Precise Charm and Bottom Quark Masses

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Outline

1. Introduction
2. α_s from $R(s)$
3. m_c
4. m_b

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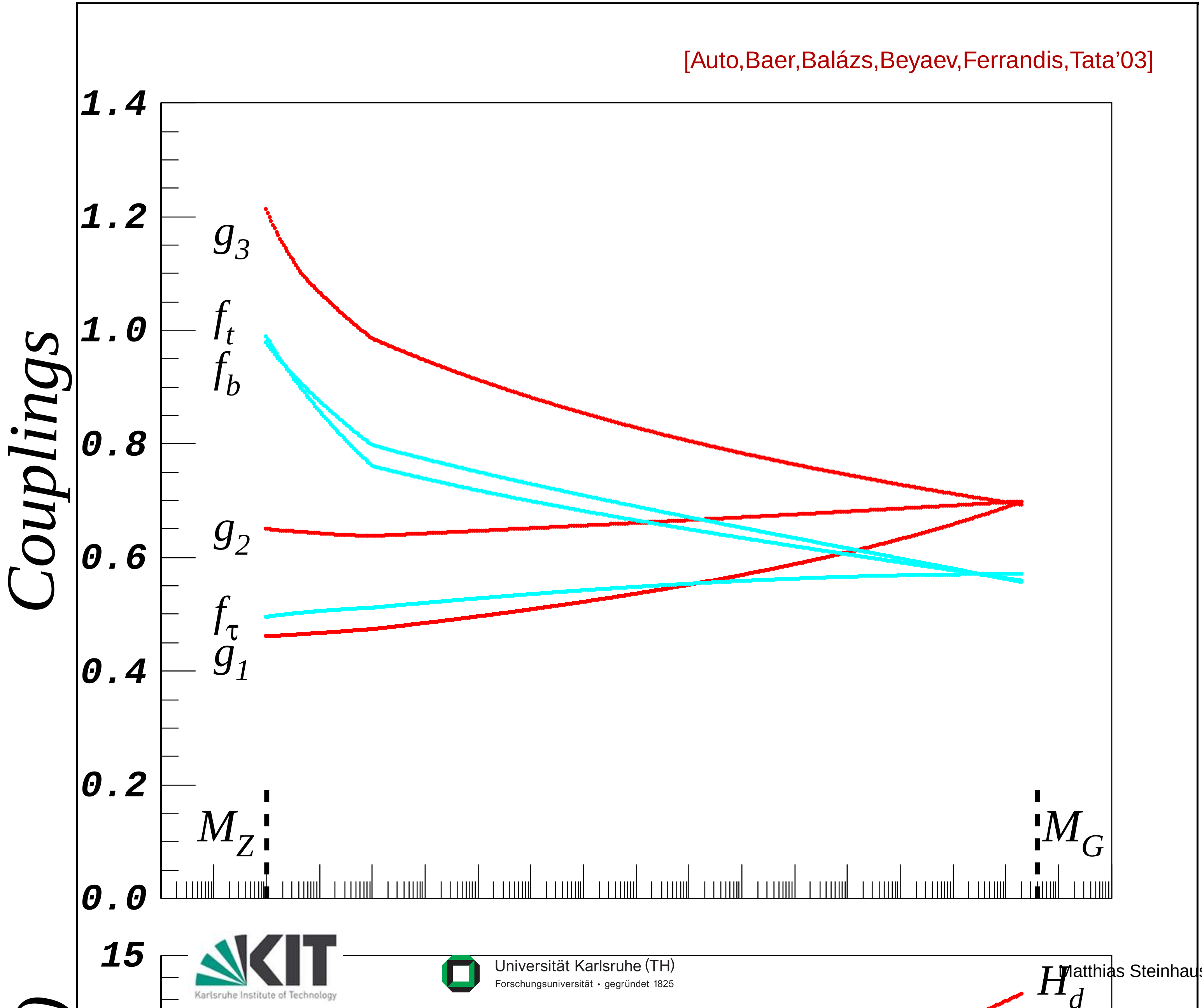
Introduction

- Quark masses
  - B decays: $\Gamma \sim m_b^5 \ldots$
  - Spectroscopy
  - Higgs decay ➪ ILC

    $\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$
  - Yukawa unification

Introduction



➪ needed:

$\frac{\delta m_t}{m_t} \approx \frac{\delta m_b}{m_b}$

$\delta m_t \approx 1$ GeV

➪

$\delta m_b \approx 25$ MeV necessary

Introduction

- Quark masses
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    $$\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$$
  - Yukawa unification
- Strong coupling α_s and quark masses

➪ Fundamental parameters of QCD/SM

Quark mass definitions

- pole mass
- $\overline{\mathrm{MS}}$ mass
- kinetic mass [Bigi,Shifman,Uraltsev,Vainshtein'97]
- 1S mass [Hoang,Smith,Stelzer,Willenbrock'99]
- PS mass [Beneke'98]
- RS mass [Pineda'01]
- ...

Light quark masses, top quark mass

PDG:

$$
\begin{aligned}
m_u &= 1.5\ldots 3.0 \text{ MeV} \\
m_d &= 3\ldots 7 \text{ MeV} \\
\overline{m} = \frac{m_u + m_d}{2} &= 2.5\ldots 5.5 \text{ MeV} \\
m_s &= 95 \pm 25 \text{ MeV}
\end{aligned}
$$

[Chetyrkin et al., Jamin et al., Lattice, . . .]

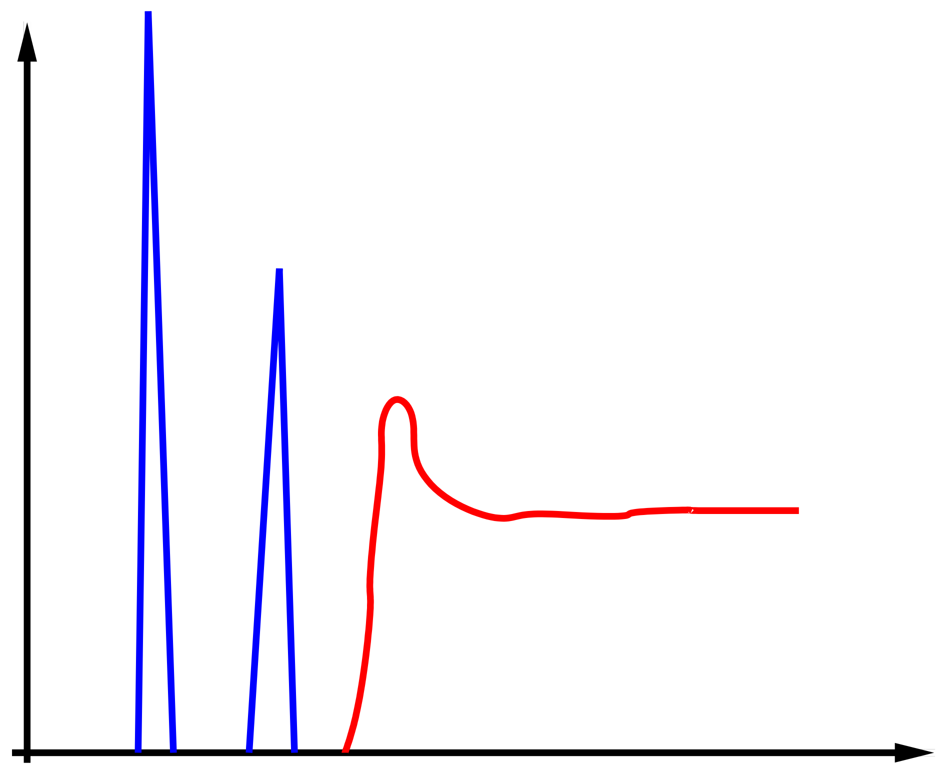
➪ less accurately known than heavy quark masses

