Electroweak Precision Physics
from LEP to ILC

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ILC Physics in Florence

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Outline

- Electroweak precision observables – Standard Model
- Theory versus data
- Perspectives
- Extensions of the SM – Supersymmetry
- Outlook
Standard Model

- the symmetry group $SU(2) \times U(1) \times SU(3)_C$
- the principle of local gauge invariance
  - fermion – vector boson interaction
  - vector boson self-interaction
- Higgs mechanism and Yukawa interactions
  - masses $M_W$, $M_Z$, $m_{\text{fermion}}$

renormalizable quantum field theory
accurate theoretical predictions

detect deviations $\rightarrow$ “new physics”?
Search for the Standard Model Higgs at LEP

Dominant production process: $e^+e^- \rightarrow ZH$

Dominant decay process: $H \rightarrow b\bar{b}$

exclusion limit (95% C.L.): $M_H > 114.4$ GeV
Precision observables – SM

Test of theory at quantum level:

Sensitivity to loop corrections

- $\mu$ lifetime: $M_W, \Delta r, G_F$
- $Z$ observables: $g_V, g_A, \sin^2 \theta_{\text{eff}}, \Gamma_Z, \ldots$

sensitivity to heavy internal particles ($X$)

Standard Model: $X = \text{Higgs, top}$
\[ \begin{align*}
M_W - M_Z \text{ correlation} \\
\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r) \\
\Delta r : \text{ quantum correction, } \Delta r = \Delta r(m_t, M_H) \\
\rightarrow M_W = M_W(\alpha, G_F, M_Z, m_t, M_H) \\
\text{complete two-loop calculation available}
\end{align*} \]
**Z resonance**

- effective $Z$ boson couplings with higher-order $\Delta g_{V,A}$

$$g^f_V \rightarrow g^f_V + \Delta g^f_V, \quad g^f_A \rightarrow g^f_A + \Delta g^f_A$$

- effective electroweak mixing angle (for $f = e$):

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \text{Re} \frac{g^e_V}{g^e_A} \right) = \kappa \cdot \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$
EW 2-loop calculations for $\Delta r$
Freitas, Hollik, Walter, Weiglein
Awramik, Czakon
Onishchenko, Veretin

EW 2-loop calculations for $\sin^2 \theta_{\text{eff}}$
Awramik, Czakon, Freitas, Weiglein
Awramik, Czakon, Freitas
Hollik, Meier, Uccirati

universal terms beyond 2-loop order (EW and QCD)
van der Bij, Chetyrkin, Faisst, Jikia, Seidensticker
Faisst, Kühn Seidensticker, Veretin
Boughezal, Tausk, van der Bij
Schröder, Steinhauser
Chetyrkin, Faisst, Kühn Chetyrkin, Faisst, Kühn, Maierhofer, Sturm
Boughezal, Czakon
charge renormalization \( e + \delta e \) involves

photon vacuum polarization

\[ \Pi^\gamma(M_Z^2) - \Pi^\gamma(0) \equiv \Delta \alpha \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1 - \Delta \alpha} \]

\[ \Delta \alpha = \Delta \alpha_{\text{lept}} + \Delta \alpha_{\text{had}}, \]

\[ \Delta \alpha_{\text{lept}} = 0.031498 \quad (3 - \text{loop}) \]

\[ \Delta \alpha_{\text{had}} = 0.02758 \pm 0.00035 \]

significant source of parametric uncertainty
\[ \Delta \alpha_{\text{had}} = -\frac{\alpha}{3\pi} M_Z^2 \Re \int_0^\infty ds' \frac{R_{\text{had}}(s')}{s'(s' - M_Z^2 - i\epsilon)} \]
Lynn, Penso, Verzegnassi, ‘87
Eidelman, Jegerlehner ‘95
Burkhardt, Pietrzyk ‘95
Martin, Zeppenfeld ‘95
Swartz ‘96
Alemany, Davier, Höcker ‘97
Davier, Höcker ‘97
Kühn, Steinhauser ‘98
Groote et al. ‘98
Erler ‘98
Davier, Höcker ‘98

\[ \Delta \alpha_{\text{had}}(M_Z^2) \times 10^{-4} \]
• **LEP1/SLC:** \( e^+ e^- \rightarrow Z \rightarrow f \bar{f} \)
  LEP1: \( \sim 4 \times 10^6 \) events/experiment
  4 experiments (1989 – 1995)

