

ILC Physics in Florence

**Neutralino Dark Matter and τ polarization:
a way to distinguish SUSY-GUT from CMSSM**

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Firenze, 12/09/2007

First part:
SUSY-GUTs, SUSY see-saw and Neutralino DM

based on L.C., Y. Mambrini, S. Vempati, arXiv:0704.3518 [hep-ph]

Neutralino Dark Matter

MSSM neutralinos:

$$\left(\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0 \right)$$

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}$$

Lightest eigenvalue:

$$\tilde{\chi}_1^0 = Z_{11}\tilde{B} + Z_{12}\tilde{W}_3 + Z_{13}\tilde{H}_d^0 + Z_{14}\tilde{H}_u^0$$

R-parity makes the LSP stable \rightarrow candidate for CDM

Relic density (WMAP):

$$0.087 \lesssim \Omega_{DM} h^2 \lesssim 0.138$$

CMSSM and three WMAP “corridors”

CMSSM: $m_0, M_{1/2}, A_0, \tan \beta \longrightarrow M_{\text{GUT}} \simeq 2 \times 10^{16} \text{ GeV}$

Mostly “Bino”:

$$\tilde{\chi}_1^0 = Z_{11} \tilde{B} + Z_{12} \tilde{W}_3 + Z_{13} \tilde{H}_d^0 + Z_{14} \tilde{H}_u^0$$

Small annihilation cross-section \rightarrow too large relic density

$$0.087 \lesssim \Omega_{DM} h^2 \lesssim 0.138$$

Peculiar conditions to enhance cross-section are needed!

CMSSM and three WMAP “corridors”

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• *Stau coannihilation*: $\tilde{\chi}_1^0 + \tilde{\tau}_1 \longrightarrow \tau + Z_0(\gamma)$

$m_{\tilde{\chi}_1^0} \simeq m_{\tilde{\tau}_1} \longrightarrow$ close to the “stau LSP” region

• *A-pole funnel*: $\tilde{\chi}_1^0 + \tilde{\chi}_1^0 \longrightarrow f + \bar{f} \quad 2 m_{\tilde{\chi}_1^0} \simeq m_{A^0}$

• *Focus point*: for small $\mu \rightarrow$ close to the “no EWSB” region ($\mu^2 < 0$)
enhancement of the higgsino components

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$$W_{\text{MSSM}} = Y^u Q u^c h_u + Y^d Q d^c h_d + Y^e L e^c h_d + \mu h_u h_d$$

$$10 = \begin{pmatrix} 0 & u^c & -u^c & -u & -d \\ -u^c & 0 & u^c & -u & -d \\ u & -u^c & 0 & -u & -d \\ u & u & u & 0 & e^c \\ d & d & d & -e^c & 0 \end{pmatrix}_L \quad \bar{5} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ e \\ \nu \end{pmatrix}_L$$

$$W_{SU(5)_{RN}} = Y^u 10 10 5_u + Y^d 10 \bar{5} \bar{5}_d + Y^\nu \bar{5} 1 5_u + M_R 1 1 + \mu 5_u \bar{5}_d$$

CMSSM vs. $SU(5)_{RN}$

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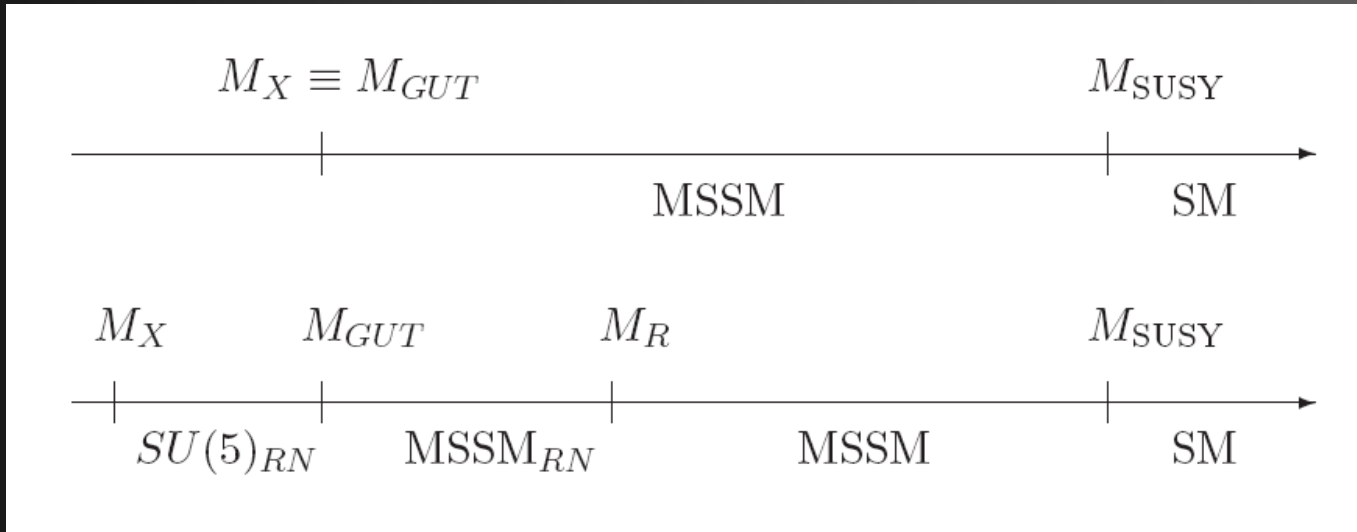
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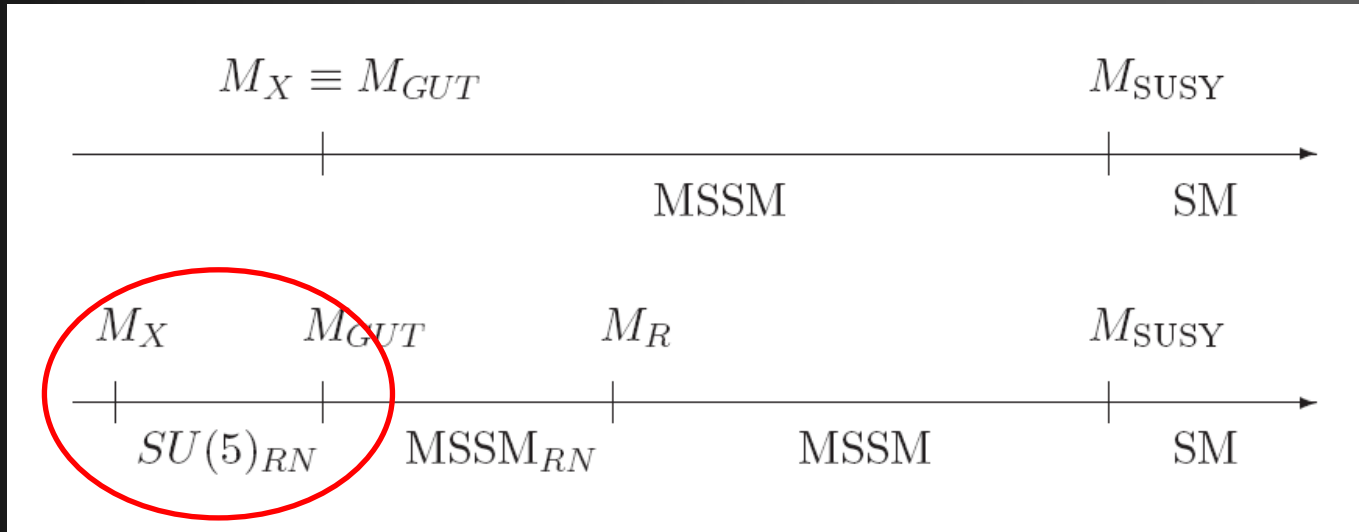
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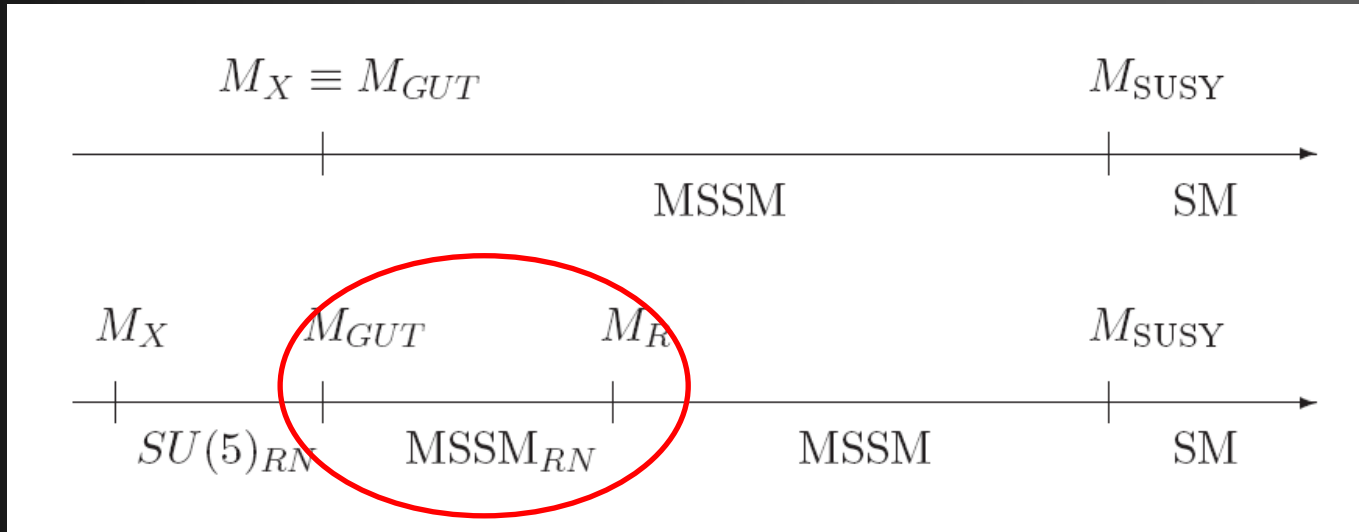
CMSSM vs. $SU(5)_{RN}$