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

[CDF,D0]

Charm/Bottom

- Consider $\sigma(e^+e^- \to$ hadrons$)$



- m_c, m_b ➪ sum rules, ("SVZ" sum rules) [Novikov et al.'78]

Charm/Bottom

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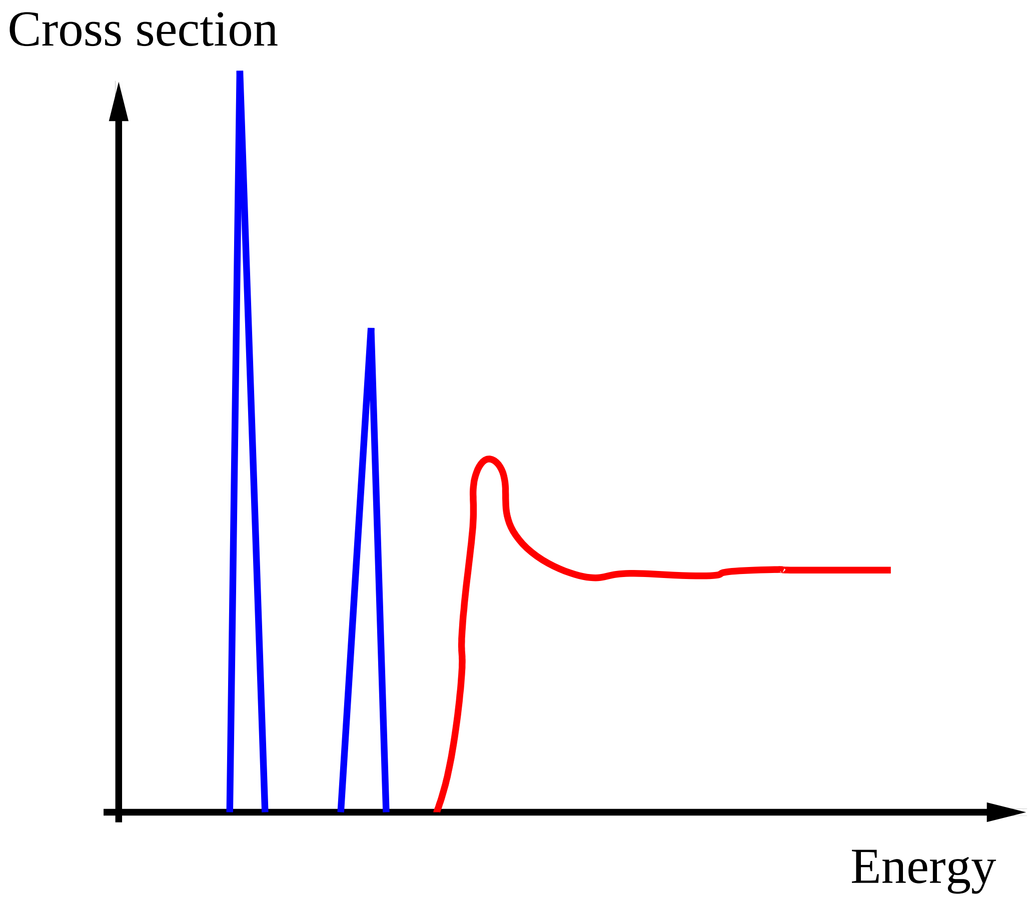
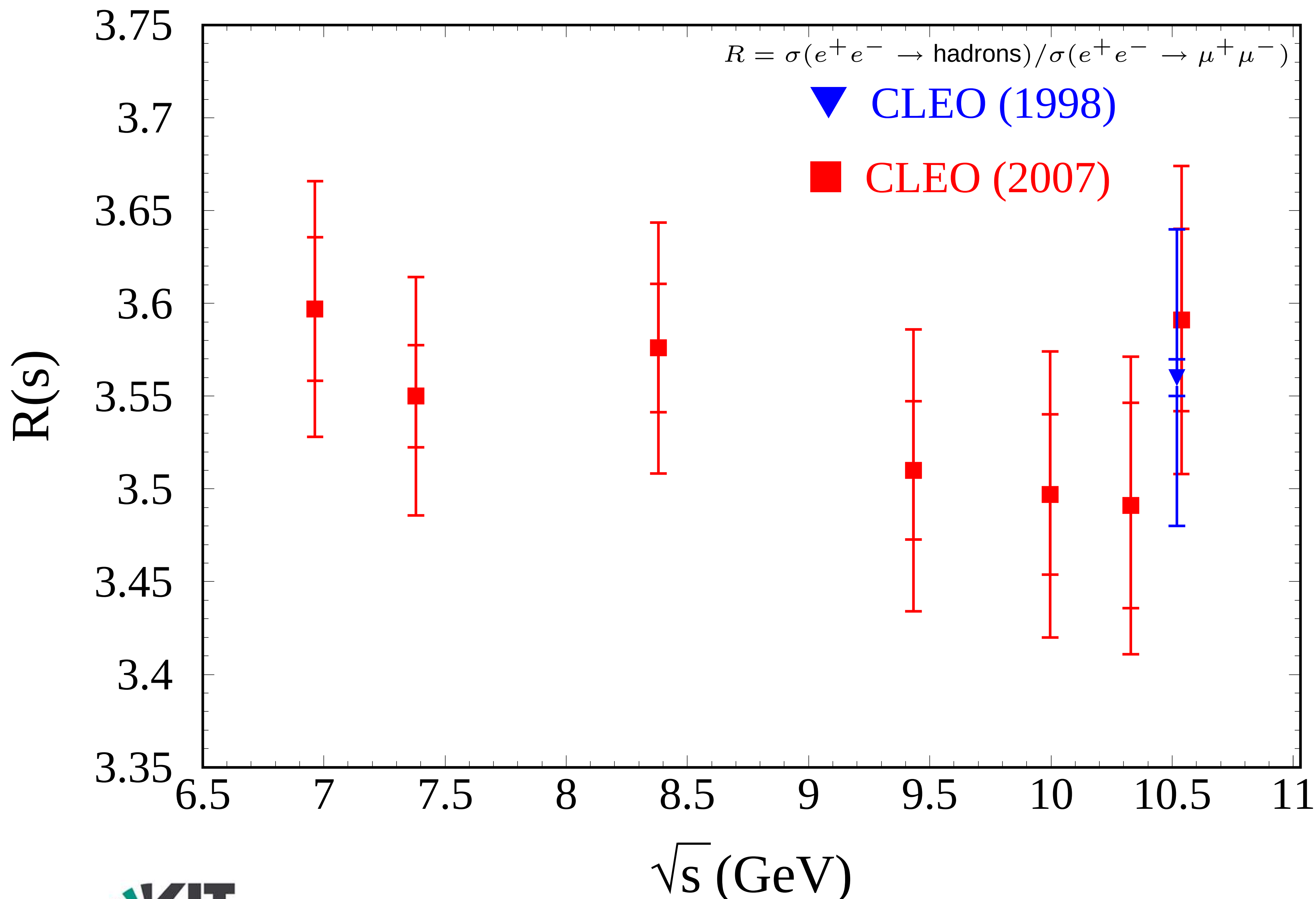


