QCD Factorization for Top Quark Mass Reconstruction

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Based on:

S. Fleming, S. Mantry, I.W. Stewart, AHH, hep-ph/0703207

I.W. Stewart, AHH, arXiv:0709.3519

… more work in progress

RadCor 2007, GGI, Florence , Oct 1-5, 2007

Outline

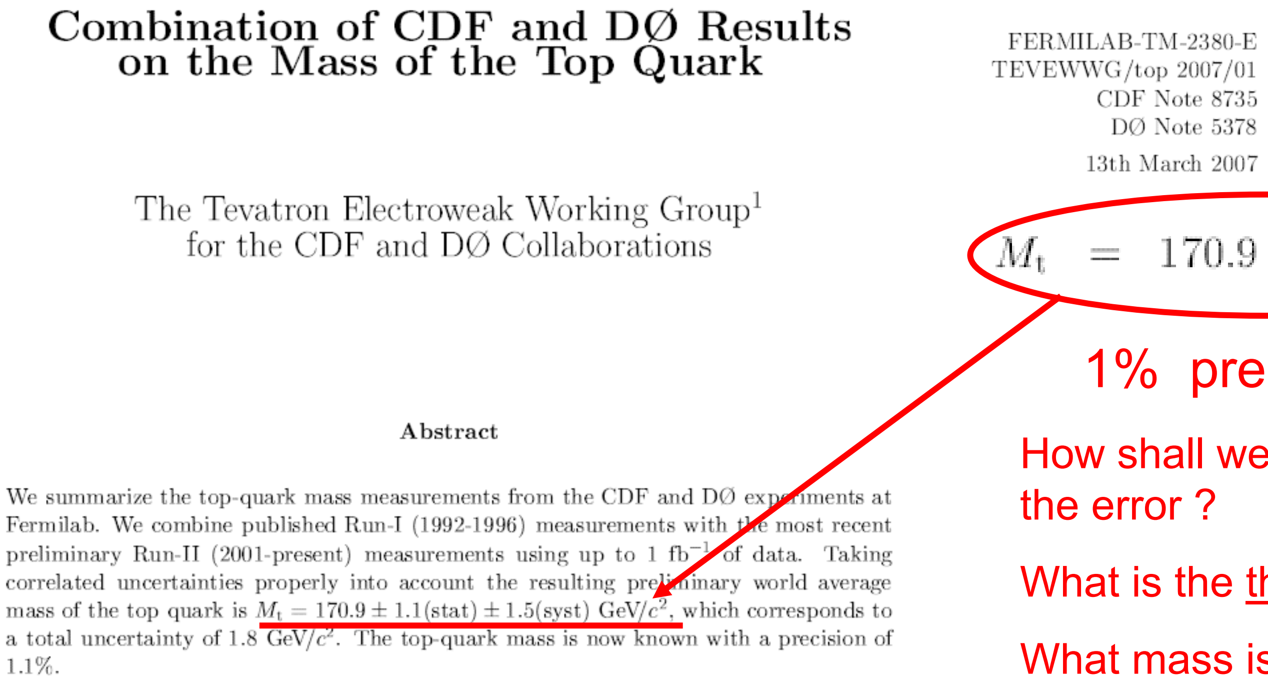
- Why do we want a precision m_t ? What kind of precision.
- Methods for top mass determinations
- Factorization theorem for t and $\bar{t}$ invariant mass distribution in electron-positron annihilation ($Q \gg m_t \gg \Gamma_t$)

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

  - EFT derivation of Factorization at LO in $m/Q,\ \Gamma_t/m$
  - Top mass determination to better than $\Lambda_{\rm QCD}$ (at least in principle)
  - Which top mass is (not) measured ?
  - Ingredients of NLL order analysis
  - LO numerical (toy) analysis
- Summary

Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays "before hadronization" ($\Gamma_t \approx 1.5 \text{ GeV}$)



Combination of CDF and DØ Results on the Mass of the Top Quark

FERMILAB-TM-2380-E
TEVEWWG/top 2007/01
CDF Note 8735
DØ Note 5378

13th March 2007

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 1 fb^{-1} of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 170.9 \pm 1.1(\text{stat}) \pm 1.5(\text{syst})\ \text{GeV}/c^2$, which corresponds to a total uncertainty of 1.8 GeV/c^2. The top-quark mass is now known with a precision of 1.1%.

$$M_t \;=\; 170.9 \pm 1.8\ \text{GeV}/c^2$$

1% precision !

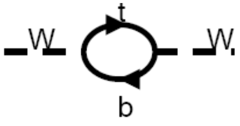
How shall we theorists judge the error ?

What is the theoretical error ?

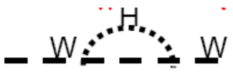
What mass is it ?

Need for a precise Top mass

Electroweak precision observables

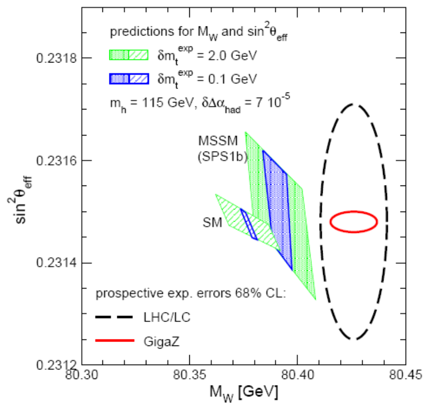


$\delta M_W \propto m_t^2$



$\delta M_W \propto \ln(M_H)$

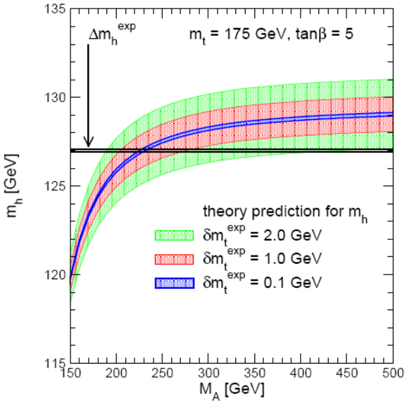
$$\sin\theta_W \times \left(1+\delta(m_t, m_H, \ldots)\right)$$
$$= 1 - \frac{M_W^2}{M_Z^2}$$



Mass of Lightest MSSM Higgs Boson

	LHC	LC
δm_h	1 GeV	50 MeV
needed δm_t	4 GeV	0.2 GeV
expected δm_t	1-2 GeV	~ 0.1 GeV

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$



- Best precision possible wanted.
- Mass definition (with small error) needs to be well defined. (Which mass is measured at Tevatron ?)

Basic Methods

Threshold Scan

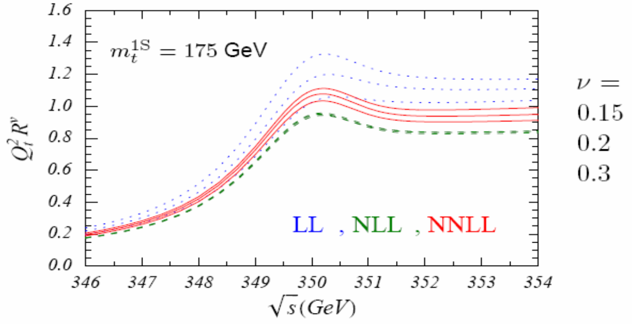
$Q \approx 2m_t$

ILC

Miquel, Martinez;
Boogert, Gounaris

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood

(renormalons, summations)



Manohar,Stewart,Teubner,AH

$\rightarrow \delta m_t^{\rm exp} \simeq 50$ MeV

$\rightarrow \delta m_t^{\rm th} \simeq 100$ MeV

What mass?