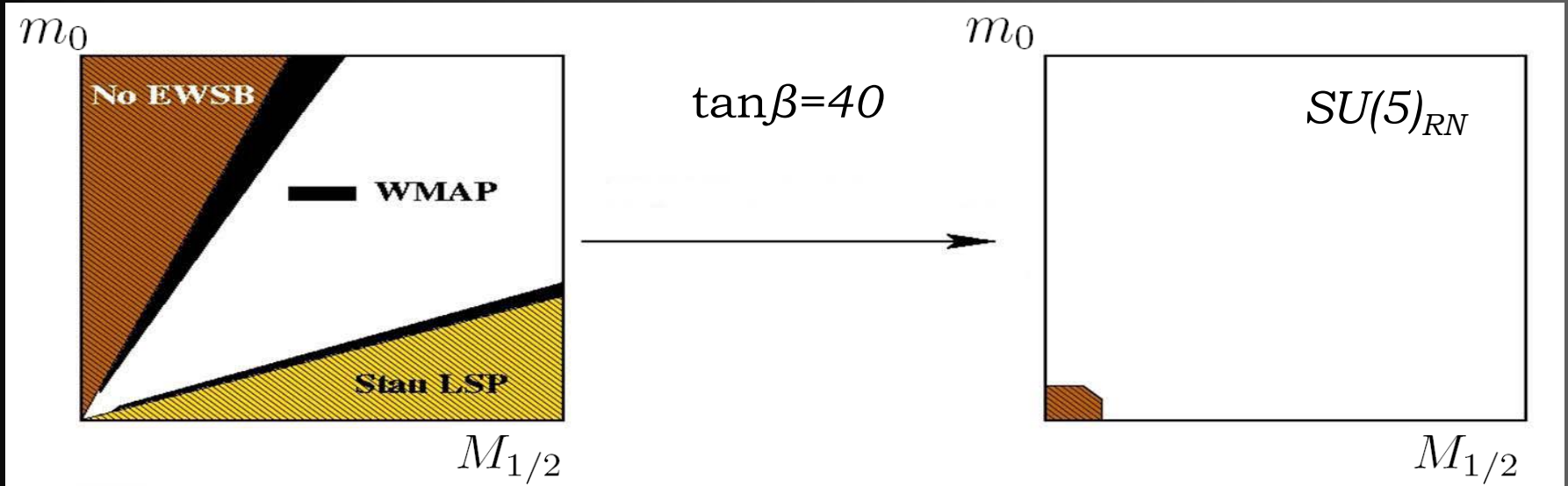
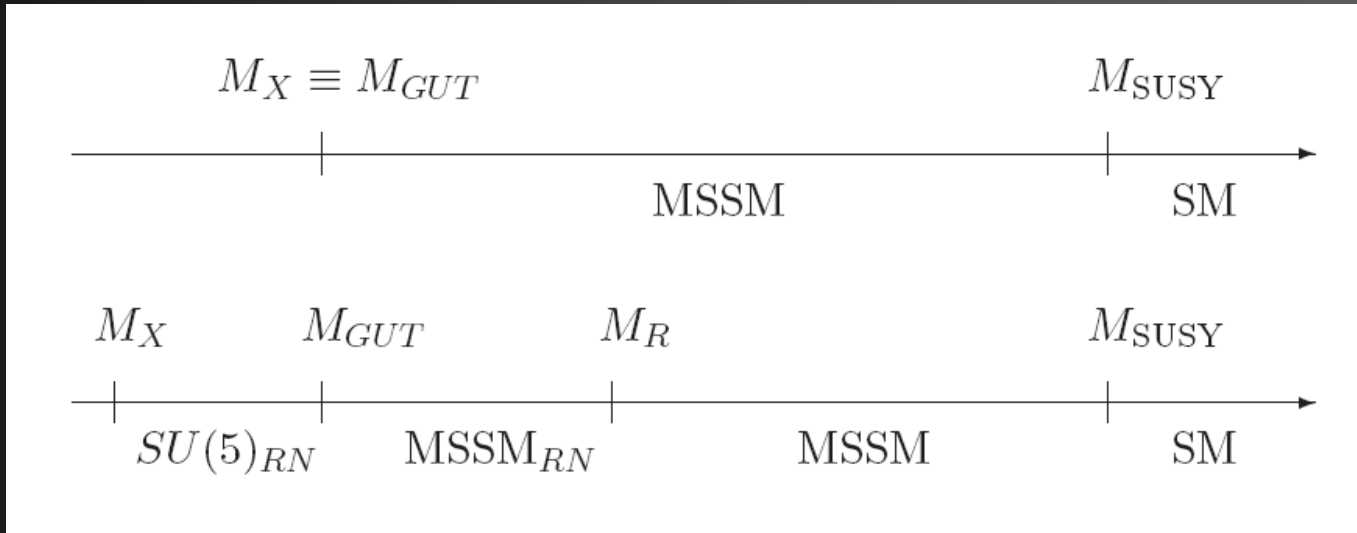
CMSSM vs. $SU(5)_{RN}$



CMSSM vs. $SU(5)_{RN}$



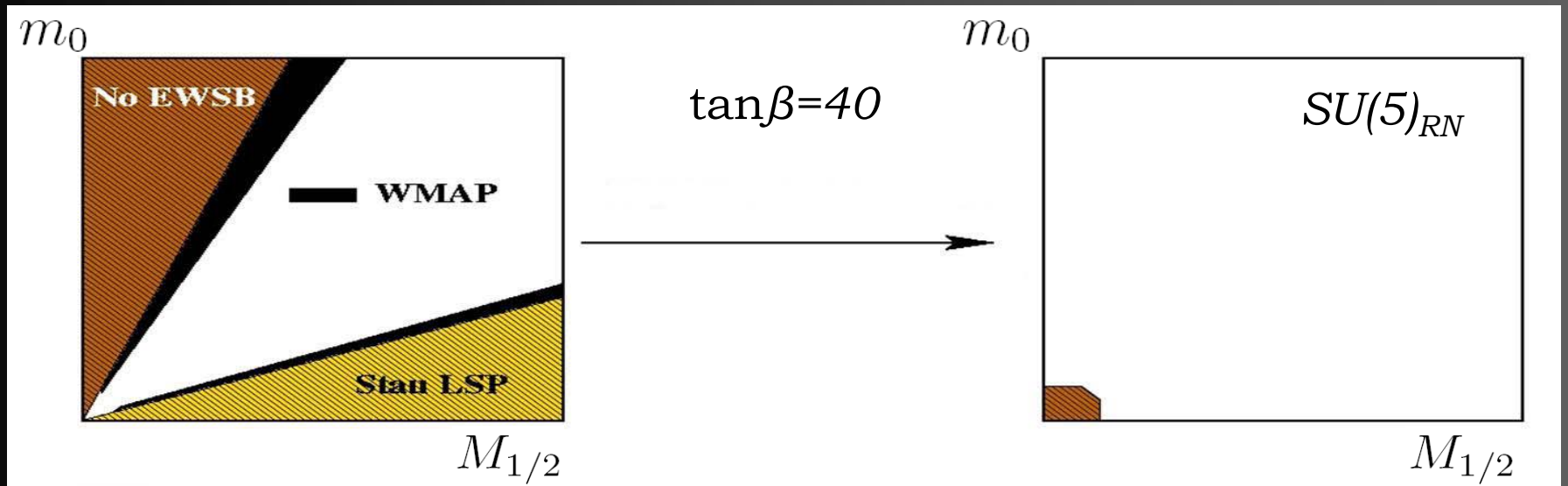
$SU(5)_{RN}$ parameter space



$SU(5)_{RN}$ parameter space

I)

II)

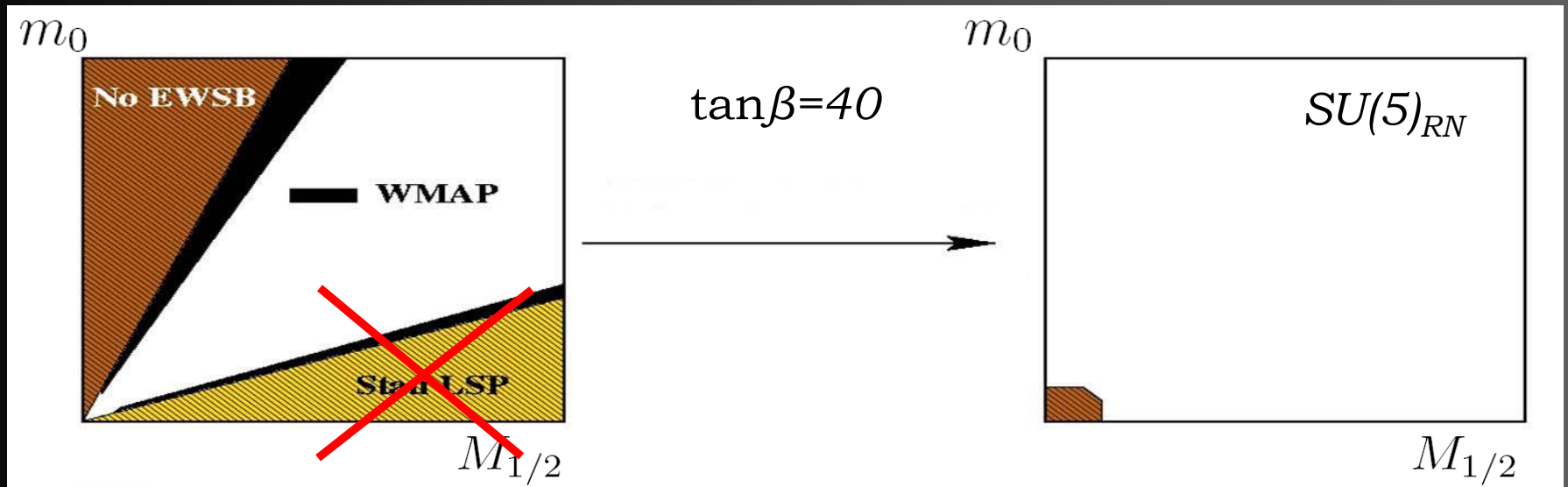


$SU(5)_{RN}$ parameter space

I) $M_X \rightarrow M_{GUT} \Rightarrow m_{\tilde{\tau}_R}^2(M_{GUT}) \simeq \frac{144}{20\pi} \alpha_5 M_{1/2}^2 \ln\left(\frac{M_X}{M_{GUT}}\right) \simeq 0.25 M_{1/2}^2$

(right stau mass for $m_0 = 0$)

II)