- m_c, m_b ➪ sum rules, ("SVZ" sum rules) [Novikov et al.'78]
- α_s from continuum

R measurement



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α_s and R

basic idea: $R^{\rm exp} = R^{\rm th}(\alpha_s, m_q) \Rightarrow \alpha_s$

(weak dependence on variation of m_q)

$R^{\rm th}(s)$:

rhad: [Harlander,MS'02]

- full quark mass dependence up to $\mathcal{O}(\alpha_s^2)$
- $\mathcal{O}(\alpha_s^3)$: $(m_q^2/s)^0$, $(m_q^2/s)^1$, $(m_q^2/s)^2$
- ...
- consistent running and decoupling of α_s

[v. Ritbergen,Larin,Vermaseren'97,Czakon'05]

[Chetyrkin,Kniehl,MS'97]

α_s and R

basic idea: $R^{\rm exp} = R^{\rm th}(\alpha_s, m_q) \Rightarrow \alpha_s$

(weak dependence on variation of m_q)

- $R^{\rm exp}(s) \Rightarrow \alpha_s^{(4)}(s) \quad (n_f = 4)$

$\sqrt{s}$ (GeV)	$\alpha_s^{(4)}(s)$	$\delta\alpha_s^{\rm stat}$	$\delta\alpha_s^{\rm sys,cor}$	$\delta\alpha_s^{\rm sys,uncor}$	$\alpha_s^{(4)}(s)\vert_{\rm CLEO}$
10.538	0.2113	0.0026	0.0618	0.0444	0.232
10.330	0.1280	0.0048	0.0469	0.0445	0.142
9.996	0.1321	0.0032	0.0516	0.0344	0.147
9.432	0.1408	0.0039	0.0526	0.0291	0.159
8.380	0.1868	0.0187	0.0461	0.0195	0.218
7.380	0.1604	0.0131	0.0404	0.0138	0.195
6.964	0.1881	0.0221	0.0386	0.0134	0.237

$\Uparrow$

massless approx.!!!

α_s and R

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(weak dependence on variation of m_q)

- $R^{\text{exp}}(s) \Rightarrow \alpha_s^{(4)}(s)$ ($n_f = 4$)
- Evolve to common scale and combine
  $\Rightarrow \alpha_s^{(4)}(9\ \text{GeV}) = 0.160 \pm 0.024 \pm 0.024$

α_s and R

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- $\alpha_s^{(4)}(9\ {\rm GeV}) \to \alpha_s^{(4)}(\mu_b^{\rm dec}) \to \alpha_s^{(5)}(\mu_b^{\rm dec}) \to \alpha_s^{(5)}(M_Z)$
  (practically) independent from $\mu_b^{\rm dec}$ (4-loop running and 3-loop decoupling)

  RunDec: [Chetyrkin,Kühn,MS'00]

  $\Rightarrow \alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$

[Kühn,MS,Teubner'07]

α_s and R

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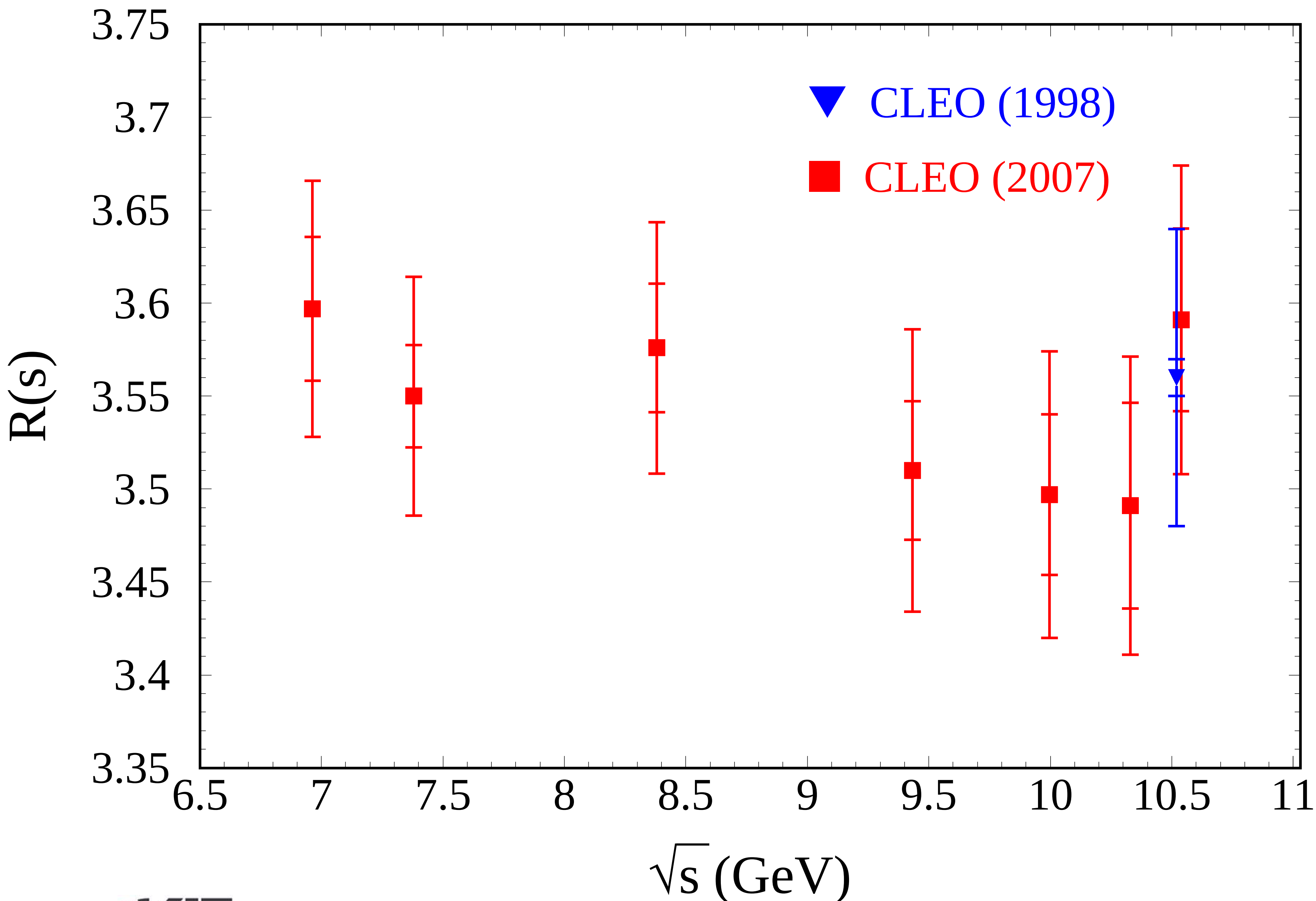
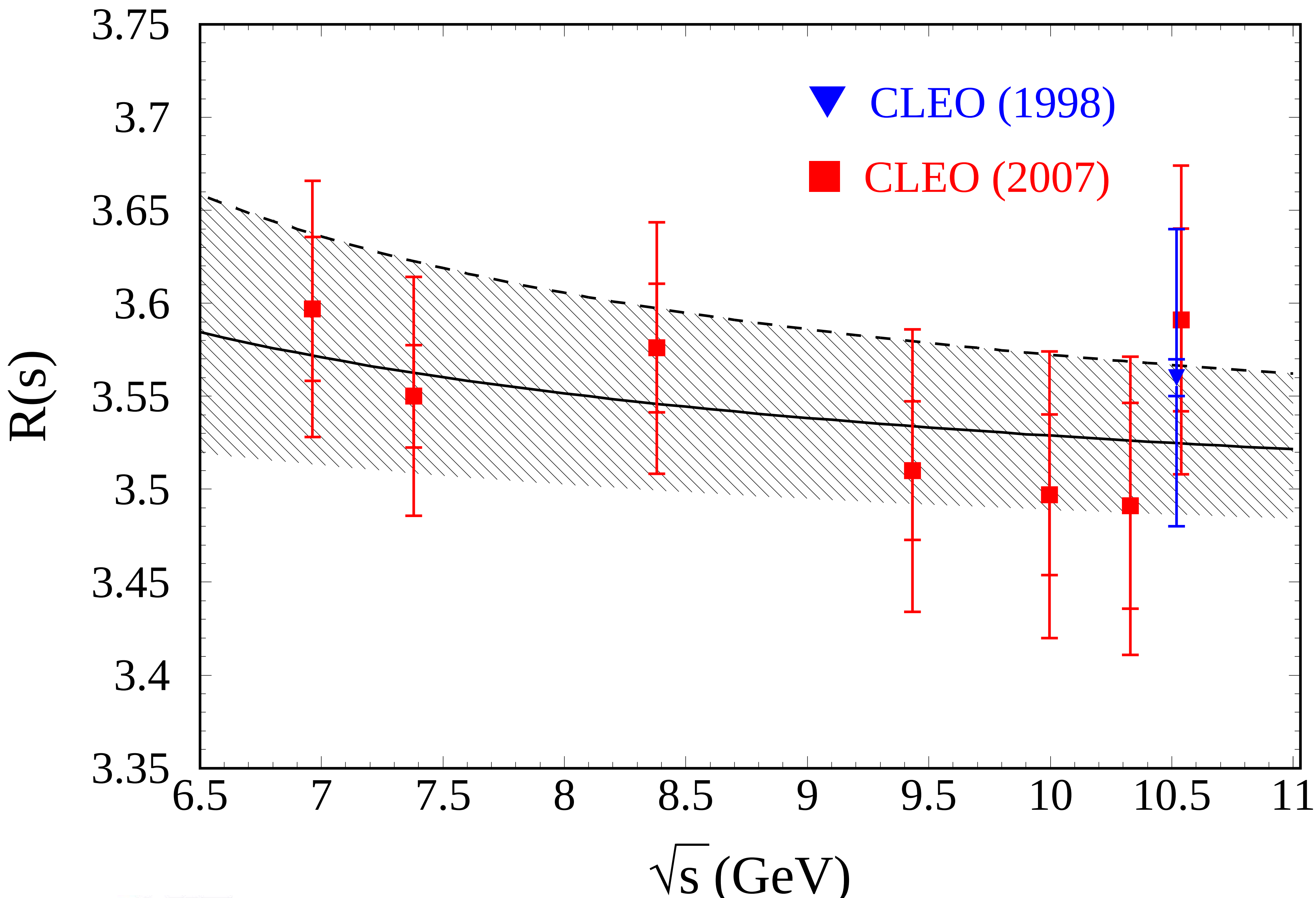
  [Kühn,MS,Teubner'07]
- CLEO analysis: $\alpha_s^{(5)}(M_Z^2)|_{\text{CLEO}} = 0.126 \pm 0.005^{+0.015}_{-0.011}$
  - massless approximation for $R(s)$
  - no decoupling of α_s

