The OPE of the B-meson light-cone wavefunction for exclusive B decays: radiative corrections and higher dimensional operators

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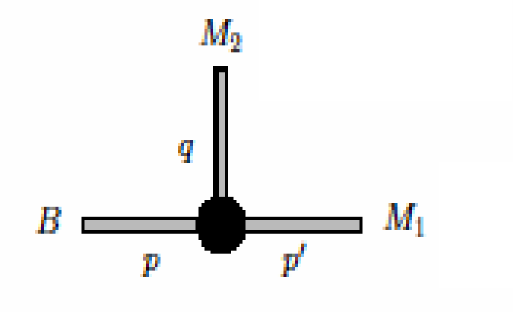
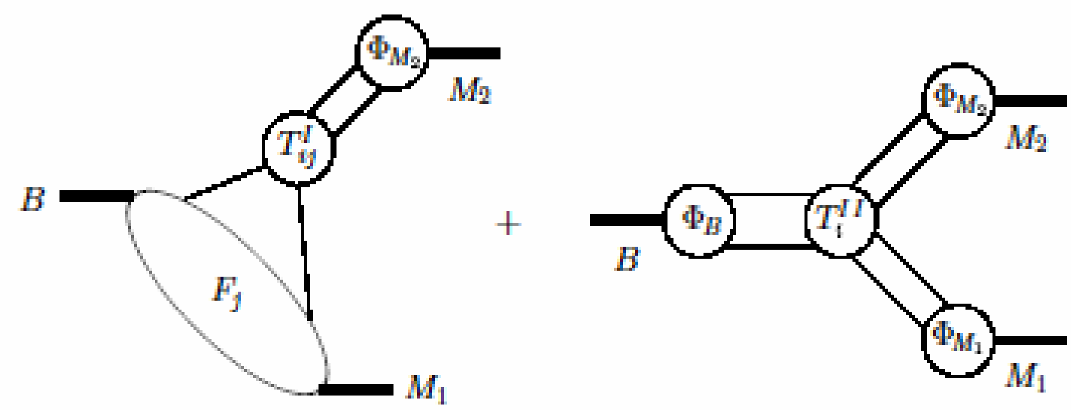
QCD factorization for Exclusive B decays

Beneke et al. ('99)
Bauer et al. ('01)

$B \to \pi\pi,\ \rho\gamma,\ \pi l \nu, \cdots$

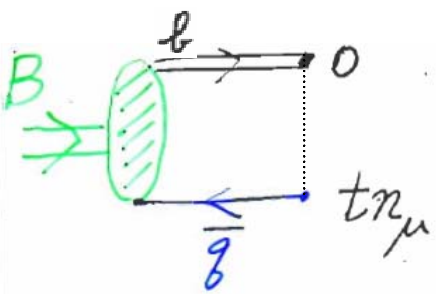
$m_B \to \infty \quad (m_b \to \infty)$





B meson's LCWF in HQET

$$b(x) = e^{-im_b v\cdot x} h_v(x) + O(1/m_b), \quad \not{v} h_v(x) = h_v(x)$$



$$\tilde{\phi}_B(t,\mu) = \langle 0 | \left[\bar{q}(tn) \mathcal{P} \exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n) \right) \not{n} \gamma_5 h_v(0) \right]_\mu | \bar{B}(v) \rangle$$

$$= \int d\omega e^{-i\omega t} \phi_B(\omega,\mu)$$

$n^\mu = (1,0,0,-1) \quad (n^2 = 0)$

$p^\mu = m_B v^\mu \quad (v^2 = 1)$

$k^+ = \omega v^+$

$$\sim \langle 0 | S \left[\bar{q}(0) D^{\nu_1} D^{\nu_2} \cdots D^{\nu_{j-1}} \not{n} \gamma_5 h_v(0) \right]_\mu | \bar{B}(v) \rangle$$

~~twist = dimension - spin~~

$t \Leftrightarrow \mu$

IR structure

Kawamura, Kodaira, Qiao, Tanaka, PLB523 ('01) 111

constraints from HQET eqs. of motion: $\bar{q}\overleftarrow{\not{D}} = v \cdot \overrightarrow{D} h_v = 0$

heavy quark symmetry: $\not{v} h_v = h_v$

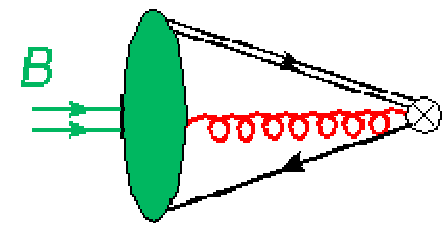
$$\phi_B(\omega) = \phi_B^{(WW)}(\omega) + \phi_B^{(g)}(\omega)$$

$$\bar{\Lambda} = m_B - m_b$$

$$\langle 0 | \bar{q} \not{n} \gamma_5 h_v | \bar{B}(v) \rangle = iF$$

$$\phi_B^{(WW)}(\omega) = iF \frac{\omega}{2\bar{\Lambda}^2} \theta(2\bar{\Lambda} - \omega)$$

