshitzer, M.Olsson, G.Turnock, B.Webber



$e^+e^- \to 3$ jets and event shapes

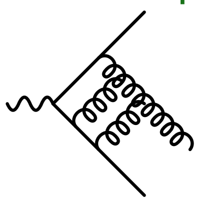
Current status: NLO and NLL

- NLO calculations of event shapes and $3j$
  R.K. Ellis, D.A. Ross, A.E. Terrano; Z. Kunszt
  J. Vermaseren, K.F. Gaemers, S.J. Oldham; L. Clavelli, D. Wyler
  K. Fabricius, I. Schmitt, G. Kramer, G. Schierholz
- NLO parton level event generators for $3j$
  EVENT: Z. Kunszt, P. Nason
  EERAD: W. Giele, E.W.N. Glover
  EVENT2: S. Catani, M. Seymour
- NLO parton level event generators for $4j$
  MenloParc: L.D. Dixon, A. Signer
  EERAD2: J. Campbell, M. Cullen, E.W.N. Glover
  Debrecen: Z. Nagy, Z. Trocsanyi
  Mercurito: D. Kosower, S. Weinzierl
- NLL resummation
  S. Catani, L. Trentadue, G. Turnock, B. Webber
- Power corrections
  G. Korchemsky, G. Sterman; Y. Dokshitzer, B.R. Webber

Ingredients to NNLO $e^+e^- \to$ 3-jet

- Two-loop matrix elements

$|\mathcal{M}|^2_{\text{2-loop},3 \text{ partons}}$



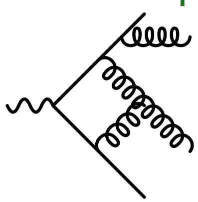
explicit infrared poles from loop integrals

L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG (RADCOR 00/02);

S. Moch, P. Uwer, S. Weinzierl

- One-loop matrix elements

$|\mathcal{M}|^2_{\text{1-loop},4 \text{ partons}}$



explicit infrared poles from loop integral and

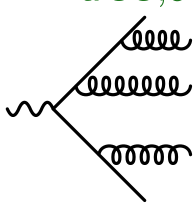
implicit infrared poles due to single unresolved radiation

Z. Bern, L. Dixon, D. Kosower, S. Weinzierl;

J. Campbell, D.J. Miller, E.W.N. Glover

- Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree},5 \text{ partons}}$



implicit infrared poles due to double unresolved radiation

K. Hagiwara, D. Zeppenfeld;

F.A. Berends, W.T. Giele, H. Kuijf;

N. Falck, D. Graudenz, G. Kramer

Infrared Poles cancel in the sum

NNLO Infrared Subtraction

Structure of NNLO m-jet cross section:

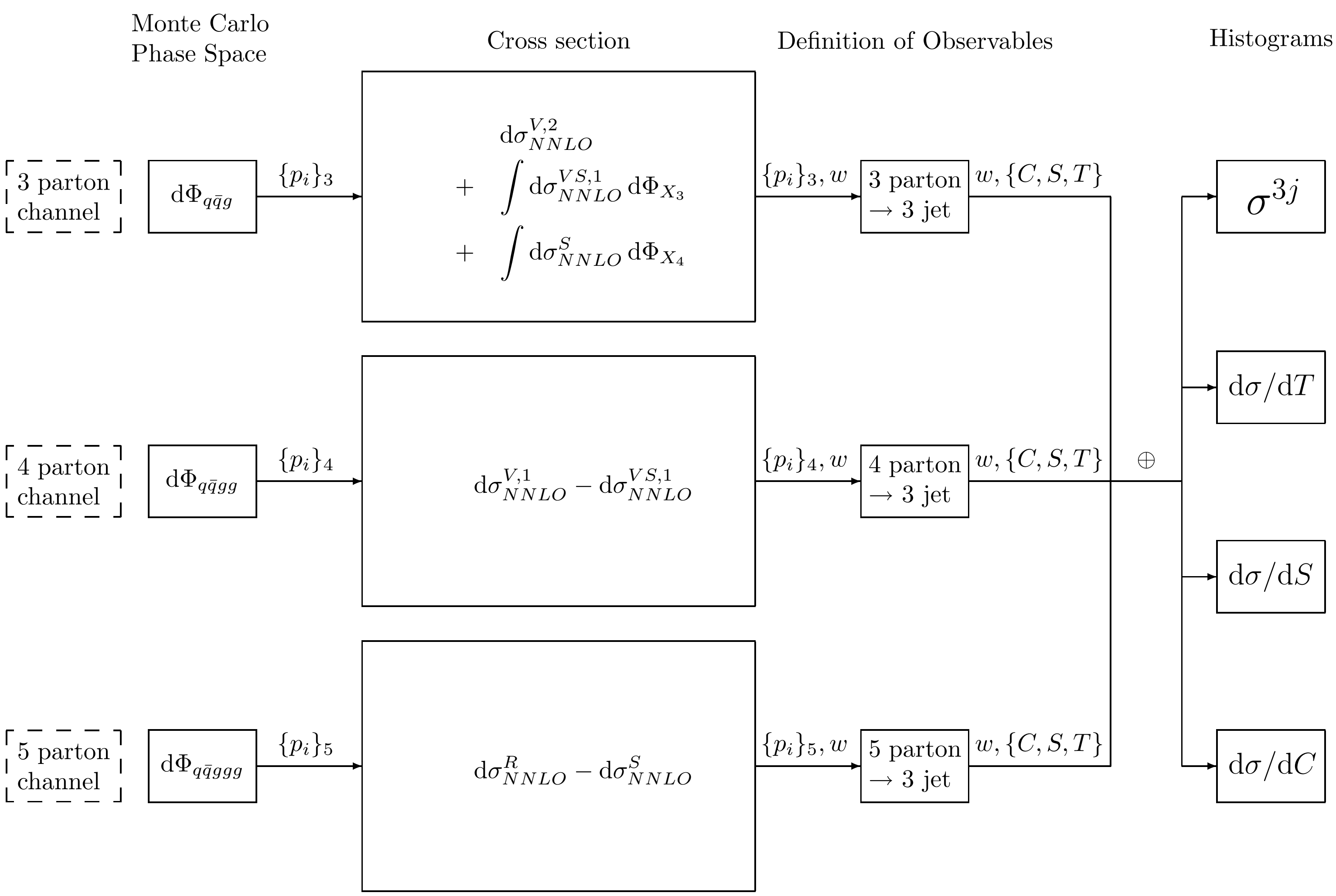
$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} \quad = \quad & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \, ,
\end{aligned}
$$

- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

Each line above is finite numerically and free of infrared ϵ-poles $\longrightarrow$ numerical programme

Numerical Implementation

Structure of $e^+e^- \to 3$ jets program:



Numerical Implementation

Antenna subtraction

NLO: M. Cullen, J. Campbell, E.W.N. Glover; D. Kosower; A. Daleo, D. Maitre, TG

NNLO: A. Gehrmann-De Ridder, E.W.N. Glover, TG (RADCOR 05)

- construct subtraction terms from physical $1 \to 3$ and $1 \to 4$ matrix elements
- each antenna function interpolates between all limits associated to one or two unresolved partons
- integrated subtraction terms cancel infrared pole structure of two-loop matrix element

  S. Catani; G. Sterman, M.E. Yeomans-Tejeda

Checks

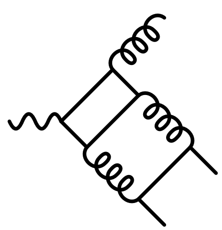
- cancellation of infrared poles in 3-parton and 4-parton channel
- convergence of subtraction terms towards matrix elements along phase space trajectories
- distributions in raw phase space variables
- independence on phase space cut y_0

Colour structure at NNLO

Decomposition into leading and subleading colour terms

$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} \quad = \quad & (N^2-1)\Bigg[N^2\, A_{NNLO} + B_{NNLO} + \frac{1}{N^2}\, C_{NNLO} + N N_F\, D_{NNLO} \\
& + \frac{N_F}{N}\, E_{NNLO} + N_F^2\, F_{NNLO} + N_{F,\gamma}\left(\frac{4}{N} - N\right)\, G_{NNLO}\Bigg]
\end{aligned}
$$

- last term: closed quark loop coupling to vector boson



$$
N_{F,\gamma} = \frac{\left(\sum_q e_q\right)^2}{\sum_q e_q^2}
$$

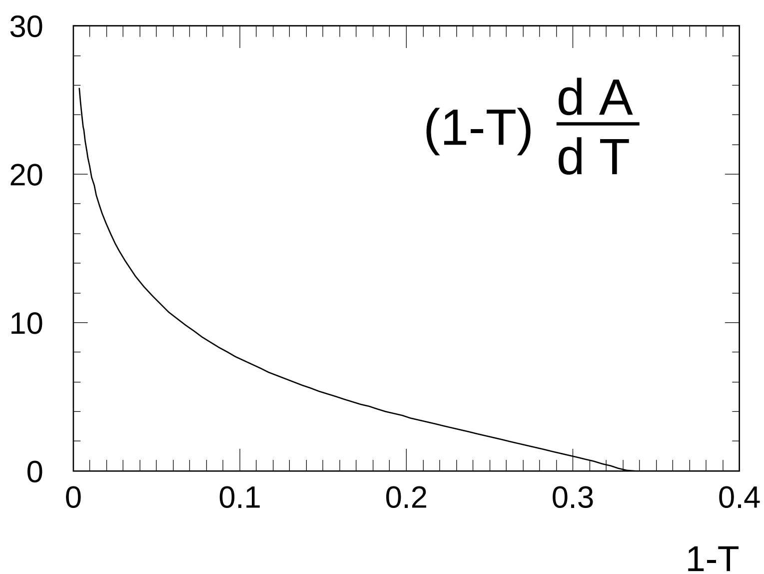
- was found to be $\mathcal{O}(10^{-4})$ in NLO $4j$ final states
  L.D. Dixon, A. Signer
- will be negelected here