R: experiment + theory



R: experiment + theory



α_s from R

- $\alpha_s^{(5)}(M_Z) = 0.110^{+0.010+0.010}_{-0.012-0.011} = 0.110^{+0.014}_{-0.017}$ [Kühn,MS,Teubner'07]
- Combine with $\alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$ [Kühn,MS'01]
  R measurements between 2 and 10.5 GeV from BES'01, MD-1'96, CLEO'97

⇨ $\alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$

- Compare: $\alpha_s^{(5)}(M_Z) = 0.1189 \pm 0.0010$ [Bethke'06]

Sum rules

$$R_Q = \frac{\sigma(e^+e^- \to Q\bar{Q} + \ldots)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s) \qquad \text{(moments)}$$

$$R_Q = 12\pi \,\mathrm{Im}\left[\Pi_Q(q^2 = s + i\varepsilon)\right]$$

$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\bigg|_{q^2=0}$$

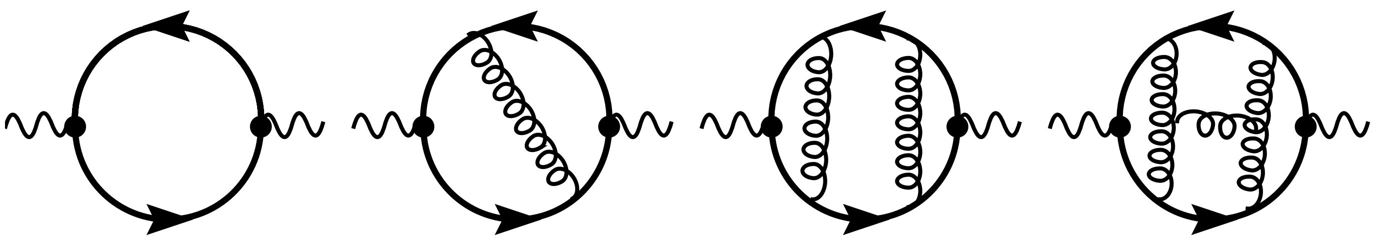
(dispersion relation)

$\mathcal{M}_n^{\rm th}$

$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{\rm d}{{\rm d}q^2}\right)^n \Pi_Q(q^2)\Big|_{q^2=0}$$

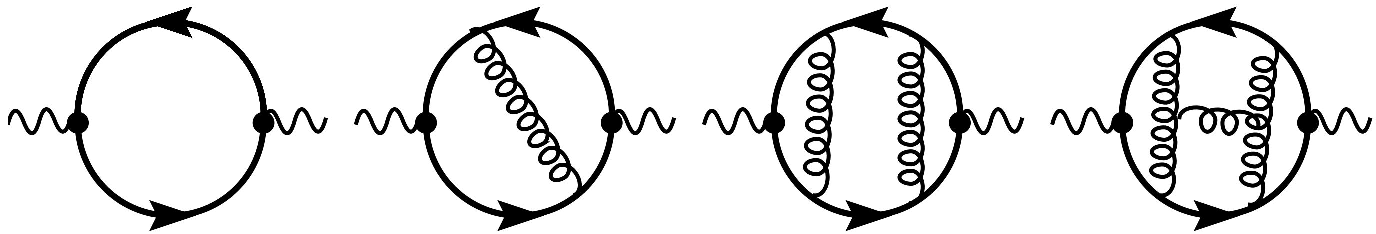
➪ compute Taylor expansion

$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2}\sum_{n\geq 0} \bar{C}_n \left(\frac{q^2}{4m_Q^2}\right)^n$$



$$\mathcal{M}_n^{\rm th} = \frac{9}{4} Q_Q^2 \left(\frac{1}{4m_Q^2}\right)^n \bar{C}_n \qquad \frac{\delta m_Q}{m_Q} = \frac{1}{2n}\frac{\delta \mathcal{M}_n}{\mathcal{M}_n}$$

C_n to 4 loops



- 1, 2 and 3 loops: MATAD [MS'96-'00]
- 4 loops:
  - method: 1. reduce to master integrals [Laporta,Remiddi'96; Laporta'01]
    2. compute masters
  - several Million equations; several GB tables
  - all steps cross-checked
    1. [Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]
    2. [Schröder,Vuorinen'05; Chetyrkin,Faisst,Sturm,Tentyukov'06],. . .