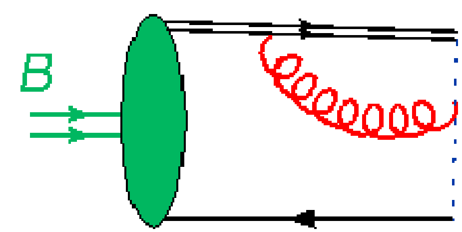
$$\phi_B^{(g)} \sim \langle 0 | \bar{q} G h_v | \bar{B}(v) \rangle$$



UV structure

Lange, Neubert,PRL523 ('03) 102001

radiative corrections from hard loops



$$\phi_B(\omega) \sim -iF\alpha_s \frac{\log(\omega/\mu)}{\omega}$$

"radiation tail"

$$\int_0^\infty d\omega\, \omega^j \phi_B(\omega) = -\infty$$

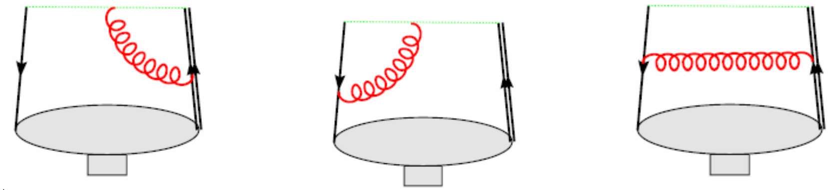
$$\bar{q}(tn) \not{n} \gamma_5 \mathrm{Pexp}\left(ig \int_0^t d\lambda\, n_\mu A^\mu(\lambda n) \right) h_v(0)$$

$$\parallel$$

$$\mathrm{Pexp}\left(ig \int_{-\infty}^0 ds\, v_\mu A^\mu(sv) \right) h_v(-\infty v)$$

cusp singularity

Radiative corrections from hard and soft/collinear loops



$d = 4 - 2\varepsilon$

$$\tilde{\phi}_{\pi}^{\text{1-loop}}(t,\mu) = \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}\right)\int_0^1 d\xi \int_0^{\xi} d\eta K(\xi,\eta)\left\langle 0\left|\bar{q}(\xi t n)\not{n}\gamma_5 q(\eta t n)\right|\pi(p)\right\rangle$$

Brodsky-Lepage pot.

- **analytic at t=0**
- **UV~IR "scaleless"** $\int d^{4-2\varepsilon}q \frac{1}{q^4} \sim \frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}$

double log

$$\tilde{\phi}_{B}^{\text{1-loop}}(t,\mu) = \frac{\alpha_s C_F}{2\pi}\int_0^1 d\xi \left[\left\{-\left(\frac{1}{2\varepsilon_{UV}^2} + \frac{\log(it\mu)}{\varepsilon_{UV}} + \log^2(it\mu) + \frac{5\pi^2}{24}\right)\delta(1-\xi)\right.\right.$$

$$\left. + \left(\frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}}\right)\left[\frac{\xi}{1-\xi}\right]_+ - \left(\frac{1}{2\varepsilon_{IR}} + \log(it\mu)\right)\right\}\left\langle 0\left|\bar{q}(\xi t n)\not{n}\gamma_5 h_v(0)\right|\bar{B}(v)\right\rangle$$

$$\left. -t\left(\frac{1}{\varepsilon_{IR}} + 2\log(it\mu) - 1 - \xi\right)\left\langle 0\left|\bar{q}(\xi t n) v\cdot\overleftarrow{D}\not{n}\gamma_5 h_v(0)\right|\bar{B}(v)\right\rangle\right] + \cdots$$

$$\mu = e^{\gamma_E}\mu_{\overline{MS}}$$

- nonanalytic at t=0 $\log^2(it\mu),\ \log(it\mu)$: nontrivial dependence on $t\mu$

- UV ⟷ IR

$\frac{1}{\varepsilon_{UV}^2},\ \frac{1}{\varepsilon_{UV}}$ 1/t $\frac{1}{\varepsilon_{IR}}$ with many higher dim. operators

