Dynamical μ Term in Gauge Mediation

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Based on: *A. Delgado, G.F. Giudice and P.S., arXiv:0706.3873*

The μ problem in SUSY theories

In SUSY extensions of the SM we must introduce two Higgs doublets with opposite hypercharge:

- To give mass to both up- and down-type quarks
- To allow for a higgsino mass term
- To cancel anomalies

Higgs/higgsino mass term in the superpotential

$$\mathcal{L} \supset \mu \int d^2\theta \; H_d \, H_u$$

There are also soft SUSY-breaking mass terms for the Higgses in the scalar potential

$$V_{\rm soft} \supset m^2_{H_d} \, |H_d|^2 + m^2_{H_u} \, |H_u|^2 - B_\mu \, (H_d \, H_u + {\rm h.c.})$$

In the MSSM, μ is the only superpotential term with the dimension of a mass

The μ problem: if μ is allowed in the SUSY limit, why is it not of $\mathcal{O}(M_P)$?

The *Giudice-Masiero* mechanism: (1988)

μ is forbidden in the SUSY limit, and is generated in the low-energy theory by SUSY-breaking effects

Parametrize the SUSY-breaking sector with a chiral superfield X that acquires a vev

$$\langle X \rangle \;=\; M + \theta^2 F$$

The SUSY-breaking spurion couples to the Higgses in a non-minimal Kahler potential

$$\mathcal{L} \quad \supset \quad \int d^4\theta \, H_d \, H_u \left(\frac{X^\dagger}{M} + \frac{X^\dagger X}{M^2} + \; \ldots \right)$$

$$\sim \quad \frac{F}{M} \int d^2\theta \, H_d \, H_u \; + \; \left(\frac{F}{M} \right)^2 (H_d \, H_u + \mathrm{h.c.}) \; + \; \ldots$$

Therefore, $\mu \sim \dfrac{F}{M}$, $B_\mu \sim \left(\dfrac{F}{M} \right)^2 \longrightarrow \dfrac{B_\mu}{\mu} \sim \dfrac{F}{M}$

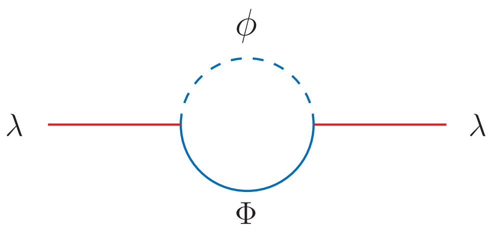
In gravity-mediated SUSY-breaking $\tilde{m} \;\sim\; \dfrac{F}{M_P} \;\sim\; \mathrm{TeV}$ is the typical soft mass

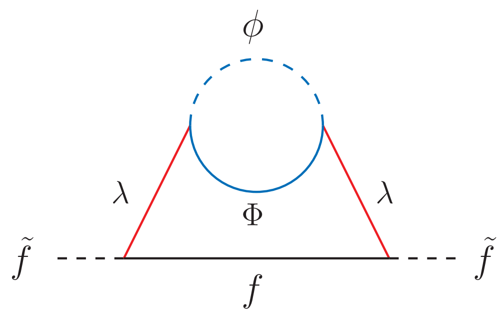
When the soft terms are loop-induced (GMSB, AMSB) the GM mechanism has a problem

In gauge mediation the SUSY-breaking sector couples only to heavy messenger fields

$$\mathcal{L} \supset \kappa \int d^2\theta \; X \, \Phi \, \bar{\Phi} \, , \qquad m_\Phi^2 = |\kappa \, M|^2 \, , \qquad m_\phi^2 = |\kappa \, M|^2 \pm |\kappa \, F|$$

The soft masses for the MSSM fields are generated at loop level by the gauge interactions





$$M_\lambda \sim m_{\tilde{f}} \sim \frac{\alpha}{4\pi} \frac{F}{M}$$

$$A_{\tilde{f}} \sim \mathcal{O}(\alpha^2)$$

We also want $\mu \sim \tilde{m} \sim \frac{\alpha}{4\pi} \frac{F}{M} \sim \mathrm{TeV}$

But $\frac{B_\mu}{\mu} \sim \frac{F}{M} \longrightarrow B_\mu \sim (10 - 100 \ \mathrm{TeV})^2$ **!!!**

Such a huge B_μ would require an unacceptable fine tuning in the Higgs sector

NMSSM alternative: *generate μ and B_μ at the weak scale through the vev of a light singlet*

$$\mathcal{L} \supset \lambda \int d^2\theta \, N \, H_d \, H_u \longrightarrow \mu = \lambda \langle N \rangle \, , \quad B_\mu = \lambda \langle F_N \rangle$$

Is it worth the pain? a light singlet requires the introduction of several new soft terms, and it can even pick up a tadpole from the SUSY-breaking sector, destabilizing the hierarchy

- Neither of these issues is too problematic in gauge mediation, where the soft terms are calculable and the SUSY-breaking scale is relatively low
- Also, the singlet-doublet interaction can give a positive contribution to the lightest Higgs boson mass and help lifting it above the LEP bound

Does NMSSM+GMSB result in an acceptable EWSB ?

The Higgs sector of the NMSSM

Superpotential and soft SUSY-breaking interactions for the Higgses and the singlet

$$W \;\supset\; \lambda\, N\, H_d\, H_u - \frac{k}{3}\, N^3$$

$$V_{\rm soft} \;\supset\; \tilde{m}^2_{H_u} |H_u|^2 + \tilde{m}^2_{H_d} |H_d|^2 + \tilde{m}^2_N |N|^2 + \left(\lambda A_\lambda N H_d H_u - \frac{k}{3} A_k N^3 + {\rm h.c.} \right)$$

Define MSSM-like parameters: $v^2 \;\equiv\; \langle H_d \rangle^2 + \langle H_u \rangle^2\,, \quad \tan\beta \;\equiv\; \dfrac{\langle H_u \rangle}{\langle H_d \rangle}\,,$

$$\mu \;\equiv\; \lambda\, \langle N \rangle\,, \quad B_\mu \;\equiv\; \lambda\, k\, \langle N \rangle^2 - \frac{\lambda^2\, v^2}{2}\, \sin 2\beta - \lambda\, A_\lambda\, \langle N \rangle$$

In gauge mediation we have $|\tilde{m}_N|\,,\; A_\lambda\,,\; A_k \;\ll\; v$. Therefore, we get $\langle N \rangle \ll v$.

This results in a very light scalar+pseudoscalar pair, ruled out by LEP searches.

We need some mechanism to generate sizeable soft SUSY-breaking terms for the singlet

$$|\tilde{m}_N|\,,\ A_\lambda\,,\ A_k\ \sim\ \mathcal{O}(\tilde{m}) \longrightarrow \langle N\rangle \gg v$$

In the limit $\langle N\rangle \gg v$ the singlet and doublet sectors decouple from each other, and the tree-level masses for the *two* CP-odd (a_i) and *three* CP-even (h_i) neutral scalars are

$$m_{a_1}^2 = \frac{2\,B_\mu}{\sin 2\beta} + \mathcal{O}(v^2)\,, \qquad m_{a_2}^2 = \frac{3\,k^2}{w}\langle N\rangle^2 + \mathcal{O}(v^2)\,,$$

$$m_{h_1}^2 = M_Z^2\cos^2 2\beta + \lambda^2 v^2 \left\{ \sin^2 2\beta - \frac{\left[\frac{\lambda}{k} + \left(\frac{A_\lambda}{2w\,A_k} - 1\right)\sin 2\beta\right]^2}{1 - \frac{1}{4w}} \right\} + \mathcal{O}(v^4)\,,$$

$$m_{h_2}^2 = m_{a_1}^2 + \mathcal{O}(v^2)\,, \qquad m_{h_3}^2 = \frac{4w-1}{3}\,m_{a_2}^2 + \mathcal{O}(v^2)$$

where $$w \equiv \left(1 + \sqrt{1 - 8\,\frac{\tilde{m}_N^2}{A_k^2}}\right) > \frac{1}{3}$$

