Theoretical Tools and Methods for a Future e^+e^- Linear Collider

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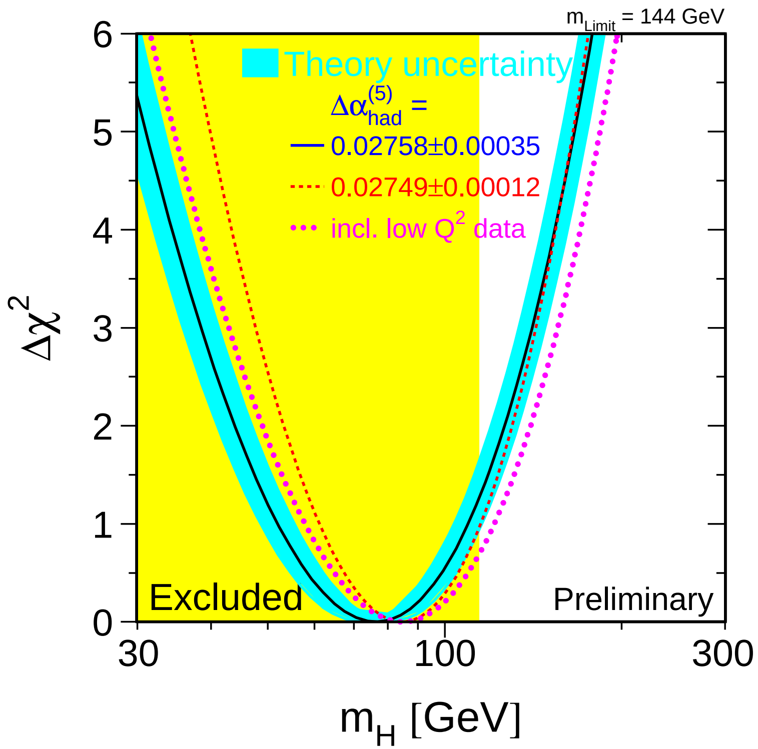
1 Introduction

Experiments at LEP/SLC/Tevatron

- confirmation of Standard Model as quantum field theory (quantum corrections significant)
- top mass m_t indirectly constrained by quantum corrections
  $\leftrightarrow$ in agreement with m_t measurement of Tevatron
- Higgs mass M_H indirectly constrained by quantum corrections
  $\hookrightarrow$ impact on Higgs searches

Great success of precision physics

- $M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)
  $\mathrm{e}^+\mathrm{e}^- \not\to \mathrm{ZH}$ at LEP2
- $M_\mathrm{H} < 144\,\mathrm{GeV}$ (LEPEWWG '07)
  fit to precision data
  i.e. via quantum corrections



The role of precision at LHC and ILC

LHC: the discovery machine (Higgs & EWSB, SUSY, etc.?)

- QCD corrections (at least NLO) are substantial parts of predictions

  typical LO uncertainties $\sim$ several $10\%-100\%$

  corrections needed for signals and many background processes

- EW corrections also important for many observables

  (precision physics, searches at high scales, particle reconstruction, etc.)

ILC: the high-precision machine (precision $\rightarrow$ window to higher energy)

- old and new physics with high accuracy (typically $\delta\sigma/\sigma \lesssim 1\%$)

  $\hookrightarrow$ QCD and EW corrections required

- the ultimate precision at GigaZ/MegaW:

  precision increases by factor ~ 10 w.r.t. LEP/SLC

  EXP: $\Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} \sim 0.00001, \qquad \Delta M_{\mathrm{W}} \sim 7\,\mathrm{MeV}$

  TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams

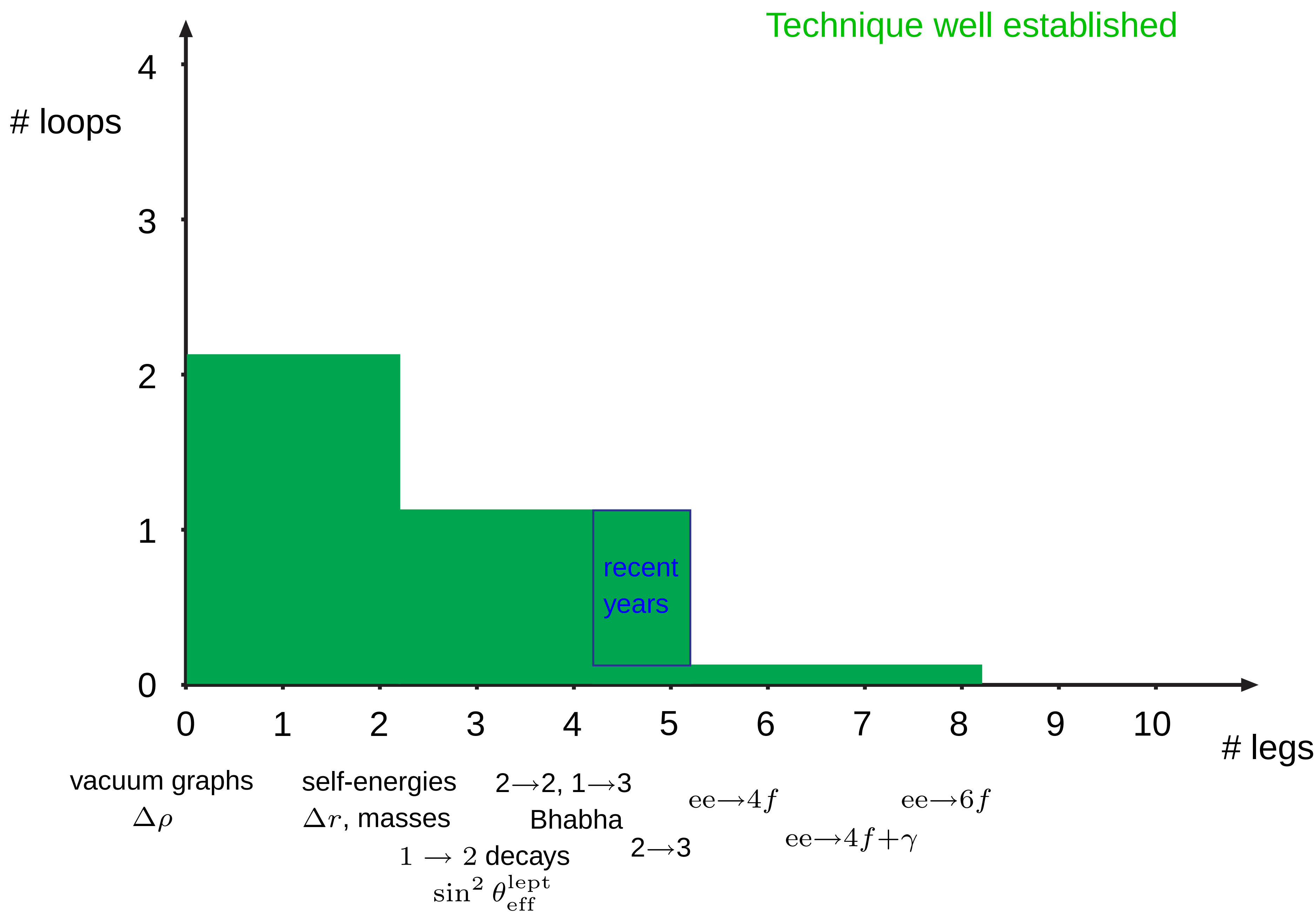
$\Rightarrow$ Precision calculations mandatory for LHC and ILC !