$SU(5)_{RN}$ parameter space

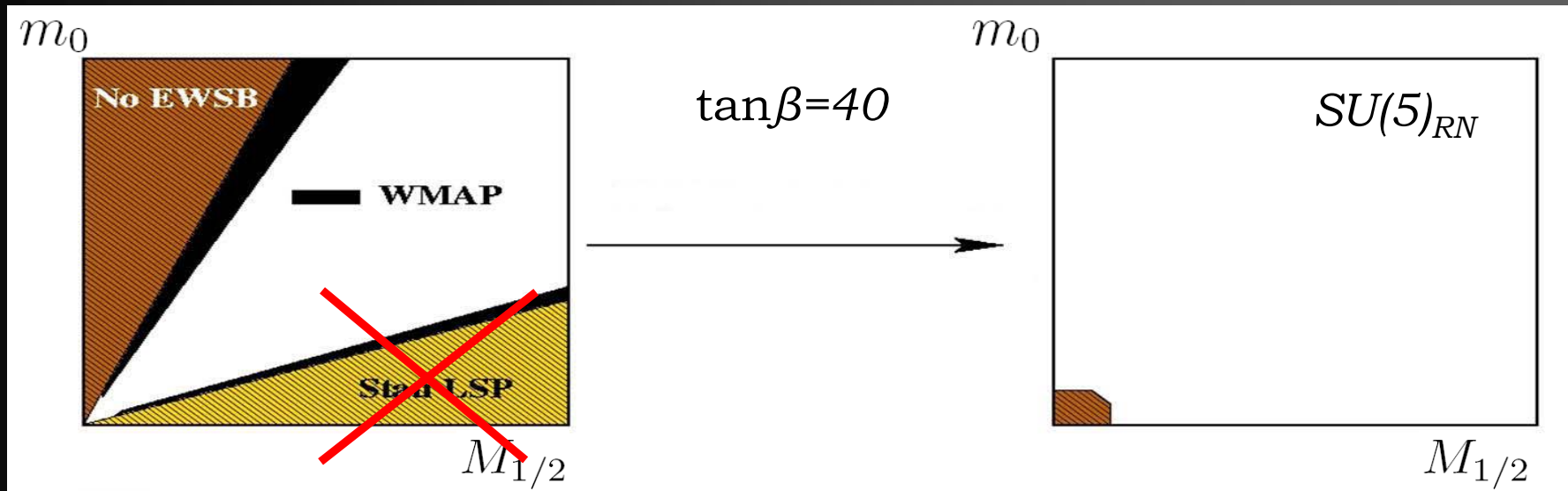
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(right stau mass for $m_0 = 0$)

II)

$$|\mu|^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_Z^2$$

$$\sin 2\beta = \frac{2B\mu}{m_{H_u}^2 + m_{H_d}^2 + 2\mu^2}$$



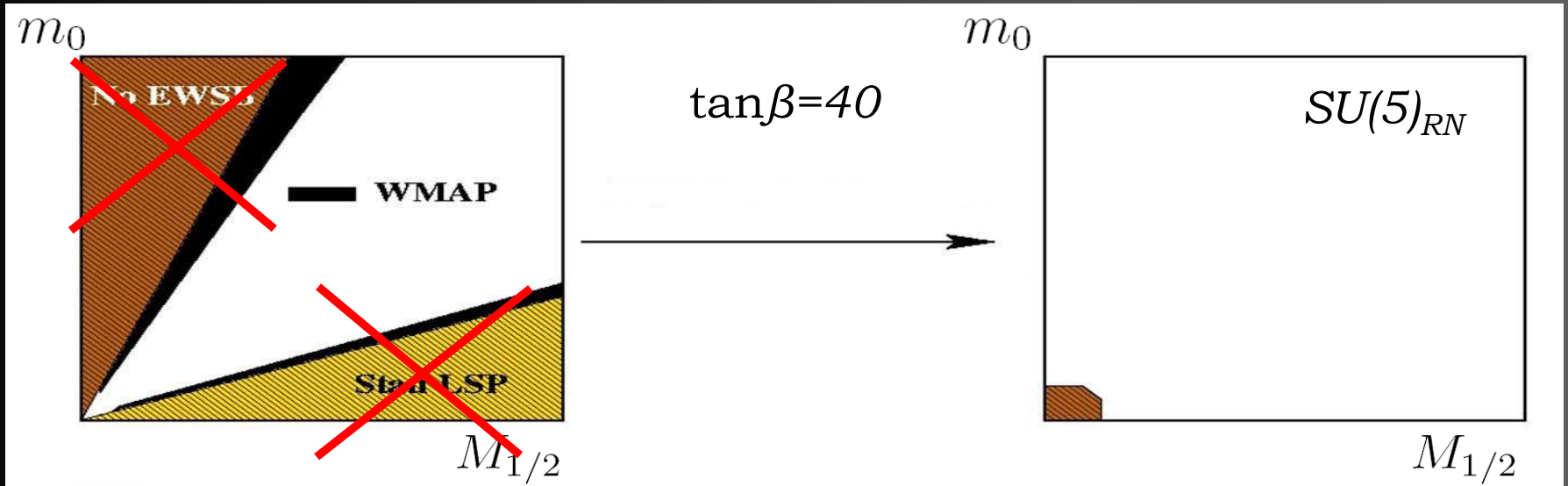
$SU(5)_{RN}$ parameter space

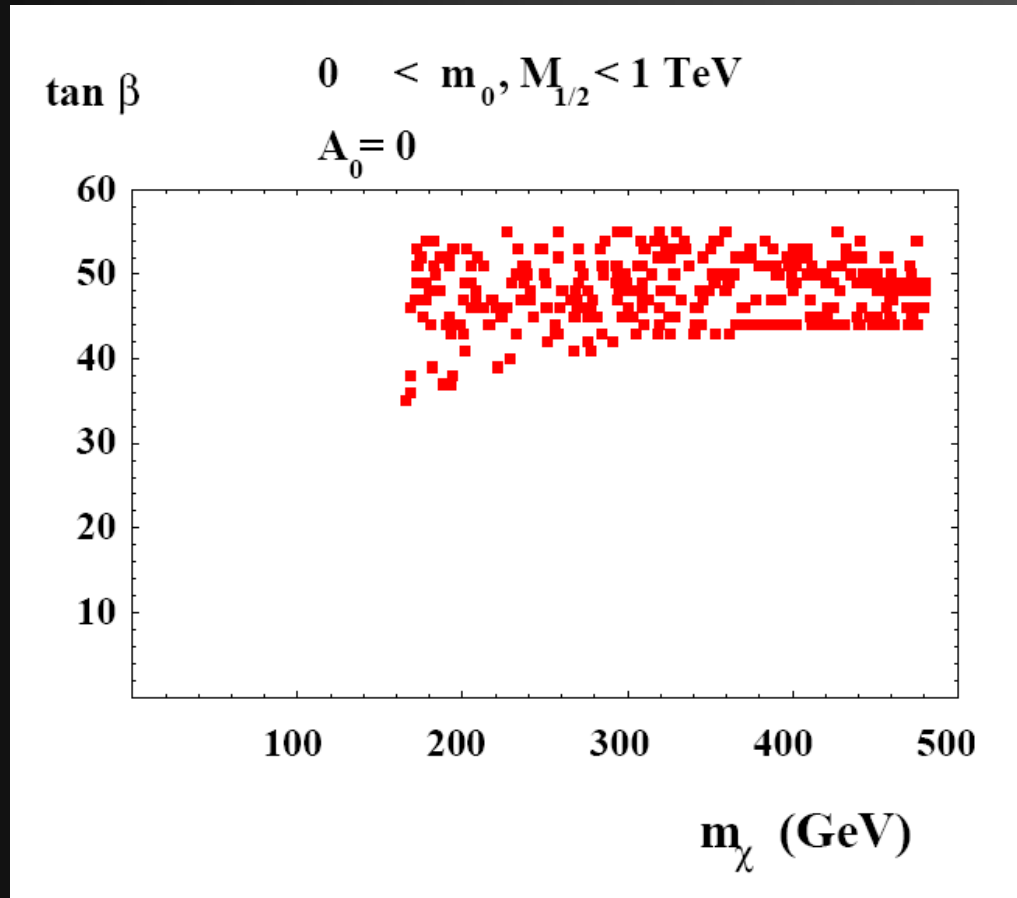
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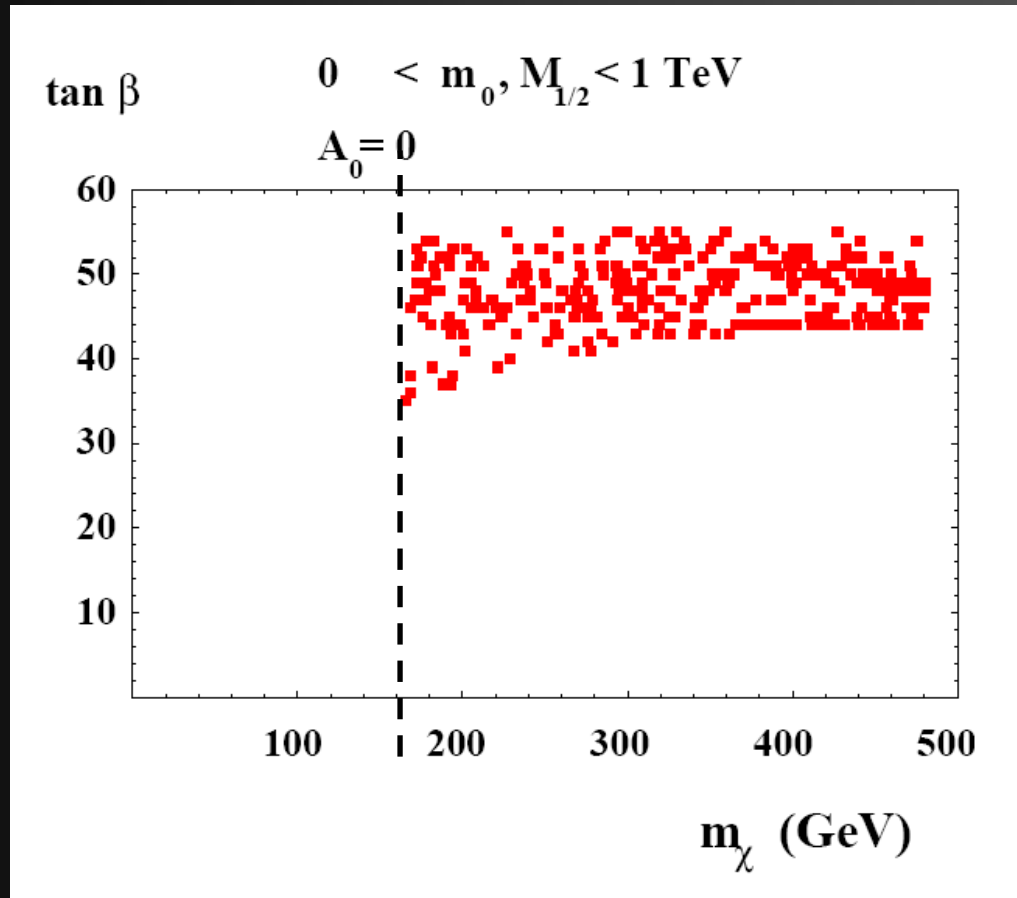
II)

$$(4\pi)^2 \frac{\partial m_{H_u}^2}{\partial \ln(\tilde{\mu}/M_X)} \simeq 6y_t^2(m_{H_u}^2 + \underbrace{m_{\tilde{U}_3}^2 + m_{\tilde{Q}_3}^2}_{\text{red bracket}} + A_t^2) + \underbrace{2y_\nu^2}_{\text{red circle}}(m_{H_u}^2 + m_{\tilde{N}}^2 + m_{\tilde{L}_3}^2 + A_\nu^2)$$