• **LEP2:** \( e^+ e^- \rightarrow W^+ W^- \)
  \( \mathcal{O}(10^4) \) \( W \) pairs (1996 – 2000)

• **Tevatron:** \( q \bar{q}' \rightarrow W \rightarrow l\nu, q \bar{q}' \)
  \( (p\bar{p}) \quad q \bar{q}' \rightarrow t\bar{t}, t \rightarrow W^+ b \rightarrow \ldots \)

• **low-energy experiments** (\( \mu \) decay, \( \nu N \) scattering, \( \nu e \) scattering, atomic parity violation, \( \ldots \) )
Theory versus Data

experimental results (selection)

\[ M_Z \ [\text{GeV}] = 91.1875 \pm 0.0021 \quad 0.002\% \]
\[ \Gamma_Z \ [\text{GeV}] = 2.4952 \pm 0.0023 \quad 0.09\% \]
\[ \sin^2 \theta_{\text{eff}} = 0.23148 \pm 0.00017 \quad 0.07\% \]
\[ M_W \ [\text{GeV}] = 80.392 \pm 0.029 \quad 0.04\% \]
\[ m_t \ [\text{GeV}] = 170.9 \pm 1.8 \quad 1.05\% \]
\[ G_F \ [\text{GeV}^{-2}] = 1.16637(1)10^{-5} \quad 0.001\% \]

quantum effects at least one order of magnitude larger than experimental uncertainties
$M_W^{\text{exp}} = (80.426 \pm 0.034) \text{ GeV}$

$M_H = 114.4 \text{ GeV}$

$M_W$ vs. $M_H$

$M_t = 174.3 \pm 5.1 \text{ GeV}$

$\delta M_W^{\text{theo}} \approx 4 \text{ MeV}$

$\delta \sin^2 \theta_{\text{eff}}^{\text{theo}} \approx 5 \cdot 10^{-5}$
\[ A_{0,1}^{\text{fb}} = 0.23099 \pm 0.00053 \]
\[ A_{1}(P_{\tau}) = 0.23159 \pm 0.00041 \]
\[ A_{1}(\text{SLD}) = 0.23098 \pm 0.00026 \]
\[ A_{0,b}^{\text{fb}} = 0.23221 \pm 0.00029 \]
\[ A_{0,c}^{\text{fb}} = 0.23220 \pm 0.00081 \]
\[ Q_{\text{had}}^{\text{fb}} = 0.2324 \pm 0.0012 \]

Average: 0.23153 \pm 0.00016

\[ \Delta \alpha_{\text{had}}^{(5)} = 0.02758 \pm 0.00035 \]

\[ m_{t} = 172.7 \pm 2.9 \text{ GeV} \]

\[ \chi^2/\text{d.o.f.}: 11.8 / 5 \]
LEP Electroweak Working Group

\[ \sin^2 \theta_{\text{eff}} \leq 0.233 \]

\[ m_t = 170.9 \pm 1.8 \text{ GeV} \]
\[ m_H = 114 \ldots 1000 \text{ GeV} \]
Z PHYSICS AT LEP 1

Edited by
Guido Altarelli, Ronald Kleiss and Claudio Verzegnassi

Volume 1: STANDARD PHYSICS

Co-ordinated and supervised by G. Altarelli

GENEVA
1989
development of precision

1990-1992
91.1904±0.0065

1993-1994
91.1882±0.0033

1995
91.1866±0.0024

average
91.1874±0.0021

m_Z [GeV]
importance of two-loop calculations

\[
\sin^2 \theta_{\text{eff}} \quad \text{vs} \quad M_H [\text{GeV}]
\]

