Electroweak loop corrections at TeV colliders

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Outline of the talk

(1) **Electroweak corrections at the TeV scale**

(2) **One-loop calculations for LHC** [Accomando, Denner, Hollik, Kasprzik, Kniehl, Kaiser, Meier, Kühn, Kulesza, S.P., Schulze, Maina, Moretti, Nolten, Ross, Dittmaier, Krämer, Baur, Wackerot, Zykunov, Beccaria, Mirabella, Marcorini, Carloni Calame, Montagna, Piccinini, Gounaris, Layssac, Renard, Verzegnassi, Ciafaloni, Comelli, Scharf, Uwer]

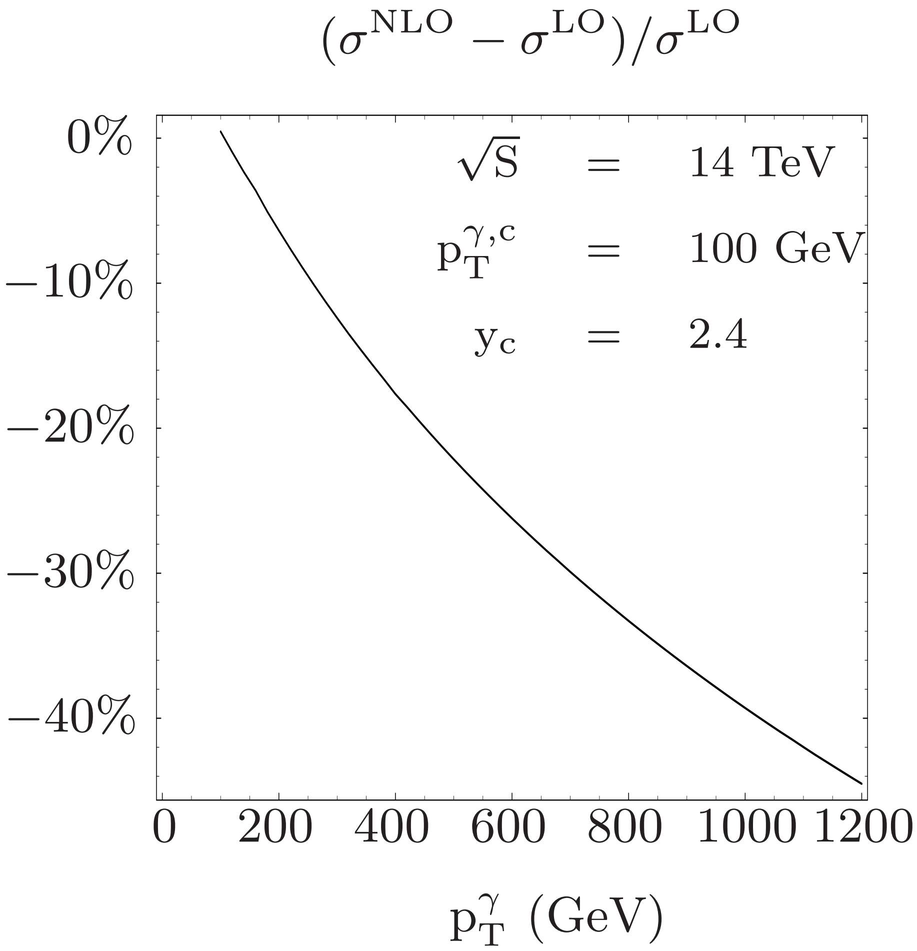
(3) **Beyond one loop with resummations** [Fadin, Lipatov, Martin, Melles, Jantzen, Kühn, Moch, Penin, Smirnov, M.Ciafaloni, P.Ciafaloni, Comelli]

(4) **Diagrammatic two-loop calculations** [Melles, Hori, Kawamura, Kodaira, Beenakker, Werthenbach, Jantzen, Kühn, Moch, Penin, Smirnov, Denner, S.P.]