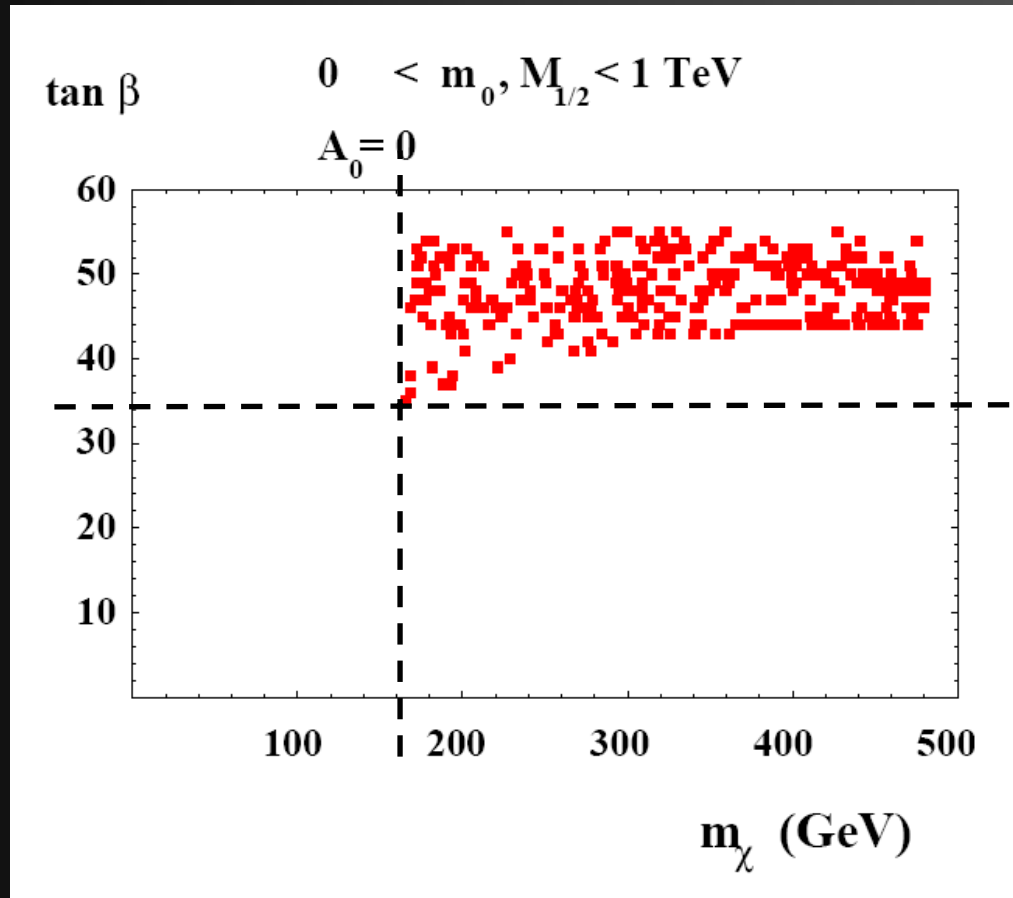
WMAP allowed points



WMAP allowed points



$$m_{\tilde{\chi}_1^0} \gtrsim 160 \text{ GeV}$$

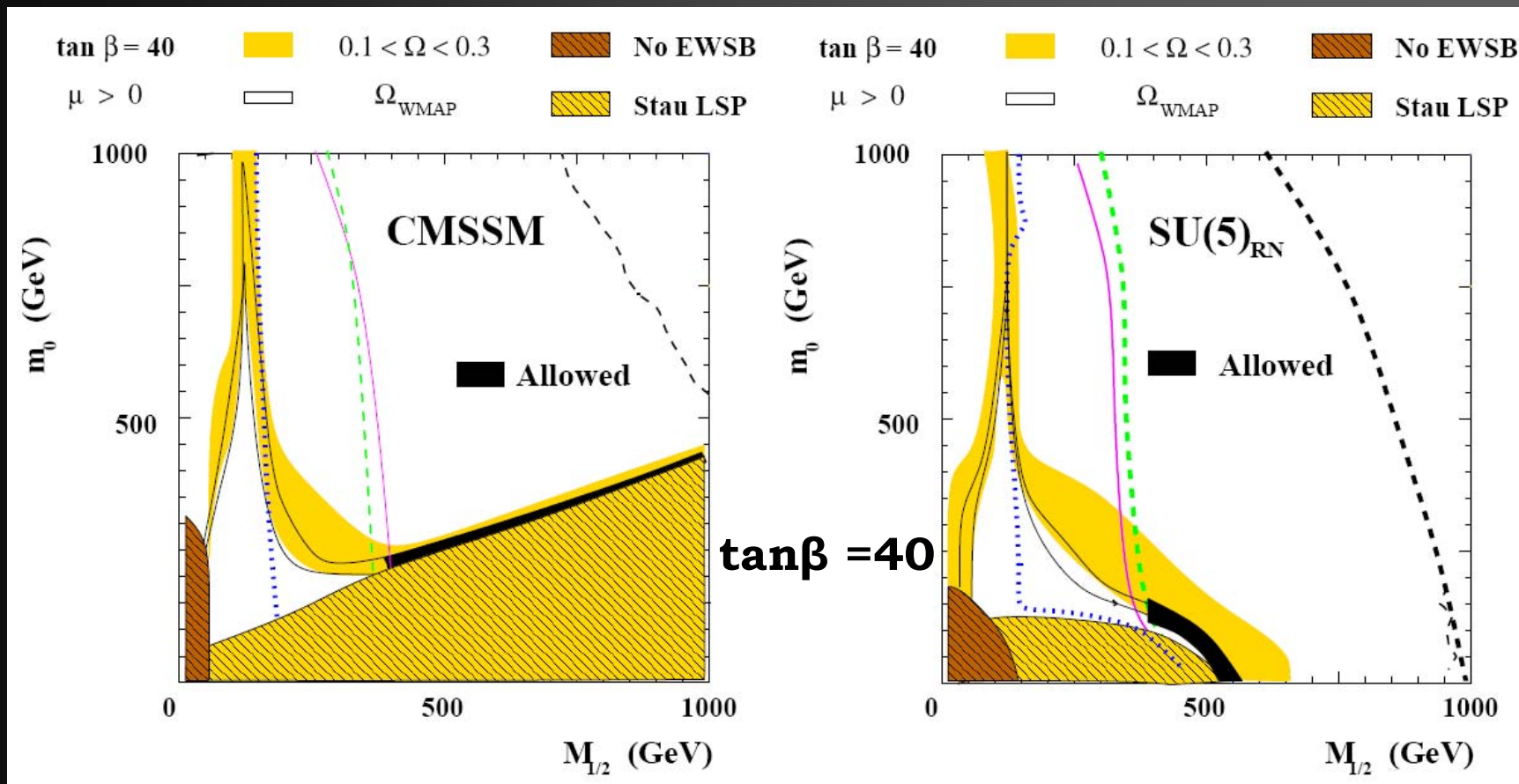


WMAP allowed points

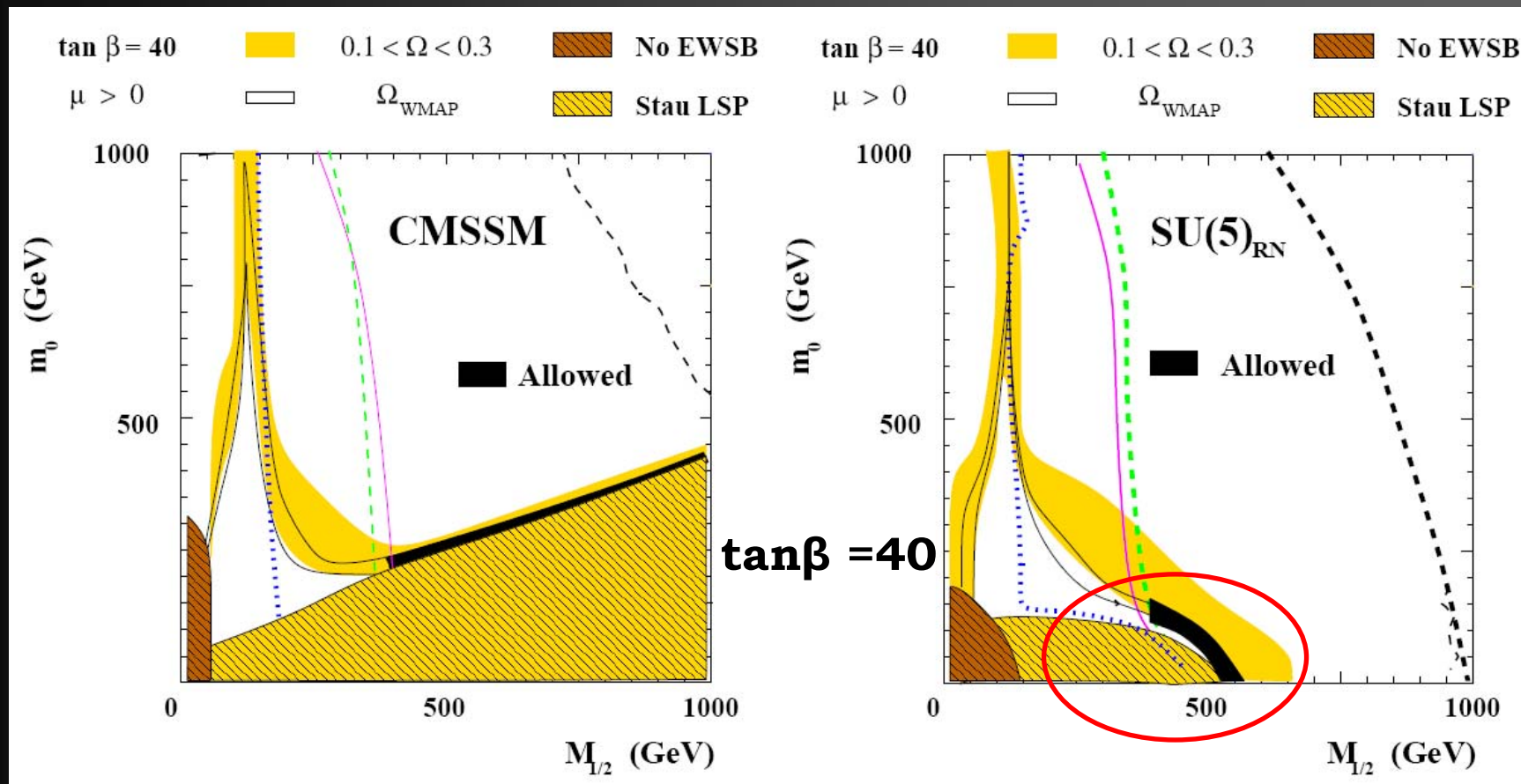
$\tan \beta \gtrsim 34$

$m_{\tilde{\chi}_1^0} \gtrsim 160 \text{ GeV}$

DM phenomenology in $SU(5)_{RN}$



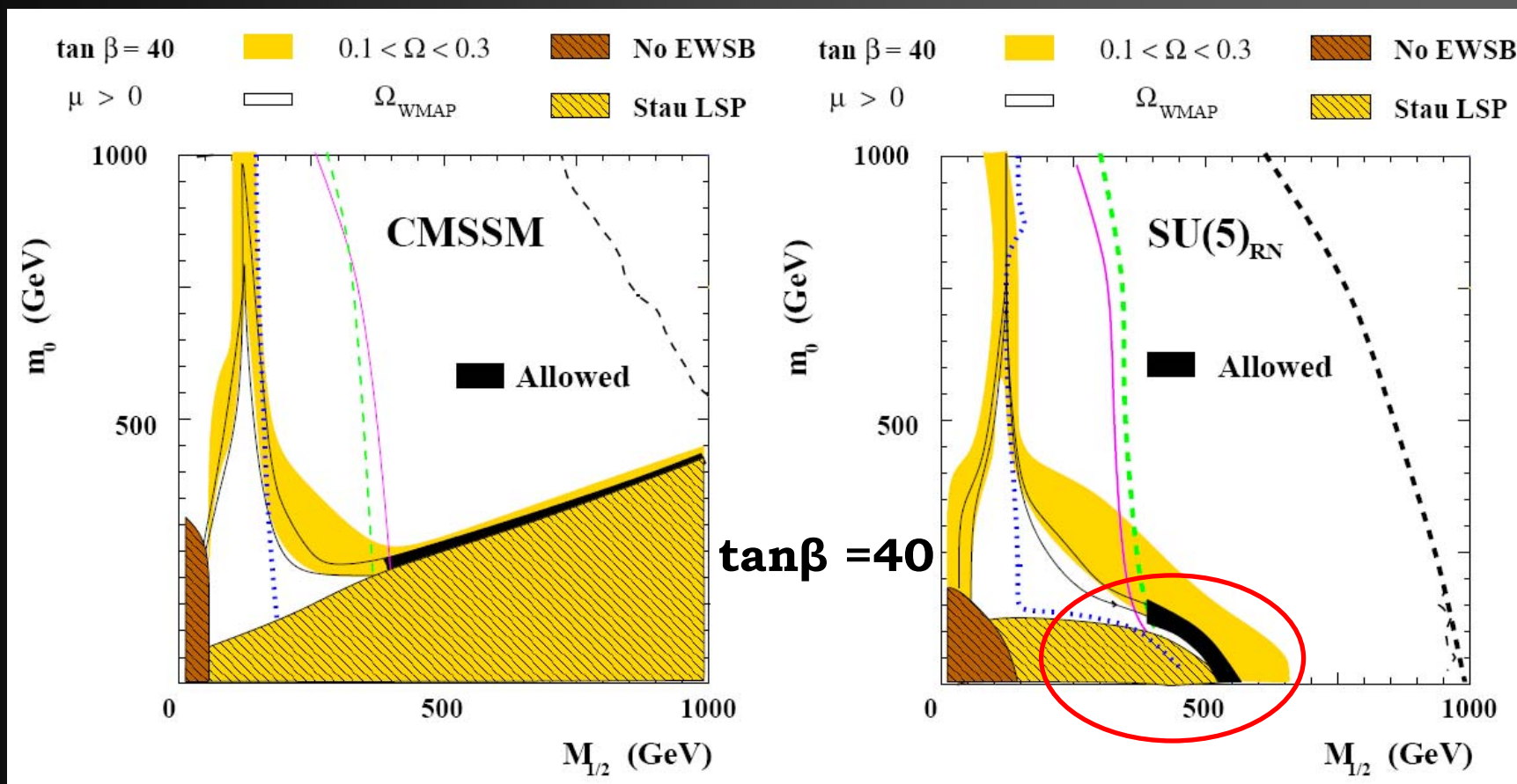
DM phenomenology in $SU(5)_{RN}$



Stau coannihilation:

$$m_{\tilde{\tau}_1}^2 \simeq m_{\tilde{\tau}_{RR}}^2 - m_\tau \mu \tan \beta$$