We must include radiative corrections. Defining the effective potential $V_{\rm eff} \;=\; V_0 + \Delta V$ the mass matrices for CP-even and CP-odd parts of $\phi_i = (H_d,\, H_u,\, N)$ become

$$(\mathcal{M}^2_{S,P})^{\rm eff} = \sqrt{Z}\,\left[\,(\mathcal{M}^2_{S,P})^0 + \Delta\mathcal{M}^2_{S,P}\right]\,\sqrt{Z}$$

$$\left(\Delta\mathcal{M}^2_S\right)_{ij} = \frac{1}{2}\;\frac{\partial^2\,\Delta V}{\partial\,{\rm Re}\,\phi_i\,\partial\,{\rm Re}\,\phi_j}\bigg|_{\rm min}\,,\qquad\qquad \left(\Delta\mathcal{M}^2_P\right)_{ij} = \frac{1}{2}\;\frac{\partial^2\,\Delta V}{\partial\,{\rm Im}\,\phi_i\,\partial\,{\rm Im}\,\phi_j}\bigg|_{\rm min}$$

We keep the $\mathcal{O}(h_t^4)$ terms in $\Delta\mathcal{M}^2_{S,P}$ and the $\mathcal{O}(h_t^2)$ terms in Z . We also include some leading-logarithmic two-loop corrections controlled by the top Yukawa and strong couplings. For m_{h_1} we agree with the code NMHDECAY (*Ellwanger, Hugonie & Gunion*) within 5 GeV

In the limit of heavy singlet the dominant $\mathcal{O}(h_t^4)$ corrections to m_{h_1} are just as in the MSSM:

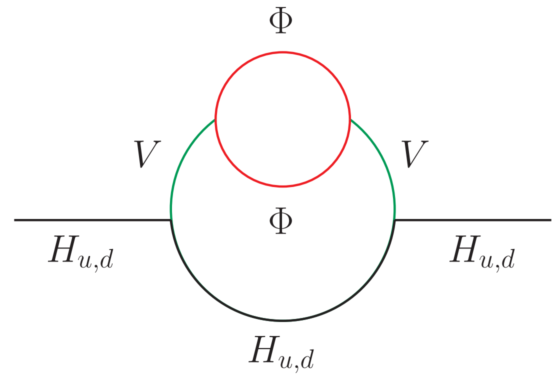
$$(\Delta m^2_{h_1})^{1-{\rm loop}} \;\simeq\; \frac{3\,m_t^4}{4\,\pi^2\,v^2}\,\left(\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12\,M_S^4}\right)\;,\qquad \left(X_t = A_t + \lambda\,\langle N\rangle\,\cot\beta\right)$$

In GMSB $A_t(M) \simeq 0$, and only a moderate weak-scale value is generated by RG evolution

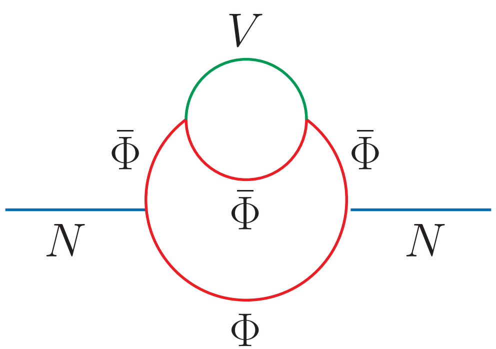
→ We will need a largish M_S (~ TeV) to evade the LEP bounds on the Higgs mass

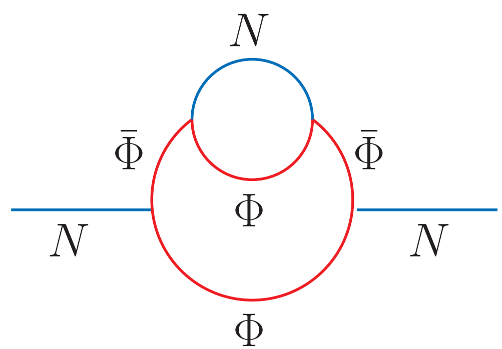
NMSSM+GMSB with singlet-messenger interactions: *N-GMSB*

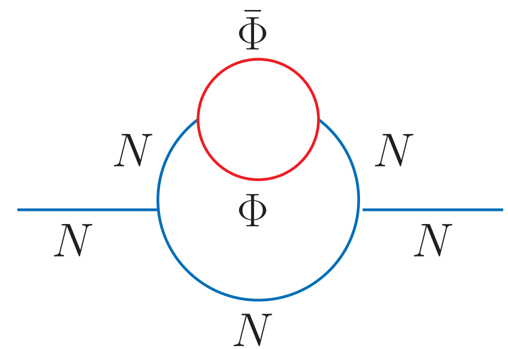
The soft masses for the Higgs doublets are mediated by the gauge interactions:



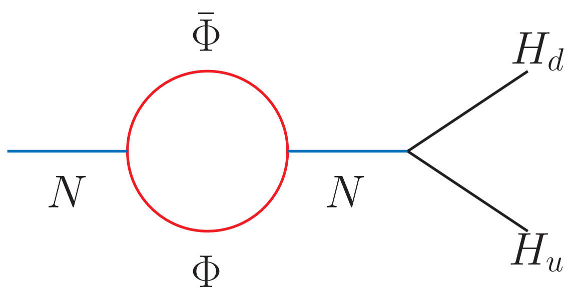
To generate a mass $\tilde{m}_N^2$ for the singlet we can couple it directly to the messengers

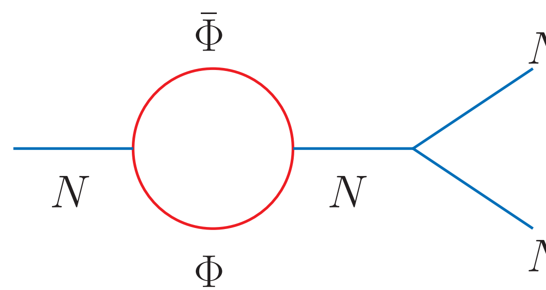






This will also generate trilinear interactions $A_\lambda,\ A_k$ (but no mass term) at one loop





This model was first proposed (without a detailed study) by Giudice & Rattazzi in 1997

We must introduce two pairs of messenger fields in the **5** and $\bar{\mathbf{5}}$ representations of SU(5)

$$W \supset X\left(\bar{\Phi}_1\,\Phi_1 + \bar{\Phi}_2\,\Phi_2\right) + \xi\,N\,\bar{\Phi}_1\,\Phi_2 + \lambda\,N\,H_d\,H_u - \frac{k}{3}\,N^3$$

($X = M + \theta^2 F$ parametrizes the SUSY-breaking sector)

A single messenger pair $(\Phi, \bar{\Phi})$ coupling to both X and N would destabilize the weak scale

$$W \;\supset\; X\,\bar{\Phi}\,\Phi \;+\; \xi\,N\,\bar{\Phi}\,\Phi \quad\longrightarrow\quad V_{\rm eff} = \frac{\xi\,d_\Phi}{16\pi^2}\,N\,\frac{F^2}{M}$$

We must also distinguish between the doublet and triplet components of the messengers

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$$W \supset X \sum_{i=1}^{2} \left(\kappa_i^D \bar{\Phi}_i^D \Phi_i^D + \kappa_i^T \bar{\Phi}_i^T \Phi_i^T\right) + N \left(\xi_D \bar{\Phi}_1^D \Phi_2^D + \xi_T \bar{\Phi}_1^T \Phi_2^T\right) + \lambda N H_d H_u - \frac{k}{3} N^3$$

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We must also distinguish between the doublet and triplet components of the messengers

We use analytical continuation in superspace to extract the soft SUSY-breaking terms at the messenger scale from the wave function renormalization of the observable fields (*we don't need to explicitly compute two-loop diagrams*)

The gaugino and sfermion soft masses are the same as in the usual GMSB

$$M_i = n\, c_i \,\frac{\alpha_i}{4\,\pi}\,\frac{F}{M}\;, \qquad m^2_{\tilde{f}} = 2\,n \sum_i c_i\, C_i^{\tilde{f}}\,\frac{\alpha_i^2}{(4\,\pi)^2}\,\frac{F^2}{M^2}\;, \qquad (n=2)$$