This talk: topical summary of recent developments in precision physics

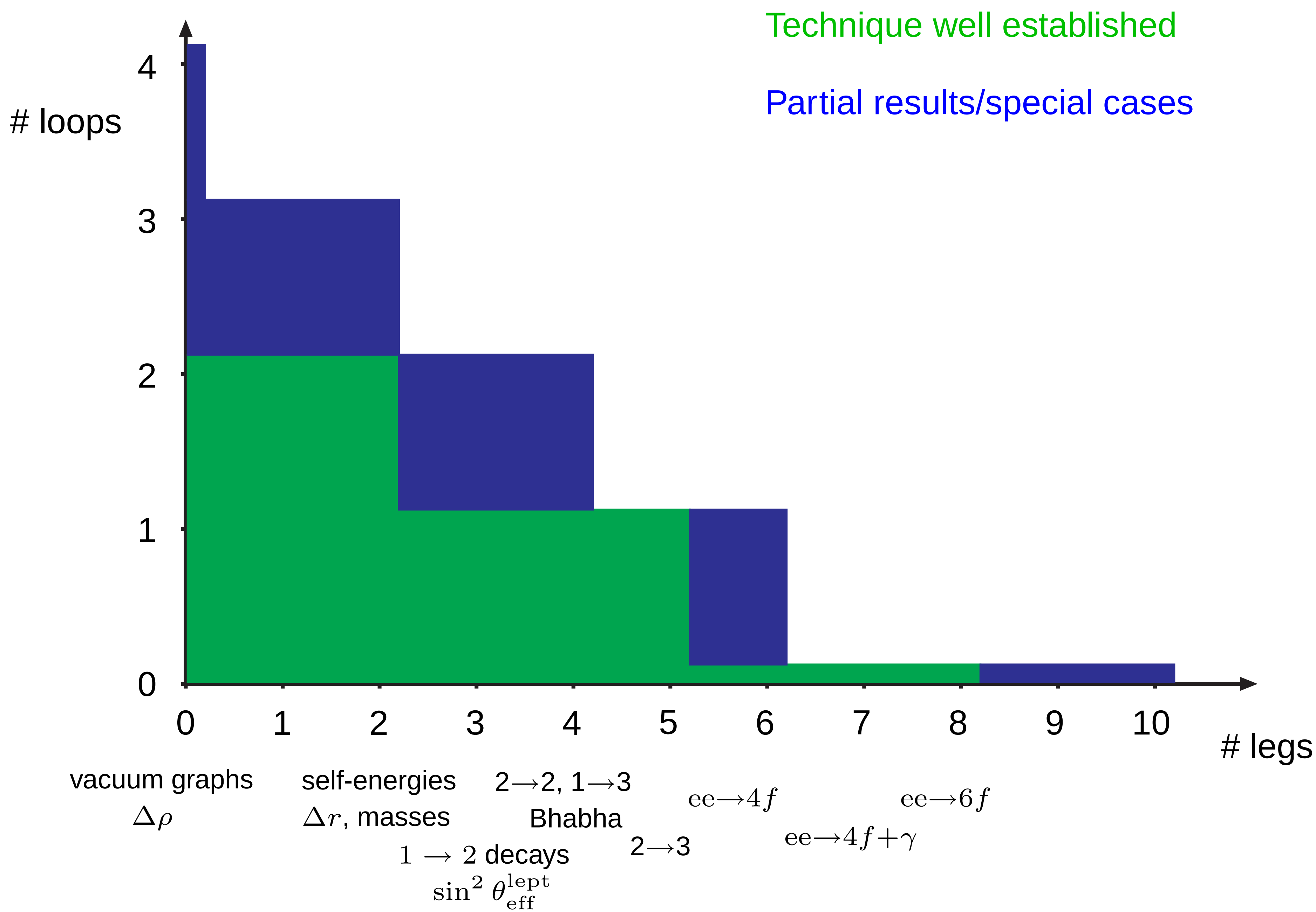
- main focus directed to phenomenological applications
  - NNLO calculations to $2 \to 2$ scattering
  - NLO corrections to many-particle processes
- necessity to develop tools & methods is highlighted in examples
- not or barely covered:
  physics beyond SM, automatization, MC and simulation tools,
  multi-loop techniques, unitarity-/twistor-inspired methods, resummation,
  topics presented in dedicated talks

$\hookrightarrow$ see, in particular, talks of P.Ciafaloni, G.Degrassi, A.Ferroglia, A.Hoang, W.Hollik, P.Mastrolia, S.Moretti, G.Passarino, S.Pozzorini, G.Zanderighi

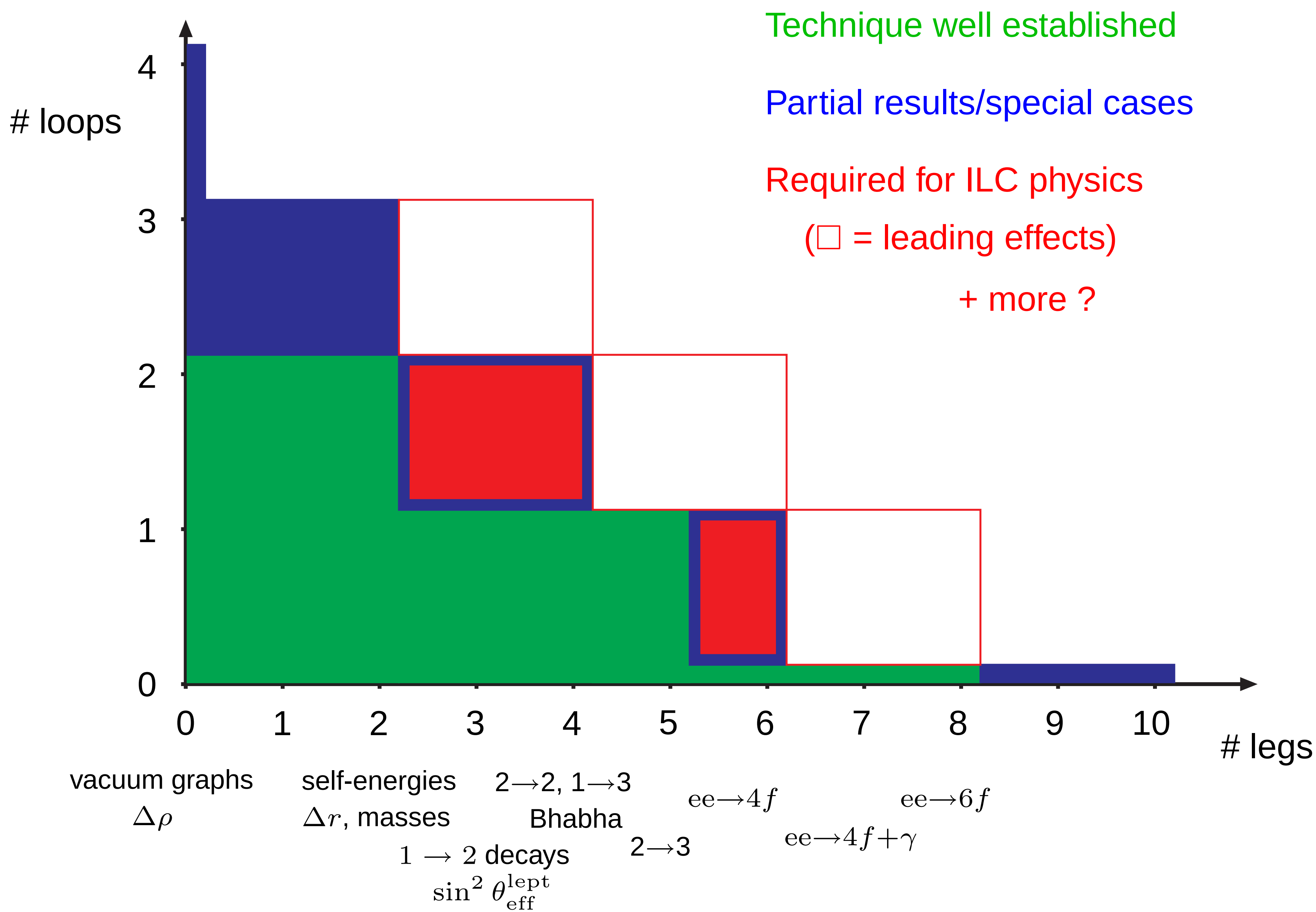
State-of-the-art in precision calculations



