Four-fermion production near the W-pair production threshold

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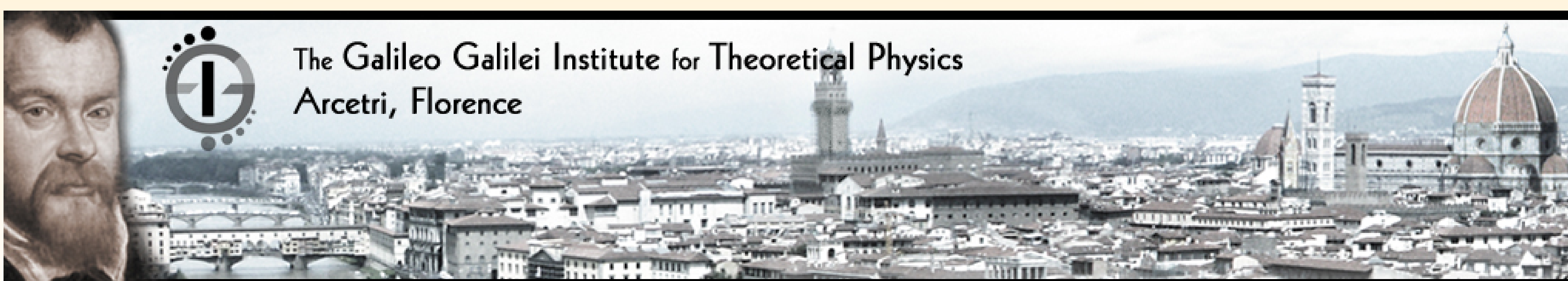

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From the high precision of the ILC we expect to

- identify the nature of new physics (discovered at the LHC?) by doing direct and indirect measurements of particle properties
- constrain new physics and model parameters (e.g. heavy masses, couplings)

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- Higgs: mass, branching ratios, width, CP, spin, couplings, [specifically top-Yukawa, Higgs self-coupling]
- anomalous couplings
- electroweak parameters
  (e.g. $M_Z, \Gamma_Z, M_W, \Gamma_W, m_t, \Gamma_t, \sin^2\theta_{W,\mathrm{eff}}, R_b, R_c, R_l, \sigma_0^{\mathrm{had}}$)
- QCD coupling and evolution (new color degrees of freedom?)
- If (SUSY) ⇒ plethora of SUSY masses and parameters
- If (ED) ⇒ measure M, δ, KK-powers
- If (XXX) ⇒ measure YYY