- 1loop
- +QCD+lead.3loop
- +2loop
**W-pair production**

\[ e^+ \rightarrow W^+ \rightarrow e^+ + \nu_e \]
\[ e^- \rightarrow W^- \rightarrow e^- + \bar{\nu}_e \]
\[ e^+ + e^- \rightarrow W^+ + W^- \]

\[ e^+ + e^- \rightarrow \gamma, Z \rightarrow W^+ + W^- \]

**LEP PRELIMINARY**

\( \sigma_{WW} \) (pb)

- YFSWW/RacoonWW
- no ZWW vertex (Gentle)
- only \( \nu_e \) exchange (Gentle)

\[ \sqrt{s} \text{ (GeV)} \]

\[ 17/02/2005 \]
blueband: theory uncertainty

“Precision Calculations at the $\mathcal{Z}$ Resonance”
CERN 95-03
[Bardin, Hollik, Passarino (eds.)]

$M_H < 144 \text{ GeV} \quad (95\% \text{C.L.})$

with renormalized probability for $M_H > 114 \text{ GeV}$:
$M_H < 182 \text{ GeV} \quad (95\% \text{C.L.})$
## Future perspectives

<table>
<thead>
<tr>
<th>error for</th>
<th>LEP/Tev</th>
<th>Tev/LHC</th>
<th>LC</th>
<th>LC/GigaZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$ [MeV]</td>
<td>29</td>
<td>15</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$</td>
<td>0.00017</td>
<td>0.00021</td>
<td>0.000013</td>
<td></td>
</tr>
<tr>
<td>$m_{\text{top}}$ [GeV]</td>
<td>1.8</td>
<td>1 - 1.5</td>
<td>0.2</td>
<td>0.13</td>
</tr>
<tr>
<td>$M_{\text{Higgs}}$ [GeV]</td>
<td>–</td>
<td>0.1</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\[
\delta M_Z = 2.1 \text{ MeV} \quad \text{(LEP)}
\]
\[
\delta G_F / G_F = 1 \cdot 10^{-5} \quad \text{($\mu$ lifetime)}
\]

GigaZ $\sim 10^9$ $Z$ bosons
MegaW $\sim 10^6$ $W$ bosons
$M_W$ from Drell-Yan at the LHC

\[ q\bar{q}' \rightarrow W^+ \rightarrow \ell^+ \nu_\ell \]

**Fig. 1.** Transverse mass distribution and $O(\alpha)$ relative correction.

[Carloni Calame et al.]
[Baur, Wackeroth]
[Dittmaier, Krämer]
[Arbuzov et al.]
$M_W$ from threshold scan in $e^+e^-$ annihilation

Figure 5.1.7: Sensitivity of the W-pair threshold scan to the W-mass. The vertical axis shows the ratio of the cross section to the predicted cross section for $M_W = 80.39$ GeV. The error bars represent the expected errors for the scan described in the text.

However, the sensitivity to the details of the resonance region can be reduced significantly, if the low energy data is used to fit the coefficients of a QCD operator product expansion instead of integrating the total cross section. If the hadronic cross section is known to 1% up to the $Z$-resonance, the uncertainties are $\sin^2 \theta_e = 0.000017$ and $M_W = 1$ MeV [24].

A $Z$-mass error of 2 MeV from LEP contributes 0.000014 to the uncertainty of the $\sin^2 \theta_e$ prediction, about the same size as the experimental error and the uncertainty from $(M_Z^2)$. For $M_W$ the direct uncertainty due to $M_Z$ is 2.5 MeV. However, if the beam energy is calibrated relative to the $Z$-mass, so that the relevant observable is $M_W = M_Z$, the error is smaller by a factor three.

An uncertainty in the top quark-mass of 1 GeV results in an uncertainty of the $\sin^2 \theta_e$ prediction of 0.00003 and in the one for $M_W$ of 6 MeV. For a top-mass error of $m_t = 100$ MeV, as it is possible from a top-threshold scan at TESLA (see section 5.2), this uncertainty is completely negligible.

Including the possible improvement on $(M_Z^2)$ very stringent tests of the Standard Model are possible. Figure 5.1.8 shows as an example the variation of the fit-$\chi^2$ as a function of the Higgs-mass for the present data and for TESLA. It can be seen that the Higgs-mass can indirectly be constrained at the level of 5% [17,26].