State-of-the-art in precision calculations



State-of-the-art in precision calculations



2 NNLO calculations

2.1 EW precision observables

Most important precision observables:

- M_W (direct measurement vs. muon decay)
  - mixed QCD/EW 2-loop corrections known — Djouadi, Verzegnassi '87; Djouadi '88; Kniehl, Kühn, Stuart '88; Kniehl, Sirlin '93 Djouadi, Gambino '94
  - complete EW 2-loop corrections known — Freitas, Hollik, Walter, Weiglein '00 Awramik, Czakon '02 Onishchenko, Veretin '02
  - improvements by 3-loop $\Delta\rho$ — Avdeev et al. '94; Chetyrkin, Kühn, Steinhauser '95 v.d.Bij et al. '00; Faisst et al. '03; Boughezal, Tausk, v.d.Bij '05

    and 4-loop QCD $\Delta\rho$ — Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal/Czakon '06

  $\hookrightarrow$ theoretical uncertainty $\Delta M_\mathrm{W} \sim 4\,\mathrm{MeV}$

- $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$ (from various asymmetries)
  - mixed QCD/EW 2-loop and 3-loop $\Delta\rho$ corrections as for M_W
  - complete EW 2-loop corrections — Awramik, Czakon, Freitas, Weiglein '04 Hollik, Meier, Uccirati '05,'06 Awramik, Czakon, Freitas '06

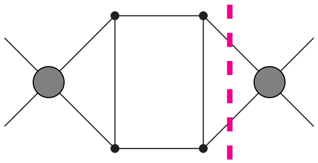
  $\hookrightarrow$ theoretical uncertainty $\Delta\sin^2\theta_\mathrm{eff}^\mathrm{lept} \sim 5\times 10^{-5}$

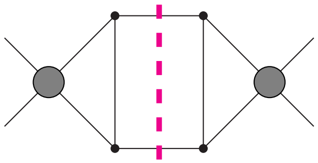
$\hookrightarrow$ Predictions in good shape for LHC, further steps desirable for ILC

2.2 NNLO calculations for $2 \to 2$ processes

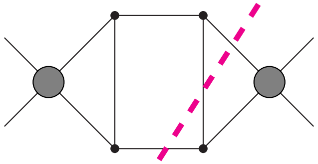
General structure of NNLO predictions:

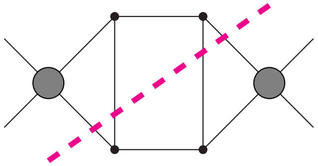
$$\Delta\sigma_{\mathrm{NNLO}} = F_{\mathrm{flux}} \int \mathrm{d}\Phi_2 \left[2\,\mathrm{Re}\left\{ \mathcal{M}^{(2\to2)}_{\text{2-loop}}\, \mathcal{M}^{(2\to2)*}_{\text{tree}} \right\} + \left| \mathcal{M}^{(2\to2)}_{\text{1-loop}} \right|^2 \right]$$





$$+ \; F_{\mathrm{flux}} \int \mathrm{d}\Phi_3\, 2\,\mathrm{Re}\left\{ \mathcal{M}^{(2\to3)}_{\text{1-loop}}\, \mathcal{M}^{(2\to3)*}_{\text{tree}} \right\} \; + \; F_{\mathrm{flux}} \int \mathrm{d}\Phi_4 \left| \mathcal{M}^{(2\to4)}_{\text{tree}} \right|^2$$





Major difficulties:

- 2-loop amplitudes $\mathcal{M}^{(2\to2)}_{\text{2-loop}}$
- extraction and cancellation of IR (soft / collinear) singularities
  $\hookrightarrow$ in particular: single and double unresolved limits in real emission amplitudes

2-loop amplitudes for $2{\to}2$ and $1{\to}3$ processes

- Algebraic reduction to master integrals — Anastasiou, Gehrmann, Glover, Laporta, Lazopoulos, Oleari, Remiddi, Smirnov, Tausk, Veretin '00–'05

  by integration by parts, Lorentz invariance identities

  $\hookrightarrow$ calculation of master integrals by Mellin–Barnes technique, — Anastasiou, Czakon, Smirnov, Tausk, Tejeda-Yeomans '99–'05

  differential equations, — Gehrmann, Remiddi '00, '01 — numerical techniques (see below)

- Direct reduction of full 2-loop amplitudes — Moch, Uwer, Weinzierl '02–'05

  $\hookrightarrow$ higher transcendental functions $\to$ nested harmonic sums

- Upcoming alternative: fully numerical approach $\to$ talk of G.Passarino
  - ◇ via sector decomposition (box master integrals, etc.) — Binoth, Heinrich '00,'03
  - ◇ via Feynman parameter integrals (all 2-/3-point integrals) — Actis, Ferroglia, Passera, Passarino, Uccirati '02–'06
  - ◇ via Mellin–Barnes representation (box master integrals, etc.) — Anastasiou, Daleo '05

- Explicit algebraic results:
  - ◇ 2-loop amplitudes for massless $2 \to 2$ processes — Anastasiou, Bern, v.d.Bij, DeFreitas, Dixon, Ghinculov, Glover, Oleari, Schmidt, Tejeda-Yeomans, Wong '01–'04
  - ◇ 2-loop QCD amplitudes for $\mathrm{e}^+\mathrm{e}^- \to 3$ jets — Garland, Gehrmann, Glover, Koukoutsakis, Moch, Remiddi, Uwer, Weinzierl '02

Towards NNLO QED corrections to Bhabha scattering → talk of A.Ferroglia

Physics motivation:

- luminosity monitor at high-energy $\mathrm{e}^+\mathrm{e}^-$ colliders (LEP/ILC)
  ↪ small-angle Bhabha scattering at LEP: BHLUMI (Jadach et al. –'97)
  (1-loop EW + higher-order QED log's)
- large cross-section → high-precision QED / EW test

Full NNLO QED prediction very important for running and future $\mathrm{e}^+\mathrm{e}^-$ colliders

Status of 2-loop and (1-loop)^2 virtual corrections

- known:
  – $m_\mathrm{e} = 0$ Bern, Dixon, Ghinculov '00
  – closed fermion loops for $m_f \neq 0$ Bonciani et al. '04; Actis, Czakon, Gluza, Riemann '07
  – $m_\mathrm{e} \to 0$ (translated $m_\mathrm{e}{=}0$ result via known IR structure) Penin '05, Becher, Melnikov '07
- in progress: $m_\mathrm{e} \neq 0$ directly from massive master integrals (MI)
  – all but few MI for boxes exist Smirnov '01; Bonciani, Mastrolia, Remiddi '02; Heinrich, Smirnov '04; Czakon, Gluza, Riemann '04–'06
  – reduction of amplitudes to MI Czakon, Gluza, Riemann '04–'06; Bonciani, Ferroglia '05

Final steps to be made:

- some missing MI for massive 2-loop boxes
- combination of 2-loop virtual with (1-loop)⊗(1γ real) and ($2\gamma/\mathrm{ee}$) real emission

Integration techniques for real radiation at NNLO

Soft/collinear singularities have very complicated overlapping structure !