We have to use OPE to separate UV and IR behaviors, in contrast to light $q\bar{q}$-meson light-cone WF!

$$\tilde{\phi}_B(t,\mu) = \sum_i \tilde{C}_i(t,\mu)\left\langle 0\left|O_i(\mu)\right|\bar{B}(v)\right\rangle \qquad \mu \le \frac{1}{t}$$

$\tilde{C}_i \sim \log^2(it\mu)$; $O_i(\mu)$: local op.

$$\tilde{\phi}_B(t,\mu_i) = \hat{U}(\mu_i,\mu)\tilde{\phi}_B(t,\mu) \qquad \mu_i = \sqrt{m_b \Lambda_{\mathrm{QCD}}}$$

$\hat{U}$: Sudakov-type [Lange, Neubert ('03)]

Cut-off scheme:

$$\tilde{\phi}_B(t,\mu,\Lambda_{\mathrm{UV}}) = \sum_i \tilde{C}_i(t,\mu,\Lambda_{\mathrm{UV}}) \langle 0 | O_i(\mu) | \bar{B}(v) \rangle$$

$\tilde{C}_i$: up to $O(\alpha_s)$

O_i : up to dim.4

$\bar{q}\Gamma h_v$ $\quad$ $\bar{q}D\Gamma h_v$

} Lee, Neubert, PRD72 ('05) 094028

$\overline{\mathrm{MS}}$ **scheme:**

$$\tilde{\phi}_B(t,\mu) = \sum_i \tilde{C}_i(t,\mu) \langle 0 | O_i(\mu) | \bar{B}(v) \rangle$$

$\tilde{C}_i$: up to $O(\alpha_s)$

O_i : up to dim.5

$\bar{q}\Gamma h_v$ $\quad$ $\bar{q}D\Gamma h_v$ $\quad$ $\{\bar{q}DD\Gamma h_v,\ \bar{q}G\Gamma h_v\}$

} **this work**

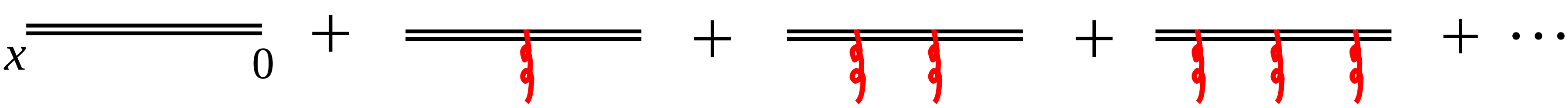
Background field method:

$$h_v \to h_v^{(\mathrm{Q})} + h_v^{(\mathrm{C})}, \quad iv \cdot D^{(\mathrm{C})} h_v^{(\mathrm{C})} = 0$$

$$q \to q^{(\mathrm{Q})} + q^{(\mathrm{C})}, \quad i\not{D}^{(\mathrm{C})} q^{(\mathrm{C})} = 0$$

$$A_\mu \to A_\mu^{(\mathrm{Q})} + A_\mu^{(\mathrm{C})}, \quad D_\mu^{(\mathrm{C})} G_{(\mathrm{C})}^{\mu\rho} = t^a \overline{q}^{(\mathrm{C})} t^a \gamma^\rho q^{(\mathrm{C})}$$

$$\underbrace{h_v^{(\mathrm{Q})}(x)\overline{h}_v^{(\mathrm{Q})}}(0) = \theta(v\cdot x)\delta^{(D-1)}(x_\perp)\frac{1+\not{v}}{2}\,\mathrm{P}\exp\left(ig\int_0^{v\cdot x} d\lambda\, v^\mu A_\mu^{(\mathrm{C})}(\lambda v)\right)$$



Fock-Schwinger gauge

$$x^\mu A_\mu^{(\mathrm{C})}(x) = 0 \;\Rightarrow\; A_\mu^{(\mathrm{C})}(x) = \int_0^1 du\, u x^\rho G_{\rho\mu}^{(\mathrm{C})}(ux)$$

OPE of B-meson's LCWF up to dim.5 ops.