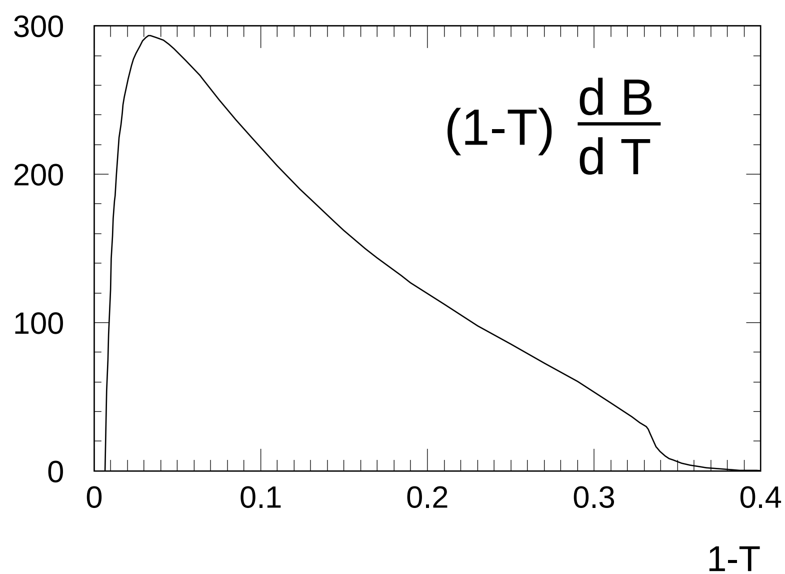
Event shapes at NNLO

NNLO expression for Thrust

$$(1-T)\,\frac{1}{\sigma_{\rm had}}\,\frac{{\rm d}\sigma}{{\rm d}T} = \left(\frac{\alpha_s}{2\pi}\right) A(T) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(B(T) - 2A(T)\right) + \left(\frac{\alpha_s}{2\pi}\right)^3 \left(C(T) - 2\,B(T) - 1.64\,A(T)\right)$$