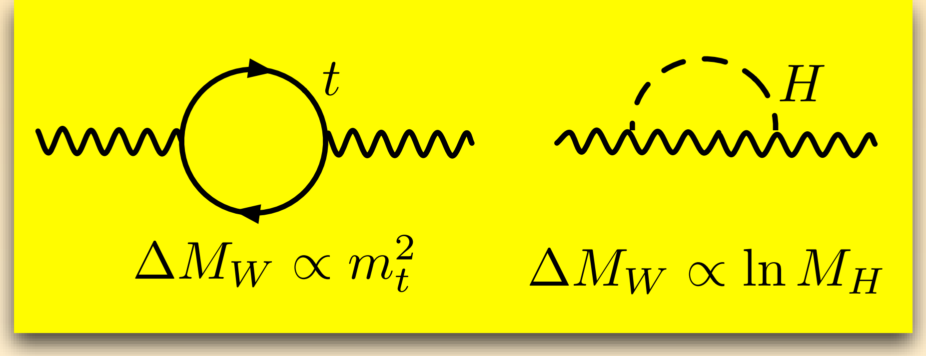
Precision measurements at the ILC

This talk

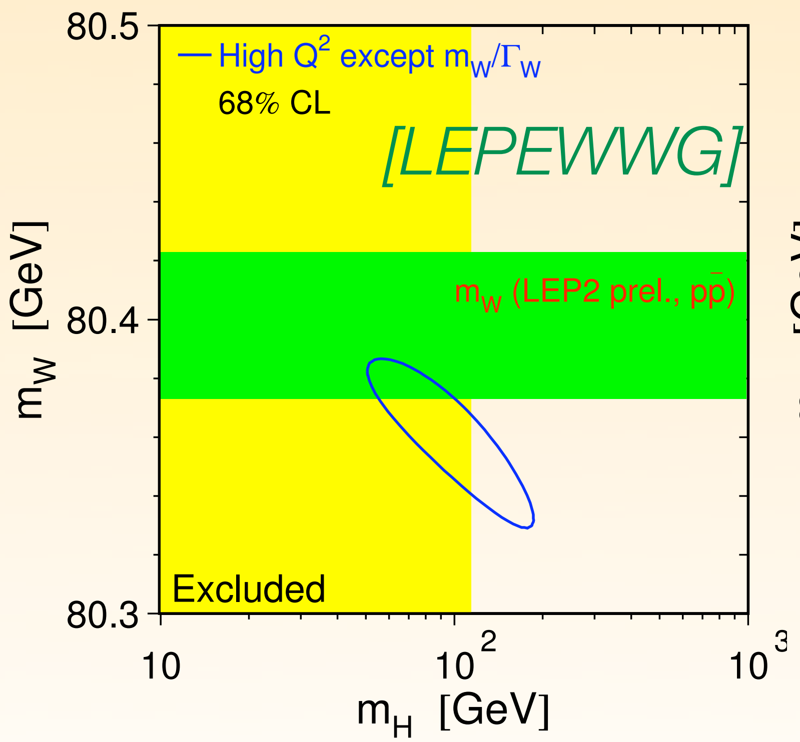
$$M_W$$

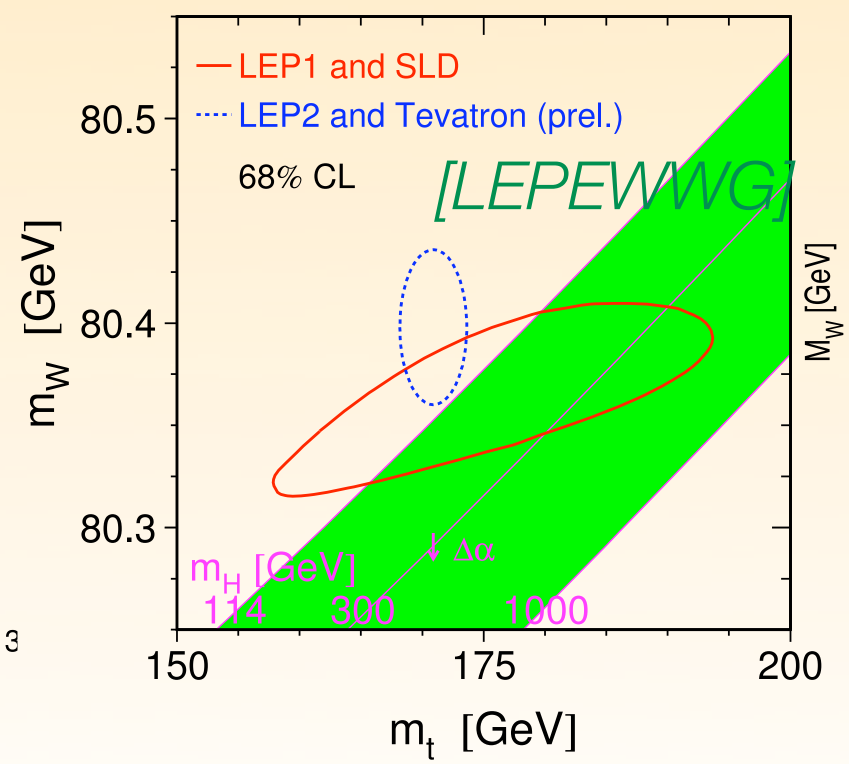
W mass

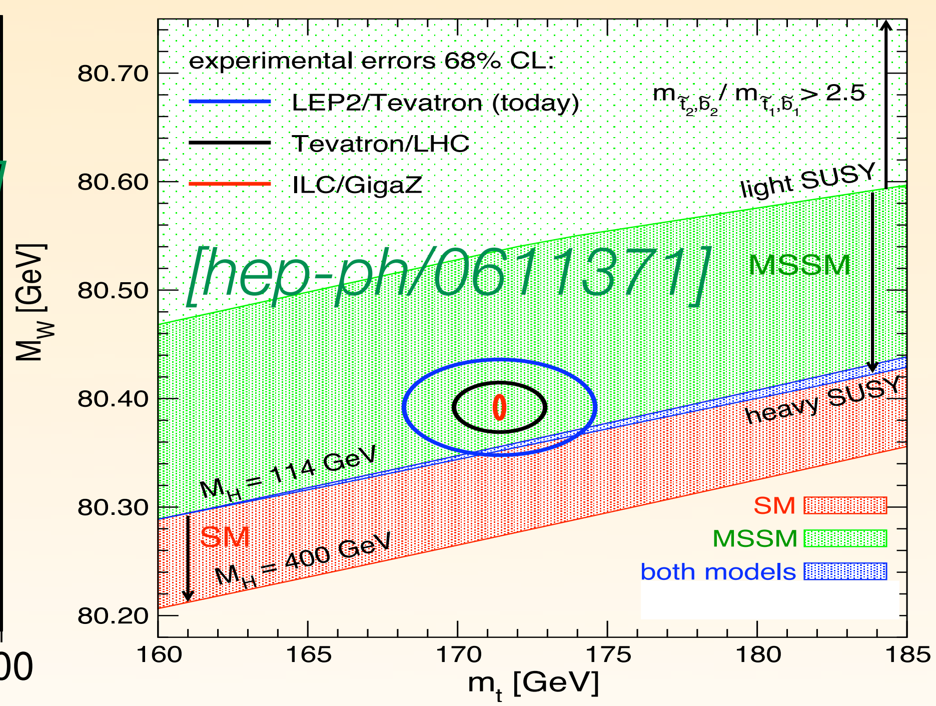
M_W is a key observable in the search of virtual-particle exchange through electroweak precision measurements



⇒ can constrain M_H by precisely measuring m_t and M_W







W mass determination

- current value: $M_W = (80.403 \pm 0.029)$ GeV determined from combination of continuum W-pair production at LEPII and single-W at the Tevatron

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- two techniques at the ILC:
  - kinematic fitting of WW production at $\sqrt{s} = 500$ GeV
    ⇒ reach 5 MeV error with $\mathcal{L} = 1000 fb^{-1}$ (several years)
  - threshold scan: exploit rapid variation of σ at threshold
    ⇒ error of 5 MeV with $\mathcal{L} = 100 fb^{-1}$ (just one year)

Some history of WW (before ’05)

NB: this is *not* a complete list of references

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- ☑ lowest order amplitudes for an onshell W-pair

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- ☑ Monte-Carlo generators: DPA+universal corrections
  [e.g. YFSWW ’00, RacoonWW ’99]

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Beyond

- accuracy of DPA in the continuum is $\sim \frac{\alpha}{\pi}\frac{\Gamma_W}{M_W} \lesssim 0.5\%$
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- want treatment which does not break down at threshold

Two approaches:

- full $\mathcal{O}(\alpha) e^+e^- \to 4$f calculation in the complex-mass scheme

  [Denner et. al '05]