$\hookrightarrow$ behaviour, e.g., described by "antenna functions" Kosower '03

Different approaches to singular integrations

- subtraction techniques
  - ◇ subtraction terms widely worked out and integrated for $\mathrm{e}^+\mathrm{e}^- \to n$jets — Weinzierl '03; Kilgore '04; Frixione, Grazzini '04; Gehrmann-DeRidder, Gehrmann, Glover '04,'05; Del Duca, Somogyi, Trocsanyi '05; Catani, Grazzini '07
  - ◇ first applications:
    - $\mathrm{e}^+\mathrm{e}^- \to 2$ jets — Gehrmann-DeRidder, Gehrmann, Glover '04; Frixione, Grazzini '04 (subtr. terms); Weinzierl '06 (full result)
    - $\mathrm{e}^+\mathrm{e}^- \to 3$ jets — Gehrmann-DeRidder, Gehrmann, Glover '05 (subtr. terms); Gehrmann-DeRidder, Gehrmann, Glover, Heinrich '07 (full result)
- direct numerical integration via sector decomposition
  - ◇ technique described in detail — Heinrich '02,'06; Gehrmann-DeRidder, Gehrmann, Heinrich '03; Gehrmann-DeRidder, Gehrmann, Glover '03; Anastasiou, Melnikov, Petriello '04; Binoth, Heinrich '04
  - ◇ first applications:

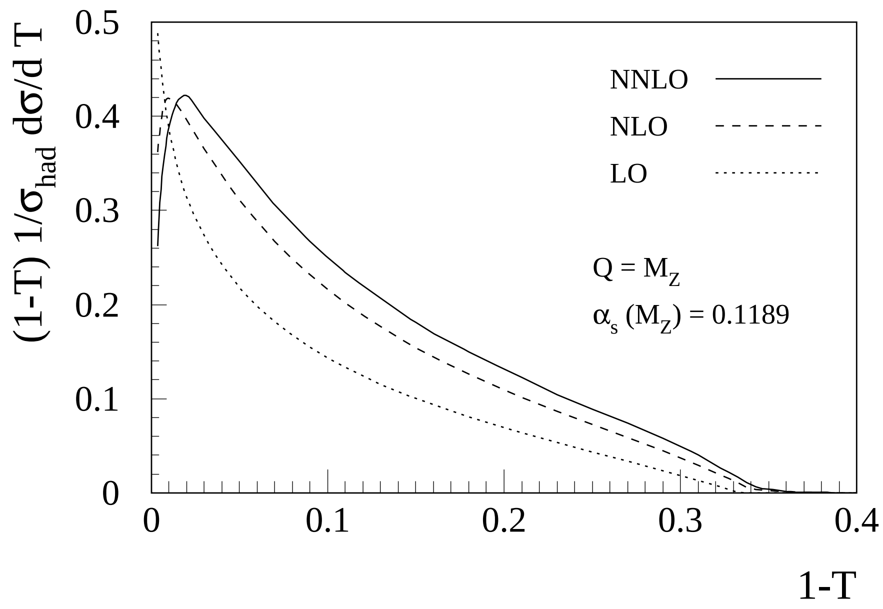
    $\mathrm{e}^+\mathrm{e}^- \to 2$ jets, $\mathrm{pp} \to \mathrm{H}+X$, $\mathrm{W}+X$ in NNLO QCD, $\mu \to \mathrm{e}\bar{\nu}_\mathrm{e}\nu_\mu$ in NNLO QED — Anastasiou, Melnikov, Petriello '04–'06

    parts of $\mathrm{e}^+\mathrm{e}^- \to 3$ jets in NNLO QCD — Heinrich '06

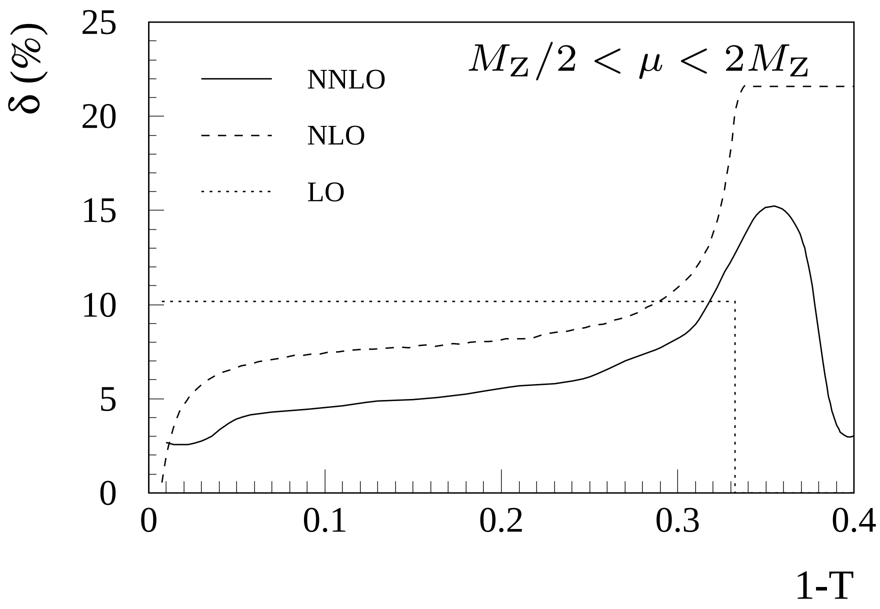
Numerical results for $\mathrm{e^+e^-} \to \mathrm{3jets}$ in NNLO QCD

Gehrmann-DeRidder, Gehrmann, Glover, Heinrich '07

Thrust distribution:



Residual ren. scale dependence:



Thrust: $$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$ ($T \to 1$ for 2-jet production)

- NNLO corrections significant $(15-20\%)$
- renormalization scale dependence (theoretical uncertainty) decreased in NNLO
- NNLO result will have impact on α_s determination from LEP event-shape data

3 NLO corrections to multi-particle production

3.1 General considerations

Existing precision calculations for many-particle processes at LHC and ILC

with up to 5-point loop diagrams:

$e^+e^- \to$ 4jets (QCD), $\nu\bar{\nu}H$, $t\bar{t}H$, $e\bar{e}H$, $\nu\bar{\nu}\gamma$, ZHH, ZZH, $\gamma\gamma \to t\bar{t}H$

NLO EW/QCD: Glover/Miller, Campbell et al., Bern et al., Dixon/Signer, Nagy/Trocsanyi, Weinzierl/Kosower, GRACE-loop, Denner et al., You et al., Chen et al., Zhang et al., Zhou et al. ’96–’06

$pp \to$ 3jets, $\gamma\gamma$+jet, V+2jets, $Q\bar{Q}H$, $t\bar{b}H^-$, $b\bar{b}V$, HHH, $t\bar{t}$+jet,
H+2jets (QCD+EW), VV+2jets (VBF)

NLO QCD: Bern et al., Kunszt et al., Kilgore/Giele, Campbell et al., Nagy, Del Duca et al., Campbell/Ellis, Beenakker et al., Dawson et al., Dittmaier et al., Peng et al., Plehn/Rauch, Febres Cordero et al., Jäger et al., Ciccolini et al. ’96–’07

$H \to 4$ fermions: NLO EW+QCD — Bredenstein et al. ’06; NLO QED — Carloni-Calame et al. ’06

Existing precision calculations for many-particle processes at LHC and ILC

with up to 6-point loop diagrams (current technical frontier)

Cross-section calculations:

$\mathrm{e^+e^-} \to 4$ fermions (CC):	NLO EW	Denner, Dittmaier, Roth, Wieders, ’05
$\mathrm{e^+e^-} \to \nu\bar{\nu}\mathrm{HH}$:	NLO EW	GRACE-loop ’05
$\gamma\gamma \to \mathrm{t\bar{t}b\bar{b}}$:	NLO QCD	Guo, Ma, Han, Zhang, Jing ’07