C_n to 4 loops

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^3\left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3\right)$$

$$l_{m_c} = \ln(m_c^2/\mu^2)$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	−0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

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$$-6.0 \leq \bar{C}_2^{(30)} \leq 7.0\,, -6.0 \leq \bar{C}_3^{(30)} \leq 5.2\,, -6.0 \leq \bar{C}_4^{(30)} \leq 3.1$$

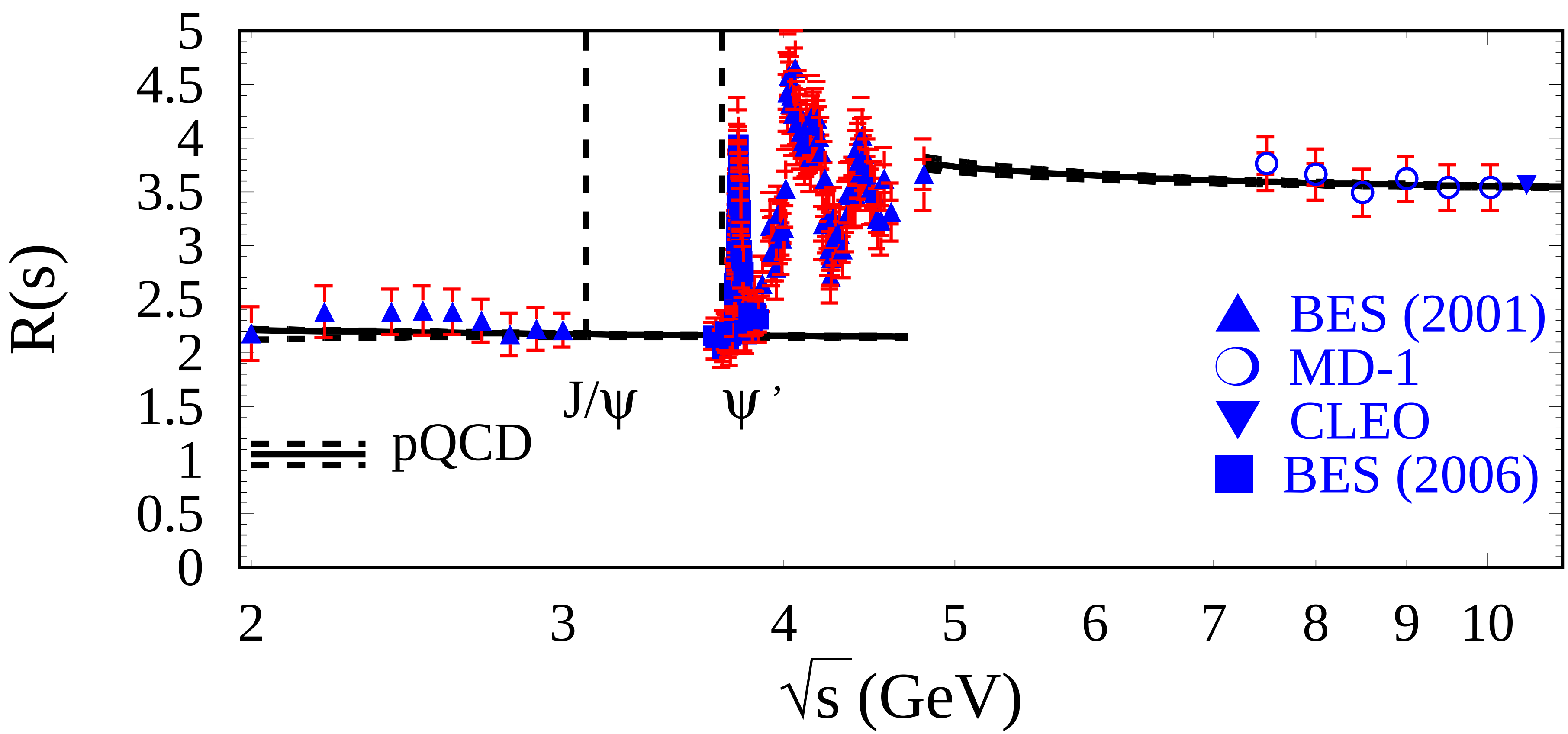
$$\mathcal{M}^{\text{exp}} = \mathcal{M}^{\text{res}} + \mathcal{M}^{\text{thresh}} + \mathcal{M}^{\text{cont}}$$

- $\mathcal{M}^{\text{res}}$: $$R^{\text{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$$

	J/Ψ	$\Psi(2S)$
M_Ψ(GeV)	3.096916(11)	3.686093(34)
Γ_{ee}(keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

$$\mathcal{M}^{\text{exp}} = \mathcal{M}^{\text{res}} + \mathcal{M}^{\text{thresh}} + \mathcal{M}^{\text{cont}}$$

- $\mathcal{M}^{\text{res}}$: $\quad R^{\text{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2}\left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$
- $\mathcal{M}^{\text{thresh}}$:



$\mathcal{M}^{\rm exp} = \mathcal{M}^{\rm res} + \mathcal{M}^{\rm thresh} + \mathcal{M}^{\rm cont}$

- $\mathcal{M}^{\rm res}$: $R^{\rm res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$
- $\mathcal{M}^{\rm thresh}$: 3.73 GeV $\leq \sqrt{s} \leq 4.8$ GeV, BES01,06
- $\mathcal{M}^{\rm cont}$: $\sqrt{s} \geq 4.8$ GeV

no data
$R^{\rm theory}$ ⇨ full mass dependence up to $\mathcal{O}(\alpha_s^2)$ rhad: [Harlander,MS'02]

$\mathcal{M}^{\mathrm{exp}}$

n	$\mathcal{M}_n^{\mathrm{res}} \times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{thresh}} \times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{cont}} \times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{exp}} \times 10^{(n-1)}$	$\mathcal{M}_n^{\mathrm{np}} \times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

m_c

$$\mathcal{M}_n^{\rm th} + \mathcal{M}_n^{\rm np} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

$$m_c(\mu) \;=\; \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp} - \mathcal{M}_n^{\rm np}}\right)^{1/(2n)}$$

- 1. set $\mu = 3$ GeV ⇨ $m_c(3$ GeV$)$
  2. RGE ⇨ $m_c(m_c)$
- Uncertainties
  - $\delta\mathcal{M}_n^{\rm exp}$
  - $\alpha_s(M_Z) = 0.1189 \pm 0.0020$ [Bethke'06]; $\delta\alpha_s \times 2$
  - $\mu = (3 \pm 1)$ GeV
  - $\delta\mathcal{M}_n^{\rm np}$

m_c

$$\mathcal{M}_n^{\rm th} + \mathcal{M}_n^{\rm np} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