DM phenomenology in $SU(5)_{RN}$



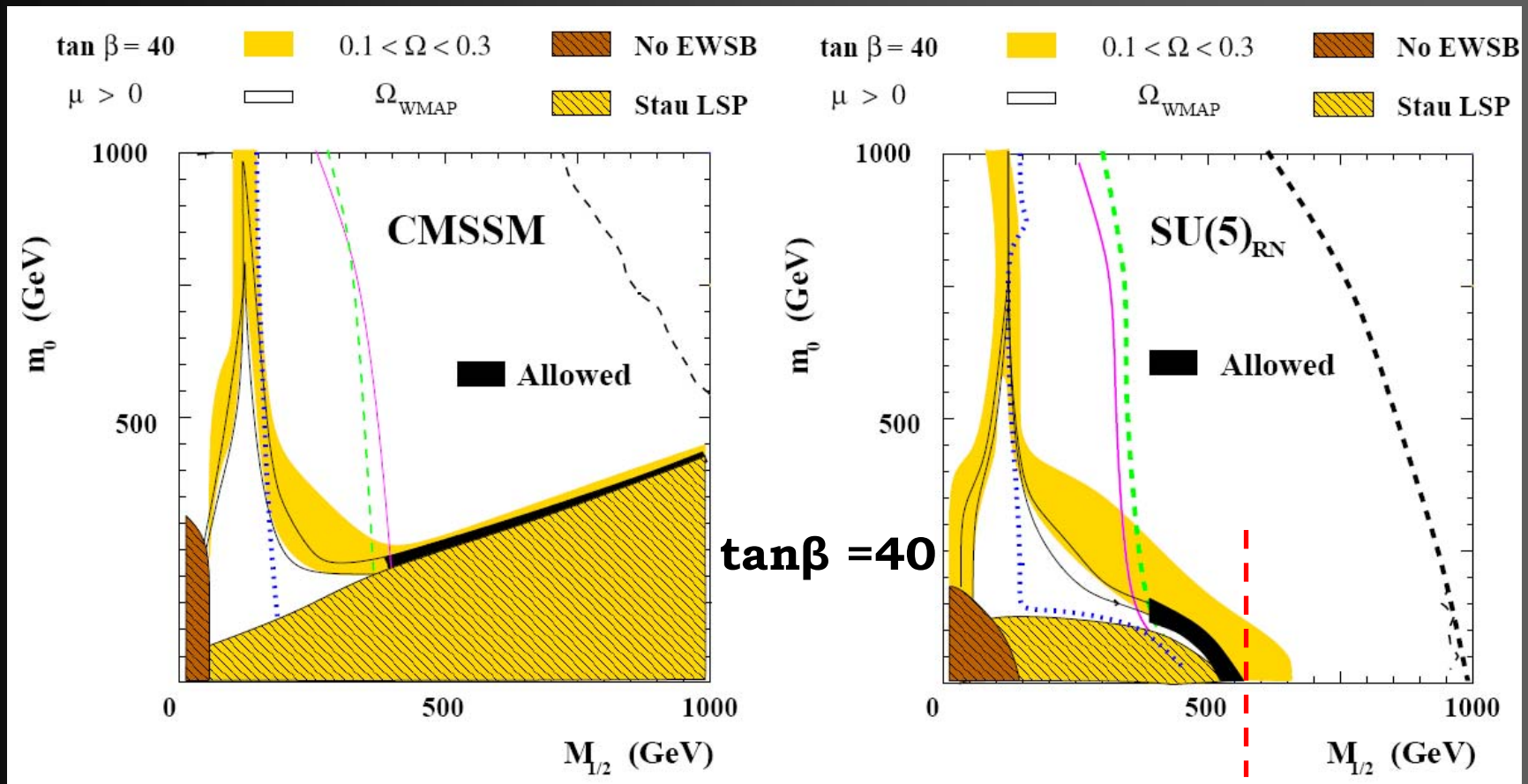
Stau coannihilation:

$$m_{\tilde{\tau}_1}^2 \simeq m_{\tilde{\tau}_{RR}}^2 - m_\tau \mu \tan \beta$$

→ lower limit on $\tan \beta$

$$\tan \beta \gtrsim 34$$

DM phenomenology in $SU(5)_{RN}$



Stau coannihilation:

$$m_{\tilde{\tau}_1}^2 \simeq m_{\tilde{\tau}_{RR}}^2 - m_\tau \mu \tan \beta$$



upper limit on the LSP mass

$$m_{\tilde{\chi}_1^0} \simeq 240 \text{ GeV}$$

$$(\tan \beta = 40, A_0 = 0)$$

The other CMSSM “corridors” ?