$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr}$ + pert.series

(short distance mass: 1S $\leftrightarrow$ $\overline{\rm MS}$)

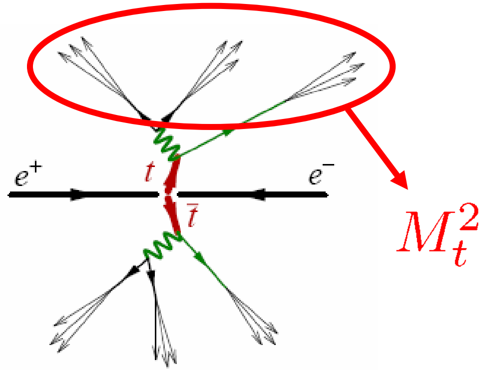
"threshold masses"

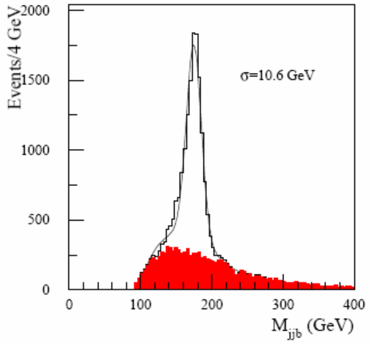
Invariant Mass Reconstruction

$Q \geq 2m_t$

Tev +LHC + ILC

- data available soon
- measures different top mass ?
- uncertainties (much) more involved
- many different methods available
- error around 1 GeV challenging





Reconstruction Methods

Lepton + Jets

Atlas study:
Borjanovic etal

Plain Method:

- W reconstruction + b tagging
- Inv. mass from M_{jjb}
- $\Delta R = 0.4$
- 100.000 events after cuts

Major error sources:

- b jet energy scale: $x \times 0.7\ \mathrm{GeV}$
- FSR jets: $1\ \mathrm{GeV}$

Kinematic Fit:

- reconstruct entire event
- impose constraints (e.g. $M_{jjb} = M_{jl\nu}$)
- vary unknows freely

- b jet energy scale: $x \times 0.7\ \mathrm{GeV}$
- FSR jets: $< 0.5\ \mathrm{GeV}$

Continuous Jet Definition:

- plain method for varying cone size: $\Delta R = 0.3 \ldots 1.0$
- take weighted average
- FSR error reduced

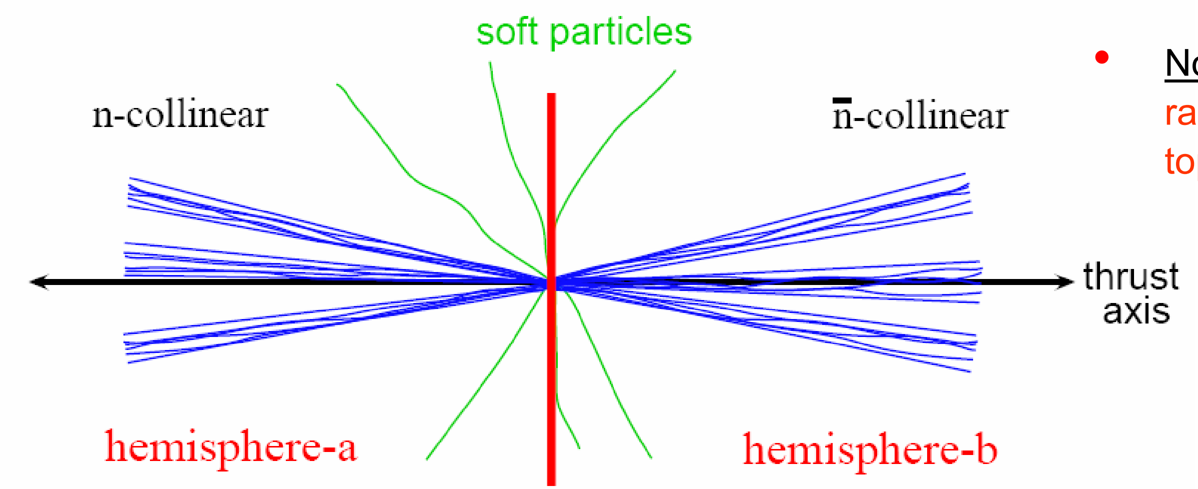
Large p_T Events:

event shape-like !

- select events with $p_T > 200\ \mathrm{GeV}$
- top pair back-to-back $\Longrightarrow$ decay products in different hemispheres
- large cone size around top/antitop jet axes: $\Delta R = 0.8 \ldots 1.8$
- M_t and $M_{\bar{t}}$ from in-cone momenta
- strong sensitivity to soft jets + Underlying Events
- Mass scale calibrations (W mass)

- UE: $1.3\ \mathrm{GeV}$
- calibration $1\ \mathrm{GeV}$

ILC: Ideal Study Case



$$M_t^2 = \Big(\sum_{i\,\epsilon\,a} p_i^\mu\Big)^2 \qquad M_{\bar t}^2 = \Big(\sum_{i\,\epsilon\,b} p_i^\mu\Big)^2$$

- No beam remnant : all soft radiation can be assigned to top or antitop

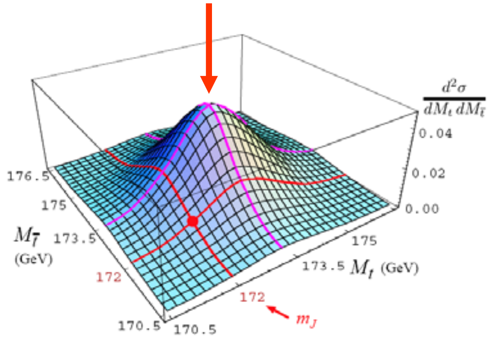
$$\frac{d^2\sigma}{dM_t\,dM_{\bar t}}$$

ILC at large c.m. energy: $Q \gg m_t$

- **Dijet limit: QCD factorization** á la Korchemsky/Sterman
- related to thrust and heavy jet mass event shapes
- set up similar to Chekanov/Morgunov experimental analysis (based on k_T jet algorithm)
- unstable particle effective theory method (Fadin, Khoze; Beenaker etal, Beneke etal, Reisser,AH)

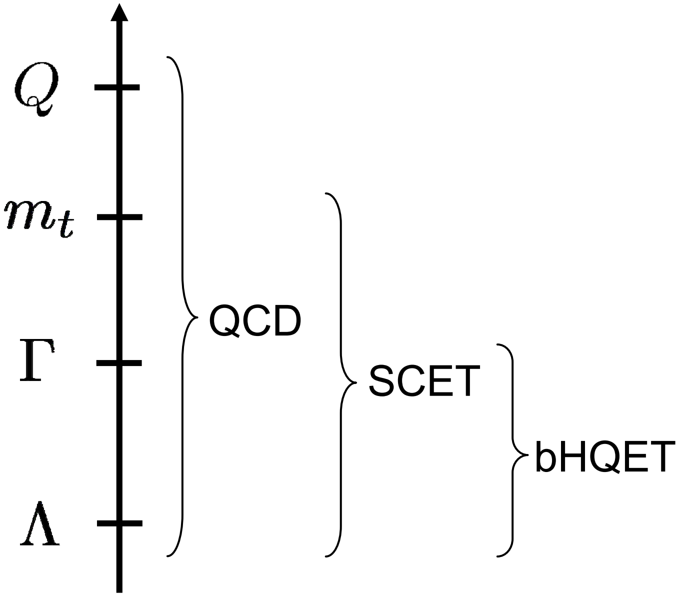
Resonance region:

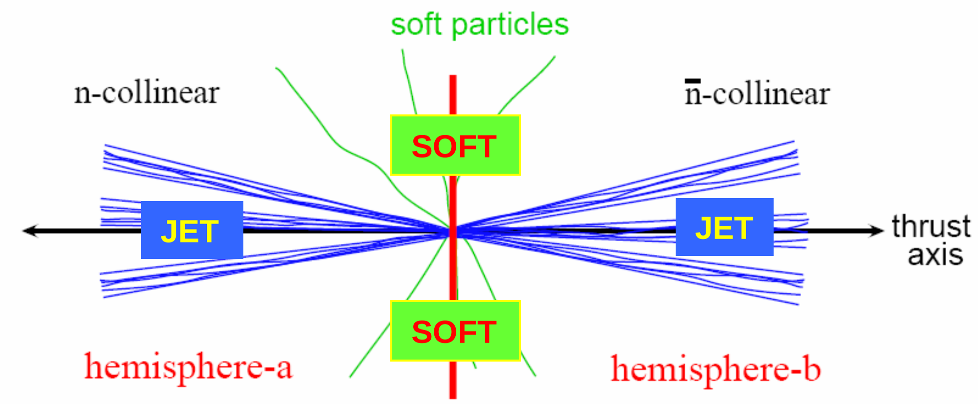
$$M_{t,\bar t} - m_t \sim \Gamma$$



Factorization Theorem

Fleming, Mantry, Stewart, AH
hep-ph/0703207





LO factorization theorem

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \qquad \hat s = \frac{M_t^2 - m_J^2}{m_J}$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \underbrace{B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)}_{\text{JET}} \underbrace{B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)}_{\text{JET}} \underbrace{S_{\text{hemi}}(\ell^+, \ell^-, \mu)}_{\text{SOFT}}$$