Amplitude calculations “only”:

$gg \to gggg$:	NLO QCD	Bern et al. ’05,’06; Britto et al. ’06; Berger,Forde ’06 (analytically) R.K.Ellis, Giele, Zanderighi ’06; R.K.Ellis, Giele, Kunszt ’07 (numerically)
$\gamma\gamma \to \gamma\gamma\gamma\gamma$:	NLO QED	Nagy, Soper ’06; Ossola, Papadopoulos, Pittau ’07 (numerically) Binoth, Heinrich, Gehrmann, Mastrolia ’07 (analytically)

Complications in corrections to many-particle processes

- huge amount of algebra, long final expressions

  $\hookrightarrow$ computer algebra / automatization

- multi-dimensional phase-space integration

  $\hookrightarrow$ Monte Carlo techniques

- complicated structure of singularities and matching of virtual and real corrections

  $\hookrightarrow$ subtraction R.K.Ellis et al. '81; S.D.Ellis et al. '89; Mangano et al. '92; Kunszt/Soper '92; Frixione et al. '96; Nagy/Z. Trócsányi '96; Campbell et al. '98; Catani/Seymour '96; Dittmaier '99; Phaf/Weinzierl '01; Catani et al. '02

  and slicing techniques

  Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01, etc.

- numerically stable evaluation of one-loop integrals with up to 5,6,... external legs

  $\hookrightarrow$ techniques to solve problems with inverse kinematical (e.g. Gram) det's

  Stuart et al. '88/'90/'97; v.Oldenborgh/Vermaseren '90; Campbell et al. 96; Ferroglia et al. '02; del Aguila/Pittau '04; Binoth et al. '02/'05; Denner/Dittmaier '02/'05; v.Hameren et al. '05; R.K.Ellis et al. '05; Anastasiou/Daleo '05; Ossola et al. '06/'07; Lazopoulos et al. '07; Forde '07

  [But: many proposed methods not (yet?) used in complicated applications]

- treatment of unstable particles, issue of complex masses

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Proposed solutions for loop calculations:

- naive fixed-width scheme

  $\hookrightarrow$ breaks gauge invariance only mildly (?),
  but partial inclusion of widths in loops screws up singularity structure

- pole expansions   Stuart '91; Aeppli et al. '93, '94; etc.

  $\hookrightarrow$ consistent, gauge invariant,
  but not reliable at threshold or in off-shell tails of resonances

- effective field theory approach   Beneke et al. '04,'07; Hoang,Reisser '04

  $\hookrightarrow$ involves pole expansions,
  can be combined with threshold expansions $\rightarrow$ talk of G.Zanderighi

- complex-mass scheme   Denner, Dittmaier, Roth, Wieders '05

  $\hookrightarrow$ gauge invariant, simple, valid everywhere in phase space

3.2 NLO EW corrections to $e^+e^- \to 4$ fermions

Denner, Dittmaier, Roth Wieders '05

Details of the calculation:

- final states: $\nu_\tau \tau^+ \mu^- \bar{\nu}_\mu,\ u\bar{d}\mu^-\bar{\nu}_\mu,\ u\bar{d}s\bar{c}$ (charged current)
- complex-mass scheme proposed for unstable particles in loop calculations
- new tensor reduction methods for numerical stabilization Denner, Dittmaier '02,'05
- real corrections $e^+e^- \to 4f+\gamma$ from RacoonWW Denner et al. '99–'01
- checks: – UV/IR/mass singularities, gauge invariance, slicing/subtraction
  – two independent calculations

Physics motivation:

Improvement over "double-pole approximation" (DPA) for $e^+e^- \to WW \to 4f$

needed for ILC: – M_W from WW threshold scan where DPA insufficient
– TGC analysis at high energies

Recent related result:

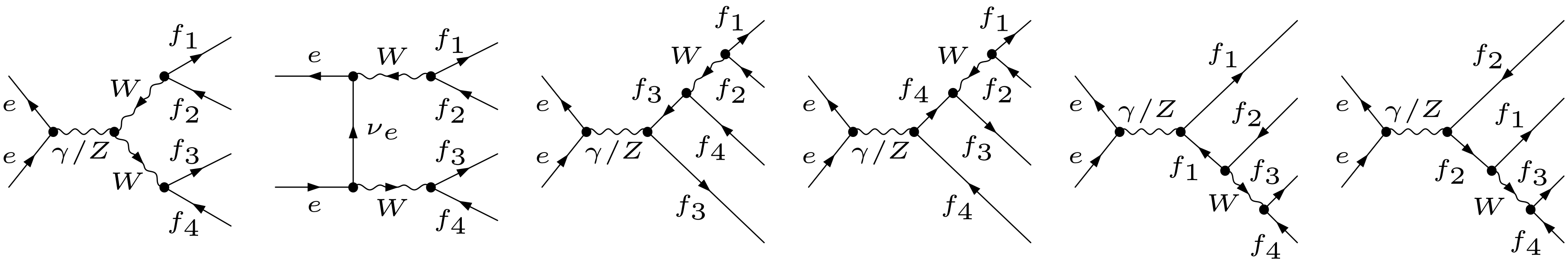
σ_{tot} for $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ via effective field theory for pole⊕threshold expansion

Beneke, Falgari, Schwinn, Signer, Zanderighi '07

$\hookrightarrow$ "continuation" of DPA to WW threshold

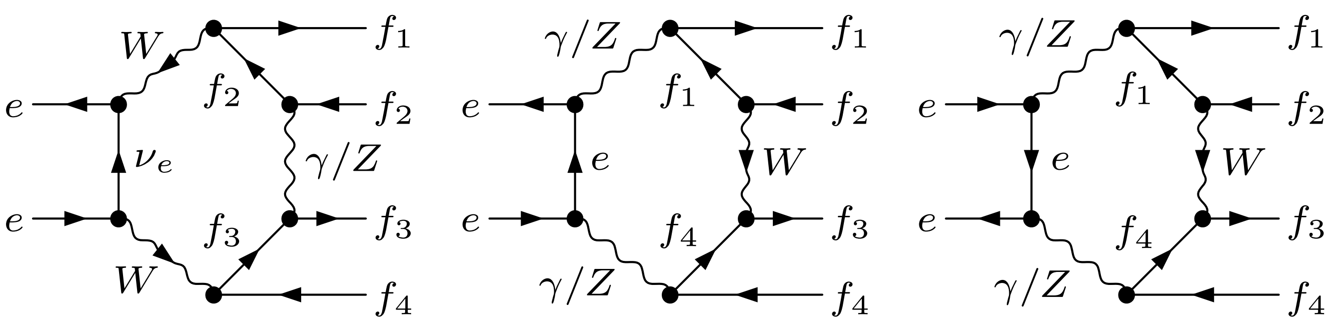
Some Feynman diagrams...