- A-pole funnel

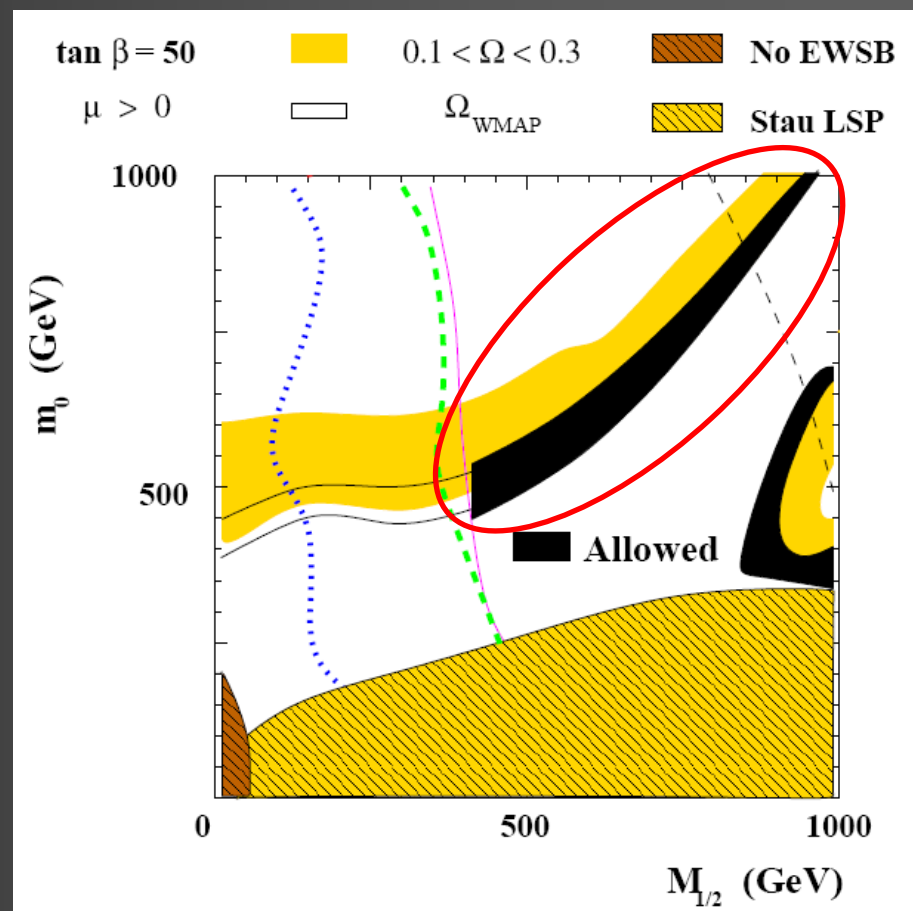
$$\tan \beta \simeq 45 - 50$$

no more upper bound on the
LSP mass

- Focus point

not present up to 5 TeV

$\tan \beta = 50$



Second part: τ polarization

Tau polarization in stau decays

Tau polarization:

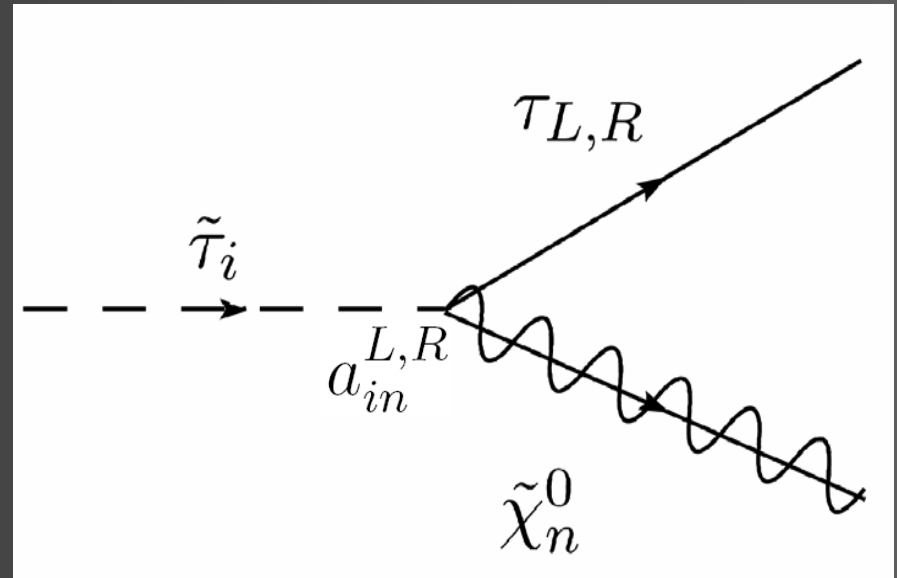
$$\mathcal{P}_\tau = \frac{\text{Br}(\tilde{\tau}_i \rightarrow \tilde{\chi}_n^0 \tau_R) - \text{Br}(\tilde{\tau}_i \rightarrow \tilde{\chi}_n^0 \tau_L)}{\text{Br}(\tilde{\tau}_i \rightarrow \tilde{\chi}_n^0 \tau_R) + \text{Br}(\tilde{\tau}_i \rightarrow \tilde{\chi}_n^0 \tau_L)}$$

If

$$m_\tau \ll m_{\tilde{\tau}_1}$$



$$\mathcal{P}_\tau = \frac{(a_{in}^R)^2 - (a_{in}^L)^2}{(a_{in}^R)^2 + (a_{in}^L)^2}$$



M. Nojiri, PRD **51** (1995) 6281 [hep/ph/9412374]

Tau polarization in stau decays

$$\tilde{\chi}_n^0 = Z_{n1}\tilde{B} + Z_{n2}\tilde{W}_3 + Z_{n3}\tilde{H}_d^0 + Z_{n4}\tilde{H}_u^0$$

$$m_{\tilde{\tau}}^2 = \begin{pmatrix} m_{\text{LL}}^2 & m_{\text{LR}}^2 \\ m_{\text{LR}}^2 & m_{\text{RR}}^2 \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} m_{\tilde{\tau}_1}^2 & 0 \\ 0 & m_{\tilde{\tau}_2}^2 \end{pmatrix} \mathcal{R}; \quad \mathcal{R} = \begin{pmatrix} \cos \theta_\tau & \sin \theta_\tau \\ -\sin \theta_\tau & \cos \theta_\tau \end{pmatrix}$$



$$\begin{aligned} \tilde{\tau}_1 &= \tilde{\tau}_R \sin \theta_\tau + \tilde{\tau}_L \cos \theta_\tau \\ \tilde{\tau}_2 &= \tilde{\tau}_R \cos \theta_\tau - \tilde{\tau}_L \sin \theta_\tau \end{aligned}$$

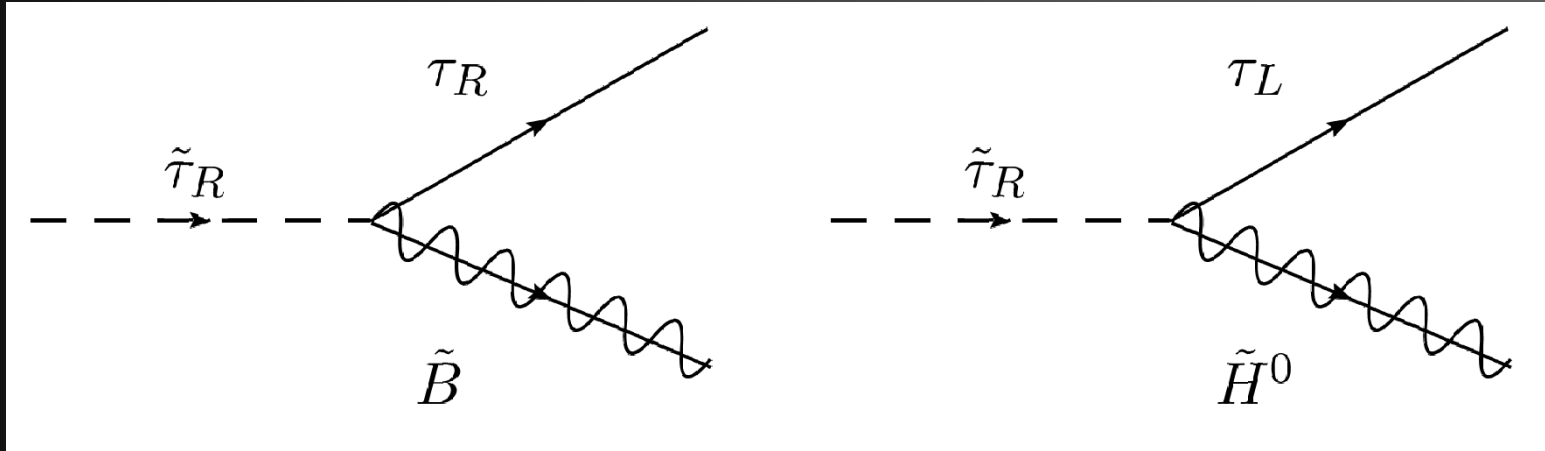
$$m_{\text{LL}}^2 \approx (1 - \rho_L)m_0^2 + c_L M_{1/2}^2$$

$$m_{\text{RR}}^2 \approx (1 - \rho_R)m_0^2 + c_R M_{1/2}^2$$

$$m_{\text{LR}}^2 = m_\tau(A_\tau - \mu \tan \beta) \approx -m_\tau \mu \tan \beta$$