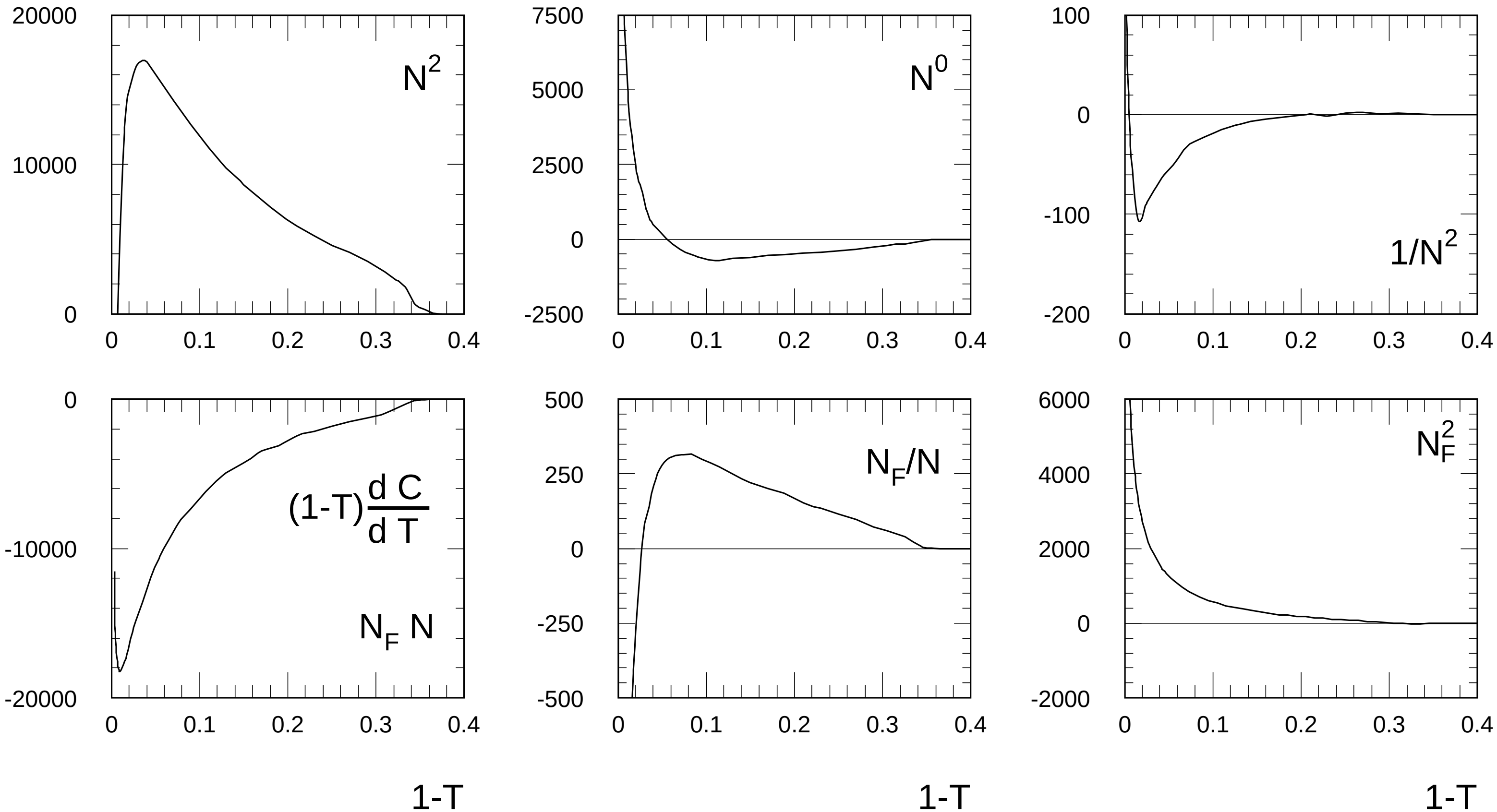
with LO contribution $A(T)$, NLO contribution $B(T)$