If the Higgs-mass is in the range predicted by the current precision data, the Higgs can be constrained at the level of 5%.
\[ M_W \quad \text{from threshold} \quad e^+e^- \rightarrow WW \rightarrow 4f \]

Some Feynman diagrams...

...for LO:

...for NLO: total number = \( \mathcal{O}(1200) \)

40 hexagons

+ graphs with reversed fermion-number flow in final state

+ 112 pentagons

+ 227 boxes ('tHF gauge) + many vertex and self-energy corrections
\( M_W \) from threshold  \( e^+e^- \rightarrow WW \rightarrow 4f \)

Denner, Dittmaier, Roth, Wieders '05

Numerical results for LEP2 energies

\[ \sigma \quad [\text{fb}] \]

\[ e^+e^- \rightarrow \nu_{\tau} \tau^+ \mu^- \bar{\nu}_\mu \]

\[ \delta \quad [\%] \]

\[ e^+e^- \rightarrow \nu_{\tau} \tau^+ \mu^- \bar{\nu}_\mu \]

IBA  \( \text{DPA} \)  \( \text{ee4f} \)

[Graphs showing cross sections and relative uncertainties for different processes.]
Figure 5.1.8: $\chi^2$ as a function of the Higgs-mass for the electroweak precision data now and after GigaZ running.

In this case the data can be used to check the consistency of the SM or to measure free parameters in by then established extensions of the model. As an example Fig. 5.1.9 shows the constraints that can be obtained in $m_A$ and $\tan \beta$ from the low energy running if other SUSY parameters, especially the stop sector, are already known or, alternatively, in $m_A$ and $m_{\tilde{t}}$ if $\tan \beta$ and the parameters, that can be measured from the light stop only, are known [26]. Further applications of GigaZ to Supersymmetry are discussed in chapter 2.3.2.

For more model independent analyses frequently reparameterisations of the radiative correction parameters are used where the large isospin-breaking corrections are absorbed into one parameter, so that the others depend only on the logarithmic terms. One example are the so called "parameters" [27].

In this parameterisation $\gamma_1$ absorbs the large isospin-splitting corrections, $\gamma_3$ contains only a logarithmic $M_H$ dependence while $\gamma_2$ is almost constant in the Standard Model and most extensions. Figure 5.1.10 a)-c) shows the the expectations in the $\gamma_i$-$\gamma_j$-planes, compared to present data and to the SM prediction. Since the prediction for $\gamma_2$ is almost constant, in Fig. 5.1.10 d) the $\gamma_1$-$\gamma_3$-plane is shown, if $\gamma_2$ is fixed to the predicted value. In this case the precision along the large axis of the ellipse is dominated by the precise measurement of the W-mass.
[Erler, Heinemeyer, Hollik, Weiglein, Zerwas]
Theoretical bounds on Higgs boson mass from

- perturbativity $\rightarrow$ upper bound
- unitarity $\rightarrow$ upper bound
- triviality (Landau pole) $\rightarrow$ upper bound
- vacuum stability $\rightarrow$ lower bound
combined effects, RGE in two-loop order:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left( 12\lambda^2 - 3 g_t^4 + 6 \lambda g_t^2 + \cdots \right)$$
SM Higgs:

- $\lambda H^4$ term ad hoc
- Higgs boson mass: free parameter $\sim \sqrt{\lambda}$
- no a-priori reason for a light Higgs boson

SUSY Standard Model avoids these questions

$$H_2 = \begin{pmatrix} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} v_1 + H_1^0 \\ H_1^- \end{pmatrix}$$

couples to $u$ couples to $d$

- SUSY gauge interaction $\rightarrow$ $H^4$ terms
- self coupling remains weak
Minimal Supersymmetric SM