PART 1

Introduction: electroweak corrections at the TeV scale

Example: electroweak corrections to $pp \to Z\gamma$ at the LHC



Hollik, Meier (2004)

At small p_T

- Corrections of $\mathcal{O}(\alpha) \sim 1\%$

At $p_T > 100$ GeV

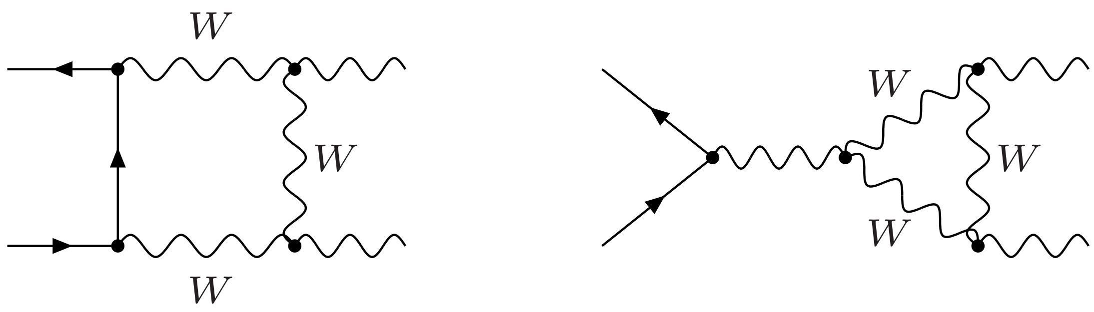
- **large negative corrections $\gg$ 1%**
- **increase with p_T**
- -40% **at $p_T \sim$ 1TeV !**

Origin: scattering energy $\gg$ characteristic scale of EW corrections

Large double logarithms

$$\frac{\delta\sigma}{\sigma} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\% \qquad \text{at} \quad \sqrt{s} \sim 1\,\mathrm{TeV}$$

from vertex and box diagrams involving virtual weak bosons



Kuroda, Moultaka, Schildknecht (1991); Degrassi, Sirlin (1992); Beenakker, Denner, Dittmaier, Mertig, Sack (1993); Denner , Dittmaier, Schuster (1995); Denner, Dittmaier, Hahn (1997), Beccaria, Montagna, Piccinini, Renard, Verzegnassi (1998); Ciafaloni, Comelli (1999)

Affect all hard scattering processes at LHC and ILC!

Asymptotic expansion of 1-loop EW corrections

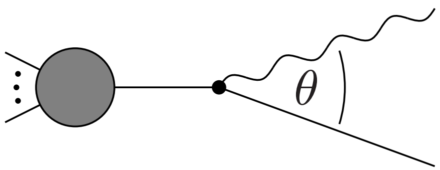
General form of M_W^2/s expansion

$$\alpha\left[C_2\ \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + C_1\ \underbrace{\ln\left(\frac{s}{M_W^2}\right)}_{\text{soft,coll}} + \tilde{C}_1\ \underbrace{\ln\left(\frac{s}{\mu_\mathrm{R}^2}\right)}_{\text{UV}} + C_0\right]$$

Terms of $\mathcal{O}(M_W^2/s)$ negligible for $s \sim 1\,\mathrm{TeV}^2$

$$C_k = \sum_{j=0}^{\infty} C_k^{(j)}\left(\frac{M_W^2}{s}\right)^j \quad\rightarrow\quad C_k^{(0)}$$

Mass singularities from **soft/collinear gauge bosons** coupling to external lines



$$\Longrightarrow \int \frac{\mathrm{d}E}{E} \int \frac{\mathrm{d}\cos\theta}{(1-\cos\theta)}$$

Analogies with **massless gauge theories** (QED,QCD)

Key to understand electroweak logarithms

$$M^2/s \ll 1 \quad \Rightarrow \quad \ln\left(\frac{s}{M^2}\right) \qquad \textbf{mass singularities!}$$

Analogous singularities in massless QCD

$$M^2 = 0 \quad \Rightarrow \quad \frac{1}{\varepsilon} \quad \text{in } D = 4 - 2\varepsilon$$