...for LO:



...for NLO: total number $= \mathcal{O}(1200)$

40 hexagons



+ graphs with reversed fermion-number flow in final state

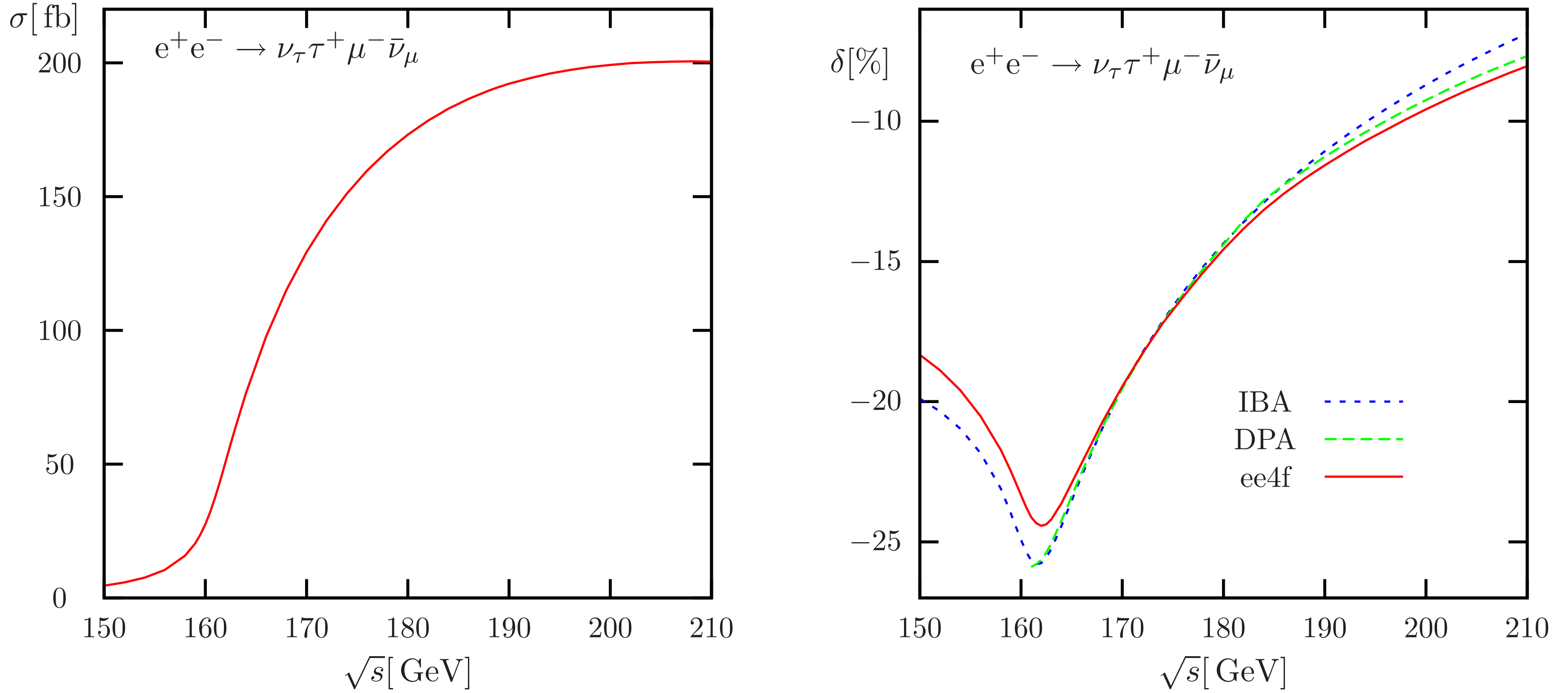
+ 112 pentagons

+ 227 boxes ('tHF gauge) + many vertex and self-energy corrections

Numerical results for LEP2 energies

Complete $\mathcal{O}(\alpha)$ corrections to the total cross section

Denner, Dittmaier, Roth, Wieders '05



- $|\mathrm{ee4f} - \mathrm{DPA}| \quad \sim \quad 0.5\%$ for $170\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 210\,\mathrm{GeV}$
- $|\mathrm{ee4f} - \mathrm{IBA}| \quad \sim \quad 2\%$ for $\sqrt{s} \lesssim 170\,\mathrm{GeV}$

$\hookrightarrow$ agreement with error estimates of DPA and "Improved Born Approximation"

3.3 NLO EW corrections to $e^+e^- \to \nu\bar{\nu}HH$

Boudjema, Fujimoto, Ishikawa, Kaneko, Kato, Kurihara, Shimizu, Yasui '05

Full $2 \to 4$ calculation performed with GRACE-LOOP package — Belanger et al. hep-ph/0308080

- number of loop diagrams (non-linear gauge, $m_e \to 0$):
  $\#(e^+e^- \to \nu_e\bar{\nu}_e HH) \sim 3400$, $\#(e^+e^- \to \nu_\mu\bar{\nu}_\mu HH) \sim 1800$
- gauge-invariance check via non-linear gauge with gauge parameters
  (for vanishing particle widths)
- REDUCE and FORM used to process interference of LO and NLO amplitudes
  $\hookrightarrow$ 5- and 6-point integrals converted into 4-point integrals
- in-house library $\oplus$ FF for loop integrals
  v.Oldenborgh '91

Physics motivation:

Higgs self-coupling enters $\underbrace{e^+e^- \to ZHH}_{\sqrt{s} \lesssim 1\,\mathrm{TeV}}$ and $\underbrace{e^+e^- \to \nu\bar{\nu}HH}_{\sqrt{s} \gtrsim 1\,\mathrm{TeV}}$ in LO

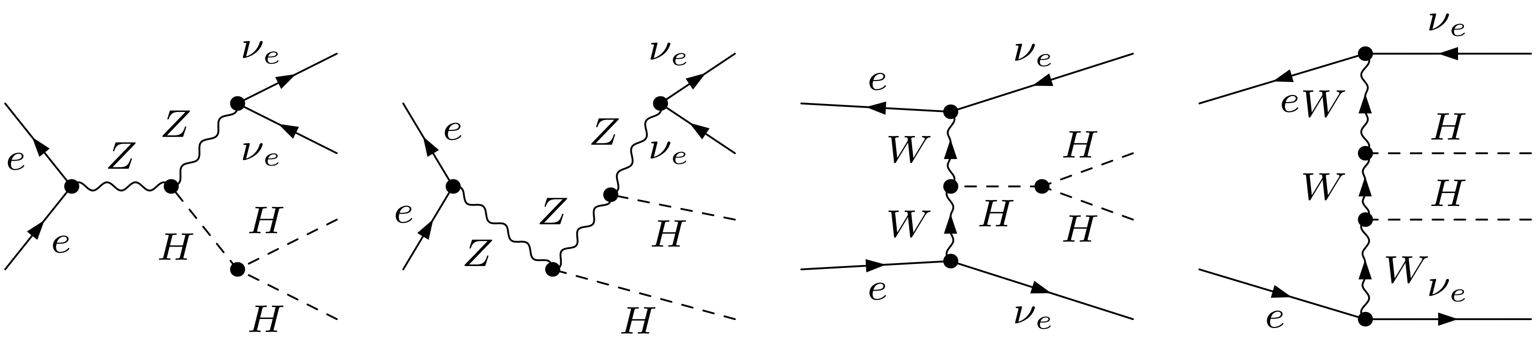
larger cross-section for $\sqrt{s} \lesssim 1\,\mathrm{TeV}$ / $\sqrt{s} \gtrsim 1\,\mathrm{TeV}$

$\hookrightarrow$ check of Higgs mechanism / information on EWSB

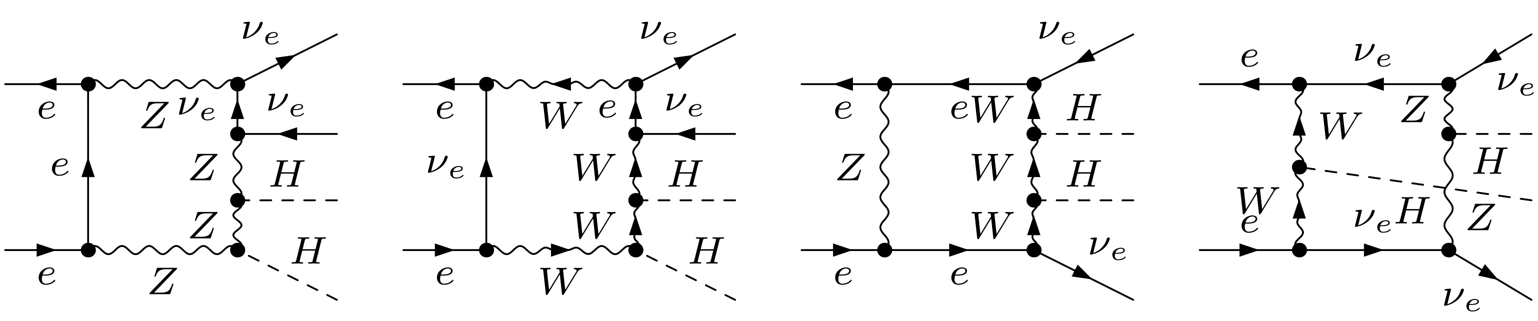
But: Both reactions have very small cross sections: $\sigma_{ZHH+\nu\bar{\nu}HH} \sim 0.1-1\,\mathrm{fb}$

Some Feynman diagrams...