Superpartners for Standard Model particles:

\[ u, d, c, s, t, b \]_{L,R} \quad \left[ e, \mu, \tau \right]_{L,R} \quad \left[ \nu_{e, \mu, \tau} \right]_L \quad \text{Spin } \frac{1}{2}

\[ \tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b} \]_{L,R} \quad \left[ \tilde{e}, \tilde{\mu}, \tilde{\tau} \right]_{L,R} \quad \left[ \tilde{\nu}_{e, \mu, \tau} \right]_L \quad \text{Spin } 0

\[ g, W^\pm, H^\pm \] \quad \gamma, Z, H_1^0, H_2^0 \quad \text{Spin } 1 / \text{Spin } 0

\[ \tilde{g}, \tilde{\chi}_{1,2}^\pm, \tilde{\chi}_{1,2,3,4}^0 \] \quad \text{Spin } \frac{1}{2}

Enlarged Higgs sector: two Higgs doublets, physical states:
\[ h^0, H^0, A^0, H^\pm \]
masses and mixing of SUSY particles through soft-breaking

model parameters

- gaugino masses: $M_1, M_2, M_3$
- sfermion masses: $M_L, M_{\tilde{u}_R}, M_{\tilde{d}_R}$ for each doublet of squarks and sleptons
- trilinear coupling: $A_{\tilde{f}}$ for each $\tilde{f}$ → $L-R$ sfermion mixing
- supersymmetric Higgsino mass parameter: $\mu$
- Higgs sector parameters: $M_A, \tan \beta = v_2/v_1$
Benchmark scenarios
“Snowmass points and slopes” (SPS), hep-ph/0202233

examples (mSUGRA):

- **SPS1a**: \(m_0 = 100 \text{ GeV}, m_{1/2} = 250 \text{ GeV}, A_0 = -100,\)
  \(\tan \beta = 10, \mu > 0.\)

- **SPS1b**: \(m_0 = 200 \text{ GeV}, m_{1/2} = 400 \text{ GeV}, A_0 = 0,\)
  \(\tan \beta = 30, \mu > 0.\)
Spectrum of Higgs bosons in the MSSM (example)

large $M_A$: $h^0$ like SM Higgs boson $\sim$ decoupling regime

$m_h^0$ strongly influenced by quantum effects, e.g.
1-loop: complete

2-loop:
- QCD corrections \( \sim \alpha_s \alpha_t, \alpha_s \alpha_b \)
- Yukawa corrections \( \sim \alpha_t^2 \)

present theoretical uncertainty:
\( \delta m_h \approx 3-4 \text{ GeV} \)

[Degrassi, Heinemeyer, WH, Slavich, Weiglein]

\[ m_{h,0} \text{ prediction at different levels of accuracy:} \]

\[ \tan \beta = 3, \quad M_Q = M_A = 1 \text{ TeV}, \quad m_{\tilde{g}} = 800 \text{ GeV} \]

\( X_t : \) top-squark mixing parameter

\[ X_t = \Lambda_t - \mu \cot \beta \]
dependent on all SUSY particles and masses/mixings through Higgs self-energies
Test of theory at quantum level:

Sensitivity to loop corrections

\[ X = \text{Higgs bosons, SUSY particles} \]

\[ \mu \text{ lifetime: } M_W, \Delta r, G_F \]

\[ Z \text{ observables: } g_V, g_A, \sin^2 \theta_{\text{eff}}, \Gamma_Z, \ldots \]


new: 2-loop improvements \( \mathcal{O}(\alpha \alpha_s, \alpha_t^2, \alpha_b^2, \alpha_t \alpha_b) \)

and complex parameters

[Heinemeyer, WH, Stöckinger, A. Weber, Weiglein 06]
[Heinemeyer, WH, A. Weber, Weiglein 07]
$M_W - M_Z$ correlation

\[ \frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r) \]

$\Delta r$: quantum correction, $\Delta r = \Delta r(m_t, X_{\text{SUSY}})$

$\rightarrow M_W = M_W(\alpha, G_F, M_Z, m_t, X_{\text{SUSY}})$

$X_{\text{SUSY}} = \text{set of non-standard model parameters}$
Z resonance

effective $Z$ boson couplings

\[ g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f \]

with higher order contributions \( \Delta g_{V,A}^f (m_t, X_{\text{SUSY}}) \)

\[ \sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \text{Re} \frac{g_V^e}{g_A^e} \right) = \kappa \cdot \left( 1 - \frac{M_W^2}{M_Z^2} \right) \]
$M_W$ and $\sin^2 \theta_{\text{eff}}$ for varied SUSY-scale