Factorization and Universality of QCD mass singularities [Kunszt,Soper,Catani]

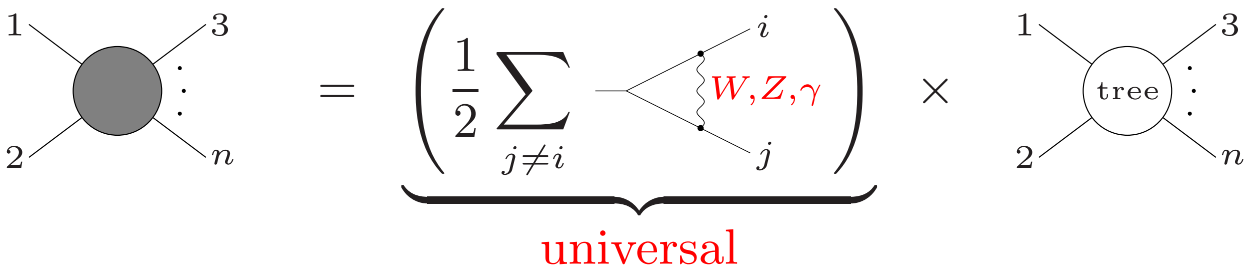
$$1 \text{ loop} \qquad \mathcal{M}^{(1)} = \underbrace{I^{(1)}(\varepsilon)}_{1/\varepsilon^2,\,1/\varepsilon} \mathcal{M}^{(0)} + \mathcal{O}(1)$$

$$2 \text{ loops} \qquad \mathcal{M}^{(2)} = \underbrace{I^{(2)}(\varepsilon)}_{1/\varepsilon^4,\,1/\varepsilon^3} \mathcal{M}^{(0)} + \mathcal{O}(\varepsilon^{-2})$$

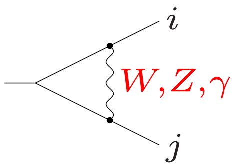
Singular factors $I^{(N)}(\varepsilon)$ process-independent!

Factorization and universality of one-loop EW logarithms [Denner, S.P. (2001)]

For **arbitrary processes** $(e, \nu, u, d, t, b, \gamma, Z, W^{\pm}, H, g)$



proven with **collinear Ward identities** for spontaneously broken YM theories



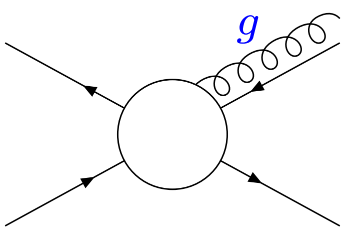
$$= \frac{\alpha}{4\pi}\left\{ \sum_{V=\gamma,Z,W} I_i^V I_j^V \ln^2\frac{r_{ij}}{M_W^2} + 2 I_i^Z I_j^Z \ln\frac{r_{ij}}{M_W^2}\ln\frac{M_W^2}{M_Z^2} + \gamma_{ij}^{\mathrm{ew}} \ln\frac{s}{M_W^2} \right.$$
$$\left. + Q_i Q_j \sum_{k=i,j}\left[\ln\frac{r_{ij}}{m_k^2}\ln\frac{M_W^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{M_W^2}{m_k^2} - \ln\frac{M_W^2}{\lambda^2} - \frac{1}{2}\ln\frac{M_W^2}{m_k^2}\right]\right\}$$

Simple and general recipe for LL and NLL

Mass singularities and physical observables

Massless gauge theories

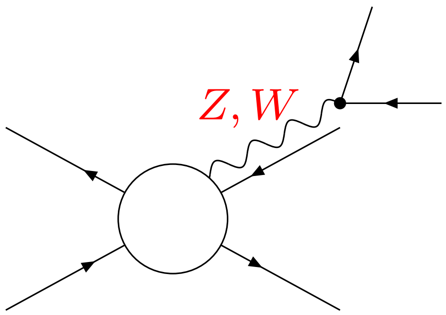
- **real emission** of soft and collinear massless particles **cannot be detected**