The singlet-messenger interactions generate A-terms at 1-loop and scalar masses at 2-loop

$$A_\lambda \;=\; \frac{A_k}{3} \;=\; -\,\frac{1}{16\pi^2}\,\left(2\,\xi_D^2 + 3\,\xi_T^2\right)\frac{F}{M}\;,$$

$$\begin{aligned}\tilde{m}_N^2 \;&=\; \frac{1}{(16\pi^2)^2}\Bigg[8\xi_D^4 + 15\xi_T^4 + 12\xi_D^2\xi_T^2 - 16g_s^2\xi_T^2 - 6g^2\xi_D^2 - 2g'^2\left(\xi_D^2 + \frac{2}{3}\xi_T^2\right) \\ &\qquad - 4k^2\left(2\xi_D^2 + 3\xi_T^2\right)\Bigg]\frac{F^2}{M^2}\end{aligned}$$

$$\tilde{m}^2_{H_u} = \tilde{m}^2_{H_d} = \frac{1}{(16\pi^2)^2}\left[n\left(\frac{3\,g^4}{2} + \frac{5\,g'^4}{6}\right) - \lambda^2\left(2\,\xi_D^2 + 3\,\xi_T^2\right)\right]\frac{F^2}{M^2}$$

Phenomenology of the N-GMSB

Three new parameters w.r.t. the usual GMSB: $\xi_U \equiv \xi_{D,T}(M_{\rm GUT})\,,\ \lambda,\ k$ (but no $\mu\,,\, B_\mu$)

The size of the soft SUSY-breaking parameters is determined by M and F. We choose them such as to maximize the radiative correction to the light Higgs mass

- Large F/M generates a sizeable stop mass scale $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$
- Large M generates a sizeable $A_t(M_S)$ through RG evolution

Take $M = 10^{13}$ GeV and $F/M = 1.72 \times 10^5$ GeV (such that $M_S \approx 2$ TeV, $A_t \approx -1.4$ TeV)

Conditions on the parameters are imposed at different scales ($M_t\,,\ M_S\,,\ M\,,\ M_{\rm GUT}$)

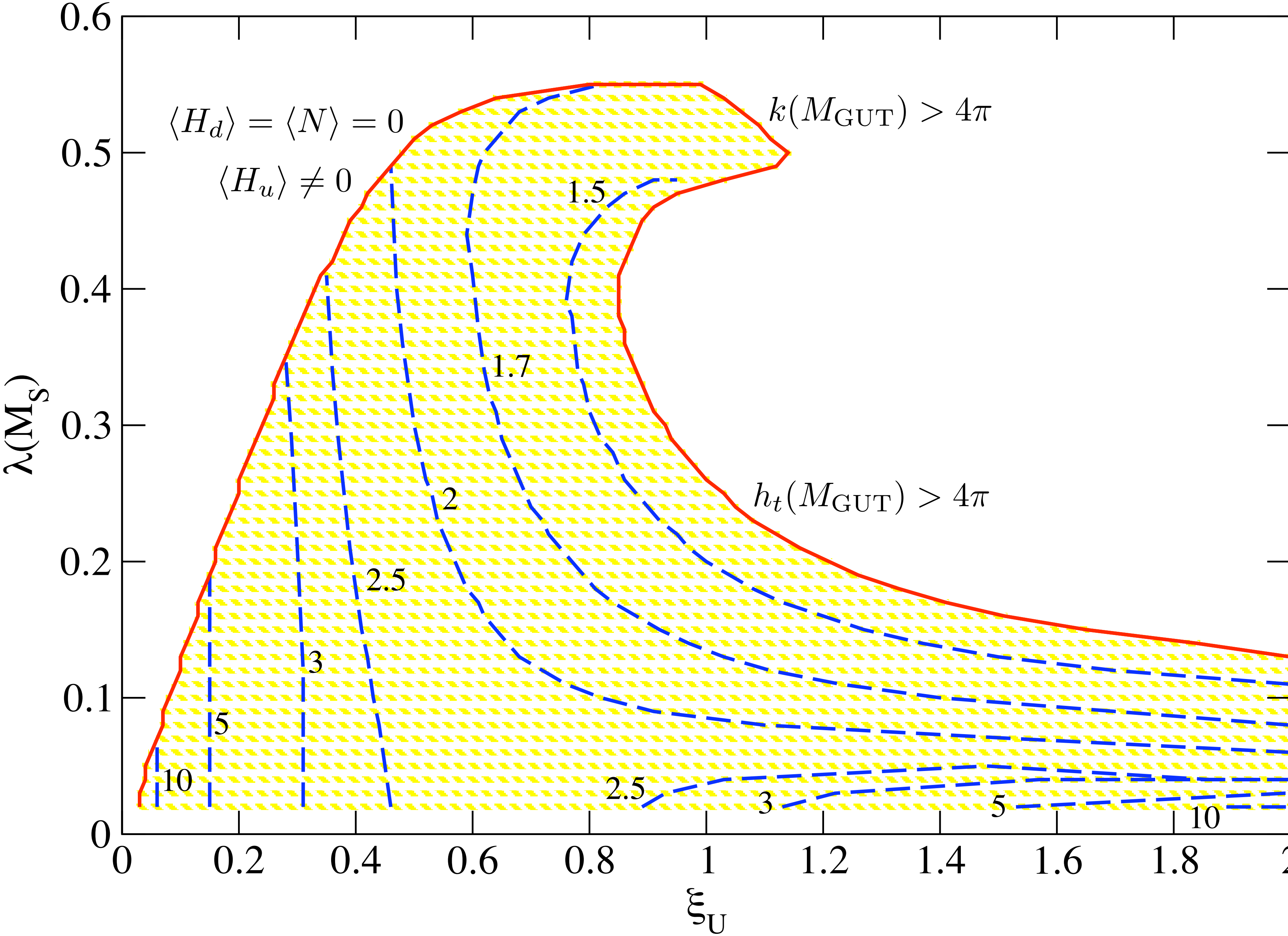
We solve the RGE of a tower of effective theories and get all the parameters at M_S

The EWSB conditions imposed at the scale M_S determine $\langle H_d\rangle\,,\ \langle H_u\rangle$ and $\langle N\rangle$.

Fixing $v^2 = \langle H_d\rangle^2 + \langle H_u\rangle^2$ as input, we can use them to determine $\tan\beta\,,\ \langle N\rangle$ and k