Step 1: Factorization in SCET

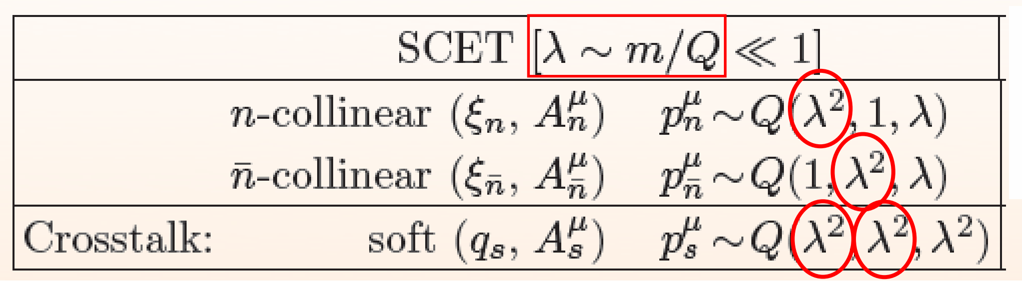
$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+,\mu)\, J_{\bar{n}}(s_{\bar{t}} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

- Integrate out Q
- Invariant mass fluctuations $s_{t,\bar{t}} = M_{t,\bar{t}}^2 - m_t^2 \sim m_t^2 \;\rightarrow\; \Gamma_t$
- Jet functions still contain large logs in peak region: $J_{n,\bar{n}}(s, m_t, \Gamma_t, \mu)$
- top decay products (b,W) still appear as explicit d.o.f.
- applicable to massless jet event shapes (thrust, etc.)

Soft-Collinear Effective Theory

(Bauer, Fleming, Luke, Pirjol, Stewart)

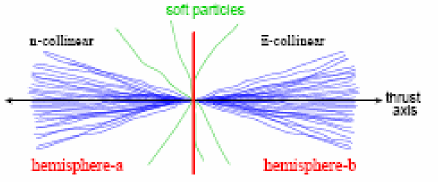
Degrees of freedom



	SCET $[\lambda \sim m/Q \ll 1]$	
	n-collinear (ξ_n, A_n^μ)	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar n$-collinear $(\xi_{\bar n}, A_{\bar n}^\mu)$	$p_{\bar n}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft (q_s, A_s^μ)	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

quark fields

gluon fields



Light-cone coordinates

$$p^\mu = (+, -, \perp)$$

$$p_n^2 = p_n^+ p_n^- + p_\perp^2 \sim m^2 \ll Q^2$$

$$(p_n + p_s)^2 \sim m^2 \ll Q^2$$

Leading order Lagrangian (n-collinear)

$$\mathcal{L}_{qn}^{(0)} = \bar\xi_n \Big[in\cdot D_s + gn\cdot A_n + (i\not{D}_c^\perp - m) W_n \frac{1}{\bar n\cdot\mathcal{P}} W_n^\dagger (i\not{D}_c^\perp + m)\Big] \frac{\not{\bar n}}{2} \xi_n$$

collinear-soft coupling

collinear Wilson line

massive SCET:
Leibovich etal,
Boos, Feldmann, Mannel etal

$$iD_s^\mu = i\partial^\mu + g A_s^\mu$$

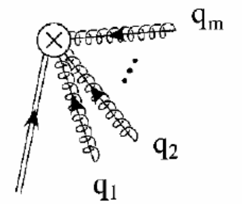
$$W_n(x) = P\exp\left(ig\int_0^\infty ds\, \bar n\cdot A_n(s\bar n)\right)$$

Soft-Collinear Effective Theory

Top pair production current

$$J^\mu_i(0) = \int d\omega\, d\bar\omega\, C(\omega,\bar\omega,\mu)\,\mathcal{J}^\mu_i(\omega,\bar\omega,\mu)$$

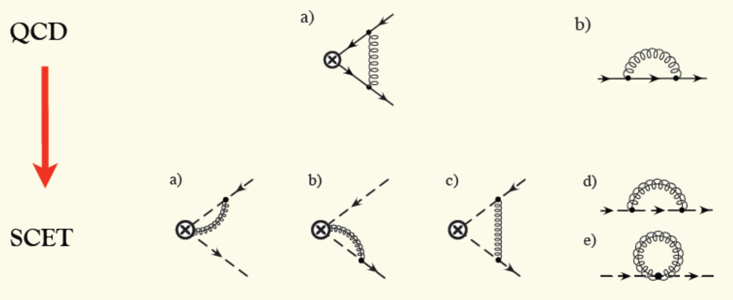
QCD ↑ Wilson Coeff. ↑ SCET ↑



$$W = \sum_{m=0}^{\infty} \sum_{\text{perms}} \frac{(-g)^m}{m!} \frac{\bar n\cdot A^{a_1}_{n,q_1} \cdots \bar n\cdot A^{a_m}_{n,q_m}}{\bar n\cdot q_1\, \bar n\cdot(q_1+q_2)\cdots \bar n\cdot(\sum_{i=1}^m q_i)} T^{a_m}\cdots T^{a_1}$$

$$\mathcal{J}^\mu_i(\omega,\bar\omega,\mu) = \bar\chi_{n,\omega}(0)\Gamma^\mu_i\chi_{\bar n,\bar\omega}(0)\ , \quad \chi_{n,\omega}(0) = \delta(\omega-\bar{\mathcal{P}})(W^\dagger\xi_n)(0)$$

→ one-loop matching



$$C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\ln\frac{-Q^2}{\mu^2} - \ln^2\frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

- agrees with massless SCET
- known to $\mathcal{O}(\alpha_s^2)$

Kramer, Lampe ’87

Matsuura, v.d. March, v. Neerven ’88, ‘89

Gehrmann, Huber, Maitre ‘05

Soft-Collinear Effective Theory

QCD Cross Section

$$\sigma = \sum_X^{res.} (2\pi)^4 \, \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \, \langle 0 | \mathcal{J}_i^{\nu\dagger}(0) | X \rangle \langle X | \mathcal{J}_i^{\mu}(0) | 0 \rangle$$

Integrate out hard fluctuations at Q

$$\mathcal{J}_i^\mu(0) = \int d\omega \, d\bar\omega \, C(\omega, \bar\omega, \mu) \, J_i^{(0)\mu}(\omega, \bar\omega, \mu)$$

SCET Cross Section (LO in m/Q)

$$\sigma = \sum_{\vec n} \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4 \, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \sum_i L^{(i)}_{\mu\nu} \int d\omega \, d\bar\omega \, d\omega' \, d\bar\omega'$$
$$\times C(\omega, \bar\omega) C^*(\omega', \bar\omega') \langle 0 | \bar\chi_{\bar n, \bar\omega'} \bar\Gamma^\nu_j \chi_{n,\omega'} | X_n X_{\bar n} X_s \rangle \langle X_n X_{\bar n} X_s | \overline{\chi}_{n,\omega} \Gamma^\mu_i \chi_{\bar n, \bar\omega} | 0 \rangle$$

factorization of asymptotic final states

$$|X\rangle = |X_n X_{\bar n} X_s\rangle = \underbrace{|X_n\rangle}_{\text{Collinear: } n} \otimes \underbrace{|X_{\bar n}\rangle}_{\text{Collinear: } \bar n} \otimes \underbrace{|X_s\rangle}_{\text{Soft}}$$

Soft-Collinear Effective Theory

Factorization: soft-collinear decoupling

$$\sigma \;=\; \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4\,\delta^4(q-P_{X_n}-P_{X_{\bar n}}-P_{X_s}) \sum_{ij} L_{\mu\nu}^{(ij)} \int d\omega\, d\bar\omega\, d\omega'\, d\bar\omega'$$
$$\times C(\omega,\bar\omega) C^\dagger(\omega',\bar\omega') \langle 0|\bar\chi_{n,\omega}\Gamma_i^\mu \chi_{\bar n,\bar\omega}|X_n X_{\bar n} X_s\rangle\langle X_n X_{\bar n} X_s|\bar\chi_{\bar n,\bar\omega'}\bar\Gamma_j^\nu \chi_{n,\omega'}|0\rangle$$

$$\mathcal{L}_{qn}^{(0)} = \bar\xi_n\, in.D_s\, \frac{\bar n}{2}\, \xi_n$$

$$\Downarrow$$

$$\mathcal{L}_{qn}^{(0)} = \bar\xi_n\, in.\partial_s\, \frac{\bar n}{2}\, \xi_n$$

