Matching Coefficients in NRQCD and HQET

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Outline

Fermionic corrections to the three-loop matching coefficient of the vector current.

Nucl.Phys.B758:144-160,2006;
P.M., J.Piclum, D. Seidel, M. Steinhauser

Introduction

- matching coefficients: Connection between QCD and non-relativistic QCD (NRQCD)

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- important for threshold behaviour of bottom and top system
- top production at threshold at a future linear collider $\Rightarrow$ determination of top quark mass
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- decay of heavy mesons
- fermionic non-singlet contribution with one light fermion loop

Calculation

$$j_v^\mu = \bar{Q}\gamma^\mu Q \qquad \leftrightarrow \qquad \tilde{j}^i = \phi^\dagger \sigma^i \chi$$

$$j_v^k = c_v(\mu)\tilde{j}^k + \frac{d_v(\mu)}{6m_Q^2}\phi^\dagger \sigma^k \vec{D}^2 \chi + \dots$$

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \dots$$

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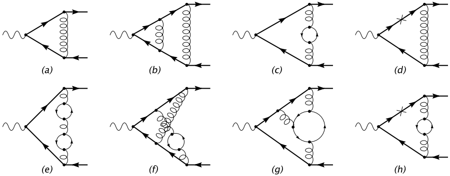
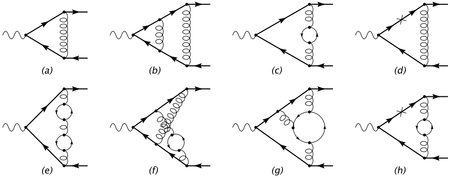
- c_v independent of external momenta $\Rightarrow$ threshold expansion $\Rightarrow$ evaluate Γ_v for $s = 4m^2$
- $\tilde{\Gamma}_v$ tree level
- $\tilde{Z}_2 = 1$
- $\tilde{Z}_v$ calculated
- Z_2 n_l part recalculated and agreement with known result found

Calculation

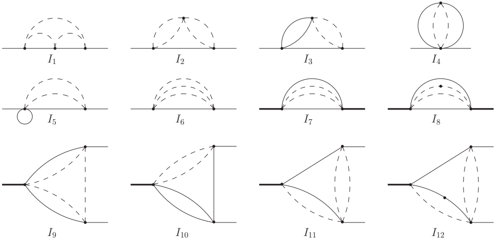


Calculation



- diagrams generated with qgraf
- 4 three-loop topologies identified and mapped with the help of q2e and exp
- calculation done with form
- reduction to 12 master integrals with crusher
- master integrals calculated/checked with Mellin-Barnes (MB.m)

Master Integrals



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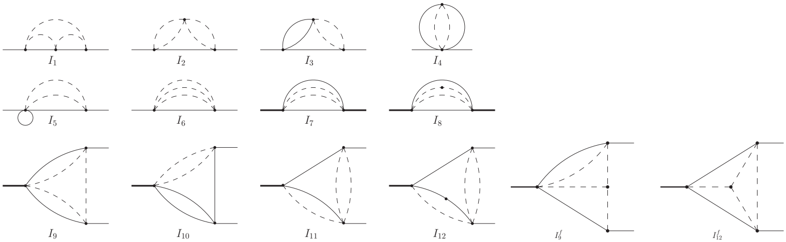
Master Integrals



- Some coefficients of the ϵ expansion of the master integrals only known numerically
- Try to avoid expansion of master to more than $\mathcal{O}(\epsilon^0)$ $\Rightarrow$ ϵ-finite basis
- ϵ-inite integrals more complicated $\Rightarrow$ lower numerical accuracy
- but helpful to determine the analytical results for some of the "normal" master integrals

Bottom System

Decay rate of $\Upsilon(1S)$ into leptons

$$\Gamma(\Upsilon(1S) \to l^+ l^-) = \Gamma_{\mathrm{LO}} \rho_1 \left[c_v^2(m_b) + \frac{C_F^2 \alpha_s^2(\mu_s)}{12} c_v(m_b) (d_V(m_b) + 3) \right]$$