$\longrightarrow$ Two free parameters to play with: ξ_U and $\lambda(M_S)$

$\tan\beta$ in the $\xi_U - \lambda(M_S)$ plane



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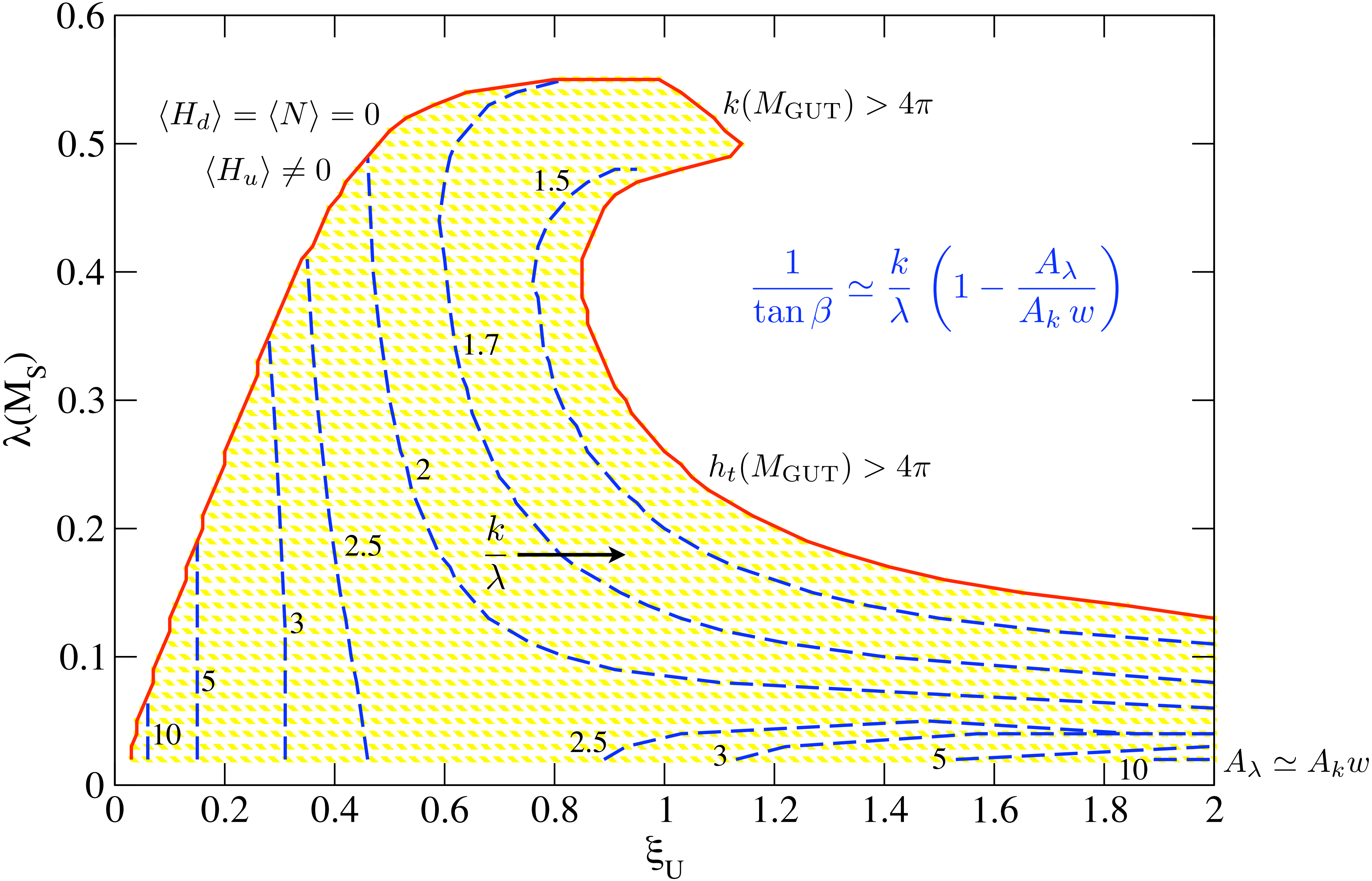
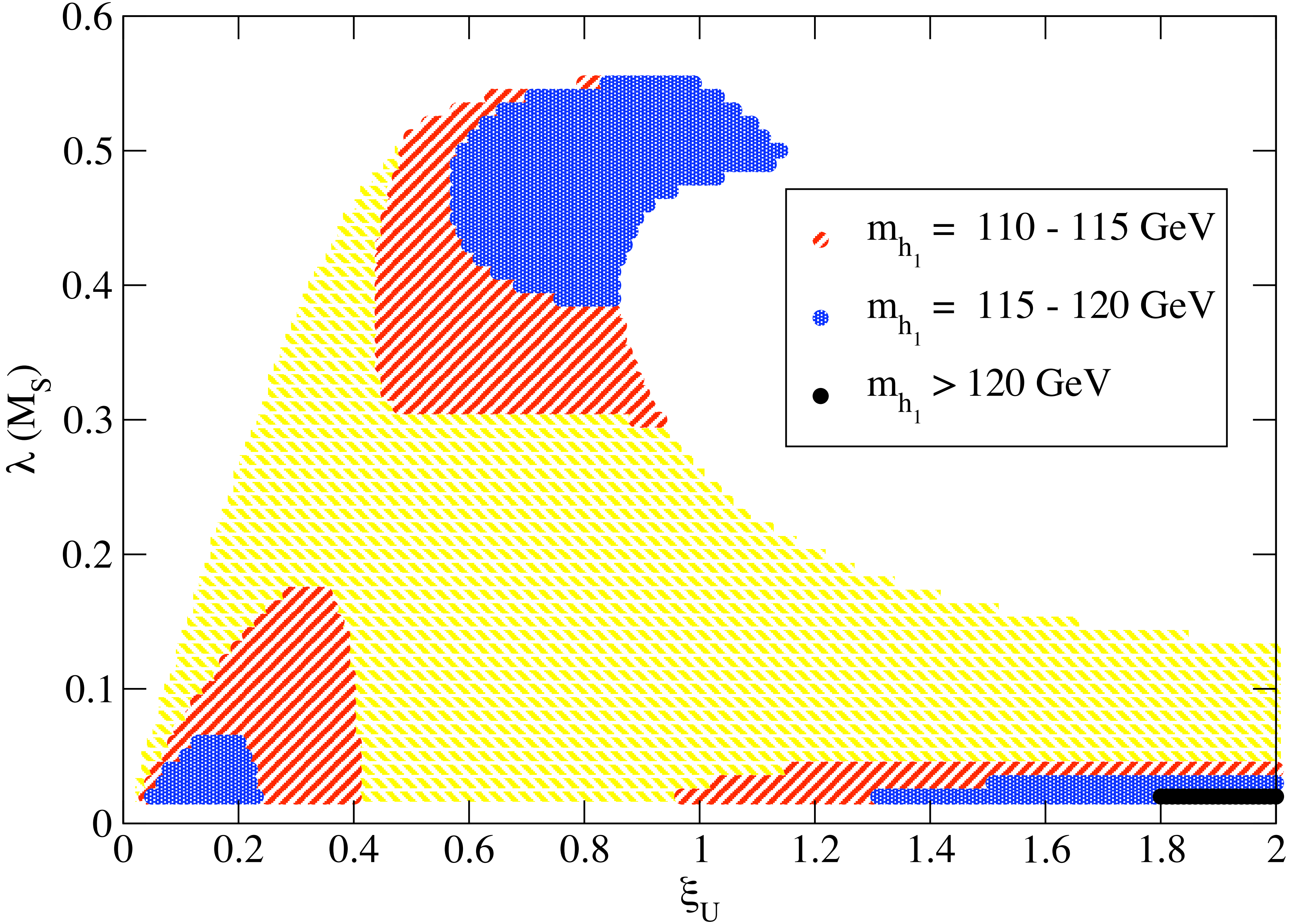


m_{h_1} in the $\xi_U - \lambda(M_S)$ plane



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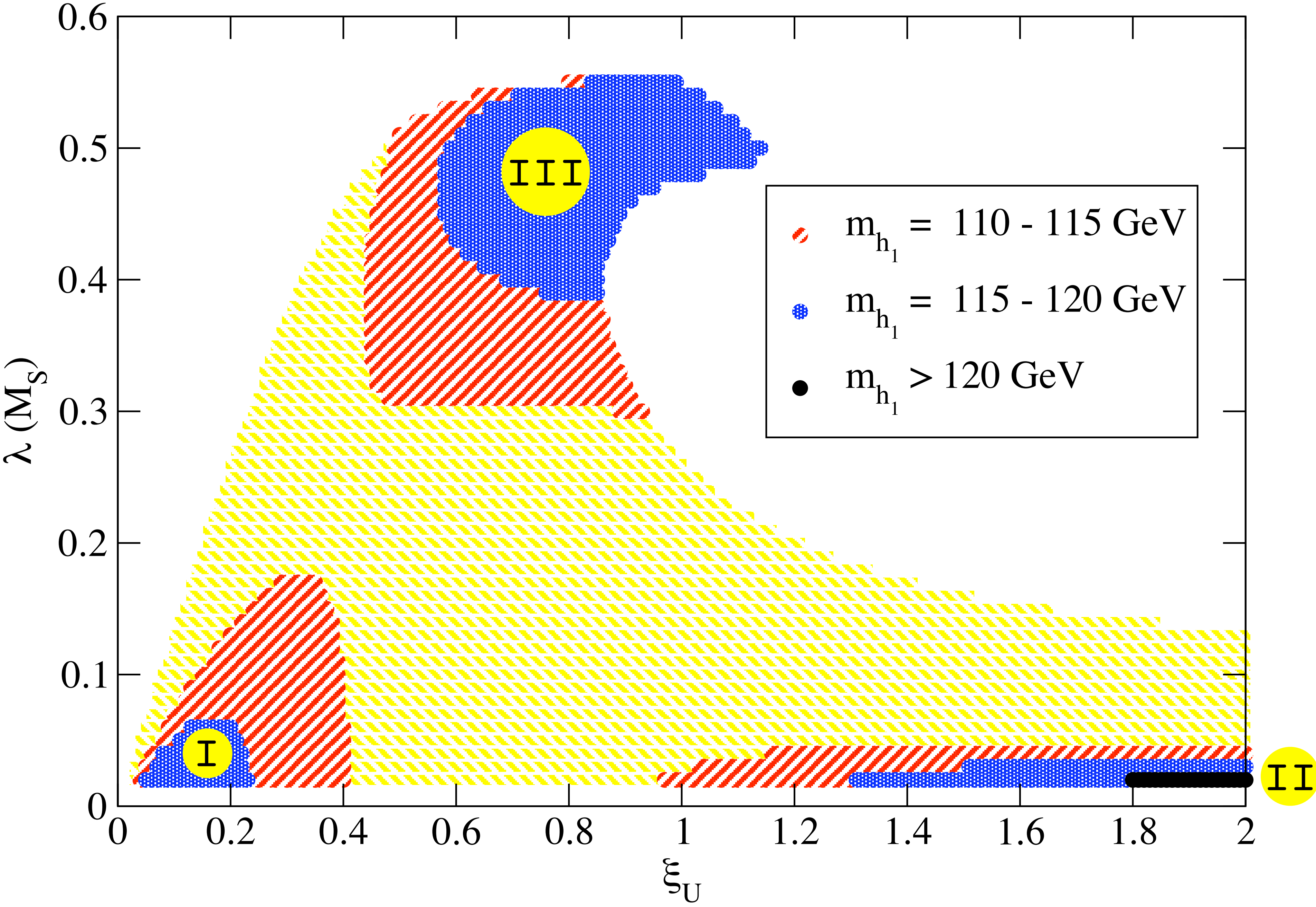