- $1/\varepsilon$ mass singularities cancel (KLN theorem)

Electroweak theory

- **real emission** of massive W and Z bosons **can be detected**



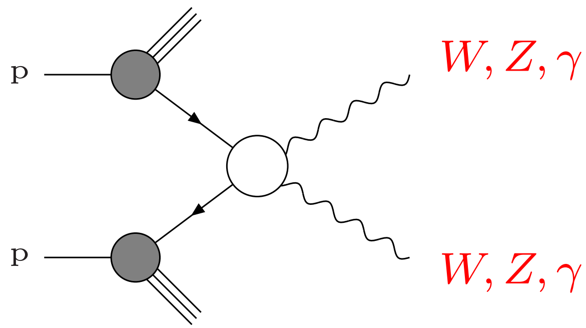
- $\ln\left(\frac{s}{M^2}\right)$ corrections remain present in exclusive observables
- and even in fully inclusive observables [M. Ciafaloni, P. Ciafaloni, Comelli (2000)]

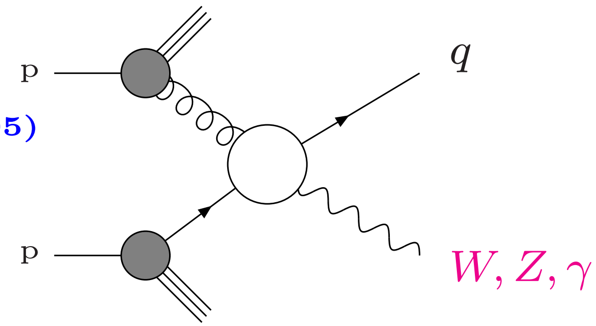
PART 2

One-loop calculations for LHC

Recent pp → VV and pp → Vj calculations

process	calculation	
Wγ, WZ	1,3	**Accomando, Denner, S.P. (2002)**
Zγ	2,4	**Hollik, Meier (2004)**
WW, WZ, ZZ	1,2,3	**Accomando, Denner, Kaiser (2005)**
Wγ, Zγ	1,2,3,4	**Accomando, Denner, Meier (2005)**
γγ		
Z,γ	4	**Maina, Moretti, Ross (2004)**
Z,γ	3,4,5	**Kühn, Kulesza, S.P., Schulze (2004,2005)**
W	2,3,4,5	**Kühn, Kulesza, S.P., Schulze (2007)**
W	2,4	**Hollik, Kasprzik, Kniehl (2007)**





[1]decay of weak bosons

[2]real photon bremsstrahlung

[3]one-loop NLL approximation

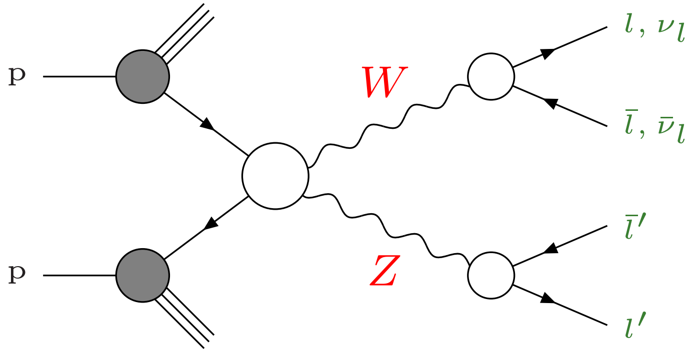
[4]exact one-loop predictions

[5]two-loop NLL approximation

Double-pole and NLL approximations

Factorization and separation of scales

$$\mathcal{M}_{\mathrm{DPA}} = \frac{-\sum_{\lambda,\lambda'} \mathcal{M}^{qq'\to W_\lambda Z_{\lambda'}} \mathcal{M}^{W_\lambda\to l\nu_l} \mathcal{M}^{Z_{\lambda'}\to l'\bar{l}'}}{(p_W^2 - M_W^2 + \mathrm{i}M_W\Gamma_W)\,(p_Z^2 - M_Z^2 + \mathrm{i}M_Z\Gamma_Z)}$$