Event shapes at NNLO

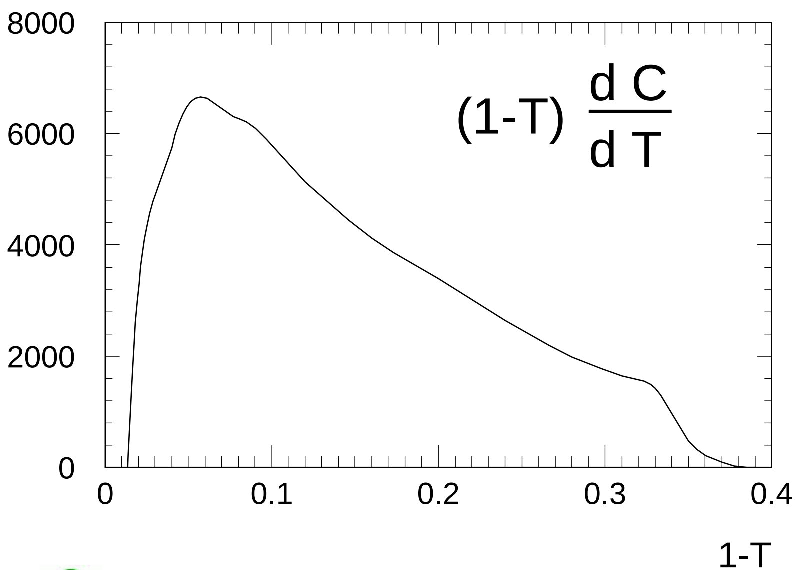
Individual colour structures

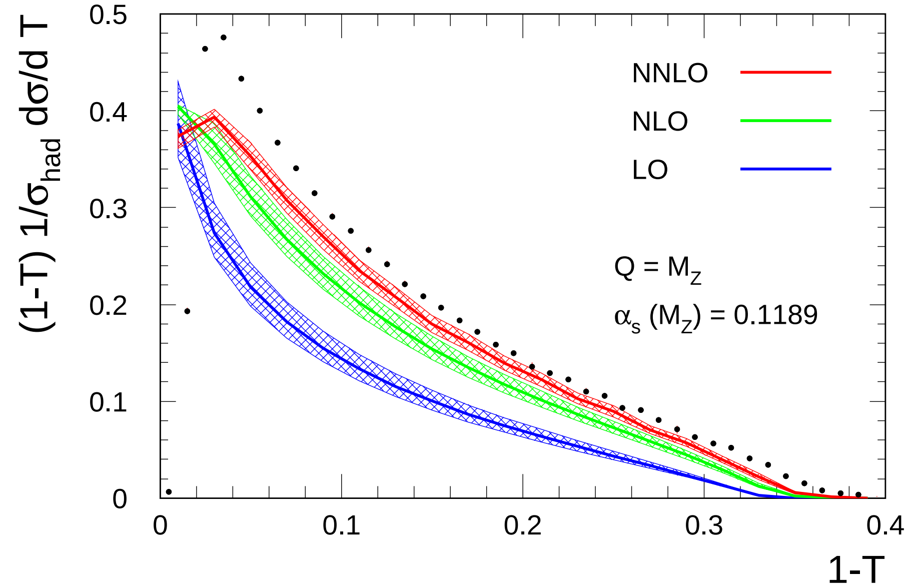


- dominated by leading colour N^2 and $N_F\,N$
- sizable contributions from N^0, N_F/N and N_F^2
- negligible contribution from $1/N^2$

Results

NNLO thrust distribution





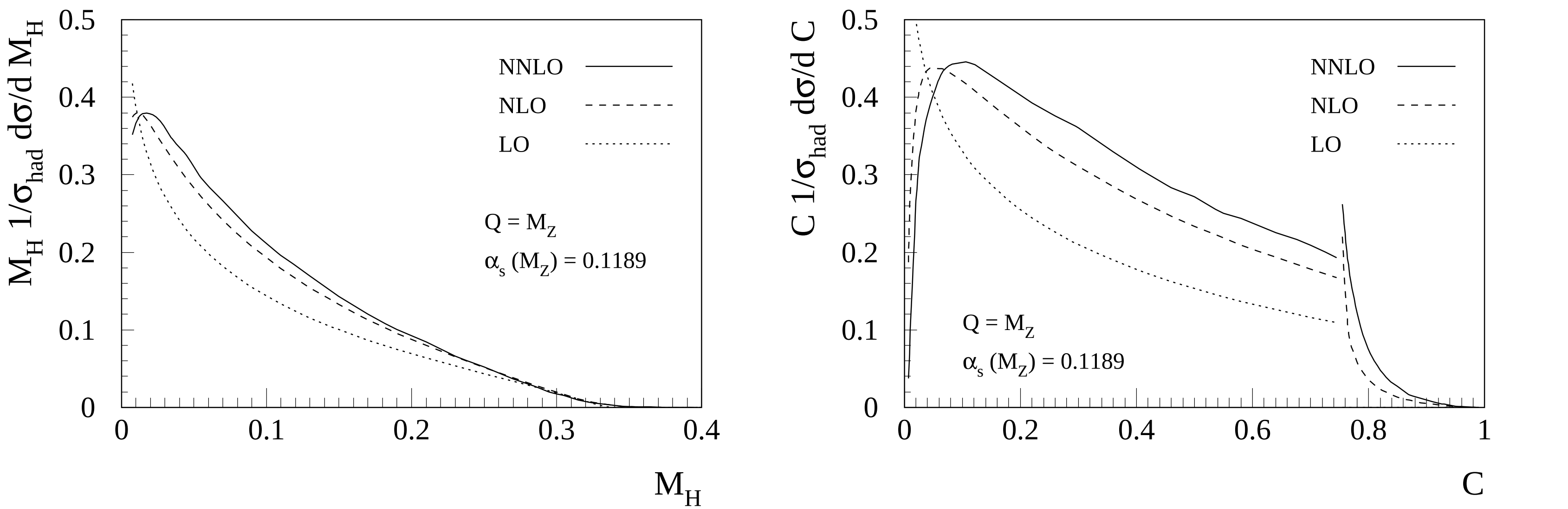
- NNLO corrections sizable
- theory error reduced by 30–40 %
- large $1-T$: need hadronization corrections
- small $1-T$: two-jet region, need matching onto NLL resummation
  Work in progress: G. Luisoni, TG
- mean value $\langle 1-T\rangle$: $\mathcal{A} = 2.101$ $\qquad \mathcal{B} = 44.98$ $\qquad \mathcal{C} = 1095 \pm 130$