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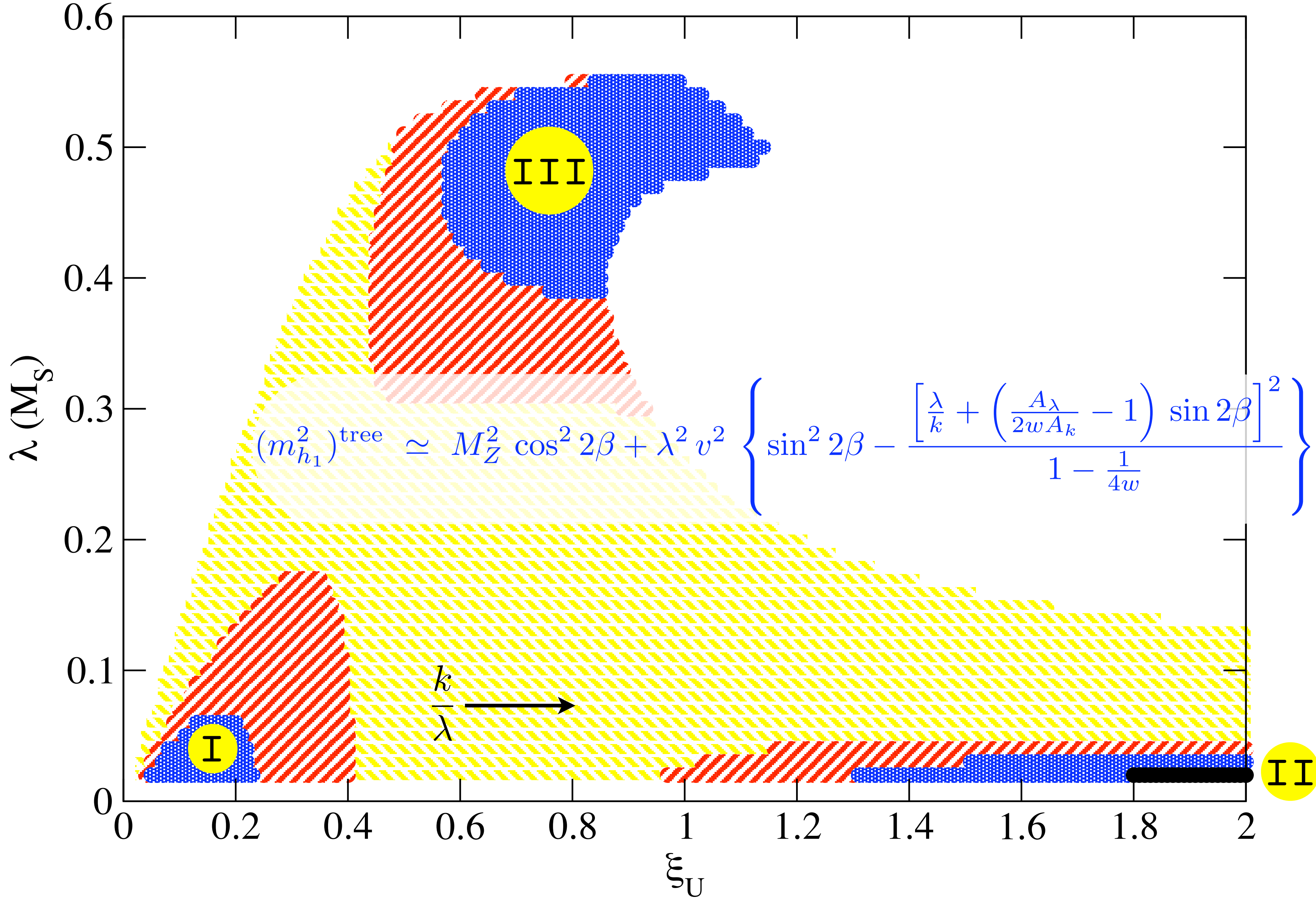
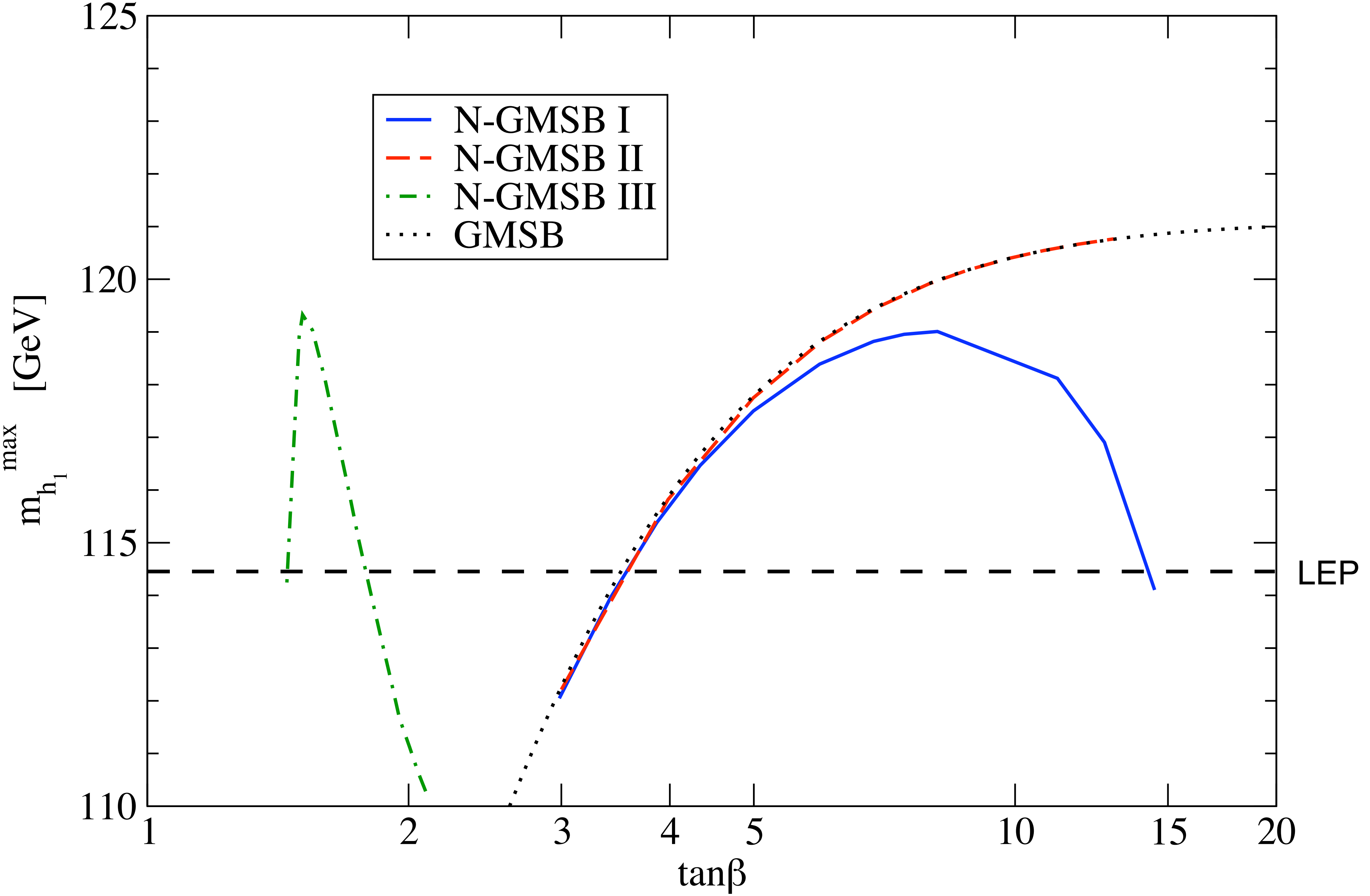