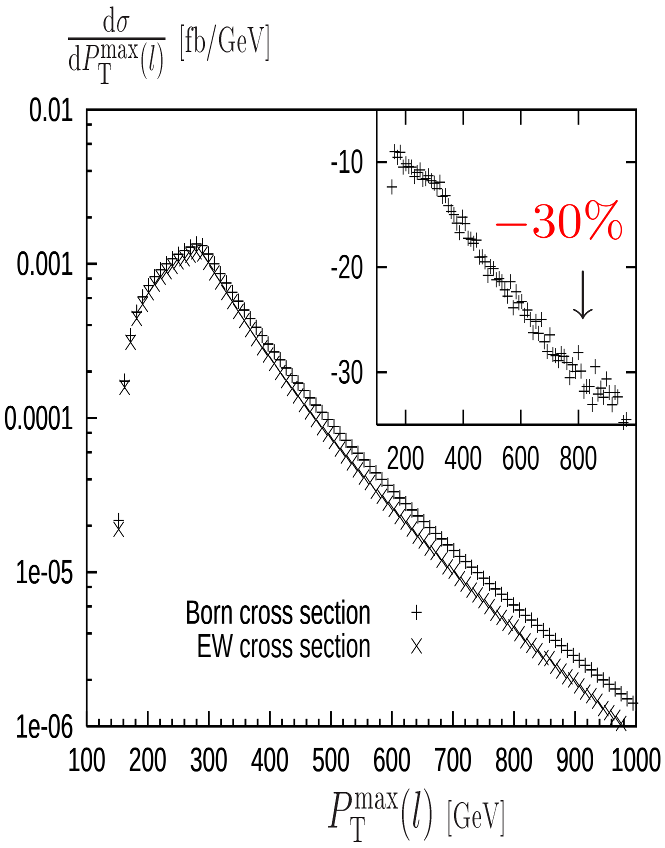
NLL corrections for T/L gauge-boson production

$$\mathcal{M}^{\bar{q}q'\to W_\mathrm{T} Z_\mathrm{T}} = \mathcal{M}_{\mathrm{Born}}^{\bar{q}q'\to W_\mathrm{T} Z_\mathrm{T}} \left\{ 1 - \frac{\alpha}{8\pi}\left\{ 2C_{q_\mathrm{L}}^{\mathrm{ew}} + C_W^{\mathrm{ew}}\left[1 + \frac{c_\mathrm{W}^2 \cos\hat{\theta}}{c_\mathrm{W}^2\cos\hat{\theta} - s_\mathrm{W}^2 Y_{q_\mathrm{L}}}\right]\right\}\ln^2\frac{\hat{s}}{M_\mathrm{W}^2}\right.$$
$$\left. + \frac{\alpha}{4\pi}\left\{3C_{q_\mathrm{L}}^{\mathrm{ew}} - \frac{1}{s_\mathrm{W}^2}\left[\ln\frac{\hat{t}\hat{u}}{\hat{s}^2} + \frac{c_\mathrm{W}^2 \ln(\hat{t}/\hat{u})}{c_\mathrm{W}^2\cos\hat{\theta} - s_\mathrm{W}^2 Y_{q_\mathrm{L}}}\right]\right\}\ln\frac{\hat{s}}{M_\mathrm{W}^2}\right\}$$