$$\langle 1-T\rangle(\alpha_s = 0.1189) = 0.0398 + 0.0146 + 0.0068$$

Results

NNLO heavy jet mass and C-parameter

heavy jet mass M_H^2/s $\qquad$ C-parameter

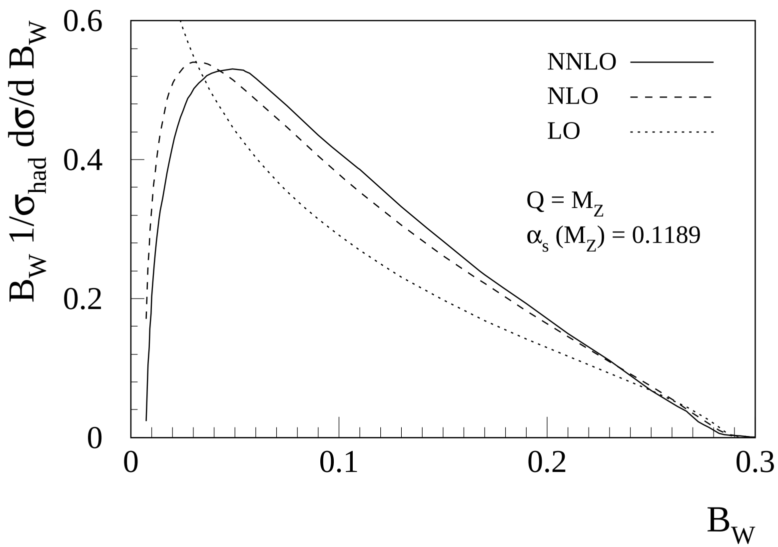


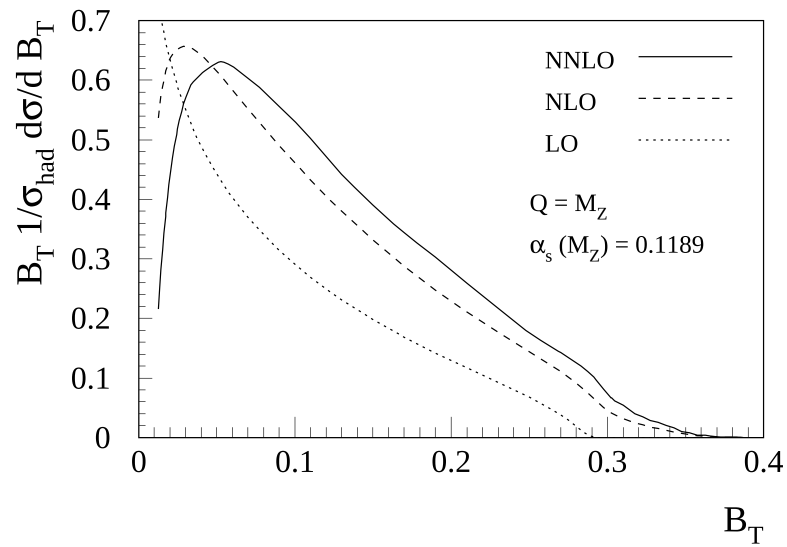
- heavy jet mass (closely related to thrust) has very small NNLO corrections
- NNLO corrections for C large
- again require matching onto NLL resummation and hadronization corrections
- Sudakov shoulder in $C = 0.75$ also requires resummation
  S. Catani, B. Webber

Results

NNLO jet broadenings

wide jet boadening B_W total jet boadening B_T





- relative magnitude of NNLO corrections smaller than for thurst
- NNLO corrections for B_W smaller than for B_T
- again require matching onto NLL resummation and hadronization corrections