$(m_{h_1})^{\max}$ vs $\tan\beta$ in the three regions



The other NMSSM particle masses:

$$m_{a_1}\,,\ m_{h_2}\ \sim\ \mu\,, \qquad m_{a_2}\,,\ m_{h_3}\,,\ M_{\tilde N}\ \sim\ \frac{k}{\lambda}\,\mu$$

- **Region I** $\quad \mu \lesssim M_S\,, \quad \lambda \ll 1\,, \quad \dfrac{k}{\lambda} \ll 1$

  - The singlet-like scalars and the singlino are much lighter than the MSSM-like particles
  - The singlino can be the NLSP. Peculiar decay chain $\tilde B \longrightarrow \tilde N\, h_1 \longrightarrow \tilde G\, a_2\, h_1$

- **Region II** $\quad \mu \lesssim M_S\,, \quad \lambda \ll 1\,, \quad \dfrac{k}{\lambda} \gg 1$

  - The singlet-like scalars and the singlino are much heavier and essentially decoupled
  - This region corresponds to the *MSSM limit* of the NMSSM

- **Region III** $\quad \mu \gtrsim M_S\,, \quad \lambda \sim 0.5\,, \quad \dfrac{k}{\lambda} \sim 1$

  - All the scalars except h_1 , as well as the singlino, are quite heavy
  - m_{h_1} can be pushed to ~160 GeV if we give up perturbativity up to the GUT scale

Summary

- The B_μ problem of gauge mediation can be solved by adding a new singlet
- For an acceptable EWSB we must introduce singlet-messenger interactions that generate sizeable soft SUSY-breaking terms for the singlet
- As usual in GMSB, satisfying the LEP bound on the Higgs mass requires large values of the stop masses, and the model is somewhat fine-tuned
- Still, there are three distinct regions of the parameter space with acceptable Higgs mass spectrum and potentially interesting collider signatures

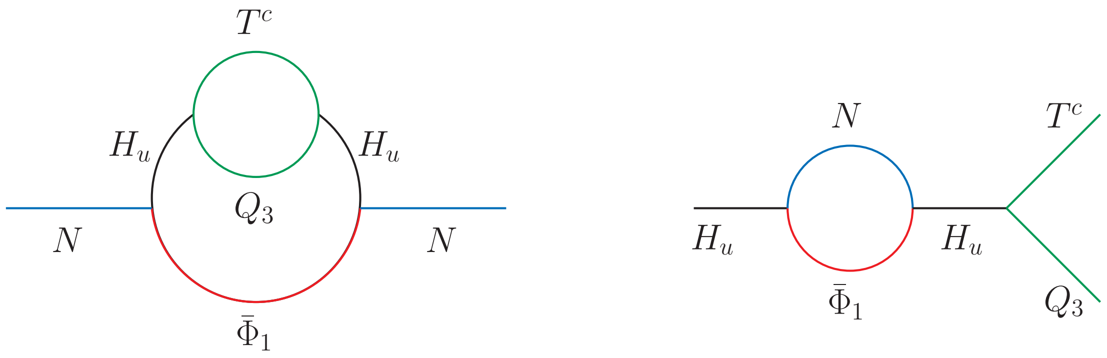
Spare Parts

Variations & Alternatives

- *"Yukawa Deflected Gauge Mediation"* *(Chacko, Katz, Perazzi & Ponton 2002)*

$$W \supset \lambda N H_d H_u - \frac{k}{3} N^3 + \xi N \bar{\Phi}_1^D H_u + h_t H_u Q_3 T^c$$

Additional messenger-Yukawa contributions to the soft mass parameters:



- *Add extra vector-like quarks* *(Dine & Nelson 1993, Agashe & Graesser 1997, de Gouvea, Friedland & Murayama 1997)*

$$W \supset \lambda N H_d H_u - \frac{k}{3} N^3 + \xi N Q \bar{Q}$$

The soft squark masses give a large and negative contribution to the running of $\tilde{m}_N^2$

New matter (squarks & quarks) at the TeV scale. Watch out for contributions to electroweak precision observables and FCNCs

Interlude: *a smart way to extract the soft terms at the messenger scale from the wave function renormalization of the observable fields* (*Giudice & Rattazzi 1997*)

$$\mathcal{L} \supset \int d^4\theta \, Z_Q(X, X^\dagger)\, Q^\dagger Q \;+\; \left(\int d^2\theta \, W(Q) \;+\; \text{h.c} \right)$$

Expand the w.f.r. of the matter superfields around the origin in superspace

$$\mathcal{L} \supset \int d^4\theta \left(Z_Q + \frac{\partial Z_Q}{\partial X} F\,\theta^2 + \frac{\partial Z_Q}{\partial X^\dagger} F^\dagger\,\bar\theta^2 + \frac{\partial^2 Z_Q}{\partial X \partial X^\dagger} F F^\dagger\,\theta^2\bar\theta^2 \right)\Bigg|_{X=M} Q^\dagger Q$$

Redefine the superfields so that they are canonically normalized:

$$Q' \equiv Z_Q^{\frac{1}{2}} \left(1 + \frac{\partial \ln Z_Q}{\partial X} F\,\theta^2 \right)\Bigg|_{X=M} Q$$

This kills the terms linear in F and leaves a soft (mass)² for the scalar component:

$$\tilde m_Q^2 \;=\; -\,\frac{\partial^2 \ln Z_Q}{\partial \ln X \partial \ln X^\dagger}\Bigg|_{X=M} \frac{F F^\dagger}{M M^\dagger}$$

The redefinition of the superfields in W also induces A-terms in the scalar potential

$$V \;=\; \sum_i A_i\, Q_i \frac{\partial W}{\partial Q_i} \;+\; \text{h.c.}\,, \qquad\qquad A_i \;=\; \frac{\partial \ln Z_{Q_i}}{\partial \ln X}\Bigg|_{X=M} \frac{F}{M}$$

The question is: how does the w.f.r. $Z_Q(X, X^\dagger)$ *depend on* X *and* $X^\dagger$*???*

The Lagrangian is invariant under the symmetry $X \to e^{i\varphi}\, X\,, \quad \bar{\Phi}\Phi \to e^{-i\varphi}\, \bar{\Phi}\Phi$

→ Z_Q can only depend on the combination $X^\dagger X$

Analytical continuation in superspace: determine how Z_Q depends on the messenger mass M, then replace $M \to \sqrt{X^\dagger X}$

$$\frac{\partial \ln Z_Q(X, X^\dagger)}{\partial \ln X}\bigg|_{X=M} = \frac{\partial \ln Z_Q(M)}{2\,\partial \ln M}\,, \qquad \frac{\partial^2 \ln Z_Q(X, X^\dagger)}{\partial \ln X \partial \ln X^\dagger}\bigg|_{X=M} = \frac{\partial^2 \ln Z_Q(M)}{4\,\partial (\ln M)^2}$$