- construct an effective theory designed to exploit the hierarchy between the physical scales (M, Γ ,v)

  [Beneke et. al '07]

The ee4f calculation

[Denner et. al '05]

Essential ingredients:

(1) extension of the complex mass scheme to one-loop

- split bare mass into complex renormalized mass (⇒ resummed) and complex counterterms (⇒ not resummed) in the Lagrangian
- use complex masses everywhere (e.g. in $\cos^2\theta_W$) (⇒ spurious terms)
- unitarity violations, but only at the next order in PT

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(2) three external fermion pairs ⇒ non-trivial spinor structure

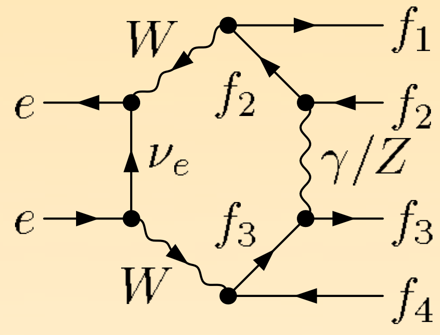
- algorithm to reduce $\mathcal{O}(10^3)$ spinor chains to only $\mathcal{O}(10)$ independent spinor structures

The ee4f calculation

Essential ingredients:

[Denner et. al '05]

(3) develop new numerical techniques to compute one-loop six-point tensor integrals with complex masses in the loop



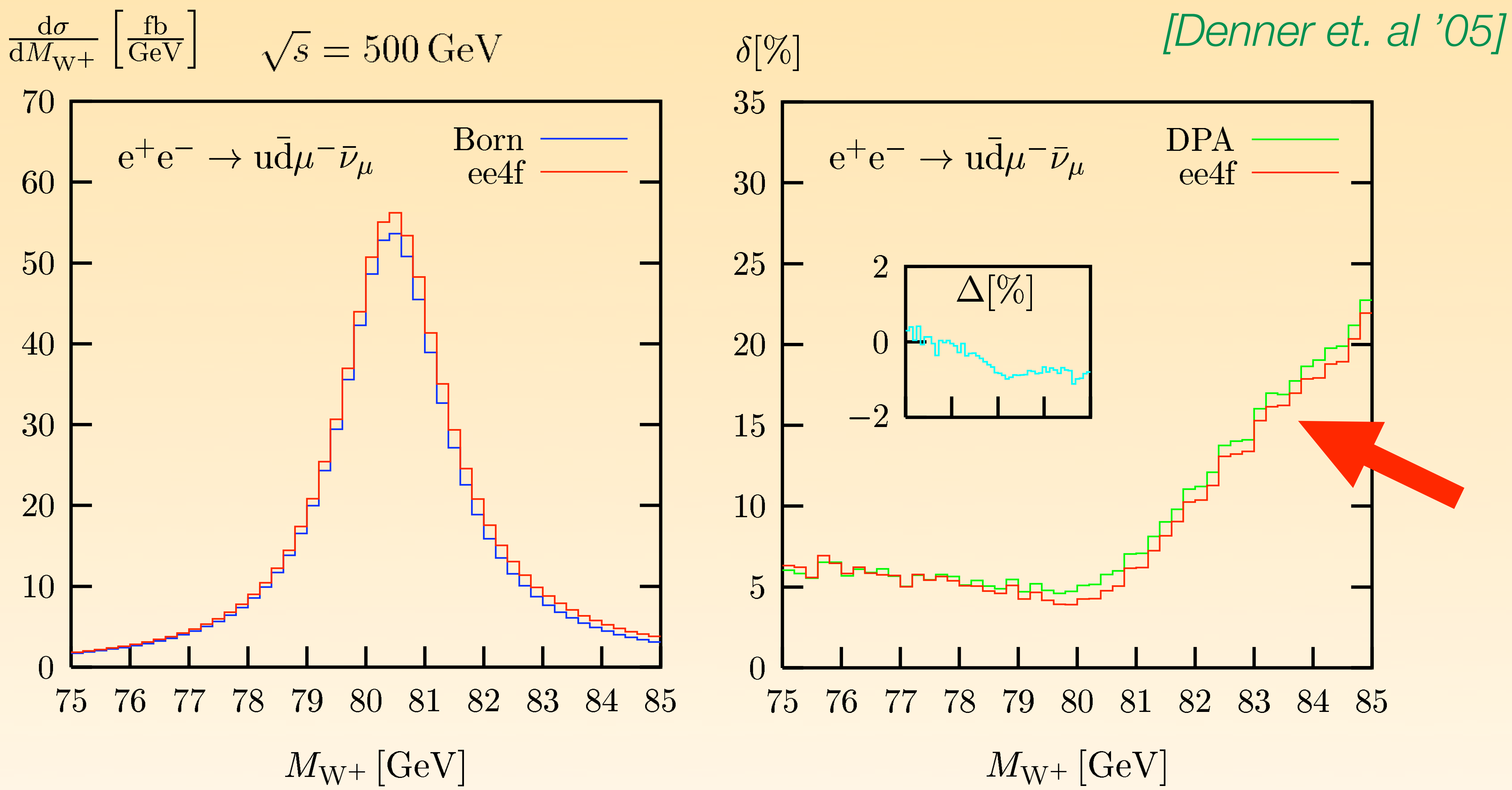
- based on numerical Passarino-Veltman reduction
- need rescue systems do deal with vanishing inverse Gram determinants
- general techniques can be used for other processes [e.g. used for H → 4f and pp → ttj]
- one-loop multi-particle processes very important both for LHC&ILC