Tree level matching:

Grozin, Neubert, PRD55 ('97) 272
Kawamura, Kodaira, Qiao, Tanaka, PLB523 ('01) 111

$$\tilde{\phi}_B^{\text{tree}}(t) = \left\langle 0 \left| \bar{q}^{(C)}(tn) \mathrm{P} \exp\left(ig \int_0^t d\lambda n^\mu A_\mu^{(C)}(\lambda n) \right) \not{n} \gamma_5 h_v^{(C)}(0) \right| \bar{B}(v) \right\rangle$$

$$= \sum_i \tilde{C}_i^{(0)}(t,\mu) \left\langle 0 \left| O_i(\mu) \right| \bar{B}(v) \right\rangle$$

$$= \left\langle 0 \left| \bar{q} \not{n} \gamma_5 h_v \right| \bar{B}(v) \right\rangle$$

$$+ t \left\langle 0 \left| \bar{q} \overleftarrow{D} \cdot n \not{n} \gamma_5 h_v \right| \bar{B}(v) \right\rangle$$

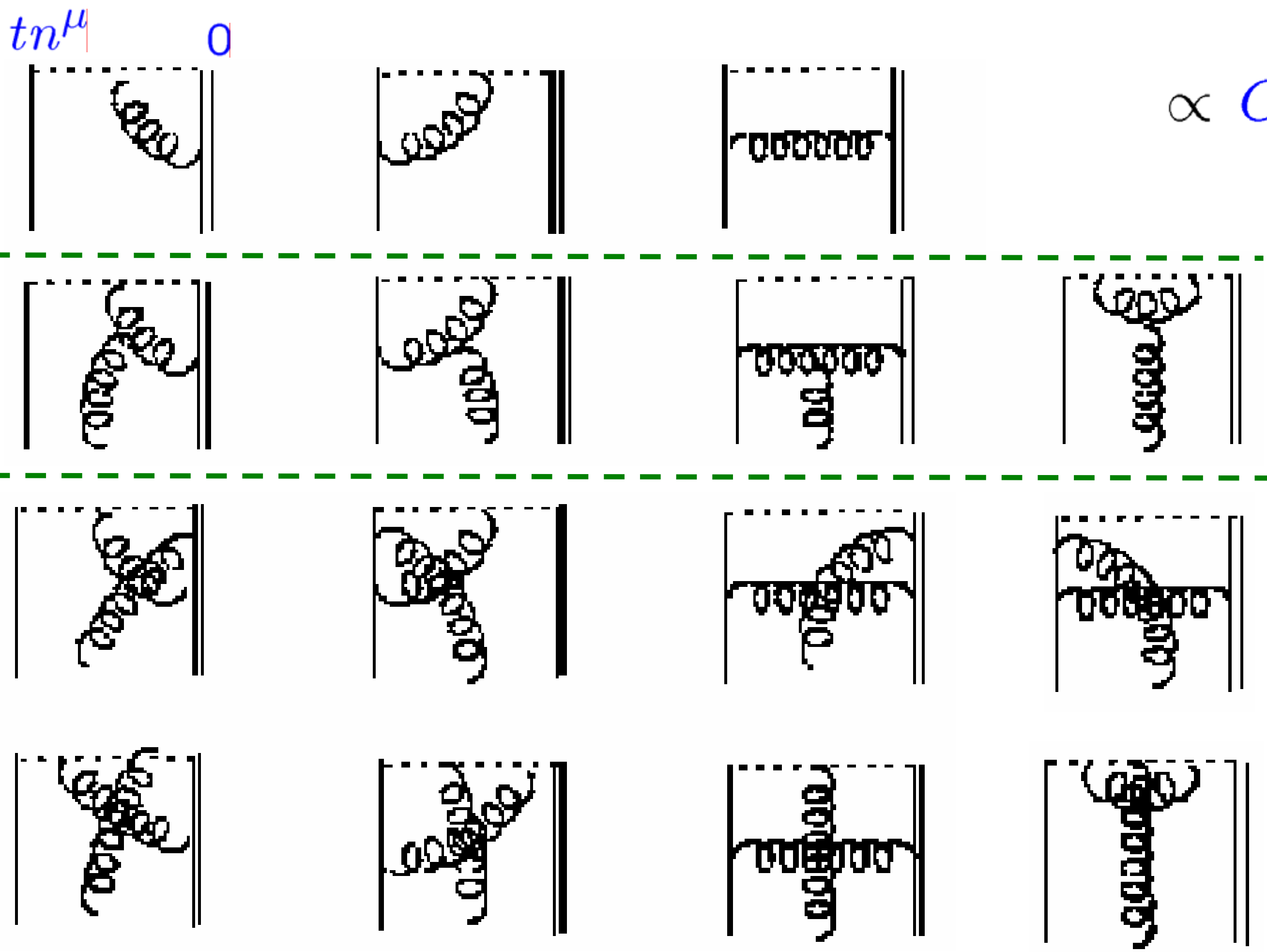
eq. of motion + HQ symmetry

$$+ \frac{2i}{3} t^2 \left\langle 0 \left| \bar{q} g \mathbf{E} \cdot \boldsymbol{\alpha} \gamma_5 h_v \right| \bar{B}(v) \right\rangle + \frac{1}{3} t^2 \left\langle 0 \left| \bar{q} g \mathbf{H} \cdot \boldsymbol{\sigma} \gamma_5 h_v \right| \bar{B}(v) \right\rangle$$