M enters Z_Q as the scale at which the messengers are integrated out of the theory, inducing discontinuities in the anomalous dimensions of the matter superfields

$$\ln \frac{Z_Q(\mu)}{Z_Q(\Lambda)} = \int_{\ln \Lambda}^{\ln M} dt\, \gamma_Q^{(+)} + \int_{\ln M}^{\ln \mu} dt\, \gamma_Q^{(-)}\,, \qquad \gamma_Q^{(\pm)} \equiv \frac{d \ln Z_Q}{d \ln \mu}\bigg|_{\substack{\mu > M \\ \mu < M}}$$

$$\ln Z_Q(\mu) \approx \text{const.} + \Delta\gamma_Q \ln M + \mathcal{O}(> 1\,\text{loop})\,, \qquad \Delta\gamma_Q = \gamma_Q^{(+)} - \gamma_Q^{(-)}$$

A-terms generated at 1-loop: $A_i(M) = \frac{\Delta\gamma_{Q_i}}{2}\,\frac{F}{M}$

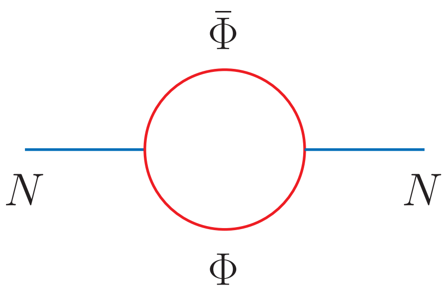
(mass)² terms generated at 2-loop:

$$\tilde{m}_Q^2(M) = -\frac{1}{4}\sum_i \left[\beta_{\lambda_i}^{(+)}\frac{\partial\left(\Delta\gamma_Q\right)}{\partial\lambda_i^2} - \Delta\beta_{\lambda_i}\frac{\partial\gamma_Q^{(-)}}{\partial\lambda_i^2}\right]_{\mu=M}\;\frac{F^2}{M^2}$$

Two-loop results just out of the one-loop RGE. No need to compute Feynman diagrams!!

One-loop contributions to (mass)² terms can be generated at higher orders in F/M^2

e.g.



$$\approx\; -\frac{\xi^2}{16\pi^2}\,\frac{F^4}{M^6}$$

For $\xi = \mathcal{O}(1)$ these contributions are negligible as long as $M > 4\pi\, F/M \quad (\simeq 10^6\,\mathrm{GeV})$

$$\beta^{(\pm)}_{\lambda_i} \equiv \left.\frac{d\lambda_i^2}{d\ln\mu}\right|_{\substack{\mu>M\\ \mu<M}}, \qquad \Delta\beta_{\lambda_i} = \beta^{(+)}_{\lambda_i} - \beta^{(-)}_{\lambda_i}$$

(mass)² terms generated at 2-loop:

$$\tilde{m}_Q^2(M) = -\frac{1}{4}\sum_i \left[\beta^{(+)}_{\lambda_i}\frac{\partial\left(\Delta\gamma_Q\right)}{\partial\lambda_i^2} - \Delta\beta_{\lambda_i}\frac{\partial\gamma_Q^{(-)}}{\partial\lambda_i^2}\right]_{\mu=M}\frac{F^2}{M^2}$$

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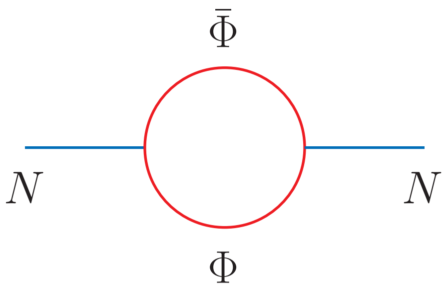

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A-terms generated at 1-loop: $A_i(M) = \frac{\Delta\gamma_{Q_i}}{2}\,\frac{F}{M}$

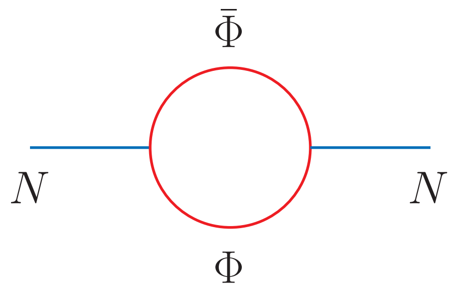
(mass)² terms generated at 2-loop:

$$\tilde{m}_Q^2(M) = -\frac{1}{4}\sum_i \left[\beta^{(+)}_{\lambda_i}\frac{\partial\,(\Delta\gamma_Q)}{\partial\lambda_i^2} - \Delta\beta_{\lambda_i}\frac{\partial\gamma_Q^{(-)}}{\partial\lambda_i^2}\right]_{\mu=M}\;\frac{F^2}{M^2}$$

Two-loop results just out of the one-loop RGE. No need to compute Feynman diagrams!!

One-loop contributions to (mass)² terms can be generated at higher orders in F/M^2

e.g.