with $\alpha_s(M_Z) = 0.118$, $m_b = 5.3$ GeV and $\mu_s = 2.0967$ GeV

$$\Gamma_1 \approx \Gamma_1^{LO}(1 - 0.446_{NLO} + 1.75_{NNLO} - 1.67_{NNNLO})$$

changes to

$$\Gamma_1 \approx \Gamma_1^{LO}(1 - 0.446_{NLO} + 1.75_{NNLO} - 1.20_{NNNLO})$$

convergence improved

Top System

normalized $t\bar{t}$ cross section at a linear collider

$$R(e^+e^- \to t\bar{t}) = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$R(e^+e^- \to t\bar{t}) = R_{\mathrm{LO}}\rho_1 \left[c_V^2(m_b) + \frac{C_F^2\alpha_s^2(\mu_s)}{12} c_V(m_b)(d_V(m_b)+3) \right]$$

with $\alpha_s(M_Z) = 0.118$, $m_t = 175$ GeV and $\mu_s = 32.625$ GeV

$$R_1 \approx \Gamma_1^{LO}(1 - 0.243_{NLO} + 0.435_{NNLO} - 0.268_{NNNLO})$$

changes to

$$R_1 \approx \Gamma_1^{LO}(1 - 0.243_{NLO} + 0.435_{NNLO} - 0.195_{NNNLO})$$

Sum of all corrections ~ 0.003

Three-Loop Chromomagnetic Interaction in HQET.

arXiv:0707.1388; submitted to Nucl. Phys, B

A. Grozin, P.M., J. Piclum, M. Steinhauser

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- responsible for breaking of the spin symmetry $\Rightarrow B - B^*$ mass splitting
- chromomagnetic matching coefficient known up to two loops
- charm mass effects at two loops known
- three-loop calculation of the matching coefficient and the anomalous dimension
- byproduct: heavy quark magnetic moments

Calculation

Lagrangian of HQET

$$\mathcal{L}_{\mathrm{HQET}} = \bar{Q}_v i v D Q_v + \frac{1}{2m_Q}\left(O_k + C_{cm}(\mu) O_{cm}(\mu)\right) + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

chromomagnetic operator

$$O_{cm} = \frac{1}{2}\bar{Q}_v G_{\mu\nu}\sigma^{\mu\nu} Q_v$$

scattering amplitude

$$\bar{u}_v(-q)\left[v^\mu - \frac{q^\mu}{2m_Q} - \frac{i}{2m_Q}\sigma^{\mu\nu} q_\nu (1+\mu_c)\right] t_a u_v(0)$$

matched with in HQET

$$\bar{u}_v(-q)\left[v^\mu - \frac{q^\mu}{2m_Q} - \frac{i}{2m_Q}\sigma^{\mu\nu} q_\nu C^0_{cm}\right] t_a u_v(0)$$

on QCD side calculate quark-antiquark-gluon vertex with onshell quarks and zero momentum transfer

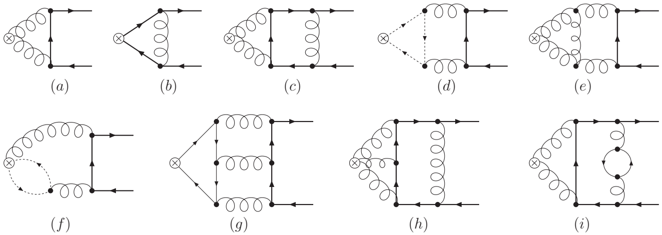
$$\Gamma^{\mu} = \gamma^{\mu} F_1(q^2) - \frac{i}{2m_Q}\sigma^{\mu\nu} q_{\nu} F_2(q^2)$$

colour charge

$$Z_2^{OS} F_1(0) = 1 \quad \Rightarrow \quad F_1(0)^{-1} = Z_2^{OS}$$

chromomagnetic moment

$$\gamma_{cm} = Z_2^{OS} F_2(0) \quad \Rightarrow \quad C_{cm}(\mu) = Z_{cm}(\alpha_s(\mu))[1 + \mu_{cm}(\alpha_s(\mu))]$$



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- Use projectors to calculate F_1, F_2

$$F_1(q^2) = \frac{1}{2(d-2)(q^2-4m_Q^2)}$$
$$\times \mathrm{Tr}\left\{(\not{p}_1 + m_Q)\left(\gamma_\mu + \frac{4m_Q(d-1)}{q^2-4m_Q^2}p_\mu\right)(\not{p}_2 + m_Q)\,\Gamma^\mu\right\}$$
$$F_2(q^2) = -\frac{2m_Q^2}{(d-2)q^2(q^2-4m_Q^2)}$$
$$\times \mathrm{Tr}\left\{(\not{p}_1 + m_Q)\left(\gamma_\mu + \frac{4m_Q^2+(d-2)q^2}{m_Q(q^2-4m_Q^2)}p_\mu\right)(\not{p}_2 + m_Q)\,\Gamma^\mu\right\}$$