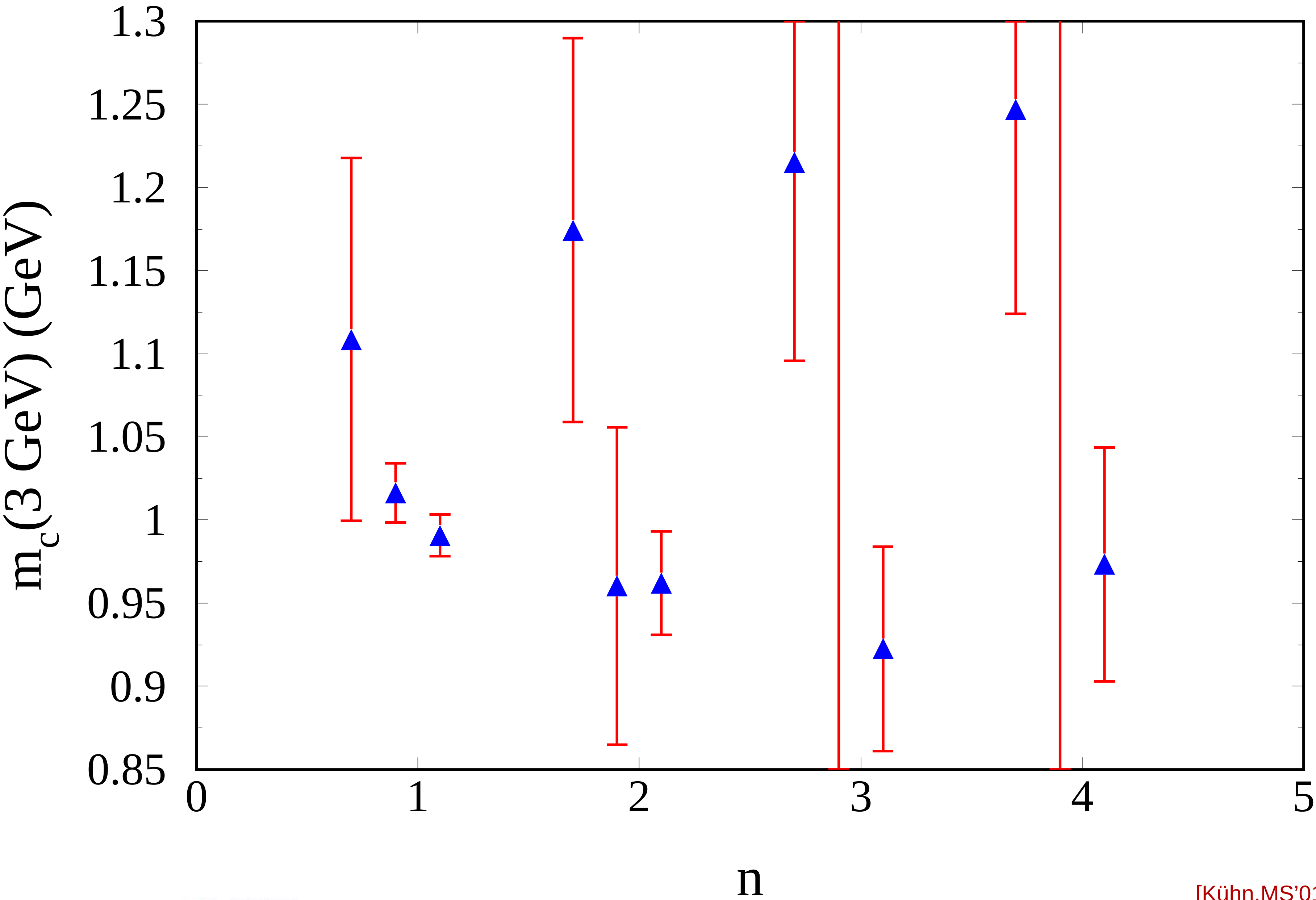
$$m_c(\mu) \;=\; \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp} - \mathcal{M}_n^{\rm np}}\right)^{1/(2n)}$$

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{(30)}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$$m_c(3\text{ GeV}) = 0.986(13)\text{ GeV}$$

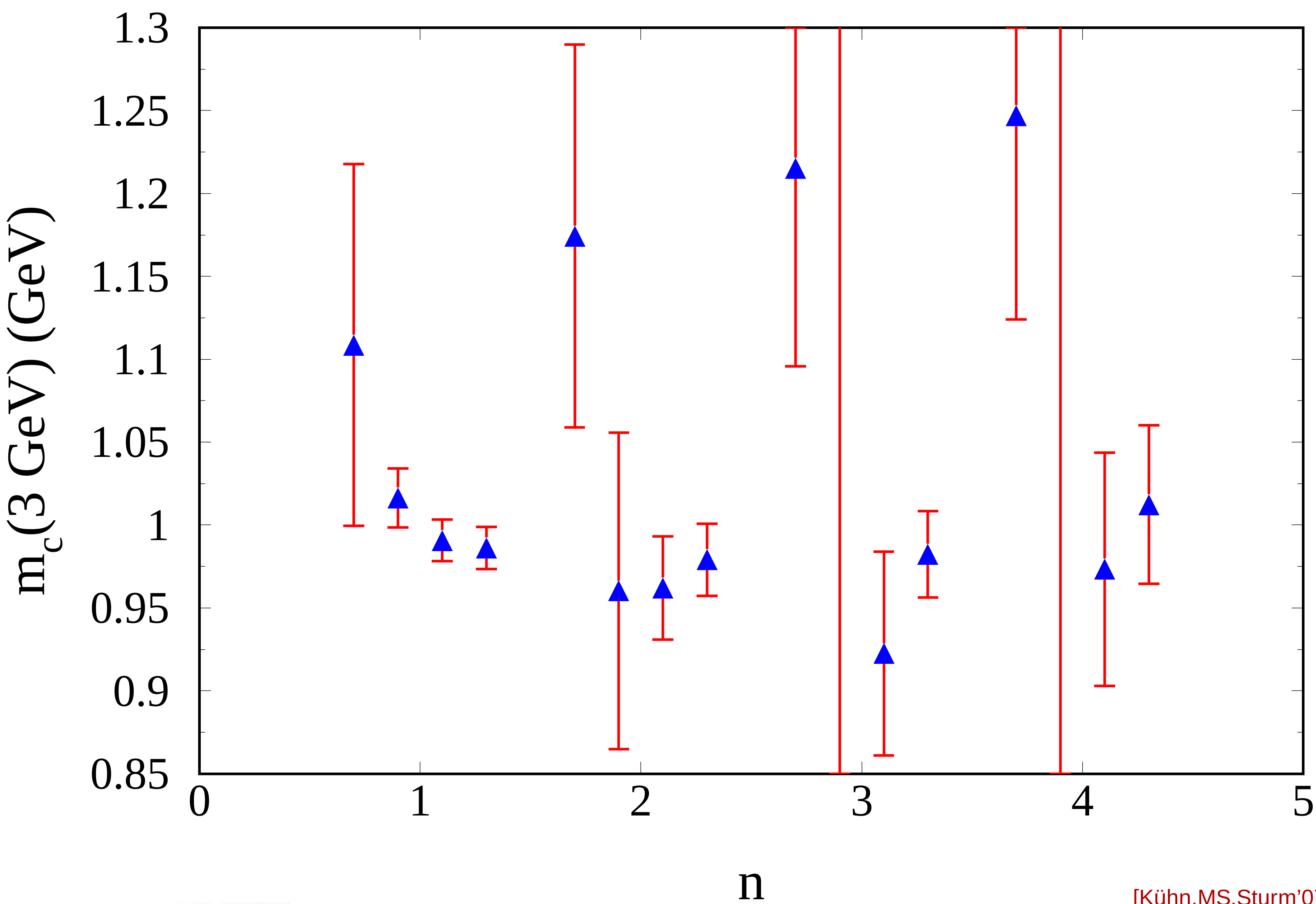
[Kühn,MS,Sturm'07]