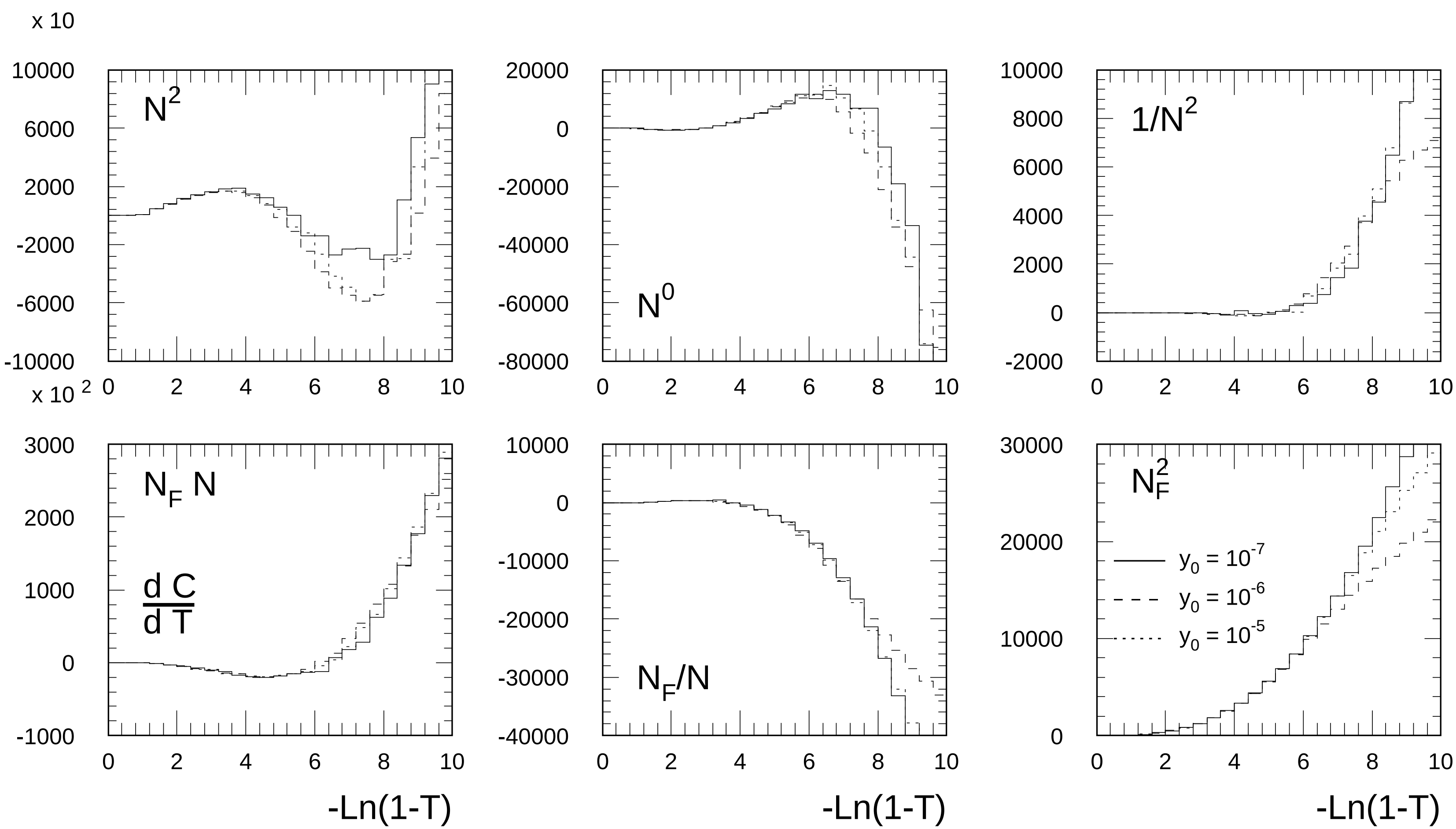
Summary and Outlook

- completed calculation of NNLO corrections to event shapes related to $e^+e^- \to 3j$
- constructed parton-level event generator, based on antenna subtraction method
- size of NNLO corrections not uniform:
  small: B_W, M_H/s, moderate: B_T, substantial: C, T
- still running: y_{23}^D, R_{3j}^D
- comparison with data just started
- next steps: matching onto resummation, hadronization corrections