Factorization and Soft field redefinition: soft-collinear decoupling

$$\Downarrow$$

$$\xi_n \to Y_n \xi_n\,, \qquad A_n^\mu \to Y_n\, A_n^\mu\, Y_n^\dagger$$

$$Y_n(x) = \overline{\mathrm{P}}\;\exp\Big(-ig\int_0^\infty ds\, n\!\cdot\! A_s(ns+x)\Big)$$

Hemisphere mass constraint

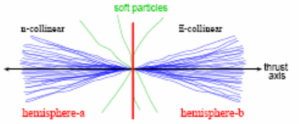
soft

$$\sigma \;=\; \sum_{X_n X_{\bar n} X_s}^{res.} (2\pi)^4\,\delta^4(q-P_{X_n}-P_{X_{\bar n}}-P_{X_s}) \sum_{ij} L_{\mu\nu}^{(ij)} \langle 0|Y_n^\dagger \Gamma_i^\mu Y_{\bar n}|X_s\rangle\langle X_s|\tilde Y_{\bar n}^\dagger \bar\Gamma_j^\nu \tilde Y_n|0\rangle$$
$$\times \int d\omega\, d\bar\omega\, |C(\omega,\bar\omega)|^2 \langle 0|\overline{\chi}_{n,\omega}|X_n\rangle\langle X_n|\chi_n|0\rangle\langle 0|\chi_{\bar n}|X_{\bar n}\rangle\langle X_{\bar n}|\overline{\chi}_{\bar n,\bar\omega}|0\rangle\,.$$

Hard Wilson coeff. **Collinear: n** **Collinear: $\bar n$**

SCET Cross Section

SCET factorization formula $s_{t,\bar t} = M^2_{t,\bar t} - m_t^2$

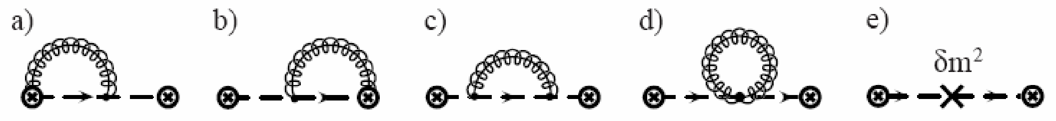


$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, J_n(s_t - Q\ell^+, \mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-, \mu)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

a la Korchemsky, Sterman
including top mass effects

Jet functions:

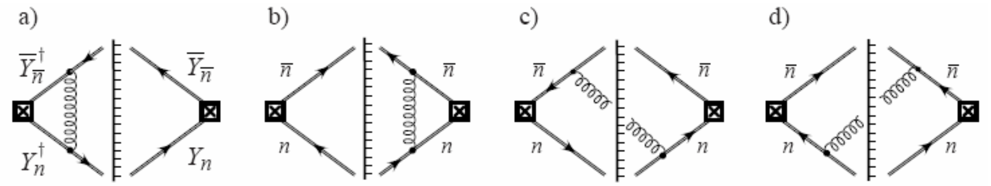
$$J_{\bar n}(Qr^-_{\bar n} - m_J^2, m_J, \Gamma_t, \mu) = \frac{1}{8\pi N_c Q}\, \mathrm{Disc} \int d^4x\, e^{ir_n\cdot x} \langle 0|\mathrm{T}\{\overline{\chi}_{\bar n,-Q}(x) \not{\bar n} \chi_{\bar n}(0)\}|0\rangle$$



- perturbative
- depends on m_t, Γ_t
- insensitive to hemisphere constraints

Soft function:

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b}) \langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}^\dagger_{\bar n}(0)|0\rangle$$



- non-perturbative
- renormalized due to UV divergences
- governs massless dijet thrust and jet mass distributions
- depends on hemisph. constr.

Step 2: Factorization in boosted HQET

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

- integrate out the top mass $\hat s_{t,\bar t} = \frac{M_{t,\bar t}^2 - m_t^2}{m_t} \sim \Gamma_t$
- Jet functions without large logs in peak region: $B_\pm(\hat s, \Gamma_t, \mu)$
- top decay “integrated out” (unstable particle EFT) $\rightarrow i\,\Gamma_t$
- soft function S unchanged (up to virtual top effects →NNLL)

boosted HQET

bHQET Lagrangian: → two copies of HQET (top+antitop)

$$\mathcal{L}_{\pm}^{\mathrm{bHQET}} = \bar{h}_{v_\pm}(iv_\pm . D_\pm - \delta m + \tfrac{i}{2}\Gamma_t)h_{v_\pm}$$

residual mass term:

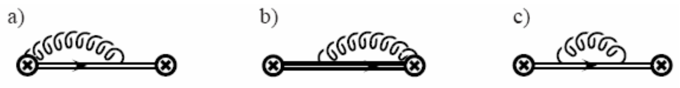
$\delta m = 0$ for pole mass

total top width:

- Inclusive treatment of top decay
- wrong mass assignment $(m/Q)^2$-suppressed

Jet functions: $$B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\,\mathrm{Disc}\int d^4x\, e^{ik\cdot x}\,\langle 0|\mathrm{T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$$

→ **one-loop matching of jet functions**



$$J_n(m\hat{s},\Gamma,\mu_m) = T_+(m,\mu_m)\,B_+(\hat{s},\Gamma,\mu_m)$$

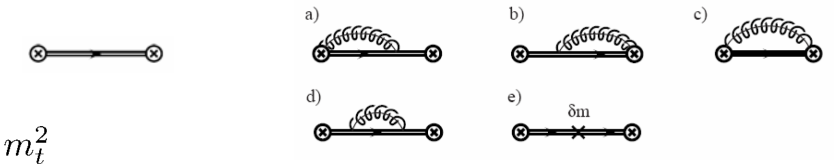
$$T_\pm(\mu,m) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right)$$

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m)\,H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat{s}_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Jet functions: $B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\,{\rm Disc}\int d^4x\; e^{ik\cdot x}\,\langle 0|{\rm T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$

- perturbative, any mass scheme
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat{s},\Gamma_t) = \frac{1}{\pi m_t}\,\frac{\Gamma_t}{\hat{s}^2+\Gamma_t^2} \qquad \hat{s} = \frac{M^2-m_t^2}{m_t}$$

Soft function: $S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\,\delta(\ell^- - k_s^{-b})\,\langle 0|\overline{Y}_{\bar n}\,Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$

- non-perturbative
- renormalized due to UV divergences
- also governs **massless dijet thrust and jet mass event distributions**

Korshemsky, Sterman, etal.

Bauer, Manohar, Wise, Lee

➡ **Short distance top mass can (in principle) be determined to better than $\Lambda_{\rm QCD}$.**

Summation of Logs

- Wilson coefficients H_Q and H_m sum <u>local</u> double logs

$$\mu \frac{d}{d\mu} H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$$

$$\gamma_H(\mu) \;=\; -\frac{\alpha_s C_F}{4\pi}\left[8\ln\frac{\mu^2}{Q^2}+12\right]$$

$$\mu \frac{d}{d\mu} H_m\Big(m,\frac{Q}{m},\mu\Big) = \gamma_{H_m}\Big(\frac{Q}{m},\mu\Big)\, H_m\Big(m,\frac{Q}{m},\mu\Big)$$

$$\gamma_{H_m}(\mu) = \frac{\alpha_s C_F}{\pi}\left[2\ln\frac{Q^2}{m^2}-2\right]$$

Summation of Logs

- low energy scales of jet functions and soft function can differ
- RG-evolution of jet and soft functions individually involve plus functions

$$\mu\frac{d}{d\mu}B_\pm(\hat{s},\mu) = \int d\hat{s}'\,\gamma_{B_\pm}(\hat{s}-\hat{s}')\,B_\pm(\hat{s}',\mu)$$

$$\gamma_B(\hat{s}-\hat{s}',\mu) = -\frac{\alpha_s C_F}{4\pi}\left\{\frac{8}{\mu}\left[\frac{\mu\,\theta(\hat{s}-\hat{s}')}{\hat{s}-\hat{s}'}\right]_+ - 4\delta(\hat{s}-\hat{s}')\right\}$$