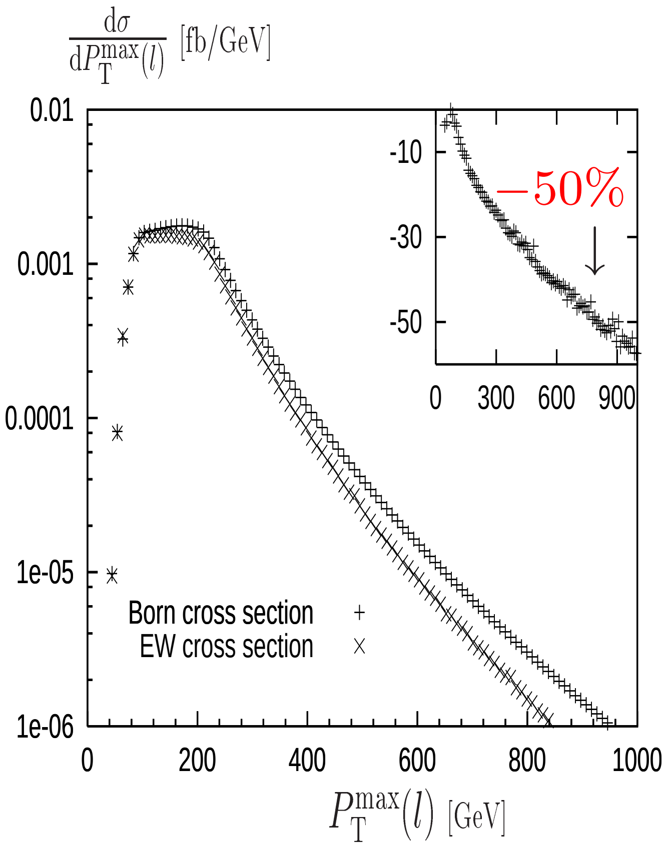
$$\mathcal{M}^{\bar{q}q'\to W_\mathrm{L} Z_\mathrm{L}} = \mathcal{M}_{\mathrm{Born}}^{\bar{q}q'\to W_\mathrm{L} Z_\mathrm{L}} \left\{ 1 - \frac{\alpha}{4\pi}\left[C_{q_\mathrm{L}}^{\mathrm{ew}} + C_\Phi^{\mathrm{ew}}\right]\ln^2\frac{\hat{s}}{M_\mathrm{W}^2} + \frac{\alpha}{4\pi}\left[-\frac{2}{s_\mathrm{W}^2}\left(\ln\frac{|\hat{u}|}{\hat{s}}\right.\right.\right.$$
$$\left.\left.\left. + \ln\frac{|\hat{t}|}{\hat{s}} - \frac{s_\mathrm{W}^2}{c_\mathrm{W}^2} Y_{q_\mathrm{L}} \ln\frac{\hat{t}}{\hat{u}}\right) 3C_{q_\mathrm{L}}^{\mathrm{ew}} + 4C_\Phi^{\mathrm{ew}} - \frac{3}{2s_\mathrm{W}^2}\frac{m_t^2}{M_\mathrm{W}^2} - b_2^{(1)}\right]\ln\frac{\hat{s}}{M_\mathrm{W}^2}\right\}$$

Accomando, Denner, S.P. (2002)

Size of electroweak NLL corrections at the LHC



$\mathrm{pp} \to \mathrm{WZ} \to e\nu_e\mu^+\mu^-$



$\mathrm{pp} \to \mathrm{ZZ} \to e^+e^-\mu^+\mu^-$

High-p_{T} region

- corrections $\gg 10\%$!
- size process-dependent

Importance

- small cross section
- important for new-physics searches

Accomando, Denner, Kaiser (2005)

Precision of NLL/NNLL approx. for $pp \to W\gamma \to l\nu_l\gamma$ [Accomando, Denner, Meier (2005)]

Complete high-energy expansion vs exact one-loop calculation

$$\underbrace{C_2 \ln^2\left(\frac{\hat{s}}{M_W^2}\right) + C_1 \ln\left(\frac{\hat{s}}{M_W^2}\right)}_{\text{NLL}} + \underbrace{B_0 + B_1 \ln\left(\frac{\hat{t}}{\hat{s}}\right) + B_2 \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)}_{\text{NNLL (not enhanced)}} + \underbrace{\mathcal{O}\left(\frac{M_W^2}{\hat{s}}\right)}_{\text{suppressed}}$$