- projectors develop pole in momentum $q \Rightarrow$ expand up to q^2 $\Rightarrow$ onshell propagators
- calculation done with arbitrary gauge
- successfully checked $Z_2^{OS} = F_1(0)^{-1}$

Results

matching coefficient

$$
\begin{aligned}
C_{cm}(m_c) &= 1 + 0.2309 + 0.1835 + 0.2362 = 1.6506 \\
C_{cm}(m_b) &= 1 + 0.1492 + 0.0676 + 0.0497 = 1.2664 \\
C_{cm}(m_t) &= 1 + 0.0741 + 0.0144 + 0.0045 = 1.0930
\end{aligned}
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anomalous dimension

$$
\begin{aligned}
\gamma_{cm}(m_c) &= 0.1599 + 0.0298 + 0.0163 = 0.2060 \\
\gamma_{cm}(m_b) &= 0.1033 + 0.0099 + 0.0029 = 0.1160 \\
\gamma_{cm}(m_t) &= 0.0513 + 0.0018 + 0.0002 = 0.0533
\end{aligned}
$$

Heavy Quark Mass Splittings

mass splittings

$$m_{B^*}^2 - m_B^2 = \frac{4}{3} C_{cm}^{(4)}(\mu) \mu_{G(4)}^2(\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

$$R = \frac{m_{B^*}^2 - m_B^2}{m_{D^*}^2 - m_D^2} = 0.88 \quad \text{(exp.)}$$

resummed logs

$$R = 0.8517 - 0.0696 - 0.0908 + [-0.1285] = 0.6914 + [-0.1285]$$

fixed order

$$R = 1 - 0.1113 - 0.0780 - 0.0755 + \ldots = 0.7352 + \ldots$$

Magnetic moments

replace external gluon by photon

$$\frac{a_Q}{Q_Q} = 0.2122\,\alpha_s^{(n_f)}(m_Q) + (0.8417 - 0.0469\,n_l)\left[\alpha_s^{(n_f)}(m_Q)\right]^2 + \left(4.5763 - 0.5856\,n_l + 0.0145\,n_l^2\right)\left[\alpha_s^{(n_f)}(m_Q)\right]^3 + \mathcal{O}\alpha_s^4$$

$$\begin{aligned} a_c &= 0.0478 + 0.0533 + 0.0758 = 0.1770 \\ a_b &= -0.0153 - 0.0103 - 0.0084 = -0.0340 \\ a_t &= 0.0152 + 0.0047 + 0.0017 = 0.0215 \end{aligned}$$

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$$a_b(m_Z) = -0.0083 - 0.0066 - 0.0056 = -0.0206$$

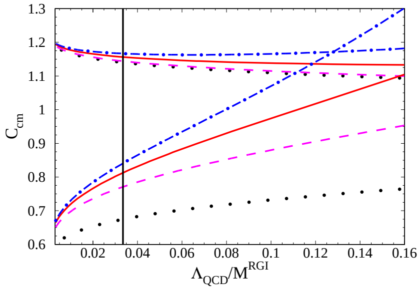
exp: $a_b > -0.135$ (68% CL)

Lattice

- matching coefficient also needed for lattice calculations
- rewrite matching coefficient in terms of Λ_{QCD}/M^{RGI}

$$M^{RGI} = \bar{m}_* \left(2\beta_0 \frac{\alpha_s(\bar{m}_*)}{\pi}\right)^{-\frac{\gamma_{m,0}}{\beta_0}} \exp\left[-\int_0^{\alpha_s(\bar{m}_*)} \left(\frac{\gamma_m}{\beta} - \frac{\gamma_{m,0}}{\beta_0}\right) \frac{d\alpha_s}{\alpha_s}\right]$$

- improved convergence, error ≈ 1 %



Conclusion

- NRQCD
  - non-singlet three loop contributions calculated
  - generally improves convergence
  - still incomplete

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- NRQCD
  - non-singlet three loop contributions calculated
  - generally improves convergence
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- HQET
  - chromomagnetic moment calculated
  - results for mass splittings not in good agreement with experimental value
  - magnetic moments calculated
  - important input for lattice calculations with improved convergence