The same parameters which are crucial for coannihilation

Tau polarization in stau decays



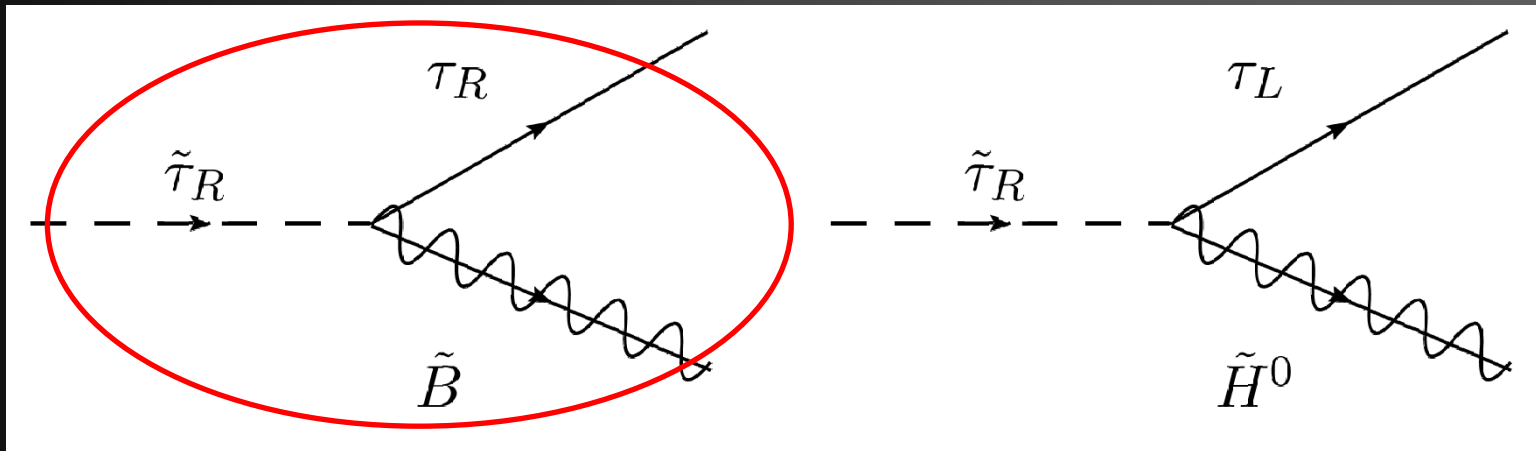
$$\tilde{\tau}_1 \rightarrow \tilde{\chi}_1^0 \tau$$

$$\mathcal{P}_\tau = \frac{(a_{11}^R)^2 - (a_{11}^L)^2}{(a_{11}^R)^2 + (a_{11}^L)^2}$$

$$a_{11}^R = -\frac{2g}{\sqrt{2}} Z_{11} \tan \theta_W \sin \theta_\tau - \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_{12} \cos \theta_\tau$$

$$a_{11}^L = \frac{g}{\sqrt{2}} [Z_{12} + Z_{11} \tan \theta_W] \cos \theta_\tau - \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_{13} \sin \theta_\tau$$

Tau polarization in stau decays



$$\tilde{\tau}_1 \rightarrow \tilde{\chi}_1^0 \tau$$

$$\mathcal{P}_\tau = \frac{(a_{11}^R)^2 - (a_{11}^L)^2}{(a_{11}^R)^2 + (a_{11}^L)^2} \rightarrow \mathbf{1}$$

CMSSM

$$a_{11}^R = -\frac{2g}{\sqrt{2}} Z_{11} \tan \theta_W \sin \theta_\tau - \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_{12} \cos \theta_\tau$$

$$a_{11}^L = \frac{g}{\sqrt{2}} [Z_{12} + Z_{11} \tan \theta_W] \cos \theta_\tau - \frac{gm_\tau}{\sqrt{2}m_W \cos \beta} Z_{13} \sin \theta_\tau$$

Measuring τ polarization at the ILC

- τ has hadronic decay modes, such as $\tau \rightarrow \pi \nu_\tau, \rho \nu_\tau, a_1 \nu_\tau$. The CM angular distribution of decay meson ($\pi; \nu = \rho, a_1$) depends on τ polarization:

$$\frac{1}{\Gamma_\pi} \frac{d\Gamma_\pi}{d\cos\theta} = \frac{1}{2}(1 + P_\tau \cos\theta)$$

$$\frac{1}{\Gamma_\nu} \frac{d\Gamma_{\nu L,T}}{d\cos\theta} = \frac{\frac{1}{2}m_\tau^2, m_\nu^2}{m_\tau^2 + 2m_\nu^2}(1 \pm P_\tau \cos\theta),$$

Hagiwara, Martin, Zeppenfeld '90

Bullock, Hagiwara, Martin '91

D.P. Roy '92

- ILC can determine τ polarization, using inclusive 1-prong channel.

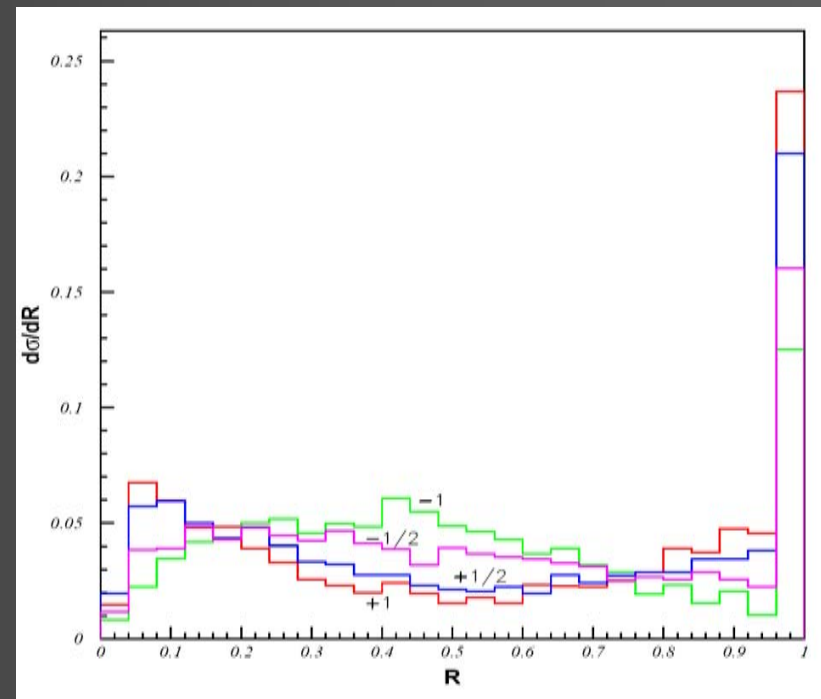
Useful variable: $R = p_{\pi^\pm} / p_{\tau\text{-jet}}$

$$\sqrt{s} = 350 \text{ GeV}, m_{\tilde{\tau}_1} = 150 \text{ GeV}, m_{\tilde{\chi}_1^0} = 100 \text{ GeV}$$

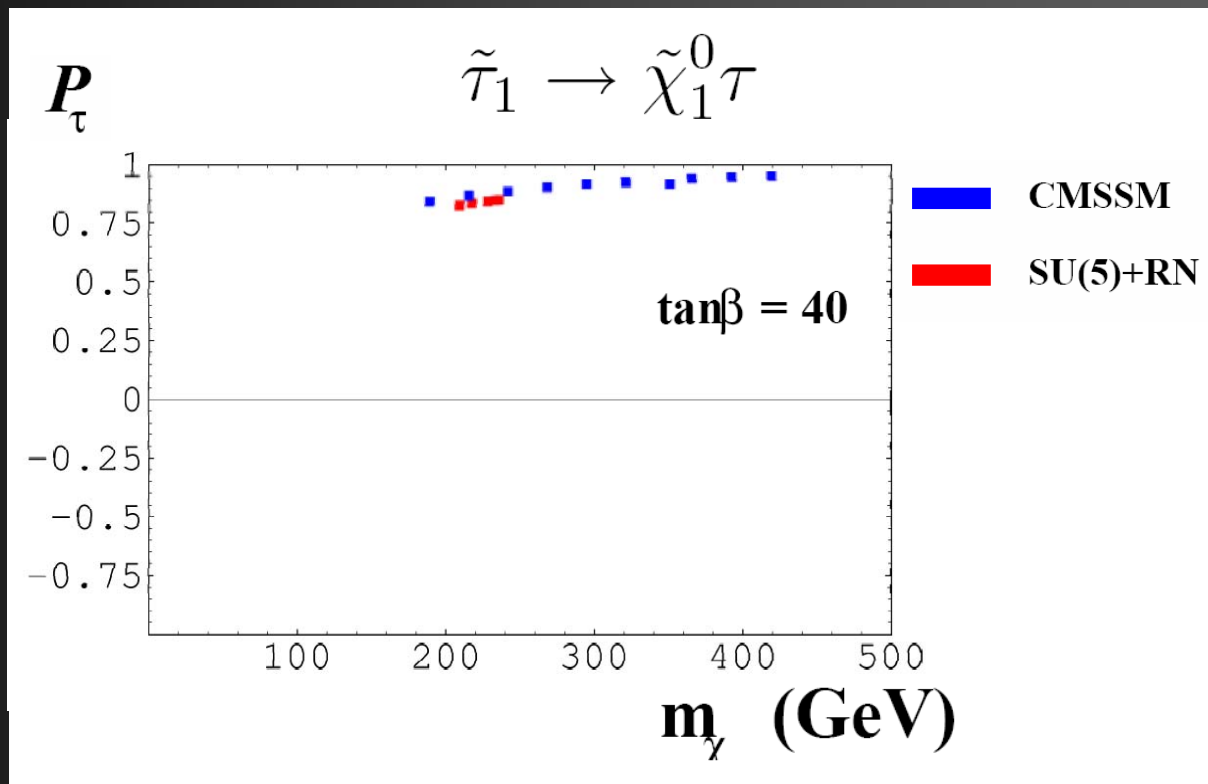
$$p_{\tau\text{-jet}}^T > 25 \text{ GeV}, \cos\theta_{\tau\text{-jet}} < 0.75$$

R. Godbole, M. Guchait, D.P. Roy,

PLB **618**, 193, 2005.