1-loop matching:

$$\bar{q}(tn)\mathrm{P}\exp\left(ig\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not{n}\gamma_5 h_v(0)$$

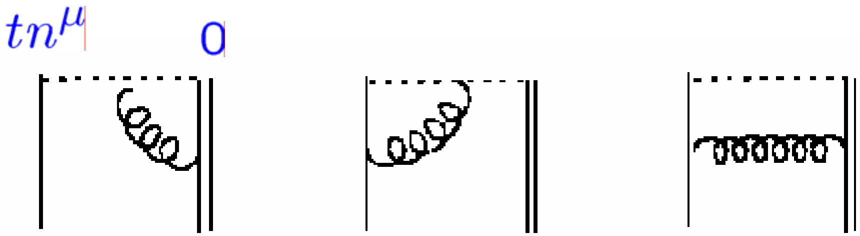
$$\Rightarrow \quad \bar{q}\Gamma h_v \qquad \bar{q}D\Gamma h_v \qquad \{\bar{q}DD\Gamma h_v, \bar{q}G\Gamma h_v\}$$



$$\propto C_F = \frac{N_C^2-1}{2N_C} \sim \frac{N_C}{2}$$

$$\propto C_G = N_C$$

$$\propto C_F - \frac{C_G}{2} \sim \frac{1}{N_C}$$

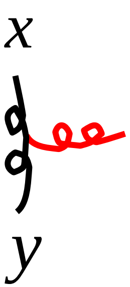


$$v_\mu \equiv \frac{n_\mu + \bar{n}_\mu}{2} \qquad \left(n^2 = \bar{n}^2 = 0,\ n\cdot\bar{n} = 2\right)$$

$$L \equiv \log\left(it\mu_{\overline{\mathrm{MS}}} e^{\gamma_E}\right)$$

double log

$$\begin{aligned}
&\left[\bar{q}(tn)\mathcal{P}\exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not{n}\gamma_5 h_v(0)\right]_{1-\text{loop}} \\
&= \frac{\alpha_s}{4\pi}C_F\Bigg[\left\{\bar{q}\not{n}\gamma_5 h_v + t\bar{q}(n\cdot\overleftarrow{D})\not{n}\gamma_5 h_v + \frac{t^2}{2}\bar{q}(n\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\right\}\left(-2L^2 - \frac{5}{12}\pi^2\right) \\
&+ \left\{t\bar{q}(n\cdot\overleftarrow{D})\not{n}\gamma_5 h_v + \frac{5}{6}t^2\bar{q}(n\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\right\}\frac{1}{\epsilon} \\
&+ \bar{q}\not{n}\gamma_5 h_v\left(-\frac{1}{\epsilon} - 2L\right) \\
&+ t\bar{q}(n\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\frac{1}{2}\left(-\frac{1}{\epsilon} - 2L\right) + t\bar{q}(v\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\left(-\frac{2}{\epsilon} - 4L + 3\right) \\
&+ t^2\bar{q}(n\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\frac{1}{6}\left(-\frac{1}{\epsilon} - 2L\right) + t^2\bar{q}(v\cdot\overleftarrow{D})(n\cdot\overleftarrow{D})\not{n}\gamma_5 h_v\left(-\frac{1}{\epsilon} - 2L + \frac{5}{3}\right) \\
&+ t^2\bar{q}(v\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v\left(-\frac{1}{\epsilon} - 2L + \frac{5}{3}\right) \\
&+ t^2\bar{q}igG_{\mu\nu}v^\mu n^\nu\not{n}\gamma_5 h_v\left(-\frac{1}{2\epsilon} - L - \frac{5}{6}\right) + t^2\bar{q}igG_{\mu\nu}\gamma^\mu n^\nu\not{\bar{n}}\gamma_5 h_v\left(-\frac{1}{6}\right)\left(\frac{1}{\epsilon} + 2L - 2\right) \\
&+ t^2\bar{q}igG_{\mu\nu}\gamma^\mu v^\nu\not{\bar{n}}\gamma_5 h_v\left(-\frac{1}{12}\right)\left(\frac{1}{\epsilon} + 2L - 2\right) + t^2\bar{q}gG_{\mu\nu}\sigma^{\mu\nu}\not{n}\gamma_5 h_v\left(-\frac{1}{24}\right)\left(\frac{1}{\epsilon} + 2L - 2\right)\Bigg]
\end{aligned}$$