$m_c(3\ \mathbf{GeV})$



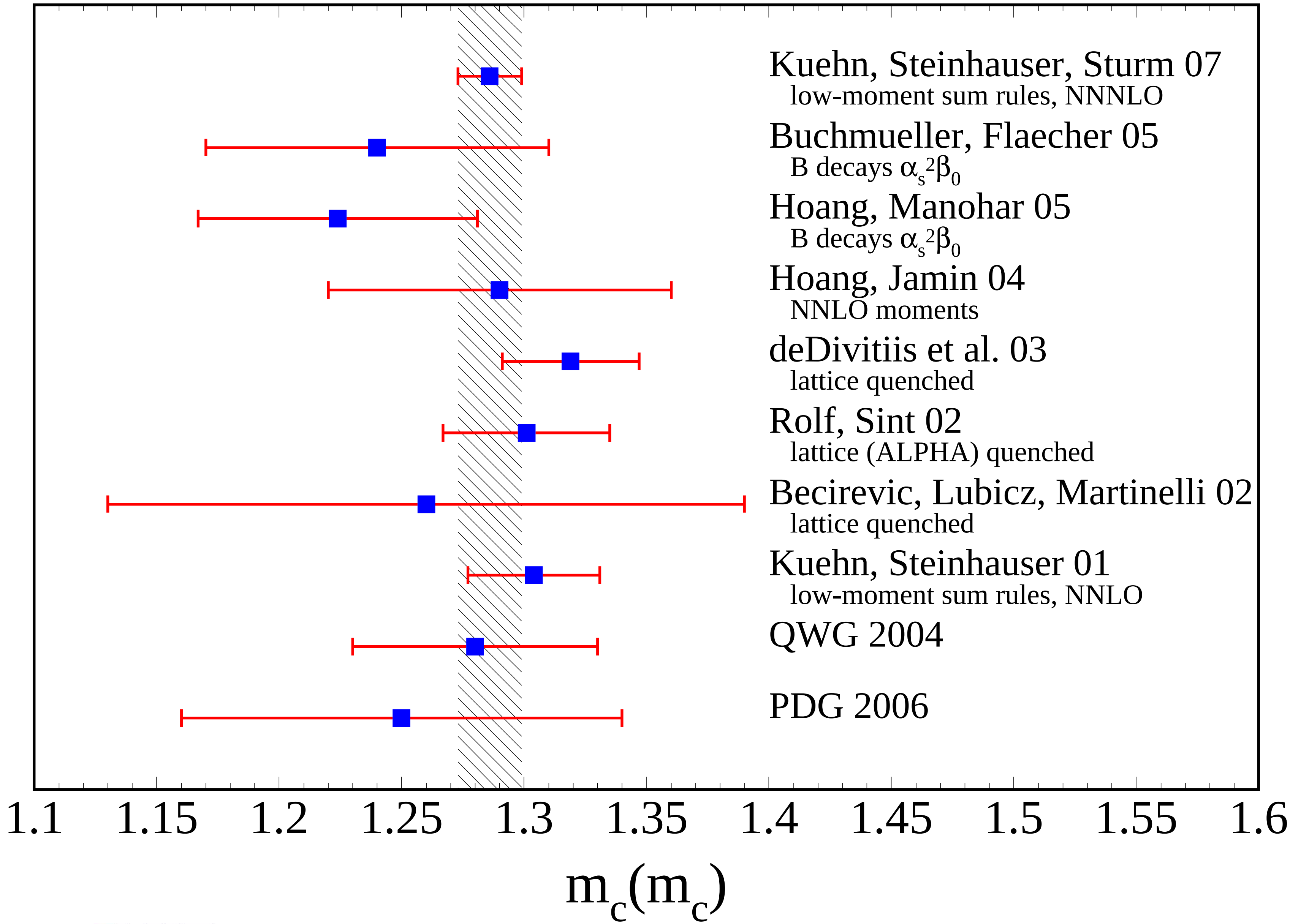
[Kühn,MS'01]

$m_c(3\ \textbf{GeV})$



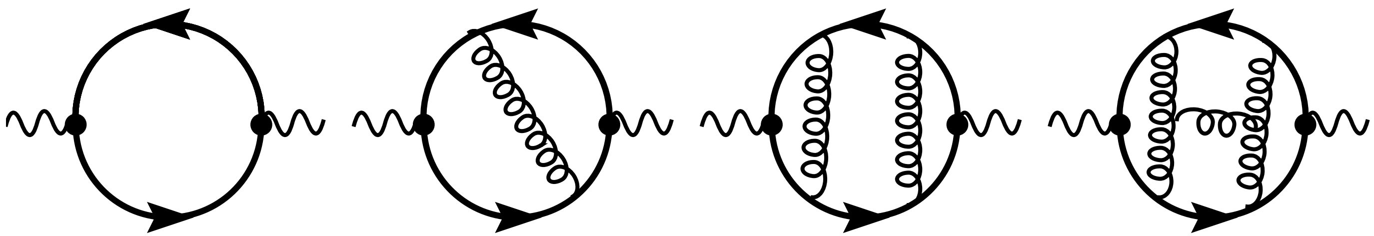
[Kühn,MS,Sturm’07]

Charm — comparison



Bottom quark

- $\mathcal{M}_n^{\rm th}$: see charm, $n_f = 5$



- $\mathcal{M}_n^{\rm np}$: negligible
- $\mathcal{M}^{\rm res}$: $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$
- $\mathcal{M}^{\rm thresh}$: CLEO data up to 11.24 GeV
- $\mathcal{M}^{\rm cont}$: pQCD above 11.24 GeV