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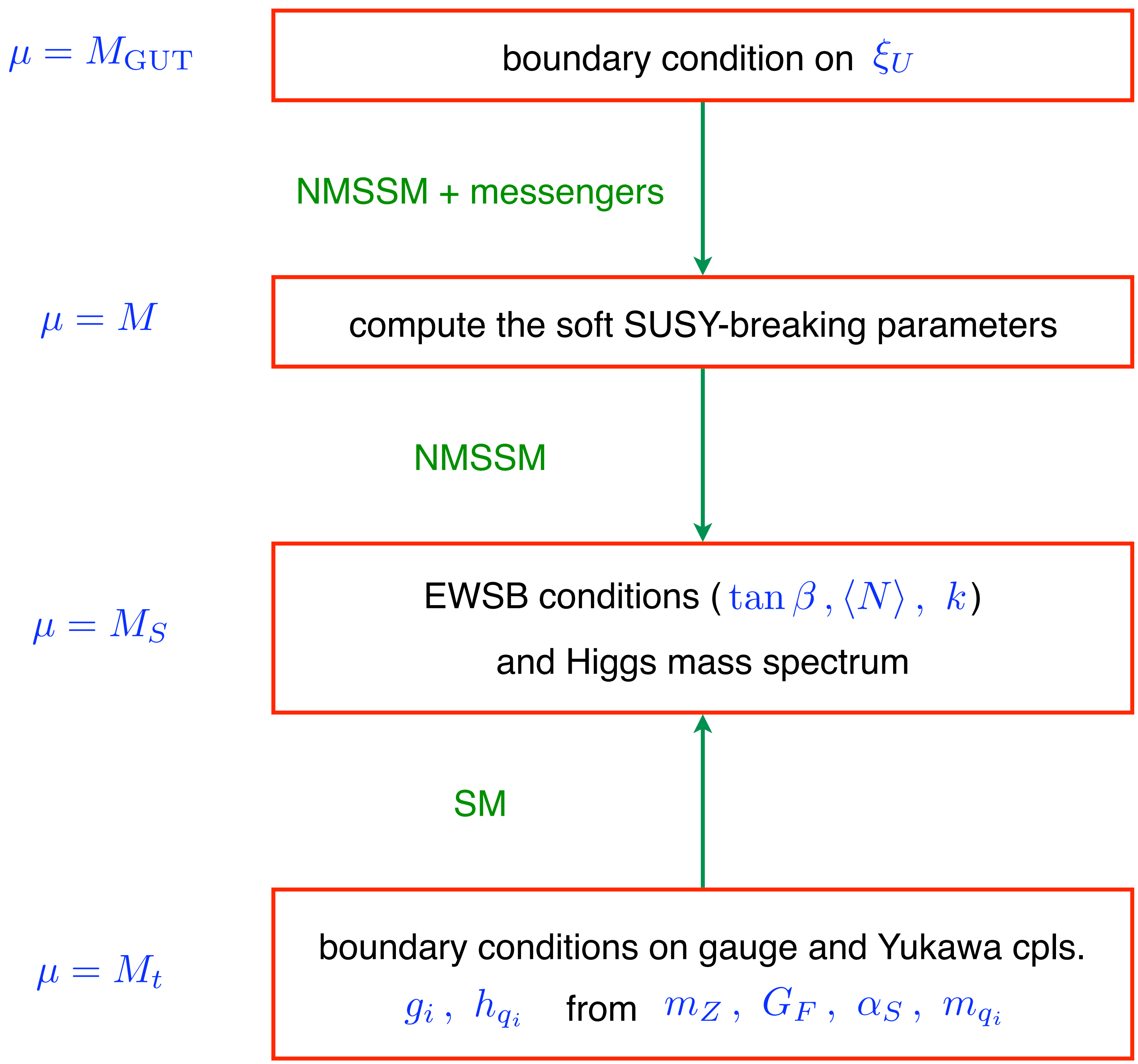
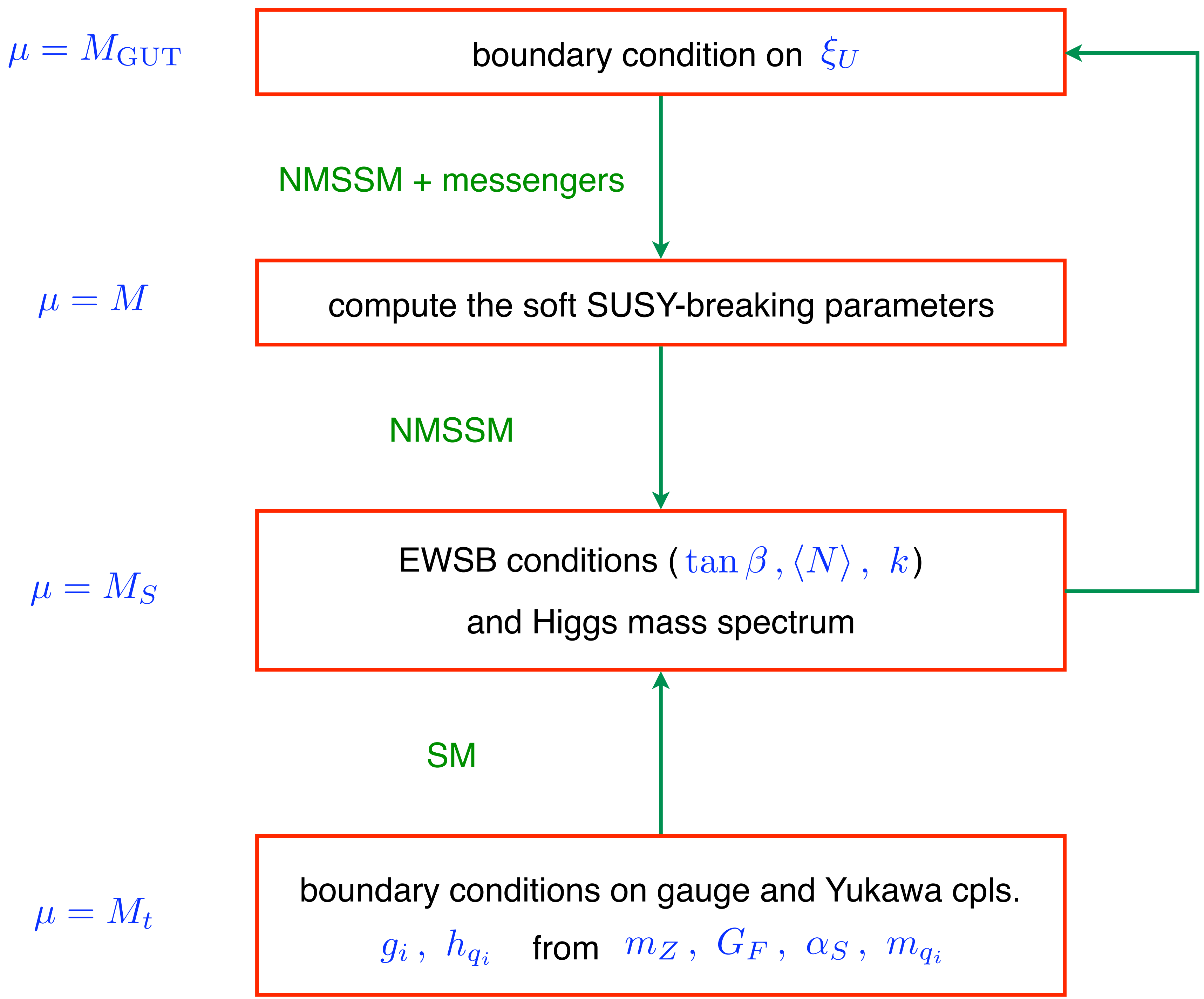
For $\xi = \mathcal{O}(1)$ these contributions are negligible as long as $M > 4\pi\,F/M \quad (\simeq 10^6\,\mathrm{GeV})$


$\mu = M_{\rm GUT}$
boundary condition on ξ_U
NMSSM + messengers
$\mu = M$
compute the soft SUSY-breaking parameters
NMSSM
$\mu = M_S$
EWSB conditions ($\tan\beta\,,\langle N\rangle\,,\ k$)
and Higgs mass spectrum
SM
$\mu = M_t$
boundary conditions on gauge and Yukawa cpls.
$g_i\ ,\ h_{q_i}$ from $m_Z\ ,\ G_F\ ,\ \alpha_S\ ,\ m_{q_i}$



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g_i , h_{q_i} from m_Z , G_F , α_S , m_{q_i}

Representative mass spectra

- **Region I** $\xi_U \;=\; 0.06\,, \quad \lambda(M_S) \;=\; 0.02$

  $M_S \;\approx\; 2\,\mathrm{TeV}\,, \quad \mu \;\approx\; -1.4\,\mathrm{TeV}\,, \quad A_t \;\approx\; -1.5\,\mathrm{TeV}\,, \quad \dfrac{k}{\lambda} \;\approx\; \dfrac{1}{7}\,, \quad \tan\beta \;\approx\; 11$

  $M_1 \;\approx\; 480\,\mathrm{GeV}\,, \quad M_2 \;\approx\; 880\,\mathrm{GeV}\,, \quad M_3 \;\approx\; 2.3\,\mathrm{TeV}\,, \quad M_{\tilde{N}} \;\approx\; 400\,\mathrm{GeV}\,,$

  $m_{h_1} = 118\,\mathrm{GeV}\,, \quad m_{h_2} \approx m_{a_1} \;\approx\; 1.8\,\mathrm{TeV}\,, \quad m_{h_3} \;\approx\; 380\,\mathrm{GeV}\,, \quad m_{a_2} \;\approx\; 210\,\mathrm{GeV}$

- **Region II** $\xi_U \;=\; 2\,, \quad \lambda(M_S) \;=\; 0.02$

  $M_S \;\approx\; 2\,\mathrm{TeV}\,, \quad \mu \;\approx\; -1.4\,\mathrm{TeV}\,, \quad A_t \;\approx\; -1.5\,\mathrm{TeV}\,, \quad \dfrac{k}{\lambda} \;\approx\; 5\,, \quad \tan\beta \;\approx\; 13$

  $M_1 \;\approx\; 480\,\mathrm{GeV}\,, \quad M_2 \;\approx\; 880\,\mathrm{GeV}\,, \quad M_3 \;\approx\; 2.3\,\mathrm{TeV}\,, \quad M_{\tilde{N}} \;\approx\; 14\,\mathrm{TeV}\,,$

  $m_{h_1} = 121\,\mathrm{GeV}\,, \quad m_{h_2} \approx m_{a_1} \;\approx\; 1.7\,\mathrm{TeV}\,, \quad m_{h_3} \;\approx\; 7\,\mathrm{TeV}\,, \quad m_{a_2} \;\approx\; 21\,\mathrm{TeV}$

- **Region III** $\xi_U \;=\; 1\,, \quad \lambda(M_S) \;=\; 0.5$

  $M_S \;\approx\; 2\,\mathrm{TeV}\,, \quad \mu \;\approx\; -2.6\,\mathrm{TeV}\,, \quad A_t \;\approx\; -1.2\,\mathrm{TeV}\,, \quad \dfrac{k}{\lambda} \;\approx\; 0.8\,, \quad \tan\beta \;\approx\; 1.5$

  $M_1 \;\approx\; 480\,\mathrm{GeV}\,, \quad M_2 \;\approx\; 880\,\mathrm{GeV}\,, \quad M_3 \;\approx\; 2.3\,\mathrm{TeV}\,, \quad M_{\tilde{N}} \;\approx\; 4.3\,\mathrm{TeV}\,,$

  $m_{h_1} = 119\,\mathrm{GeV}\,, \quad m_{h_2} \approx m_{a_1} \;\approx\; 3\,\mathrm{TeV}\,, \quad m_{h_3} \;\approx\; 3.6\,\mathrm{TeV}\,, \quad m_{a_2} \;\approx\; 4\,\mathrm{TeV}$