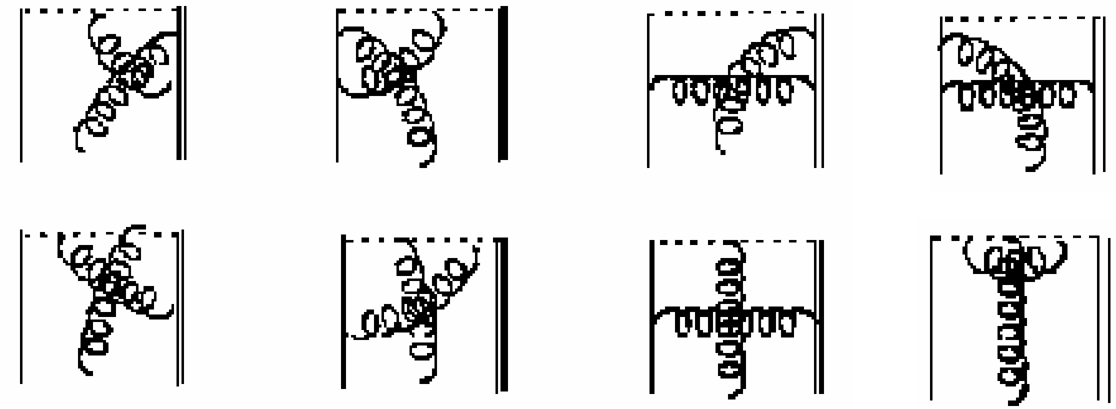
$$= \frac{\Gamma(d/2-1)}{4\pi^{d/2}} \frac{-g_{\mu\nu}}{\left[-(x-y)^2 + i\varepsilon\right]^{d/2-1}} i\int_0^1 du \int_0^1 d\alpha\alpha g G_{\eta\rho}^{(C)}(\alpha u x + \alpha\bar{u} y) y^\eta x^\rho$$

$$+ \frac{\Gamma(d/2-2)}{16\pi^{d/2}} \frac{-1}{\left[-(x-y)^2 + i\varepsilon\right]^{d/2-2}} 2igG_{\mu\nu}^{(C)}(ux+\bar{u}y) + \cdots$$

$$\left[\bar{q}(tn)\mathcal{P}\exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right) \not{n}\gamma_5 h_v(0)\right]_{1-\text{loop}}$$

$$= \frac{\alpha_s}{4\pi} C_G \left[t^2 \bar{q} i g G_{\mu\nu} v^\mu n^\nu \not{n}\gamma_5 h_v \left(-\frac{3}{4\epsilon} - \frac{3}{2}L + \frac{7}{4}\right)\right.$$

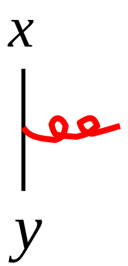
$$\left. + t^2 \bar{q} i g G_{\mu\nu} v^\mu n^\nu \not{n}\gamma_5 h_v \left(-\frac{1}{4\epsilon} - \frac{1}{2}L + \frac{1}{4}\right) + t^2 \bar{q} i g G_{\mu\nu} \gamma^\mu v^\nu \not{\bar{n}}\gamma_5 h_v \left(-\frac{1}{8\epsilon} - \frac{1}{4}L + \frac{1}{4}\right)\right]$$



$$\mathrm{P}\exp\left(ig\int_0^t d\lambda n^\mu A_\mu(\lambda n)\right)$$

$$\Rightarrow\ ig\int_0^t d\lambda n^\mu A_\mu^{(\mathrm{C})}(\lambda n)=0$$

$$\left(x^\mu A_\mu^{(\mathrm{C})}(x)=0\right)$$



$$= \frac{i\Gamma(d/2)}{2\pi^{d/2}}\frac{\not x-\not y}{\left[-(x-y)^2+i\varepsilon\right]^{d/2}}\, i\int_0^1 du\int_0^1 d\alpha\,\alpha\, gG_{(\mathrm{C})}^{\mu\rho}(\alpha u x+\alpha\bar{u}y)y_\mu x_\rho$$

$$+\frac{\Gamma(d/2-1)}{16\pi^{d/2}}\frac{\not x-\not y}{\left[-(x-y)^2+i\varepsilon\right]^{d/2-1}}\, i\int_0^1 du\, gG_{(\mathrm{C})}^{\mu\rho}(ux+\bar{u}y)\sigma_{\mu\rho}+\ \cdots$$