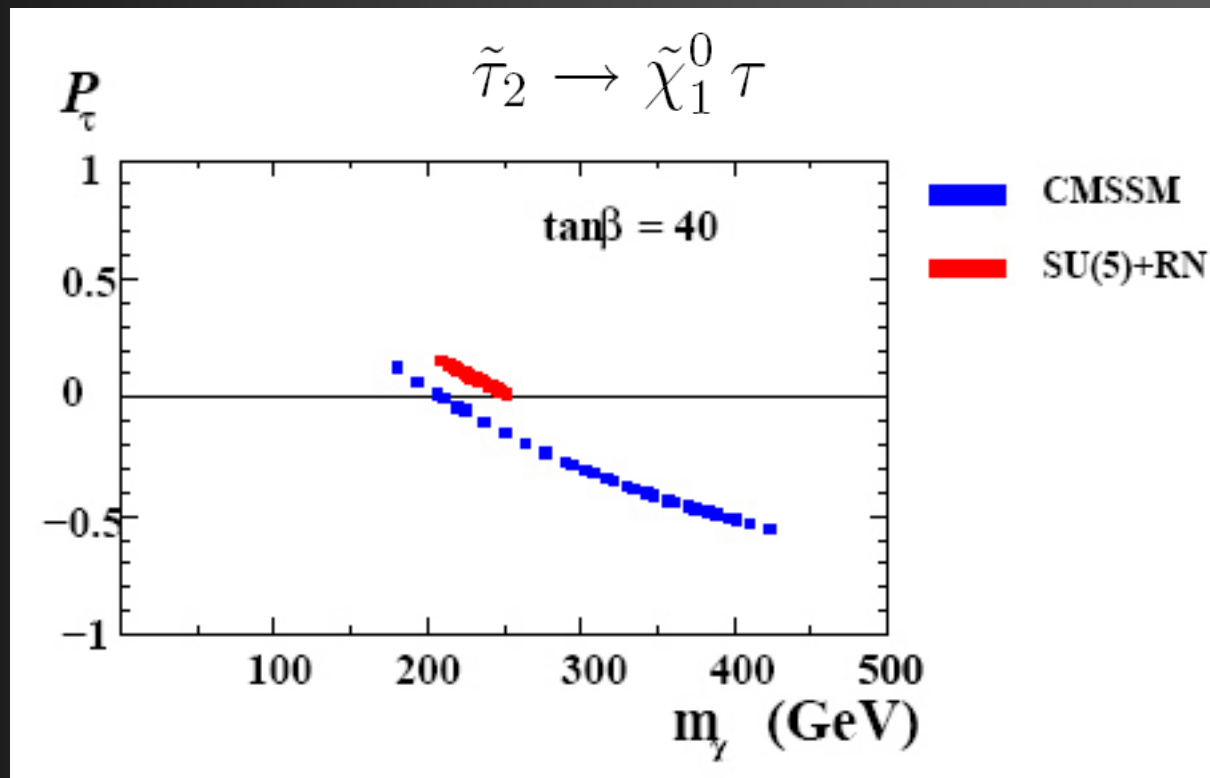
Distinguishing $SU(5)_{RN}$ from CMSSM



- τ polarization for the points lying in the stau coannihilation region
- Polarization not able to distinguish the two models
- Only possibility perhaps the upper limit on the neutralino-stau masses

L.C., R. Godbole, Y. Mambrini, S. Vempati in preparation

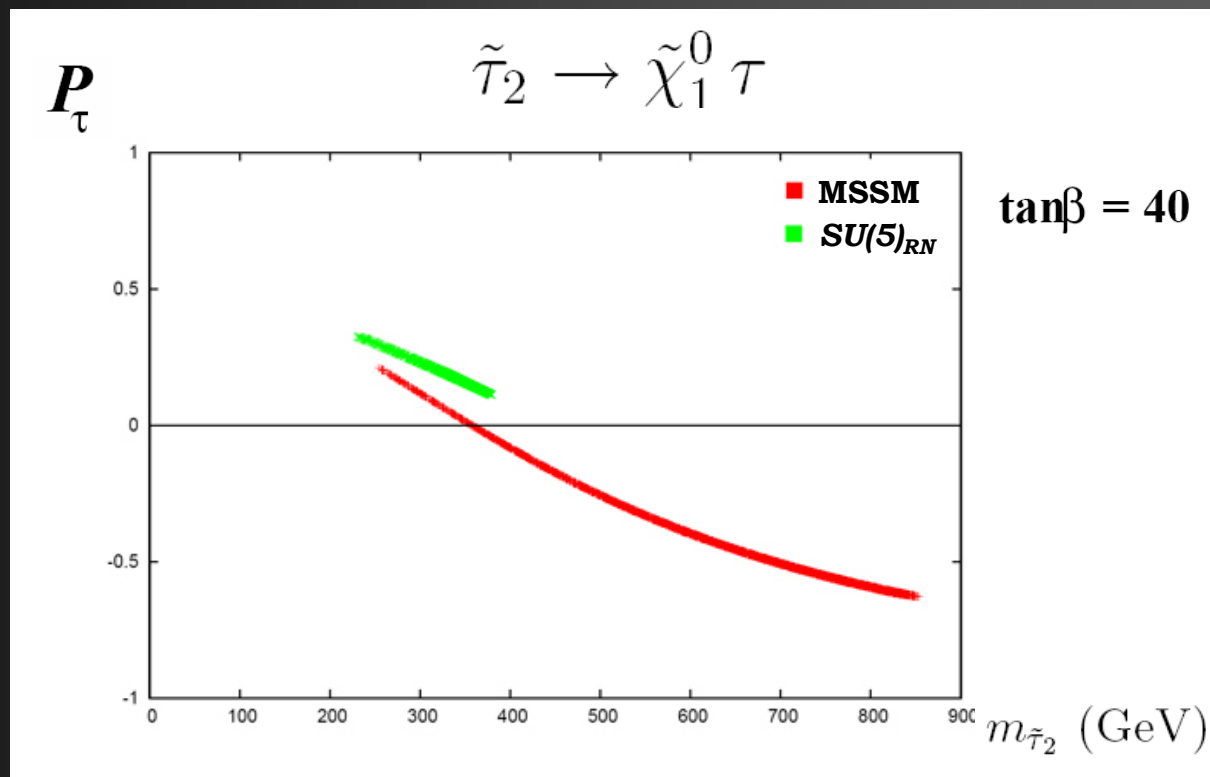
Distinguishing $SU(5)_{RN}$ from CMSSM



- τ polarization changes sign in CMSSM, while in $SU(5)_{RN}$ is always positive
- This is due to a small enhancement of the polarization in $SU(5)_{RN}$, together with the upper bound on the neutralino mass
- At least, it is possible to exclude $SU(5)_{RN}$ if negative polarization is measured

L.C., R. Godbole, Y. Mambrini, S. Vempati in preparation

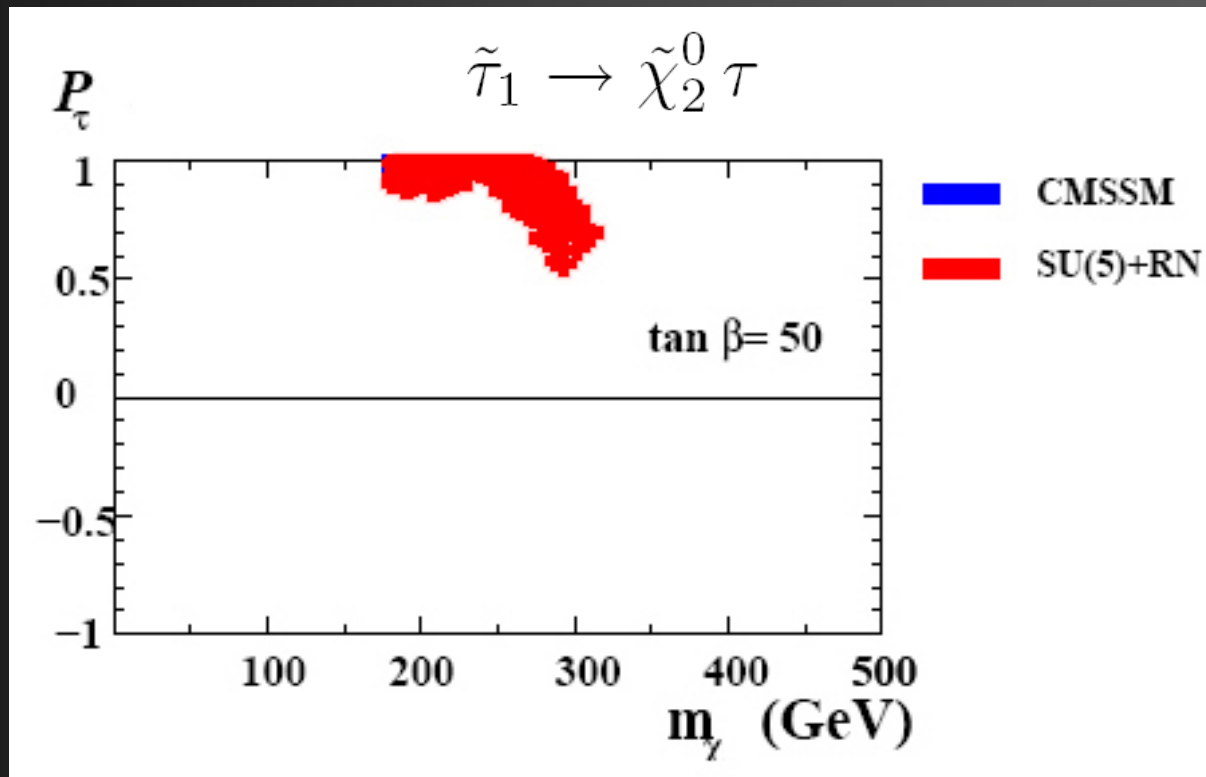
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L.C., R. Godbole, Y. Mambrini, S. Vempati in preparation

Distinguishing $SU(5)_{RN}$ from CMSSM



- A-pole funnel region: we cannot distinguish CMSSM and $SU(5)_{RN}$ using the channels above
- In $SU(5)_{RN}$ a channel forbidden in CMSSM is open: lightest stau decaying in the second neutralino. This is again a consequence of the GUT enhancement of the stau mass

L.C., R. Godbole, Y. Mambrini, S. Vempati in preparation

Summary

- In CMSSM, some peculiar relations among parameters are needed to have the correct relic density
- GUT running and/or presence of RH neutrinos can destabilize such relations
- In $SU(5)_{RN}$ relic density requirements put severe constraints on the allowed range of $\tan\beta$ (> 35); coannihilations branch shows a peculiar phenomenology and upper bound on the LSP mass in some regions of the parameter space
- The polarization of the τ from the decay of the staus can be measured at the ILC
- In the stau coannihilation region τ polarization can distinguish between CMSSM and $SU(5)_{RN}$. The difference is a pure GUT effect
- In the A-funnel region of $SU(5)_{RN}$, the decay $\tilde{\tau}_1 \rightarrow \tilde{\chi}_2^0 \tau$ is possible, while it is kinematically forbidden in CMSSM.