Input parameters

$$
\begin{array}{lll}
G_\mu = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}, & \alpha(0) = 1/137.03599911, & \alpha_\mathrm{s} = 0.1187, \\
M_\mathrm{W} = 80.425\,\mathrm{GeV}, & M_\mathrm{Z} = 91.1876\,\mathrm{GeV}, & \Gamma_\mathrm{Z} = 2.4952\,\mathrm{GeV}, \\
M_\mathrm{H} = 115\,\mathrm{GeV}, & & \\
m_\mathrm{e} = 0.51099892\,\mathrm{MeV}, & m_\mu = 105.658369\,\mathrm{MeV}, & m_\tau = 1.77699\,\mathrm{GeV}, \\
m_\mathrm{u} = 66\,\mathrm{MeV}, & m_\mathrm{c} = 1.2\,\mathrm{GeV}, & m_\mathrm{t} = 178\,\mathrm{GeV}, \\
m_\mathrm{d} = 66\,\mathrm{MeV}, & m_\mathrm{s} = 150\,\mathrm{MeV}, & m_\mathrm{b} = 4.3\,\mathrm{GeV},
\end{array}
$$

- use G_μ -scheme for the coupling: $\alpha_{G_\mu} = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$
- use $\alpha(0)$ in radiative corrections
- QCD effects included naively multiplying cross-sections by $(1 + \alpha_s/\pi)$ per hadronically decaying W

Sample ee4f results



⇒ DPA not sufficient at LC: DPA larger then ee4f for invariant masses $> M_W$. This would give a shift in the direct reconstruction of M_W!

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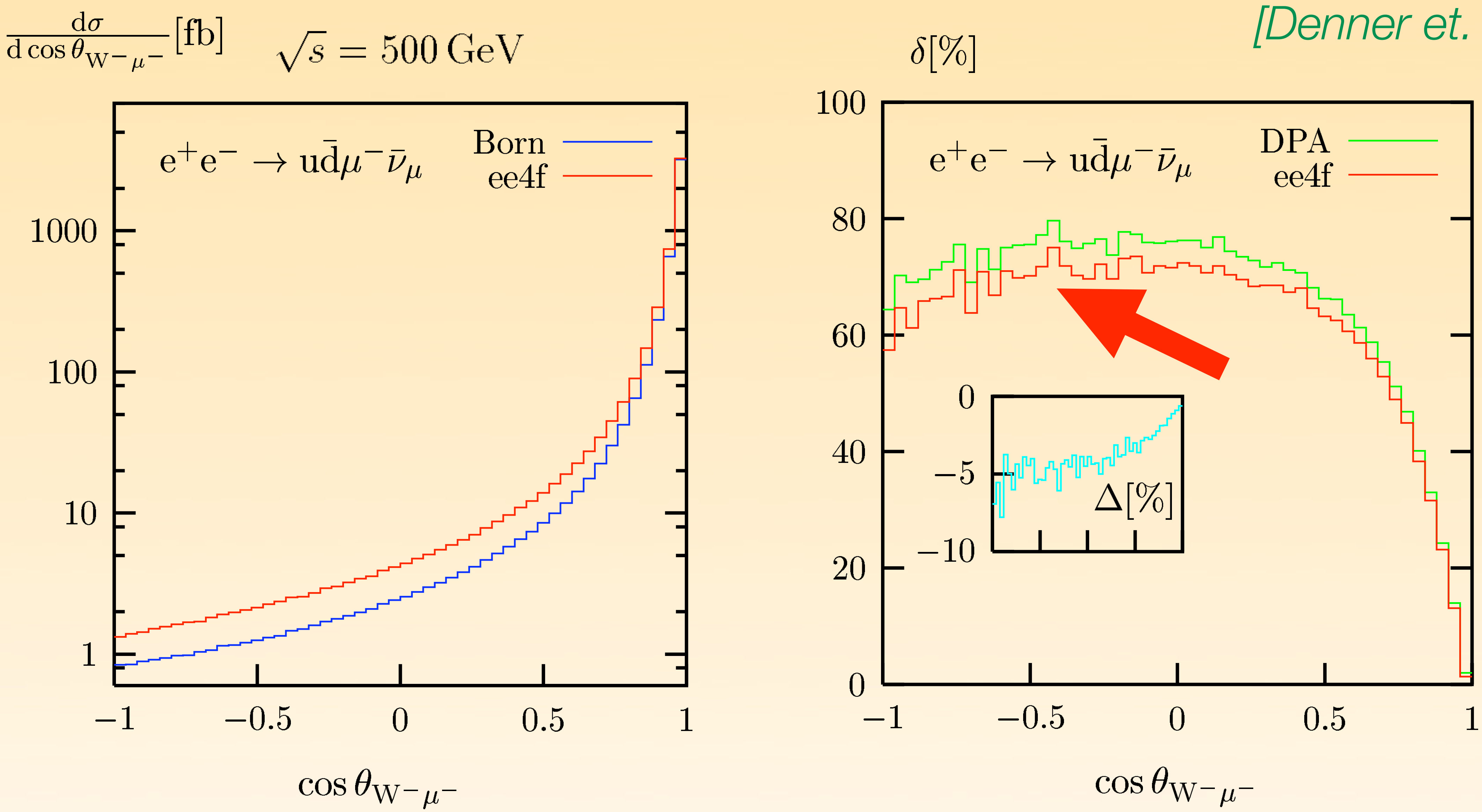
[Denner et. al '05]



⇒ DPA not sufficient at LC: distortions above 500GeV could be misinterpret as signal for anomalous triple-gauge couplings

Effective theory calculation

[Beneke, Falgari, Schwinn, Signer, GZ '07]

Exploit the hierarchy between the physical scales at threshold

$$\Gamma_W/M_W \sim \alpha_{\rm ew} \sim \alpha_s^2 \sim v^2$$

collectively called δ for power-counting

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Split physical modes into

- hard: $k_0 \sim M_W, |\vec{k}| \sim M_W$
- soft: $k_0 \sim M_W\delta, |\vec{k}| \sim M_W\delta$
- collinear: $k_\pm \sim M_W, k_\mp \sim M_W\delta, k_\perp \sim M_W\sqrt{\delta}$
- potential: $k_0 \sim M_W\delta, |\vec{k}| \sim M_W\sqrt{\delta}$

⇒ integrated out (matching coefs.)