...for LO: total number $= 18$



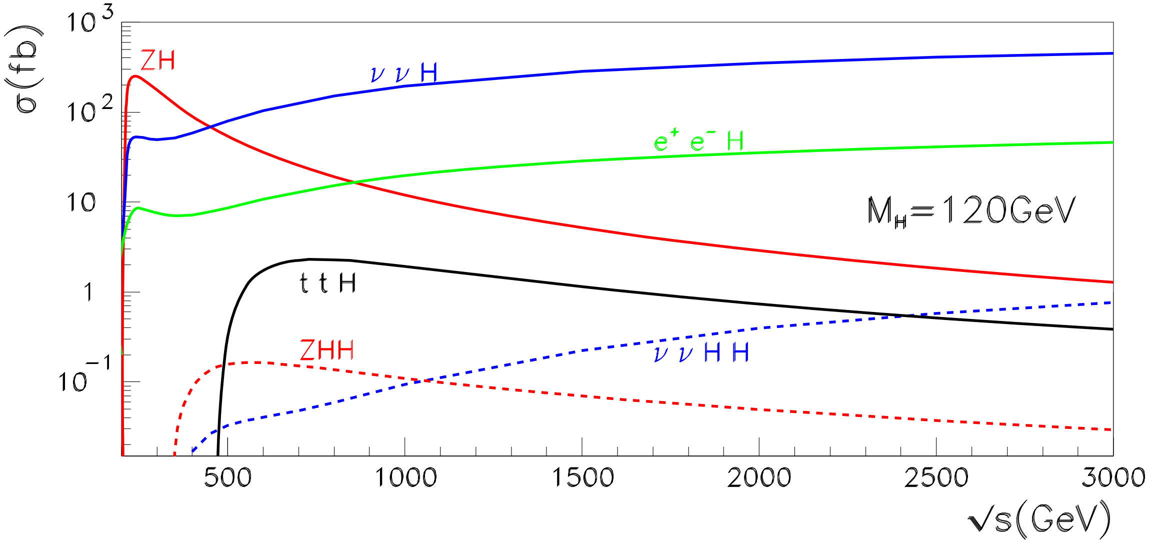
...for NLO: total number $= \mathcal{O}(4600)$ in 'tHF gauge



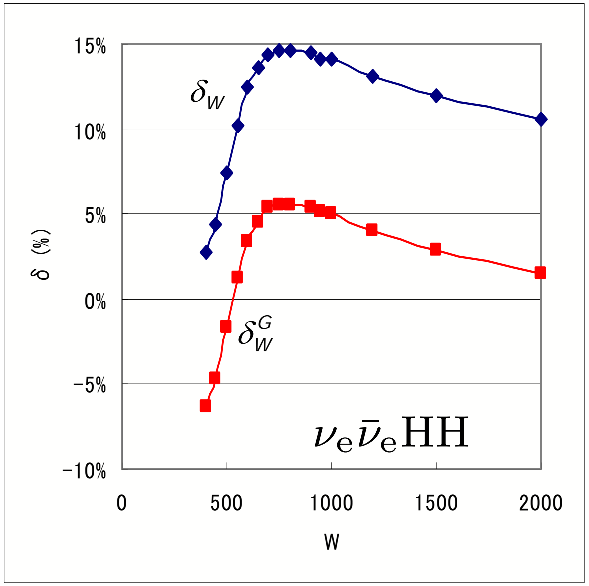
89 hexagons, 250 pentagons ('tHF gauge), etc.

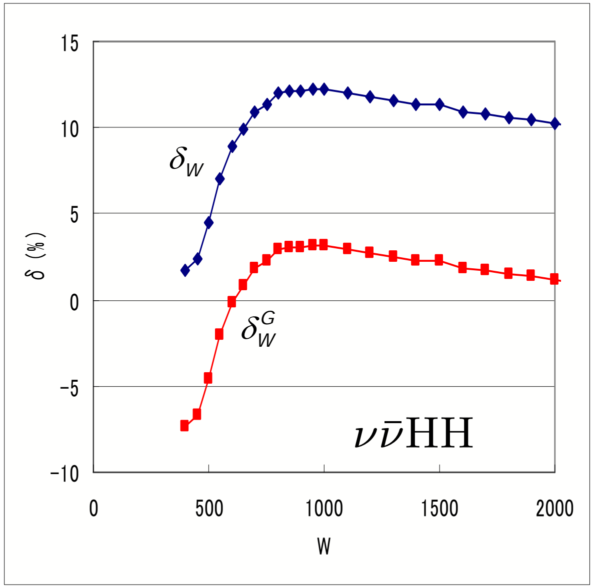
Numerical results: Boudjema et al. '05

Higgs production processes at the ILC in LO:



Weak (non-photonic) NLO corrections to $e^+e^- \to \nu\bar{\nu}HH$:





G_μ-scheme:

$$\delta_W^G = \delta_W - 4\Delta r$$

3.4 NLO QCD corrections to $\gamma\gamma \to \mathrm{t\bar{t}b\bar{b}}$ Guo, Ma, Han, Zhang, Jing '07

Details of the calculation:

- FEYNARTS for diagram generation
  Hahn '01
- FORMCALC for algebraic reduction of amplitudes
  Hahn, Perez-Victoria '99
- 5-/6-point integrals reduced with known techniques Denner, Dittmaier '02,'05; Binoth et al. '03
- up to 4-point loop integrals evaluated with LOOPTOOLS (including FF library)
  Hahn, Perez-Victoria '99 v.Oldenborgh '91
- 5-particle phase space integrated with COMPHEP Boos et al. '04
- phase-space slicing for treating IR divergence (b quarks massive)

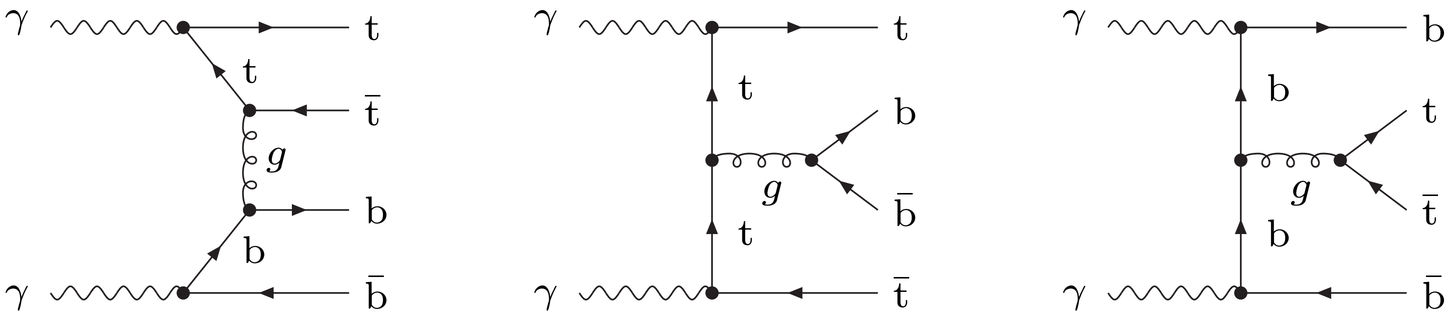
Note: consistent use of available tools and techniques !