$$\left[\bar{q}(tn)\mathcal{P}\exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not n\gamma_5 h_v(0)\right]_{1-\mathrm{loop}}$$

$$=\frac{\alpha_s}{4\pi}\left(C_F-\frac{C_G}{2}\right)\left[t^2\bar{q}igG_{\mu\nu}v^\mu n^\nu\not n\gamma_5 h_v\left(\frac{3}{2\epsilon}+3L-\frac{5}{2}\right)\right.$$

$$+t^2\bar{q}igG_{\mu\nu}v^\mu n^\nu\not n\gamma_5 h_v\left(\frac{1}{2\epsilon}+L-1\right)+t^2\bar{q}igG_{\mu\nu}\gamma^\mu v^\nu\not n\gamma_5 h_v\left(\frac{1}{4\epsilon}+\frac{1}{2}L-\frac{1}{2}\right)$$

$$\left.+\,t^2\bar{q}gG_{\mu\nu}\sigma^{\mu\nu}\not n\gamma_5 h_v\frac{1}{4}\left(\frac{1}{2\epsilon}+L-1\right)\right]$$

OPE : to dim.5 ops. & NLO corrections in the $\overline{\mathrm{MS}}$ scheme

$$\bar{q}(tn)\mathcal{P}\exp\left(\int_0^t d\lambda n_\mu A^\mu(\lambda n)\right)\not{n}\gamma_5 h_v(0)$$

$$= \bar{q}\not{n}\gamma_5 h_v \left[1 + \frac{\alpha_s}{4\pi}C_F\left(-2L^2 - 2L - \frac{5}{12}\pi^2\right)\right] \quad \text{dim.3}$$

$$+(-it)\left\{\bar{q}(in\cdot\overleftarrow{D})\not{n}\gamma_5 h_v \left[1 + \frac{\alpha_s}{4\pi}C_F\left(-2L^2 - L - \frac{5}{12}\pi^2\right)\right]\right. \quad \text{dim.4}$$

$$\left. + \bar{q}(iv\cdot\overleftarrow{D})\not{n}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}C_F(-4L+3)\right]\right\}$$

$$+\frac{(-it)^2}{2}\left\{\bar{q}(in\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v \left[1 + \frac{\alpha_s}{4\pi}C_F\left(-2L^2 - \frac{2}{3}L - \frac{5}{12}\pi^2\right)\right]\right. \quad \text{dim.5}$$

$$+ \bar{q}(iv\cdot\overleftarrow{D})(in\cdot\overleftarrow{D})\not{n}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}C_F\left(-4L + \frac{10}{3}\right)\right]$$

$$+ \bar{q}(iv\cdot\overleftarrow{D})^2\not{n}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}C_F\left(-4L + \frac{10}{3}\right)\right]$$

$$+ \bar{q}igG_{\mu\nu}v^\mu n^\nu \not{n}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-4L + \frac{10}{3}\right) + C_G\left(7L - \frac{13}{2}\right)\right\}\right]$$

$$+ \bar{q}igG_{\mu\nu}\gamma^\mu n^\nu \not{\bar{n}}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{4}{3}L + \frac{4}{3}\right) + C_G(L-1)\right\}\right]$$

$$+ \bar{q}igG_{\mu\nu}\gamma^\mu v^\nu \not{\bar{n}}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{2}{3}L + \frac{2}{3}\right) + C_G(L-1)\right\}\right]$$

$$\left. + \bar{q}gG_{\mu\nu}\sigma^{\mu\nu}\not{n}\gamma_5 h_v \left[\frac{\alpha_s}{4\pi}\left\{C_F\left(-\frac{L}{3} + \frac{1}{3}\right) + C_G\left(\frac{L}{4} - \frac{1}{4}\right)\right\}\right]\right\}$$

Matrix elements

dim.3

$$\langle 0|\bar{q}\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)$$

$F(\mu)$: decay constant

dim.4

$$\langle 0|\bar{q}(iv\cdot \overleftarrow{D})\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iv\cdot\partial\langle 0|\bar{q}\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\bar{\Lambda}$$

$$\langle 0|\bar{q}(in\cdot \overleftarrow{D})\not{v}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\frac{4}{3}\bar{\Lambda}$$

$$\bar{\Lambda} = m_B - m_b$$

dim.5 (covariant tensor formalism)