⇒ dynamical modes (effective operators)

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(soft, collinear, potential) ⇒ dynamical modes (effective operators)

⇒ NLO means $\mathcal{O}(\delta) : \mathcal{O}(\Gamma_W/M_W) \sim \mathcal{O}(\alpha_{\rm ew}) \sim \mathcal{O}(\alpha_s^2) \sim \mathcal{O}(v^2)$

[similar technique for top-pair production, Hoang et al. ’04, Hoang et al. ’07]

Effective theory calculation

Practically:

✓ compute inclusive cross-section via the imaginary part of the forward scattering amplitude

✓ split loop-integrals using the strategy of regions, i.e. expand integrands *before* doing the integration ⇒ power-counting available, e.g.

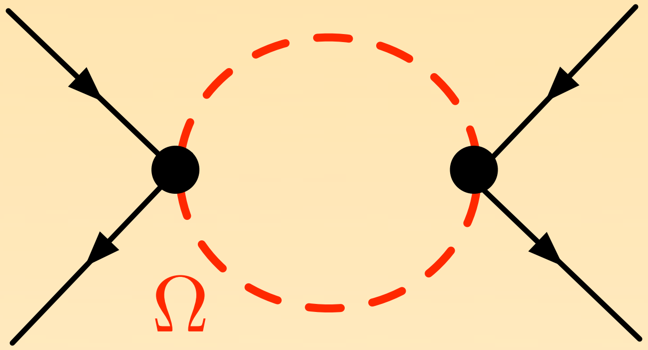
- soft: $k_0 \sim M_W \delta, |\vec{k}| \sim M_W \delta \quad \Rightarrow d^4k \sim \delta^4 M_W^4$
- potential: $k_0 \sim M_W \delta, |\vec{k}| \sim M_W \sqrt{\delta} \quad \Rightarrow d^4k \sim \delta^{5/2} M_W^4$

$$\Omega \quad \sim \frac{1}{2M_W(k_0 - \vec{k}^2 - \Delta/2)} \sim \frac{1}{M_W^2 \delta}$$

At LO: $\Delta = -i\Gamma^{(0)}$

Examples

Born



$\sim \alpha_{\rm ew}^2 \int \underbrace{d^4k}_{\delta^{5/2}} \frac{1}{(MW^2\delta)^2}$

$\sim \alpha_{\rm ew}^2 \delta^{1/2}$ ⬅ LO

Examples

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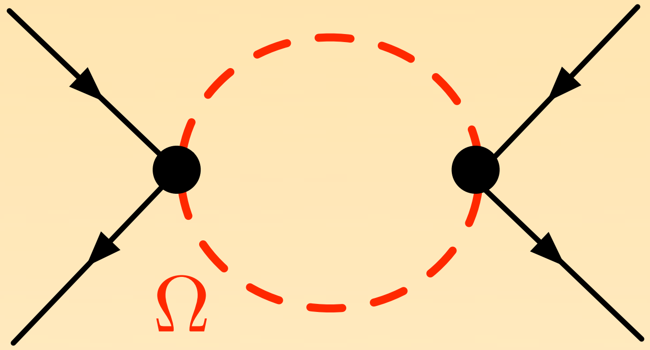

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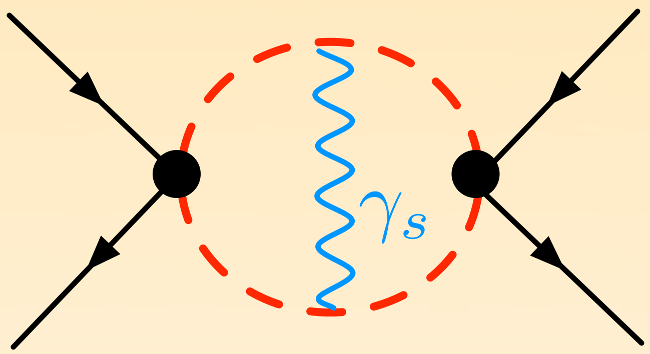
<u>*Soft photon correction*</u>



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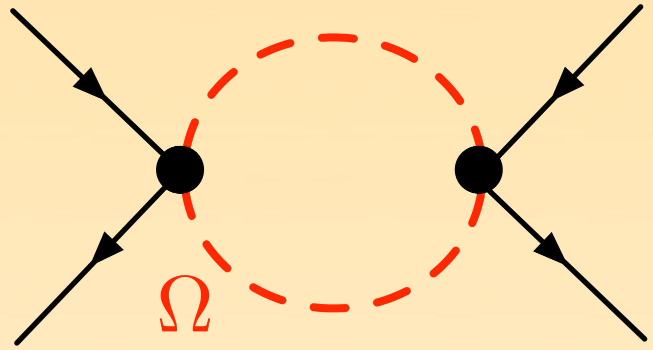