$p_{\mathrm{T}}(\gamma)/\mathrm{GeV}$	NLL	const	$\ln(\hat{t}/\hat{s})$	$\ln^2(\hat{t}/\hat{s})$
250	-6.05%	-0.26%	-1.78%	-3.95%
450	-14.4%	-0.58%	-1.97%	-3.50%
700	-22.6%	-0.74%	-1.91%	-3.05%
1000	-30.1%	-0.87%	-1.79%	-2.62%

suppr: numerically $< 0.5\%$ $\Rightarrow$ asymptotic high-energy expansion good approach

NLL: Dominant (20-30%). Agreement with Denner, S.P. (2001).
Correct description of energy dependence.

NNLL.: Less important (-5.5%) but larger than expected ($\alpha \sim 1\%$) and not neglibible.
(Analytic expression? Other processes?)

Precision of NLL/NNLL approx. for pp $\rightarrow$ Zj [Kühn, Kulesza, S.P., Schulze (2005)]

Asymptotic expansion ($|\hat{s}|, |\hat{t}|, |\hat{u}| \gg M_W^2$) for $q\bar{q} \rightarrow Zg$ amplitude

$$\overline{|\mathcal{M}_1|}^2 = 32\pi^2\alpha^2\alpha_{\mathrm{S}} \sum_{\lambda=\mathrm{R,L}} \Bigg\{ \left(I^Z_{q_\lambda}\right)^2 \sum_{V=\mathrm{Z,W^\pm}} \left(I^V I^{\bar{V}}\right)_{q_\lambda} \Bigg\{ \Bigg[-\ln^2\left(\frac{-\hat{s}}{M_V^2}\right) + 3\ln\left(\frac{-\hat{s}}{M_V^2}\right) + \frac{3}{2}[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right)$$

$$+\ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right)] + \frac{7\pi^2}{3} - \frac{5}{2}\Bigg] \left(\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\right) + \frac{1}{2}\left[3\ln\left(\frac{\hat{u}}{\hat{s}}\right) - 3\ln\left(\frac{\hat{t}}{\hat{s}}\right) - \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)\right]$$

$$\times\left(\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}}\right) + 2\left[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right) + 2\pi^2\right]\Bigg\} + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}} T^3_{q_\lambda} I^Z_{q_\lambda} \Bigg\{ \Bigg[\frac{4}{4-D} - 2\gamma_{\mathrm{E}}$$

$$+2\ln\left(\frac{4\pi\mu^2}{M_Z^2}\right) + \ln^2\left(\frac{-\hat{s}}{M_W^2}\right) - \ln^2\left(\frac{-\hat{t}}{M_W^2}\right) - \ln^2\left(\frac{-\hat{u}}{M_W^2}\right) + \ln^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2}[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right)] - \frac{20\pi^2}{9} - \frac{\pi}{\sqrt{3}}$$