→ talk by I. Scimemi

$$B_\pm(\hat{s},\mu) = \int d\hat{s}'\;U_B(\hat{s}-\hat{s}',\mu,\mu_\Gamma)\;B_\pm(\hat{s}',\mu_\Gamma)$$

$$\begin{aligned}\frac{d^2\sigma}{dM_t^2\,dM_{\bar{t}}^2} &= \sigma_0\,H_Q(Q,\mu_m)\,H_m\Big(m,\frac{Q}{m},\mu_m,\mu_\Delta\Big)\\ &\times\int_{-\infty}^{\infty} d\hat{s}_t'\,d\hat{s}_{\bar{t}}'\;U_{B_+}(\hat{s}_t,\hat{s}_t',\mu_\Delta,\mu_\Gamma)\,U_{B_-}(\hat{s}_{\bar{t}},\hat{s}_{\bar{t}}',\mu_\Delta,\mu_\Gamma)\\ &\times\int_{-\infty}^{\infty} d\ell^+ d\ell^-\;B_+\Big(\hat{s}_t'-\frac{Q\ell^+}{m},\Gamma,\mu_\Gamma\Big)\,B_-\Big(\hat{s}_{\bar{t}}'-\frac{Q\ell^-}{m},\Gamma,\mu_\Gamma\Big)\,S_{\rm hemi}(\ell^+,\ell^-,\mu_\Delta)\end{aligned}$$

Ingrediences for NLL Order

Fleming, Mantry, Stewart, AH
to appear soon

Matching Coefficient and Matrix Elements:

- massive SCET jet ⟶ bHQET jet matching $\mathcal{O}(\alpha_s)$
- bHQET jet function $\mathcal{O}(\alpha_s),\ \mathcal{O}(\alpha_s^2)$
- partonic soft function $\mathcal{O}(\alpha_s)$
- SCET current matching $\mathcal{O}(\alpha_s^{2(,3)})$

Beneke, Chapovsky, Signer, Zanderighi ‘04
→ talk by I. Scimemi

M. Schwartz

Kramer, Lampe ’87
Matsuura, v.d. March, v. Neerven ’88, ’89
Moch, Vermaseren, Vogt ‘05
Gehrmann, Huber, Maitre ’05

→ talk by T. Gehrmann

Anomalous Dimensions:

- SCET current → NLL + NNLL
- bHQET jet function → NLL + NNLL
- soft function → NLL

Moch, Vermaseren, Vogt ‘05

Soft Function Model beyond LL

Stewart, AH
arXiv: 0709.3519

partonic soft function:

$$S_{\rm part}^{\rm NLO}(\ell^\pm,\mu) = S_{\rm part}^{\rm NLO}(\ell^+,\mu)S_{\rm part}^{\rm NLO}(\ell^-,\mu)$$

$$S_{\rm part}^{\rm NLO}(\ell,\mu) = \delta(\ell) + \frac{C_F\alpha_s(\mu)}{\pi}\left\{\frac{\pi^2}{24}\delta(\ell) - \frac{2}{\mu}\left[\frac{\theta(\ell)\ln(\ell/\mu)}{\ell/\mu}\right]_+\right\}$$

- soft function non-perturbative (distribution in the peak region) → model function $S_{\rm mod}(\ell^+,\ell^-)$
- partonic contributions required for OPE for moments and predictions in the tail

⟶ absorb partonic information fully into model parameters Korchemsky,Sterman, Tafat ‘00

⟶ glue partonic tail to model function (absorbs partonic information partially into $S_{\rm mod}$)

Bosch, Lange, Neubert, Paz ‘04

Soft Function Model beyond LL

Stewart, AH
arXiv: 0709.3519

partonic soft function:

$$S^{\rm NLO}_{\rm part}(\ell^\pm,\mu) = S^{\rm NLO}_{\rm part}(\ell^+,\mu)\,S^{\rm NLO}_{\rm part}(\ell^-,\mu)$$

$$S^{\rm NLO}_{\rm part}(\ell,\mu) = \delta(\ell) + \frac{C_F\alpha_s(\mu)}{\pi}\left\{\frac{\pi^2}{24}\delta(\ell) - \frac{2}{\mu}\left[\frac{\theta(\ell)\ln(\ell/\mu)}{\ell/\mu}\right]_+\right\}$$

soft function model:

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde\ell^+ \int_{-\infty}^{+\infty} d\tilde\ell^-\; S_{\rm part}(\ell^+ - \tilde\ell^+, \ell^- - \tilde\ell^-, \mu)\; S_{\rm mod}(\tilde\ell^+,\tilde\ell^-)$$

normalized model function

$$\int_0^{+\infty} d\ell^+ \int_0^{+\infty} d\ell^-\, S_{\rm mod}(\ell^+,\ell^-) = 1 \qquad S_{\rm mod}(\ell^+,\ell^-) = \Theta(\ell^+)\Theta(\ell^-)f(\ell^+,\ell^-)$$

- consistent OPE for predictions of moments or in the tail
- $S_{\rm mod}$ has $u = \frac{1}{2}$ renormalon: $\mathcal{O}(\Lambda_{\rm QCD})$ ambiguity in partonic zero-point

Gardi '00

$$\delta S_{\rm part}(\ell^+,\ell^-) = \Lambda_{\rm QCD}\left(\frac{\partial}{\partial\ell^+} + \frac{\partial}{\partial\ell^-}\right) S_{\rm part}(\ell^+,\ell^-)$$

→ affects first power correction

Soft Function Model beyond LL

Stewart, AH
arXiv: 0709.3519

partonic soft function:

$$S^{\rm NLO}_{\rm part}(\ell^\pm,\mu) = S^{\rm NLO}_{\rm part}(\ell^+,\mu)\,S^{\rm NLO}_{\rm part}(\ell^-,\mu)$$

$$S^{\rm NLO}_{\rm part}(\ell,\mu) = \delta(\ell) + \frac{C_F\alpha_s(\mu)}{\pi}\left\{\frac{\pi^2}{24}\delta(\ell) - \frac{2}{\mu}\left[\frac{\theta(\ell)\ln(\ell/\mu)}{\ell/\mu}\right]_+\right\}$$

soft function model:

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde\ell^+ \int_{-\infty}^{+\infty} d\tilde\ell^- \; S_{\rm part}(\ell^+-\tilde\ell^+,\ell^--\tilde\ell^-,\mu)\, S_{\rm mod}(\tilde\ell^+,\tilde\ell^-)$$

normalized model function

$$\int_0^{+\infty} d\ell^+ \int_0^{+\infty} d\ell^-\, S_{\rm mod}(\ell^+,\ell^-) = 1 \qquad S_{\rm mod}(\ell^+,\ell^-) = \Theta(\ell^+)\Theta(\ell^-)f(\ell^+,\ell^-)$$

→ gapped soft model: $S_{\rm mod}(\ell^+,\ell^-) \;\rightarrow\; S_{\rm mod}(\ell^+-\Delta,\ell^--\Delta)$

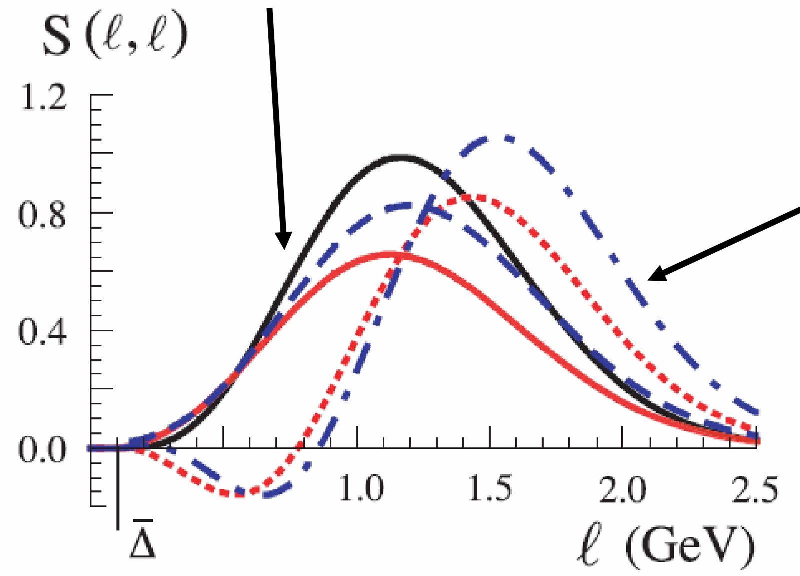
→ renormalon subtraction scheme: $\Delta \;=\; \Delta^{\rm ren.free.} + \alpha_s\delta_1 + \alpha_s^2\delta_2 + \ldots$