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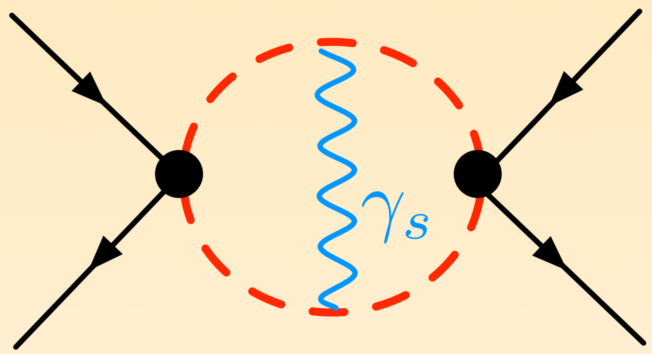

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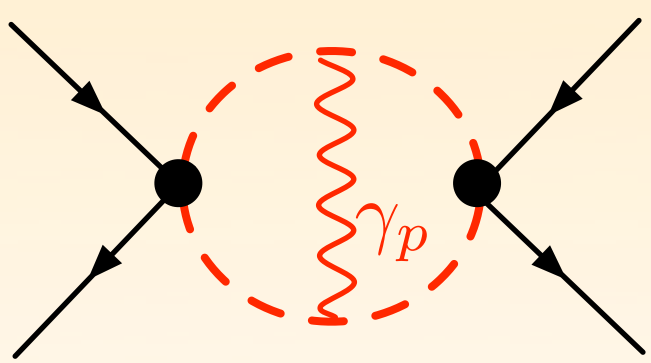
Single Coulomb exchange



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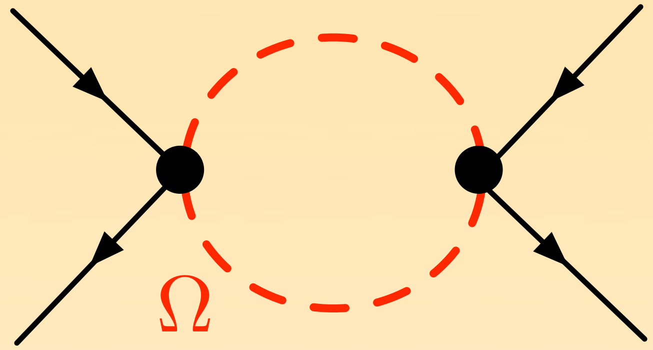

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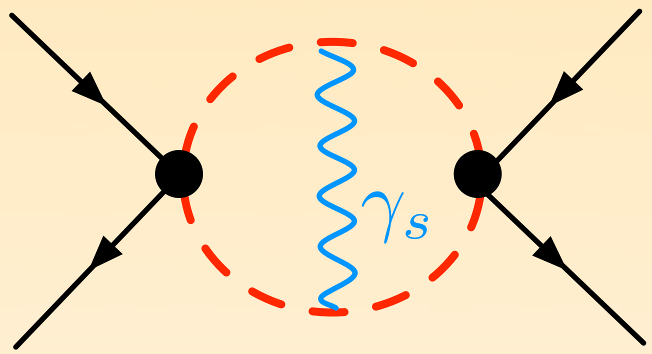

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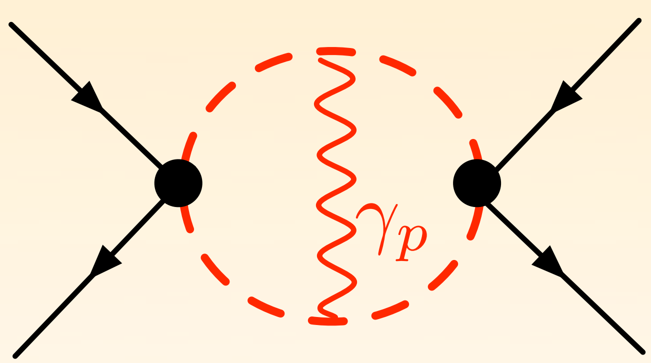

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<u>Side remarks:</u>

1) at NLO need double Coulomb exchange, not included in ee4f
2) no resummation of Coulomb photon necessary (unlike top)

Results

$$\sigma(e^+e^-) \to \mu^-\nu_\mu u\bar{d} + X \quad [\text{fb}]$$

$\sqrt{s}$ [GeV]	Born	Born+ISR	NLO
161	154.19(6)	108.60(4) [-29.6%]	117.81(5) [-22.5%]
170	481.7(2)	378.4(2) [-17.4%]	398.0(2) [-17.4%]

➡ ISR results in large negative correction ($\sim$ -30%)

➡ genuine NLO amounts to an additional $\sim$ +8% effect, much large than target accuracy of sub-percent level

Comparison between ee4f and EFT in theory

- full $\mathcal{O}(\alpha)e^+e^- \to 4$f calculation in the complex-mass scheme

[Denner et. al '05]

− technically challenging calculation

[involves one-loop hexagons to with complex masses in the loop]

+ flexible treatment of final state

+ valid in all phase space (no matching needed)

- effective theory approach

[Beneke et. al '07]

− currently inclusive cross-section only

+ technically simpler, compact analytical formulae

[most complicated loop calculation are onshell boxes]

+ formalism can be extended to higher orders

Also: proof of principle of the effective theory method to treat unstable particles

[Beneke et. al '03-'04]

Comparison between ee4f and EFT in practice

Using same input, handling ambiguities in the same way and removing double Coulomb exchange from EFT one gets:

$$\sigma(e^+e^-) \to \mu^-\nu_\mu u\bar{d} + X \quad [\mathrm{fb}]$$

$\sqrt{s}$ [GeV]	Born+ISR	DPA	NLO [EFT]	NLO [ee4f]
161	107.06(4)	115.48(7)	117.38(4)	118.12(8)
170	381.0(2)	402.1(2)	399.9(2)	401.8(2)