$$\langle 0|\bar{q}\overleftarrow{D}_\mu\overleftarrow{D}_\nu\Gamma h_v|\bar{B}(v)\rangle = \frac{iF(\mu)}{2}\mathrm{Tr}\left[\gamma_5\Gamma\frac{1+\not{v}}{2}\{c_1 v_\mu v_\nu + c_2 g_{\mu\nu} + c_3(\gamma_\mu v_\nu + \gamma_\nu v_\mu) + c_4(\gamma_\mu v_\nu - \gamma_\nu v\mu)) + c_5 i\sigma_{\mu\nu}\}\right]$$

$$\begin{aligned} c_1 &= 2\bar{\Lambda}^2 + \frac{2}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2 \\ c_2 &= -\frac{\bar{\Lambda}^2}{3} - \frac{\lambda_E^2}{3} - \frac{\lambda_H^2}{3} \\ c_3 &= -\frac{\bar{\Lambda}^2}{3} - \frac{\lambda_E^2}{6} \\ c_4 &= \frac{1}{6}(\lambda_E^2 - \lambda_H^2) \\ c_5 &= \frac{\lambda_H^2}{6} \end{aligned}$$

"Chromo-electronic"

$$\langle 0|\bar{q}\alpha\cdot \mathbf{gE}\gamma_5 h_v|\bar{B}(v)\rangle = F(\mu)\lambda_E^2(\mu)$$

"Chromo-magnetic"

$$\langle 0|\bar{q}\sigma\cdot \mathbf{gH}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\lambda_H^2(\mu)$$

$$\langle 0|\bar{q}G_{\mu\nu}\Gamma h_v|\bar{B}(v)\rangle$$

LCWF from OPE

$\overline{\text{MS}}$ scheme

$$\frac{\tilde{\phi}_B(t,\mu)}{iF(\mu)} = 1 + \frac{\alpha_s(\mu)}{4\pi}C_F\left(-2L^2 - 2L - \frac{5}{12}\pi^2\right) \quad \text{dim.3}$$

$$+(-it)\cdot\frac{4}{3}\bar{\Lambda}\left[1 + \frac{\alpha_s(\mu)}{4\pi}C_F\left(-2L^2 - 4L + \frac{9}{4} - \frac{5}{12}\pi^2\right)\right] \quad \text{dim.4}$$

$$+\frac{(-it)^2}{2}\left(2\bar{\Lambda}^2\left[1 + \frac{\alpha_s(\mu)}{4\pi}C_F\left(-2L^2 - \frac{16}{3}L + \frac{35}{9} - \frac{5}{12}\pi^2\right)\right]\right. \quad \text{dim.5}$$

$$+\frac{2}{3}\lambda_E^2(\mu)\left[1 + \frac{\alpha_s(\mu)}{4\pi}\left\{C_F\left(-2L^2 - 2L + \frac{2}{3} - \frac{5}{12}\pi^2\right) + C_G\left(\frac{3}{4}L - \frac{1}{2}\right)\right\}\right]$$

$$\left.+\frac{1}{3}\lambda_H^2(\mu)\left[1 + \frac{\alpha_s(\mu)}{4\pi}\left\{C_F\left(-2L^2 - \frac{2}{3} - \frac{5}{12}\pi^2\right) + C_G\left(-\frac{1}{2}L + \frac{1}{2}\right)\right\}\right]\right)$$

Complete OPE result with $\{\tilde{C}_i\}$ up to $O(\alpha_s)$ and $\{O_i\}$ up to dim.5

double logs due to cusp singularity

completely represented by HQET parameters $\bar{\Lambda}, \ \lambda_E^2, \ \lambda_H^2$

Cut-off Moments $\int_0^{\Lambda_{UV}} d\omega\, \omega^n \phi_B(\omega)$

dim.3&4 terms: reproduce the results in cut-off scheme by Lee & Neubert ('05)

Summary

B-meson LCWF for exclusive B decays

novel behaviors different from pion LCWF

~~twist~~ $\mu \Leftrightarrow 1/t$ **different UV & IR structures**

2-step evolution

OPE of the bilocal operator for B-meson LCWF

up to dim.5 local operators

NLO corrections for Wilson coefficients

$\sim \log^2 (i\mu t)$ **terms from cusp singularity**

B-meson LCWF from the OPE

completely expressed by 3 HQET parameters $\bar{\Lambda},\ \lambda_E^2,\ \lambda_H^2$

model-independent study for behavior of B-meson LCWF ***underway!***

nonperturbative estimates at $\mu \simeq 1\,\mathrm{GeV}$ of $\bar{\Lambda},\ \lambda_E^2,\ \lambda_H^2$

latticeQCD, QCD SR