Soft Function Model beyond LL

Stewart, AH
arXiv: 0709.3519

$$\delta = \frac{\int_{-\infty}^{L} d\ell^+ \int_{-\infty}^{L} d\ell^- \, \ell^+ \, S_{\rm part}(\ell^+,\ell^-,\mu)}{\int_{-\infty}^{L} d\ell^+ \int_{-\infty}^{L} d\ell^- \, \ell^+ \left[\frac{\partial}{\partial\ell^+} + \frac{\partial}{\partial\ell^-}\right] S_{\rm part}(\ell^+,\ell^-,\mu)}$$

→ first moment renormalon-free



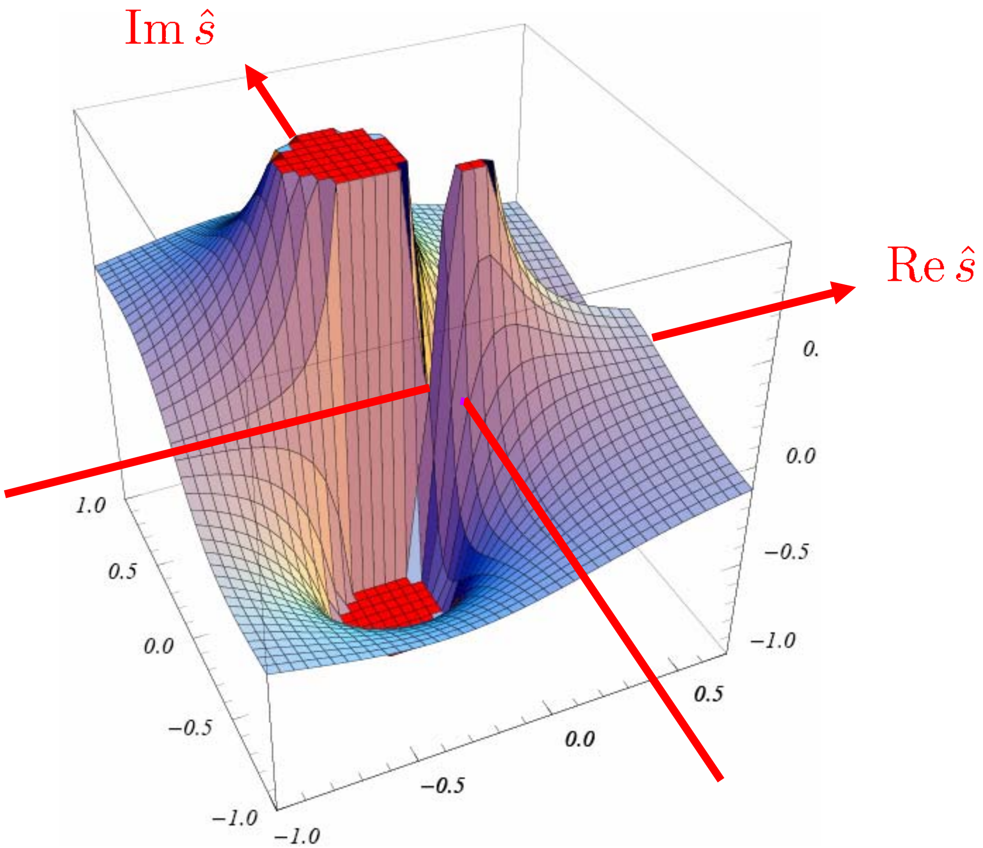
with renormalon subtraction

without renormalon subtraction

Can one measure the Pole Mass ?

$$\mathcal{B}_+(2v_+\cdot r,\Gamma_t,\mu) \;=\; \frac{-1}{4\pi N_c m}\int d^4x\, e^{ir\cdot x}\,\langle 0\big|T\{\bar h_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}\big|0\rangle$$

$$\hat{s} = \frac{M_t^2-m_t^2}{m_t}$$



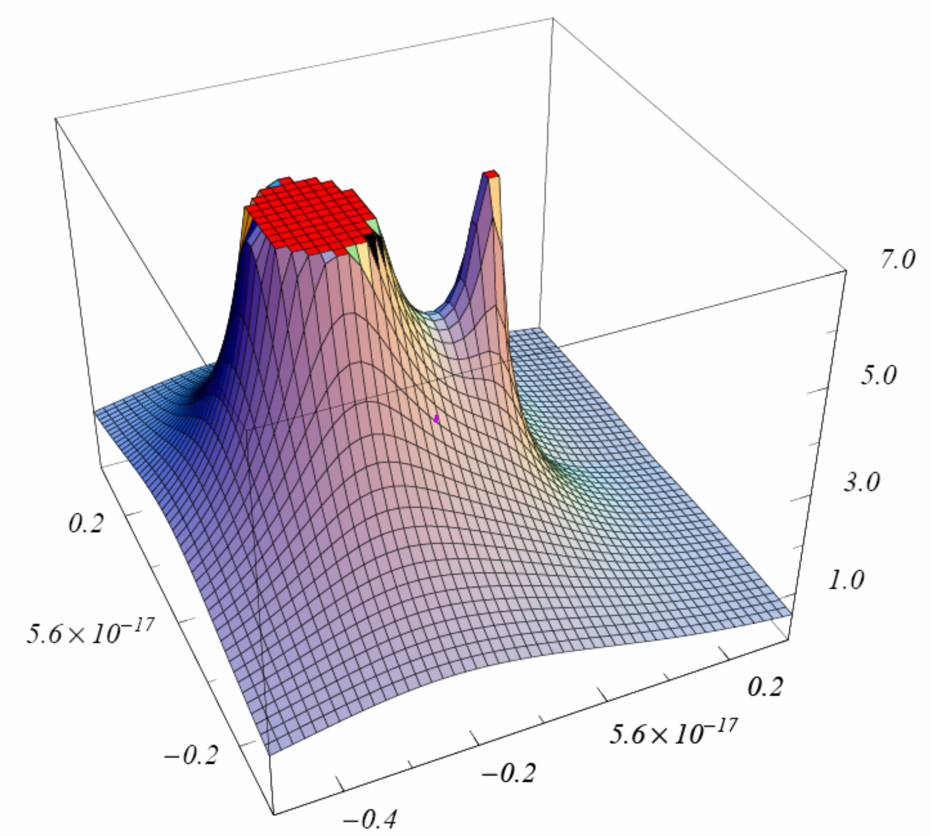
Message: The pole mass is not accessible !

Can one measure the Pole Mass ?

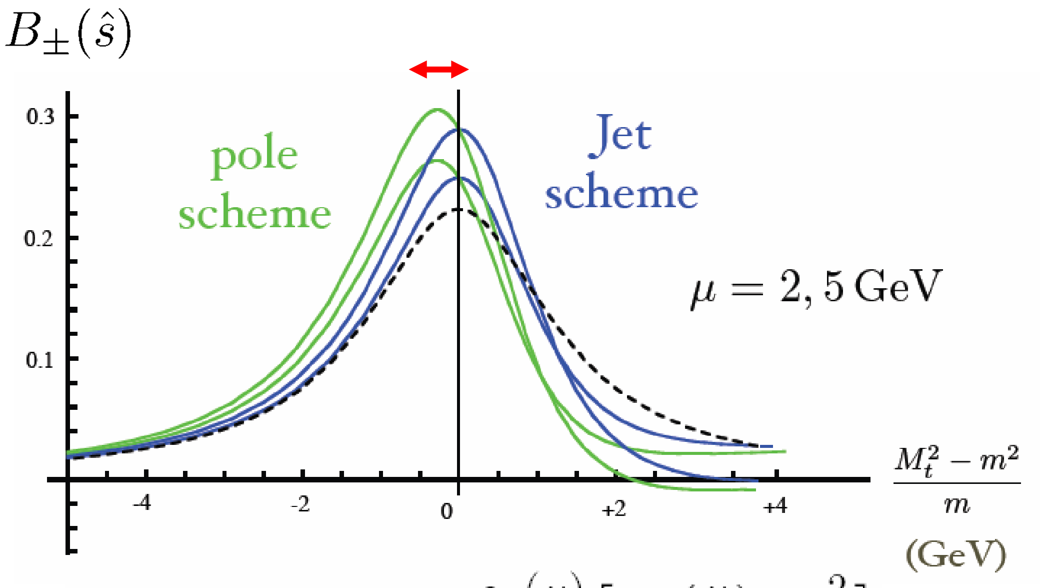
$$\mathcal{B}_+(2v_+\cdot r, \Gamma_t, \mu) \;=\; \frac{-1}{4\pi N_c m}\int d^4x\, e^{ir\cdot x}\,\langle 0\big|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}\big|0\rangle$$

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t}$$

Message: The pole mass is not accessible !