⇒ agreement between EFT and ee4f up to 0.6% at threshold

ISR & uncertainty

NLO results presented before based on

$$\sigma_{\rm v1}^{NLO} \equiv \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{\rm ee}^{\rm LL}(x_1)\Gamma_{\rm ee}^{\rm LL}(x_2) \left(\sigma^{(0)}(x_1 x_2 s) + \sigma^{(1)}(x_1 x_2 s)\right)$$

ISR & uncertainty

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$$\sigma_{\rm v1}^{NLO} \equiv \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{\rm ee}^{\rm LL}(x_1)\Gamma_{\rm ee}^{\rm LL}(x_2)\left(\sigma^{(0)}(x_1x_2s)+\sigma^{(1)}(x_1x_2s)\right)$$

since $\Gamma_{\rm ee}^{\rm LL}(x) = \delta(1-x) + \Gamma_{ee}^{\rm LL,(1)}$ one could as well do

$$\sigma_{\rm v2}^{NLO} \equiv \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{\rm ee}^{\rm LL}(x_1)\Gamma_{\rm ee}^{\rm LL}(x_2)\sigma^{(0)}(x_1x_2s)+\sigma^{(1)}(s)$$

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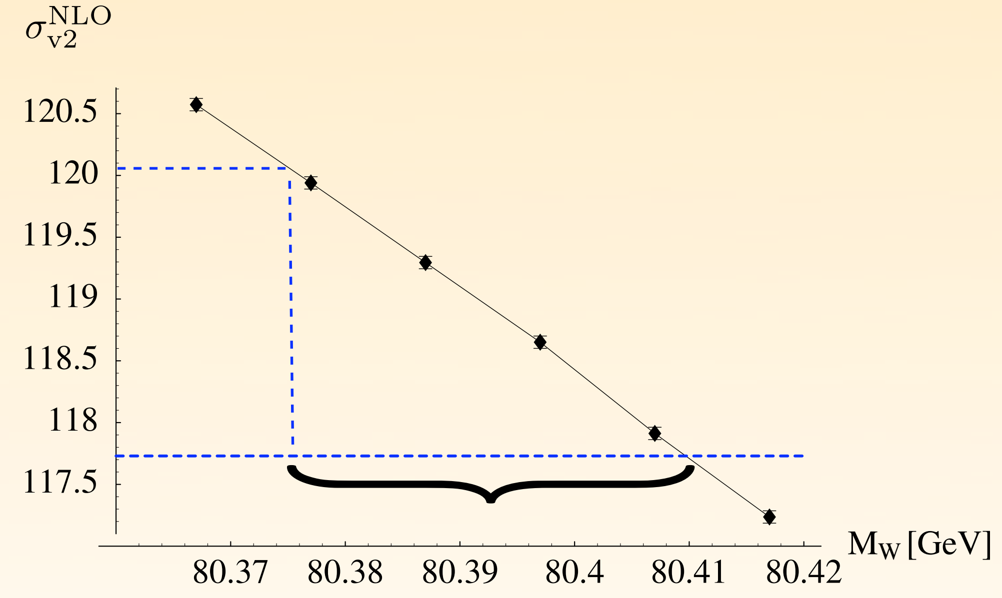
$\sqrt{s}$ [GeV]	161
NLO (v1) [fb]	117.81(5)
NLO (v2) [fb]	120.00(5)
(v1-v2)/v1	-1.9%



⇒ 30 MeV effect for M_W

Uncertainty study

Define:

- $O_i \equiv$ NLO[EFT] with $M_W = 80.077\mathrm{GeV}$ at $\sqrt{s} = 160, 161, 162, 163, 164, 170\mathrm{GeV}$

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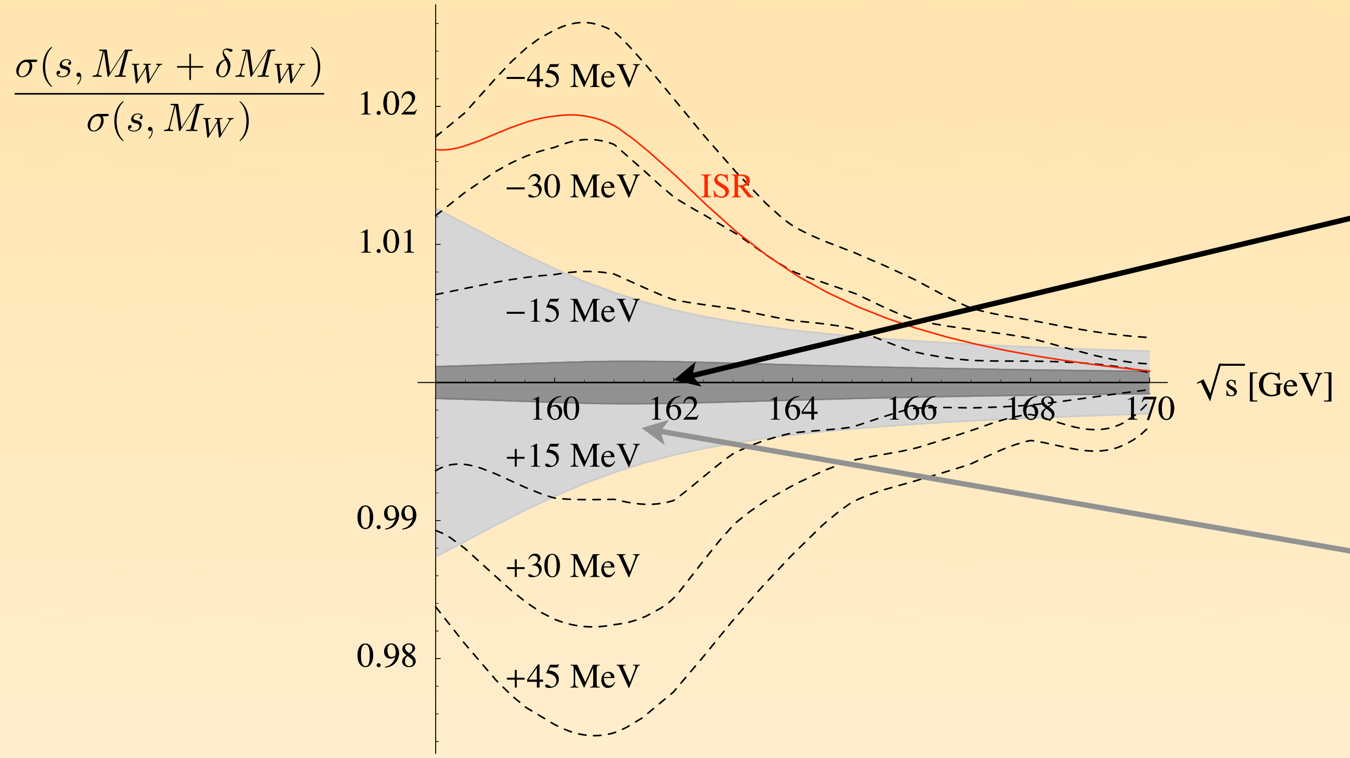
δM_W at which χ^2 is minimum gives the best estimate of the difference between true and measured mass due to missing higher orders