$M_W$ and $\sin^2 \theta_{\text{eff}}$ for varied SUSY-scale
$M_W$ and $\sin^2 \theta_{\text{eff}}$ for varied SUSY-scale

$M_W$ measurement:

$M_W^{\text{exp}} = 80.398 \pm 0.025$ GeV

$\sin^2 \theta_{\text{eff}}$ measurement:

$\sin^2 \theta_{\text{eff}} = 0.23153 \pm 0.00016$

**Conclusion:**

The MSSM is slightly favoured over the Standard Model from $M_W$.

No preference for MSSM from $\sin^2 \theta_{\text{eff}}$. 
Anomalous g-factor of the muon

\[ a_\mu^{SM} \times 10^{10} = 11659000 \]

including new \( \pi^+\pi^- \) data (CMD-2, KLOE, SND)

HMNT (06)

--- experiment

BNL

\[ e^+e^- \text{ data based SM prediction: } 3.4 \sigma \text{ below exp. value} \]

theory uncertainty from hadronic vacuum polarization

\[ 
\begin{align*}
\mu & \rightarrow \mu \gamma \\
\mu & \rightarrow \mu \gamma \gamma \\
\mu & \rightarrow \mu \gamma \gamma \gamma
\end{align*} \]

Hagiwara, Martin, Nomura, Teubner
$g - 2$ with supersymmetry

new contributions from virtual SUSY partners of $\mu, \nu_\mu$
and of $W^\pm, Z$

diagrams with chargino/sneutrino exchange
diagrams with neutralino/smuon exchange

extra terms

\[ + \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \cdot \frac{v_2}{v_1} \]

can provide missing contribution for

\[ M_{\text{SUSY}} = 200 - 600 \text{ GeV} \]

2-loop calculation [Heinemeyer, Stöckinger,...]
scan over SUSY parameters compatible with EW and $b \rightarrow s\gamma$ constraints \( (\tan \beta = 50) \)

LOSP = lightest observable SUSY particle \( (\chi_{1}^{\pm}, \chi_{2}^{0}, \cdots) \)
## CMSSM

| Variable                  | Measurement       | Fit     | $|O^\text{meas.}-O^\text{theo.}|/\sigma^\text{meas.}$ |
|--------------------------|-------------------|---------|--------------------------|
| $\Delta\alpha_{\text{em}}(m_Z)$ | 0.02758±0.00035   | 0.02774 |                         |
| $m_Z$ [GeV]              | 91.1875±0.0021    | 91.1873 |                         |
| $\Gamma_Z$ [GeV]         | 2.4952±0.0023     | 2.4952  |                         |
| $\sigma_{\text{had}}$ [nb] | 41.450±0.037     | 41.486  |                         |
| $R_t$                    | 20.767±0.025      | 20.744  |                         |
| $\lambda_{\text{at}}(P_e)$ | 0.01714±0.00095  | 0.01641 |                         |
| $\lambda_{\text{g}}(P_e)$ | 0.1465±0.0032     | 0.1479  |                         |
| $R_s$                    | 0.21629±0.00066   | 0.21613 |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.01721±0.0030    | 0.1722  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.0992±0.0016     | 0.1037  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.0707±0.0035     | 0.0741  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.923±0.020       | 0.935   |                         |
| $\lambda_{\text{g}}$     | 0.670±0.027       | 0.668   |                         |
| $\lambda_{\text{g}}(\text{SLD})$ | 0.1513±0.0021   | 0.1479  |                         |
| $\sin^2\theta_W(Q^0)$    | 0.2324±0.0012     | 0.2314  |                         |
| $m_{w}$ [GeV]            | 80.398±0.025      | 80.382  |                         |
| $m_t$ [GeV]              | 170.9±1.8         | 170.8   |                         |
| $R(b\rightarrow s\gamma)$ | 1.13±0.12         | 1.12    |                         |
| $B_s\rightarrow\mu\mu$ [×10^{-5}] | < 8.00           | 0.33    |                         |
| $\Delta\alpha_b^{(0)}$ [×10^{-5}] | 2.95±0.87        | 2.95    |                         |
| $\Omega h^2$             | 0.113±0.009       | 0.113   |                         |