Physics motivation:

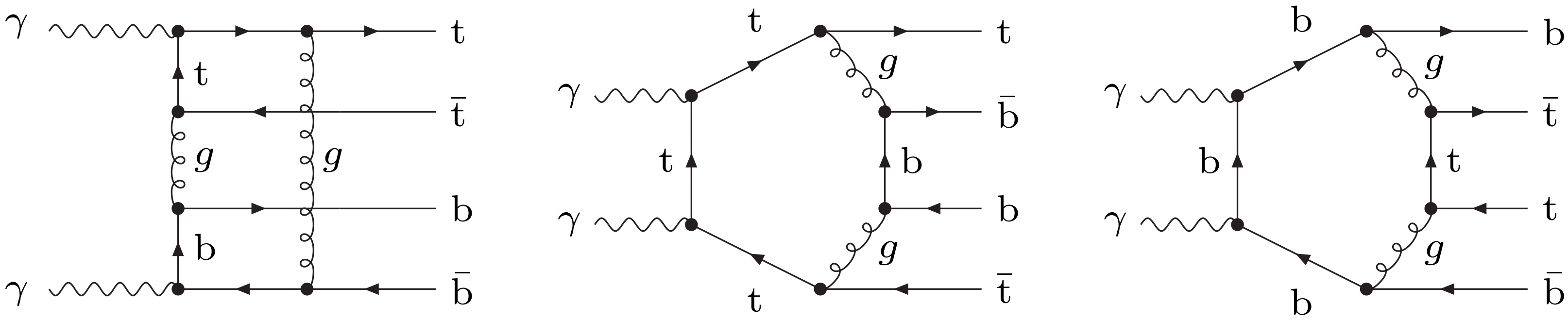
- background to $\gamma\gamma \to \mathrm{t\bar{t}H}$ at a future $\gamma\gamma$ collider
- but first step towards $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ (important background to $\mathrm{pp} \to \mathrm{t\bar{t}H}$)

Some Feynman diagrams...

...for LO: total number $= 10$



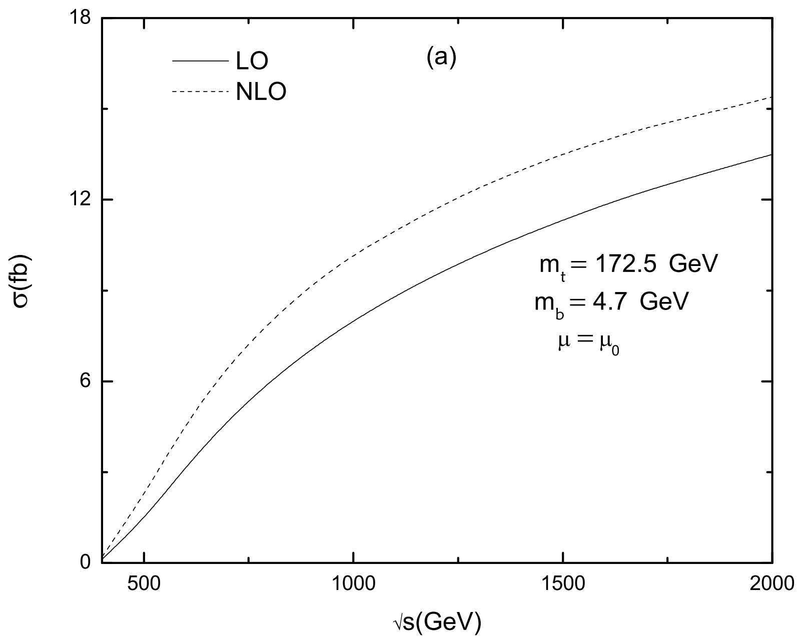
...for NLO QCD: total number $= \mathcal{O}(500)$

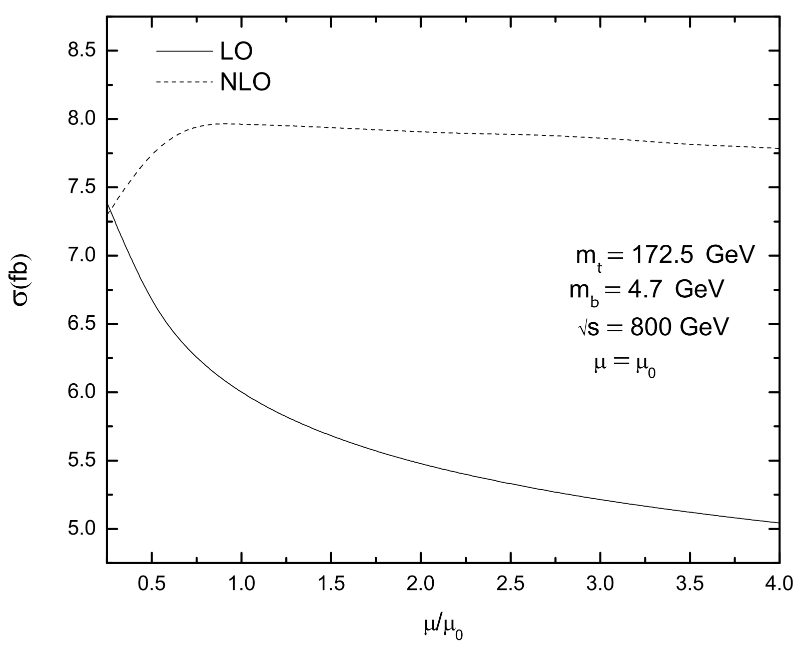


12 hexagons, 48 pentagons, etc.

Numerical results: Guo et al. ’07

Production cross section for $\gamma\gamma \to \mathrm{t\bar{t}b\bar{b}}$ and renormalization scale dependence





$(\mu_0 = m_\mathrm{t} + m_\mathrm{b})$

- K factor $\sim$ 1.55 for $\sqrt{s} = 500\,\mathrm{GeV}$
  $\sim$ 1.14 for $\sqrt{s} = 2000\,\mathrm{GeV}$
- dependence on renormalization scale μ stabilizes considerably in NLO

4 Conclusions

Recent progress on our way to the ILC:

- NNLO (and beyond) calculations for static quantities, vertices, $2{\to}2$ scattering
  ($\Delta\rho$, μ decay, $\sin^2\theta_{\rm eff}^{\rm lept}$, $gg{\to}\rm H$, Drell–Yan, Bhabha, $\rm e^+e^- \to 3jets$, etc.)
- first $2{\to}4$ processes at NLO
  ($\rm ee{\to}4\mathit{f}$, $\rm ee{\to}\nu\nu HH$, $\gamma\gamma \to \rm t\bar{t}b\bar{b}$, $6g/6\gamma$ amplitudes)
- progress in many-particle production
  (matrix elements, showers, etc.)
- great technical and conceptual progress in perturbative QFT
  (loop techniques, unitarity/twistor-inspired methods, unstable particles, etc.)
- etc.

Phenomenological progress and development of tools & methods go hand in hand.

Important tools under construction:

- subtraction formalisms for real corrections at NNLO
- automatization of / libraries for NLO multi-leg calculations
- matching of parton showers with matrix-element calculations in NLO
- etc.