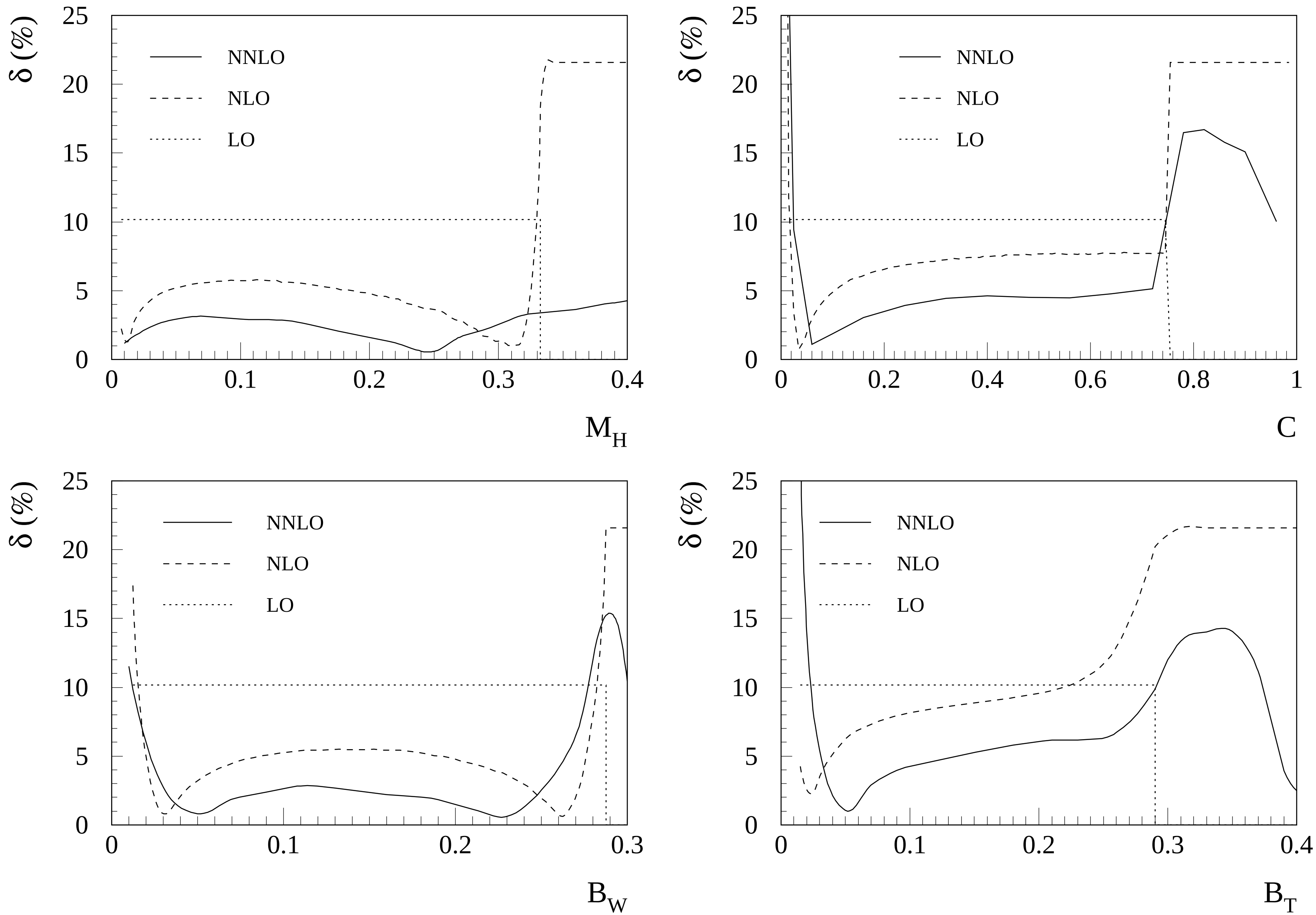
Backup

Thrust at NNLO

Checks

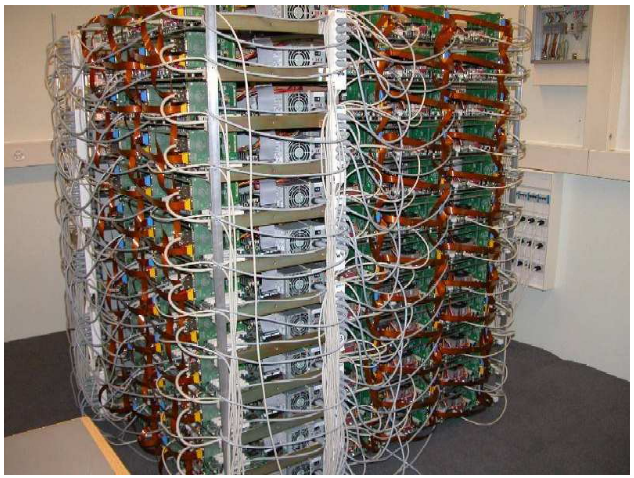


Errors at NNLO



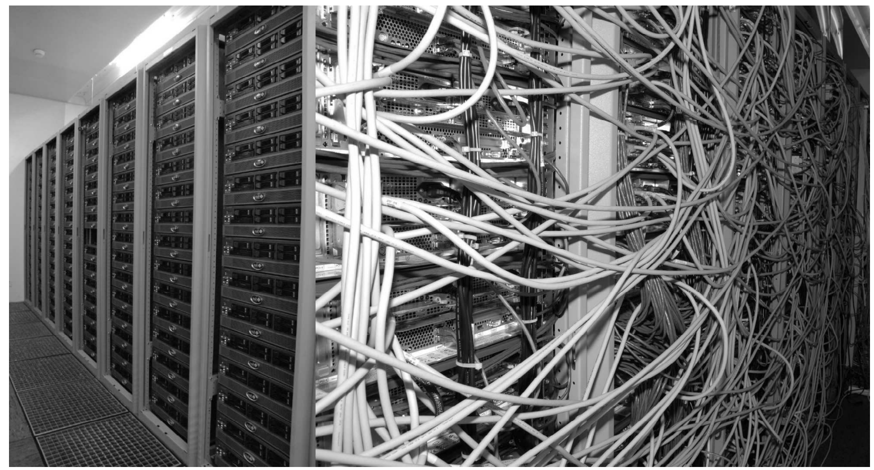
Numerical computation

zBox1 and zBox2 supercomputers



zBox1

- 288 processors,
  2.2 GHz AMD Athlon
- 0.57 TFlops
- built in-house from
  off-the-shelf components
  J. Stadel, B. Moore



zBox2

- 500 processors,
  2.6 GHz Opteron 852
- 5.2 TFlops
- built by Sun
  microsystems

used mostly by our computational astrophysics group