## Standard Model

| Variable                  | Measurement       | Fit     | $|O^\text{meas.}-O^\text{theo.}|/\sigma^\text{meas.}$ |
|--------------------------|-------------------|---------|--------------------------|
| $\Delta\alpha_{\text{em}}(m_Z)$ | 0.02758±0.00035   | 0.02768 |                         |
| $m_Z$ [GeV]              | 91.1875±0.0021    | 91.1875 |                         |
| $\Gamma_Z$ [GeV]         | 2.4952±0.0023     | 2.4957  |                         |
| $\sigma_{\text{had}}$ [nb] | 41.450±0.037     | 41.477  |                         |
| $R_t$                    | 20.767±0.025      | 20.744  |                         |
| $\lambda_{\text{at}}(P_e)$ | 0.01714±0.00095  | 0.01645 |                         |
| $\lambda_{\text{g}}(P_e)$ | 0.1465±0.0032     | 0.1481  |                         |
| $R_s$                    | 0.21629±0.00066   | 0.21586 |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.01721±0.0030    | 0.1722  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.0992±0.0016     | 0.1038  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.0707±0.0035     | 0.0742  |                         |
| $\lambda_{\text{g}}^{(0)}$ | 0.923±0.020       | 0.935   |                         |
| $\lambda_{\text{g}}$     | 0.670±0.027       | 0.668   |                         |
| $\lambda_{\text{g}}(\text{SLD})$ | 0.1513±0.0021   | 0.1481  |                         |
| $\sin^2\theta_W(Q^0)$    | 0.2324±0.0012     | 0.2314  |                         |
| $m_{w}$ [GeV]            | 80.398±0.025      | 80.374  |                         |
| $m_t$ [GeV]              | 170.9±1.8         | 171.3   |                         |
| $\Gamma_W$ [GeV]         | 2.140±0.060       | 2.091   |                         |

---

**global fit in the constrained MSSM including data from $g - 2$, $B$ physics, and cosmic relic density**

[O. Buchmueller et al., arXiv:0707.3447]
Theoretically inaccessible.

The lightest Higgs boson mass versus $m_{h} = \frac{2}{2^{\min}}$. The curve is the result of a CMSSM fit using all of the available constraints listed in Tab. 1, except the limit on $m_{h}$. The red (dark gray) band represents the total theoretical uncertainty from unknown higher-order corrections, and the dark shaded area on the right above 127 GeV is theoretically inaccessible (see text).

Right: Scan of the Higgs boson mass versus $m_{Higgs} [\text{GeV}]$ for the SM (blue/light gray), as determined by [45] using all available electroweak constraints, and for comparison, with the CMSSM scan superimposed (red/dark gray). The blue band represents the total theoretical uncertainty on the SM fit from unknown higher-order corrections.

Interestingly enough, the CMSSM prediction is consistent with the possibility that the slight excess of Higgs-like events observed by LEP [39,73] could indeed stem from a SM-like Higgs boson.

The pulls for the CMSSM, defined to be the difference between the measured value and the fit value normalized by the measurement uncertainty, are shown in the left plot of Fig. 3 (still excluding $m_{h}$ from the fit). They demonstrate that the CMSSM describes the data well, providing a $\chi^2$ of 17.0 per 13 degrees of freedom, or a 20% goodness-of-fit probability. This result may be compared with the pulls of the experimental observables used in a SM fit to electroweak data provided by [45], displayed in the right plot of Fig. 3. The SM fit results in a $\chi^2$ of 18.2 per 13 degrees of freedom, or a 15% goodness-of-fit probability [45].