Short-distance Top Jet Mass



$$m_J(\mu) = m_{\mathrm{pole}} - \Gamma\frac{\alpha_s(\mu)}{3}\left[\ln\left(\frac{\mu}{\Gamma}\right) + \frac{3}{2}\right]$$

- **One-loop: shift in the pole scheme 250 MeV**
- **shift in the pole scheme contains $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon**
- **jet mass scheme: defined such that peak located at the mass to all orders**

Top Jet mass is the scheme where we expect that a LO analysis contains the least theoretical uncertainties.

→ talk by I. Scimemi

LO Numerical Analysis

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

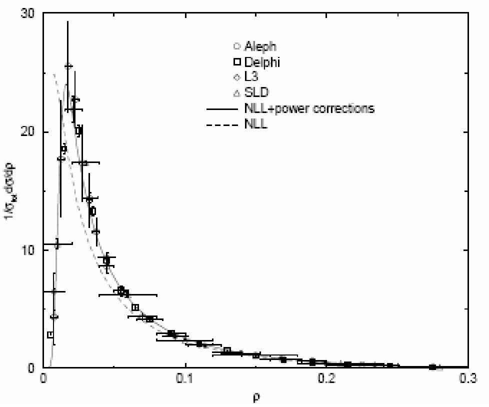
Jet functions:

$$B_\pm(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2}$$

Soft function:

$$S_{\rm hemi}^{\rm M1}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-) \frac{\mathcal{N}(a,b)}{\Lambda^2} \left(\frac{\ell^+\ell^-}{\Lambda^2}\right)^{a-1} \exp\left(\frac{-(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2}\right)$$

$$a = 2\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$



Fit to heavy jet mass distribution

Korchemsky, Tafat
hep-ph/0007005

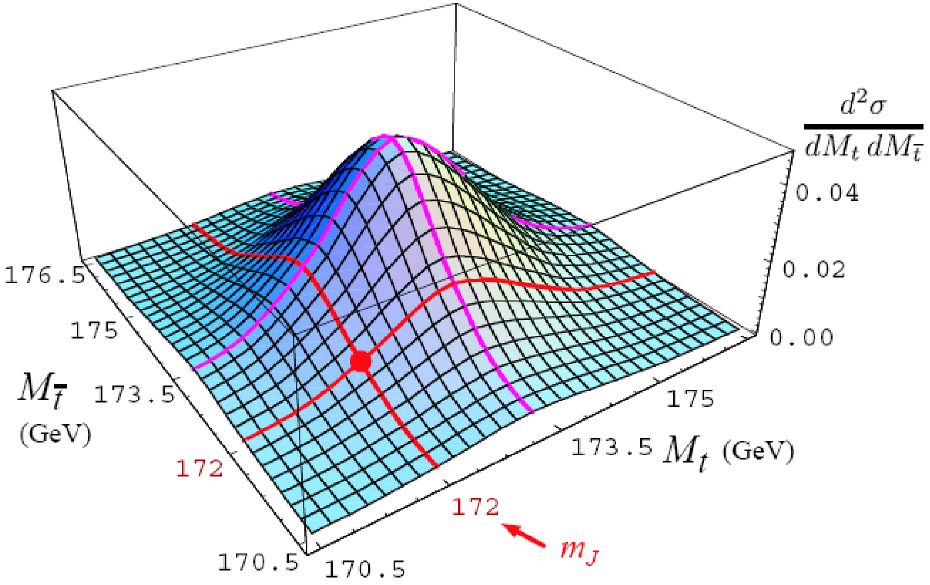
LO Numerical Analysis

Double differential invariant mass distribution:

$$Q = 745 \text{ GeV}$$

$$\Gamma = 1.43 \text{ GeV}$$

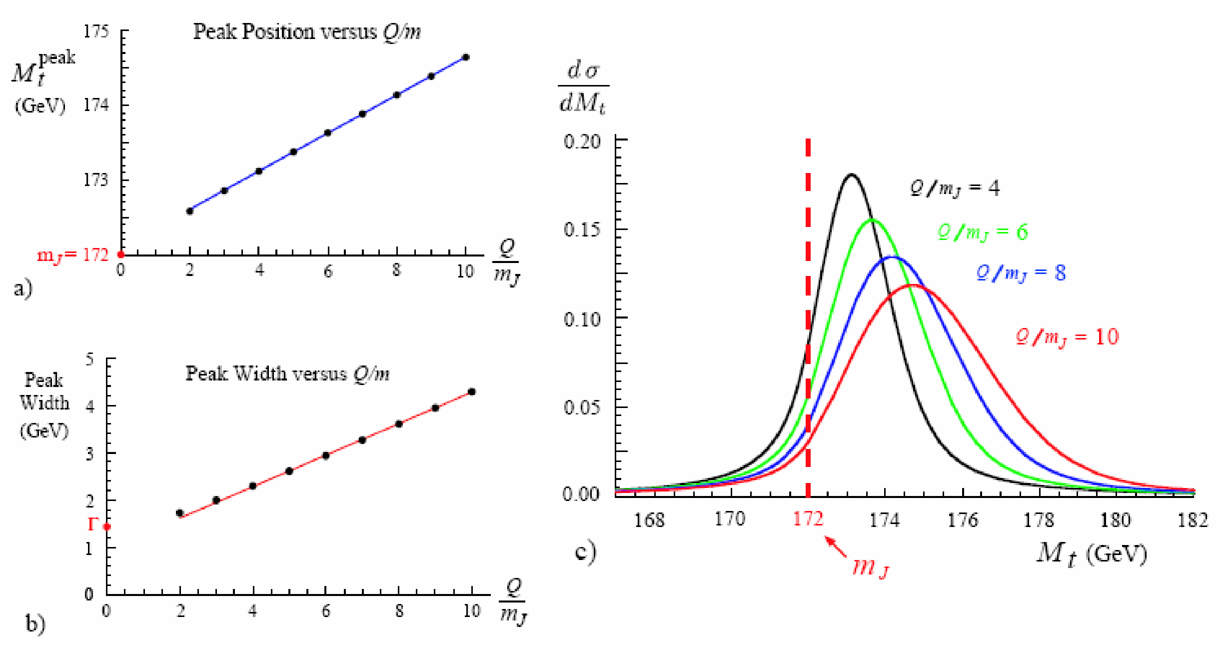
$$m_J = 172 \text{ GeV}$$



Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

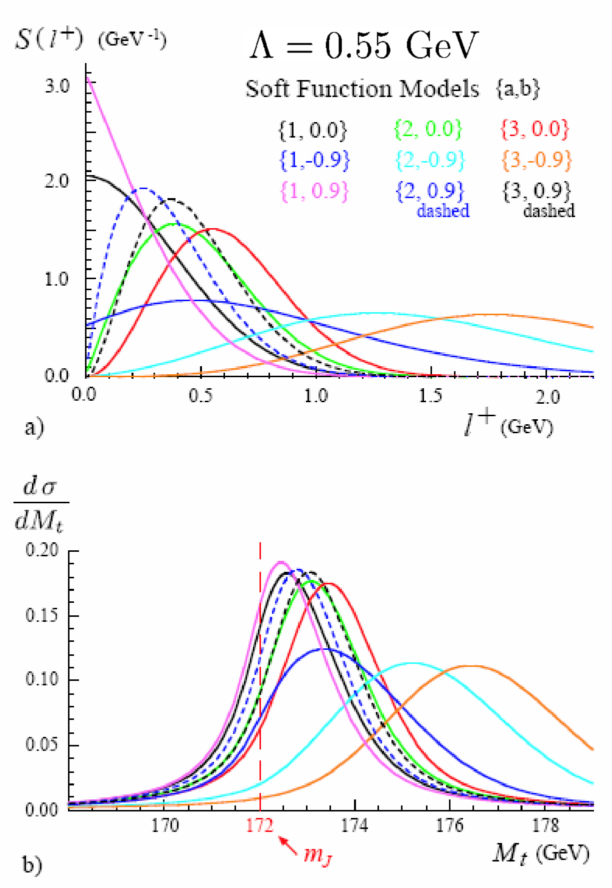
Single differential distribution:

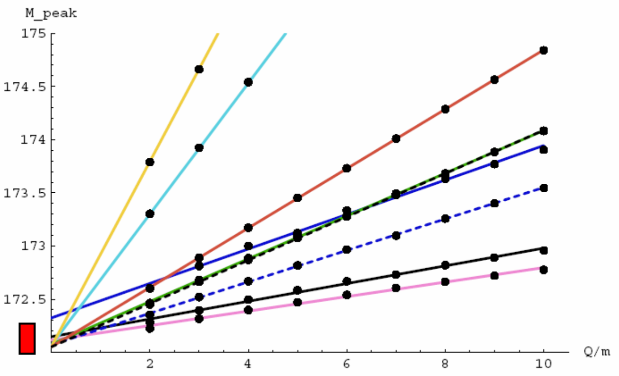


Non-perturbative effects shift the peak to higher energies and broaden the distribution.

LO Numerical Analysis

Different invariant mass prescriptions/soft functions:





$$M_t^{\text{peak}} \approx m_J + \frac{Q}{m_J}\,\text{const}$$

Fairly precise determination of jet mass from determination of Q-dependence of the peak position and extrapolation Q to zero

Theory Issues for $pp \rightarrow t\bar{t} + X$

- ★ definition of jet observables
- ★ initial state radiation
- ★ final state radiation
- • underlying events → Hadron event shapes
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

★ **Can be addressed in the framework of a LC.**

★ **Requires extensions of LC concepts and other known concepts**

Summary & Outlook

- established factorization theorem for invariant mass distributions: separation of perturbative and non-perturbative effects for the ILC
- applicable for many other systems and setups (e.g. squarks)
- exact and systematic relation of peak to a Lagrangian mass: What mass is measured ? "Jet-mass"
- resummation of large logarithms $Q \gg m_t \gg \Gamma_t$
- NLL, NNLL analysis on the way
- extension to LHC
- other applications: → massless event shapes (electron-positron)

  → hadron collider event shapes

  → …..

Backup Slides

ILC Top Mass Reconstruction

Chekanov, Morgunov

hep-ex/0301014

All-hadronic channel $e^+e^- \rightarrow t\bar{t} \rightarrow 6\ \text{jets}$

Event Selection

- k_T (Durham) jet algorithm $y_6^{cut} > \Delta_y$
  all particles are assigned to exactly 6 jets
- $| \frac{E_{vis}}{\sqrt{s}} - 1 | < \Delta_E, \quad \frac{| \sum \vec{p}_{||i} |}{\sum | \vec{p}_i |} < \Delta_{PL}, \quad \frac{| \sum \vec{p}_{Ti} |}{\sum | \vec{p}_i |} < \Delta_{PT}$

every particle is assigned to top or antitop (fully inclusive !!)

ILC special

reduces events for massless quark pair production

selects hadronic $t\bar{t}$ events

Top Reconstruction

- 6 jets with momenta $p_i, (i = 1, \ldots, 6)$
- group in pairs of 3: $M_I(1)$ and $M_I(2)$
- $|M_I(1) - M_I(2)| < \Delta_M$
  $| \vec{P}_I(1) + \vec{P}_I(2) | < \Delta_P$
- b-tagging
- massless jet pair mass: $| M_{jj} - M_W | < \Delta_W$

both invariant masses close to m_t

selects back-to-back events

reduces combinatorics

Analysis:

- $500\ \text{GeV with } 16\ \text{fb}^{-1}$
- $800\ \text{GeV with } 33\ \text{fb}^{-1}$
- $800\ \text{GeV with } 300\ \text{fb}^{-1}$

$\delta m_t^{\text{stat}} \sim 400\ \text{MeV}$ (500 GeV with 16 fb^{-1}; 800 GeV with 33 fb^{-1})

$\delta m_t^{\text{stat}} \sim 100\ \text{MeV}$ (800 GeV with 300 fb^{-1})

Relation to event shapes

- hemisphere masses are event shape variables
- related to thrust, heavy, light jet masses for massless jet production
- related to hadron event shapes for hadron colliders

e.g. relation to thrust: $1 - T = (M_t^2 + M_{\bar t}^2)/Q^2$

→ Thrust distribution in the peak region :

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t\, ds_{\bar t}\ \tilde{B}_+\Big(\frac{s_t}{m_J},\Gamma,\mu\Big)\, \tilde{B}_-\Big(\frac{s_{\bar t}}{m_J},\Gamma,\mu\Big)\, S_{\rm thrust}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar t})}{Q^2},\mu\Big)$$

Event Shapes

e.g. Thrust distribution on the peak region:

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t\, ds_{\bar{t}}\;\; \tilde{B}_+\Big(\frac{s_t}{m_J}, \Gamma, \mu\Big)\, \tilde{B}_-\Big(\frac{s_{\bar{t}}}{m_J}, \Gamma, \mu\Big)\, S_{\rm thrust}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar{t}})}{Q^2}, \mu\Big)$$

$$S_{\rm thrust}(\tau, \mu) = \int_0^{\infty} d\ell^+\, d\ell^- \delta\Big(\tau - \frac{(\ell^+ + \ell^-)}{Q}\Big) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

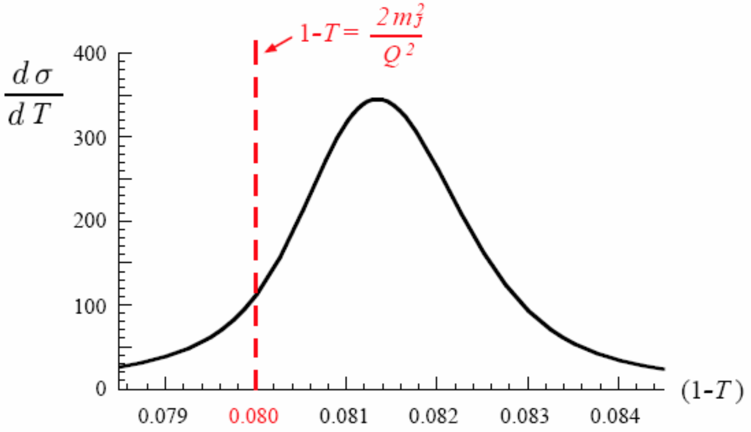
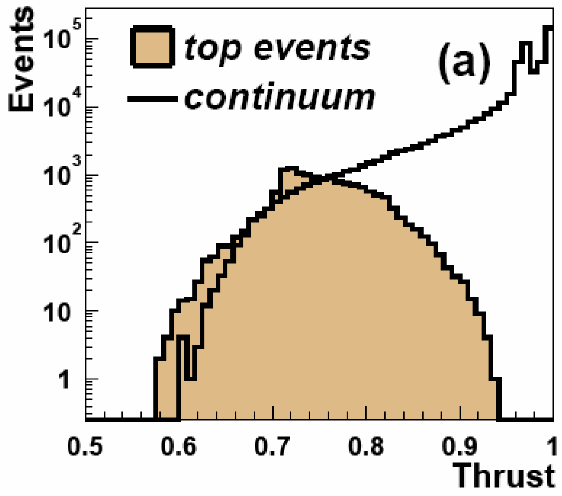


FIG. 8: Plot of the thrust distribution, $d\sigma/dT$ in units of σ_0^H, for top-initiated events in the peak region. We use $Q/m_J = 5$, $m_J = 172\,\text{GeV}$ and the soft function parameters in Eq. (115).



Chekanov; Morgunov

hep-ex/0301014