m_b

$$\mathcal{M}^{\text{th}} \stackrel{!}{=} \mathcal{M}^{\text{exp}}$$

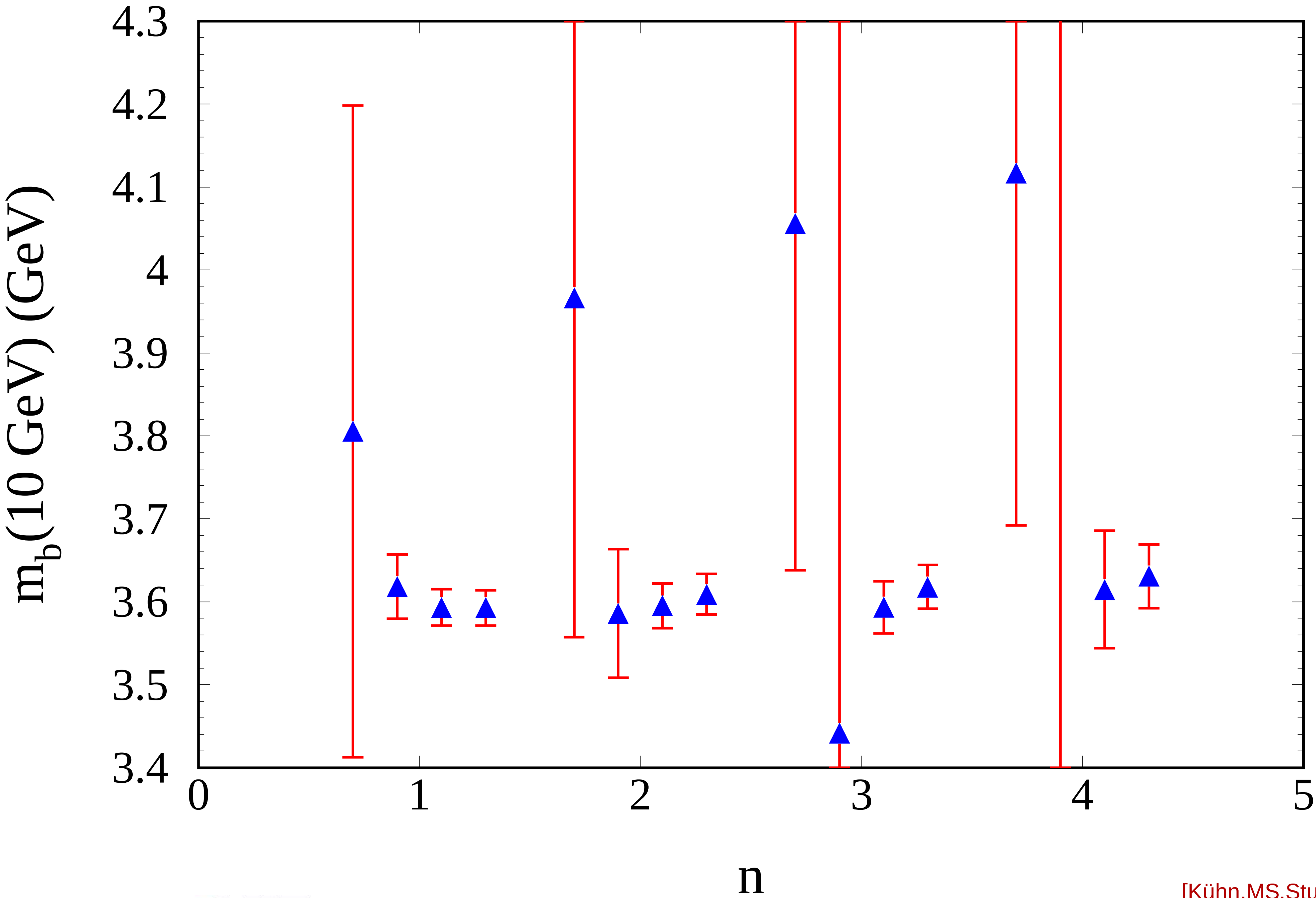
$$m_b(\mu) \quad = \quad \frac{1}{2}\left(\frac{1}{4}\frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}}}\right)^{1/(2n)}$$

n	$m_b(10$ GeV$)$	exp	α_s	μ	total	$\delta\bar{C}_n^{(30)}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$$m_b(10 \text{ GeV}) = 3.609(25) \text{ GeV}$$

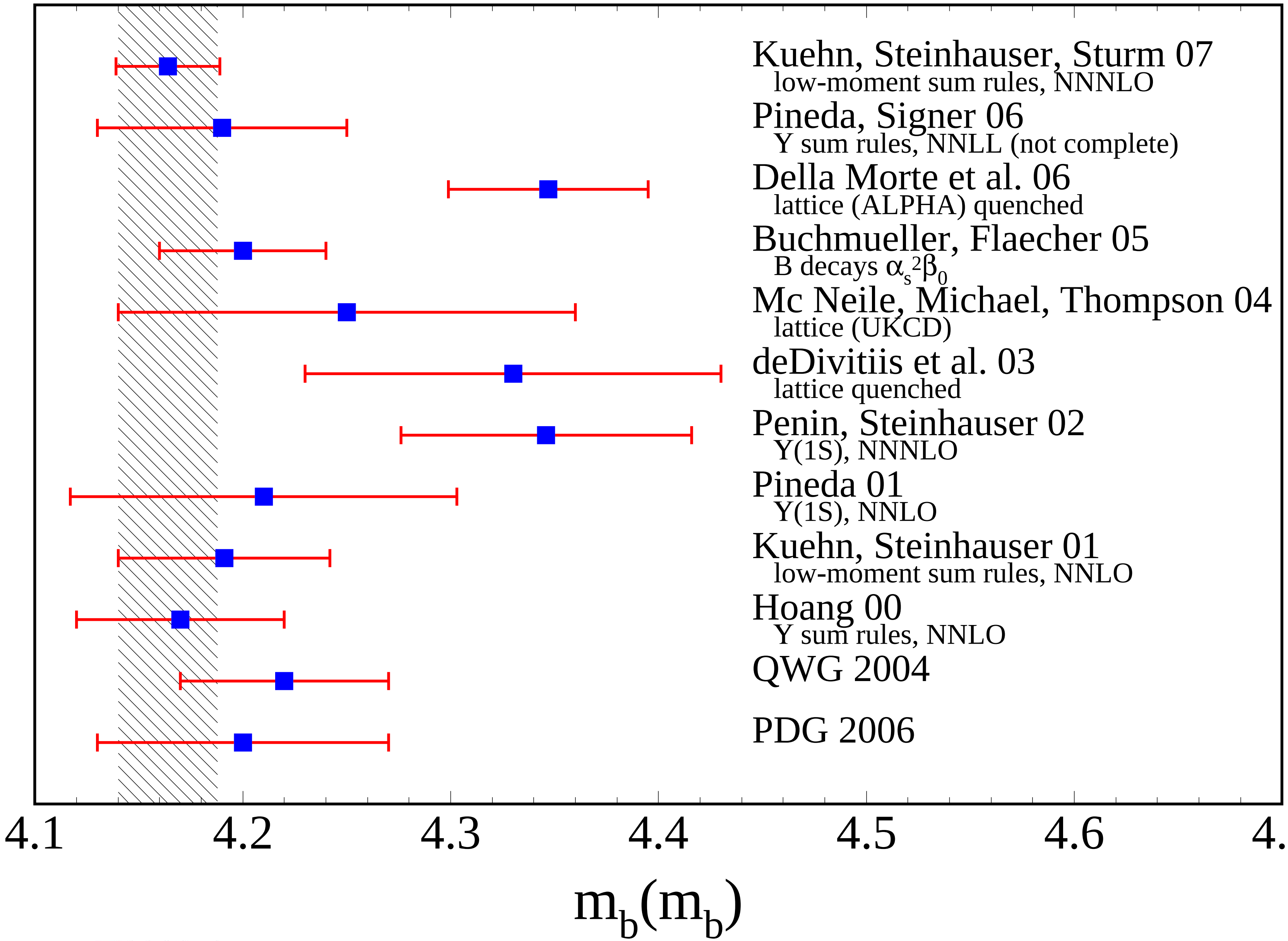
[Kühn,MS,Sturm'07]

$m_b(10\ \mathbf{GeV})$



[Kühn,MS,Sturm'07]

Bottom — comparison



Conclusions

- Most precise values for m_c and m_b
  $m_c(m_c) = 1.286(13)$ GeV $\qquad m_b(m_b) = 4.164(25)$ GeV
- NNNLO analysis
- $\overline{\mathrm{MS}}$ mass
- Possible improvements: experimental measurements:
  $R(s)$, Γ_{ee}
- $\frac{\delta m_s}{m_s} \approx 10\%$
  $\frac{\delta m_c}{m_c} \approx 1\%$
  $\frac{\delta m_b}{m_b} \approx 0.6\%$
  $\frac{\delta m_t}{m_t} \approx 1\%$
- $R^{\mathrm{exp}} \Rightarrow \alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$

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