It should be noted that a key role in the determination of CMSSM parameters is played by the CDM constraints, $a$ and $b$. As shown in Fig. 3, it is essentially impossible to distinguish between SM and CMSSM predictions in the electroweak precision observables. Indeed, because of the decoupling of virtual effects induced by sparticle loops, these observables provide mainly exclusion bounds on the sparticle spectrum. On the other hand, the three mentioned observables/constraints provide a first clue concerning the size of deviations from the SM (CDM cannot be explained in the SM, $a$ is in disagreement with the SM by more than 3 $\sigma$ using $e^{+}e^{-}$ input data for the hadronic vacuum polarization) and $b$ agrees reasonably well with the SM.
$Scatter \ plots \ for \ M_W \ & \ \sin^2 \ \theta_{\text{eff}}$

- **SUSY parameters:**
  
  - **sleptons:** $M_{\tilde{e}, \tilde{\ell}} = 100 \ldots 2000$ GeV
  - **light squarks:** $M_{\tilde{u}_{\text{up/down}}, \tilde{d}_{\text{up/down}}} = 100 \ldots 2000$ GeV
  - **$\tilde{t}/\tilde{b}$ doublet:** $M_{\tilde{t}, \tilde{b}} = 100 \ldots 2000$ GeV
    
    $A_{t,b} = -2000 \ldots 2000$ GeV
  - **gauginos:** $M_{1,2} = 100 \ldots 2000$ GeV
    
    $m_{\tilde{g}} = 195 \ldots 1500$ GeV
    
    $\mu = -2000 \ldots 2000$ GeV
  - **Higgs:** $M_A = 90 - 1000$ GeV
    
    $\tan \beta = 1.1 \ldots 60$

- **Unconstrained scan, only Higgs mass required to be in agreement with LEP data.**
$M_W(m_t)$ and $\sin^2 \theta_{\text{eff}}(m_t)$ in the MSSM

Experimental errors 68% CL:
- LEP2/Tevatron (today)
- Tevatron/LHC
- ILC/GigaZ

Preference of MSSM over SM from
- $M_W(m_t)$
- MSSM and SM equally good for $\sin^2 \theta_{\text{eff}}(m_t)$. 

MSSM

SM

light scalars

heavy scalars

$m_{\tilde{t}_2}/m_{\tilde{t}_1} > 2.5$

$M_H = 400$ GeV

$M_H = 114$ GeV

$m_{\tilde{t}_2}/m_{\tilde{t}_1} > 2.5$

Both models

- p.50
The diagram illustrates the experimental errors at the 68% confidence level for both the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM). The SM is represented by the red shaded area, with the Higgs mass range of 114 to 400 GeV. The MSSM is shown in the green shaded area, with the squark masses constrained by the condition $m_{\tilde{b}_{2}}/m_{\tilde{t}_{1}} \geq 2.5$. The experimental points are indicated by the blue, black, and red circles, representing data from LEP2/Tevatron (today), Tevatron/LHC, and ILC/GigaZ, respectively. The combination of $M_W$ and $\sin^2 \theta_{\text{eff}}$ slightly favors the MSSM over the SM.
Outlook – Possible scenarios

- A single light Higgs boson
  - SM Higgs boson?
  - SUSY light Higgs boson?
    \( H, A, H^\pm \) heavy (decoupling scenario) \( h \sim H_{\text{SM}} \)

- A light Higgs boson + more \((H, A, H^\pm)\)
  - SUSY Higgs?
  - non-SUSY 2-Higgs-Doublet model?

- A single heavy Higgs boson \((\gg 200 \text{ GeV})\)
  - SUSY ruled out
  - SM + (?) strong interaction?

- No Higgs boson
  - strongly interacting weak interaction
    new strong force \( \sim \) TeV scale
Conclusions

Electroweak precision physics
→ sensitive to quantum structure
→ constraints on unknown parameters

Precision tests of the Standard Model have established the SM as a quantum field theory

MSSM is competitive to the SM
→ global fits of similar quality (even better)
→ natural: light Higgs boson $h^0$

Future experiments at colliders
→ discovery of Higgs and SUSY at LHC (?)
→ precision studies at $e^+ e^-$ Linear Collider