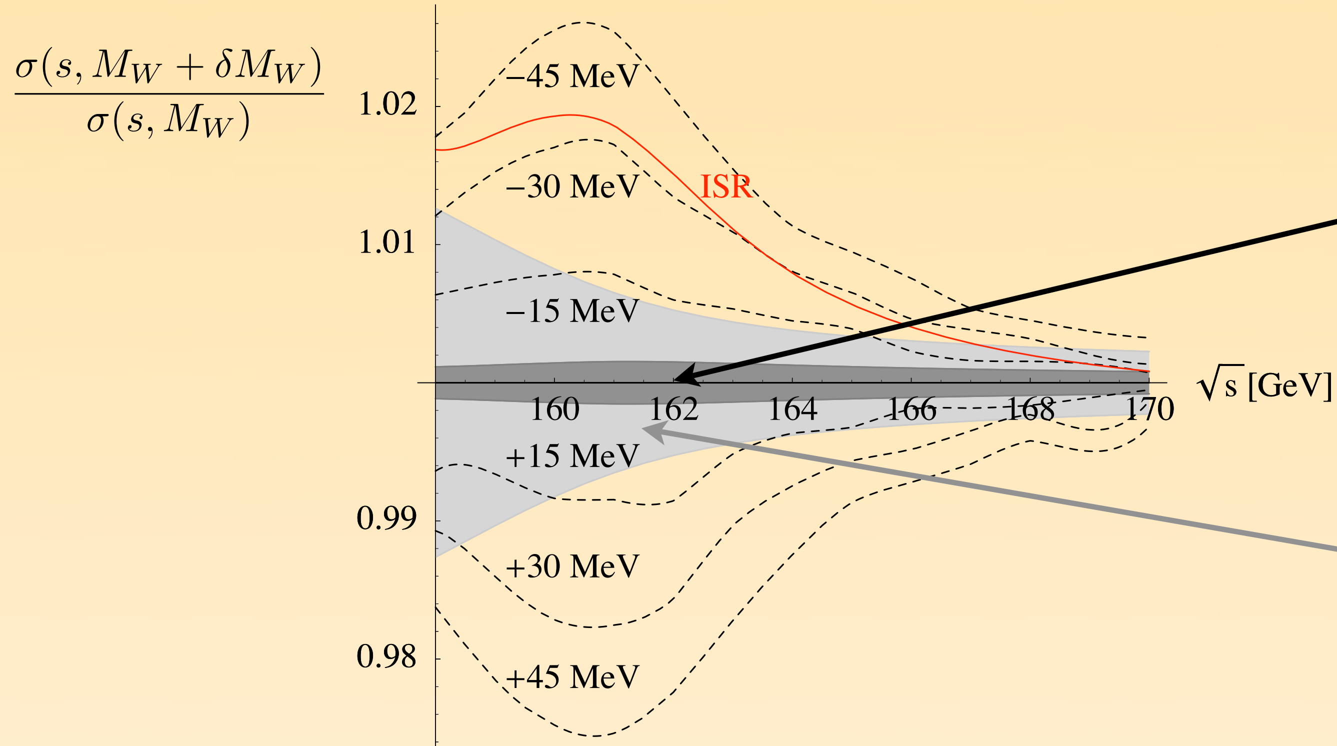
Uncertainty study



single Coulomb +
soft or hard photon

single Coulomb +
soft or hard photon
+ non-resonant $(N^{3/2}LO)$

Uncertainty study



single Coulomb + soft or hard photon

single Coulomb + soft or hard photon + non-resonant $(N^{3/2}LO)$

main uncertainties	δM_W	remedy ?
ISR-treatment	31MeV	NLL ISR resummation
choice of	15 MeV	make best choice
non-resonant	8 MeV	already in ee4f
single Coulomb + soft or hard photon	-5 MeV	not needed

Conclusions

M_W measurement at ILC with an error $\sim$ 6 MeV needs σ at threshold to an accuracy of $\sim$ 0.6%

Two independent NLO calculations of

✓ full EW $e^+e^- \longrightarrow 4$f in the complex mass scheme

✓ effective theory calculation

⇒ *results agree up to 0.6% at threshold*

However large ($\sim$2%) ambiguities from ISR treatment

⇒ resummation of NLO collinear logs mandatory to reduce error below 30 MeV