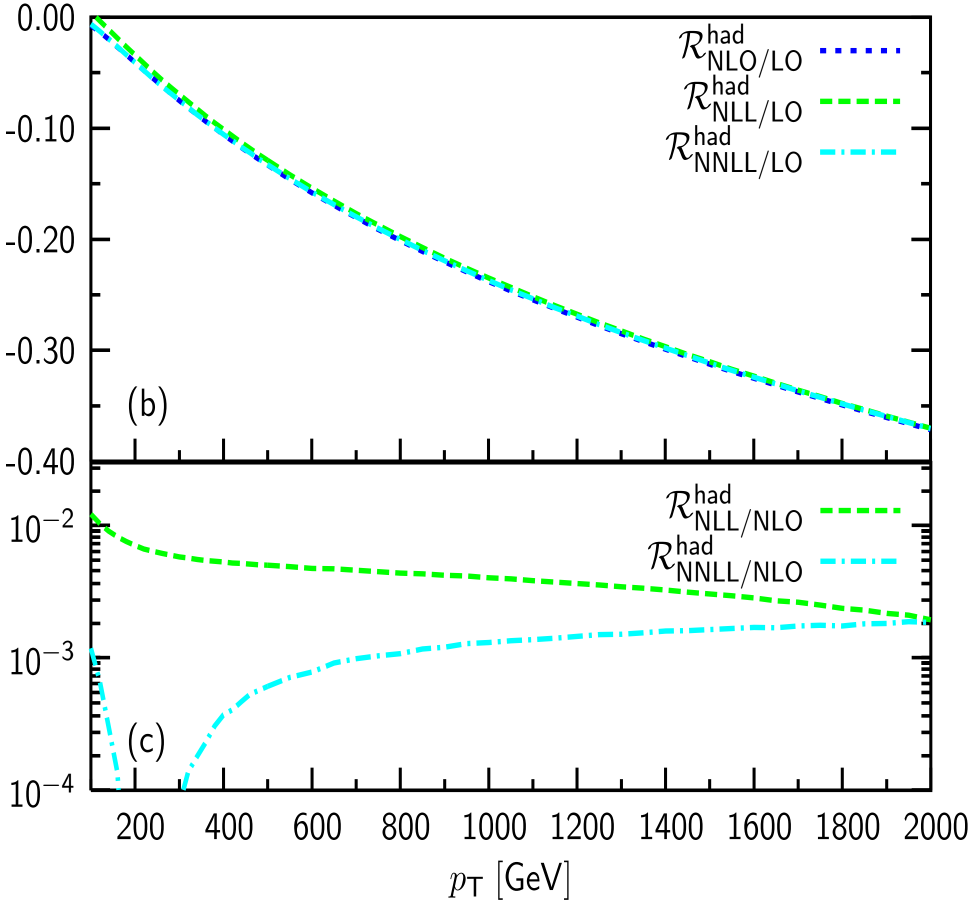
$$+2\Bigg]\left(\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\right) + \frac{1}{2}\left[\ln^2\left(\frac{\hat{u}}{\hat{s}}\right) - \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)\right]\left(\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}}\right) - 2\left[\ln^2\left(\frac{\hat{t}}{\hat{s}}\right) + \ln^2\left(\frac{\hat{u}}{\hat{s}}\right) + \ln\left(\frac{\hat{t}}{\hat{s}}\right) + \ln\left(\frac{\hat{u}}{\hat{s}}\right) + 2\pi^2\right]\Bigg\}\Bigg\}$$

Very compact expressions

- NLL predicted by process-independent formula [Denner, S.P. (2001)]
- **NNLL** consist of $\pi^2, \pi/\sqrt{3}$, $\ln(\hat{t}/\hat{u}), \ldots$ not growing with energy

NLL/NNLL approximations vs exact calculation

$\mathrm{d}\sigma/\mathrm{d}p_T$ **for pp→ Zj**



Large negative corrections

- -25% at $p_{\mathrm{T}} \sim 1$ TeV
- NLO, NLL, NNLL overlap!

Precision of NNLL approximation

- better than 0.2%

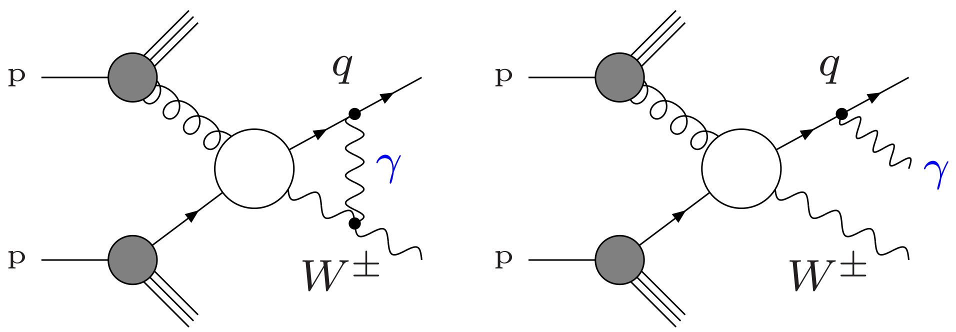
Precision of NLL approximation

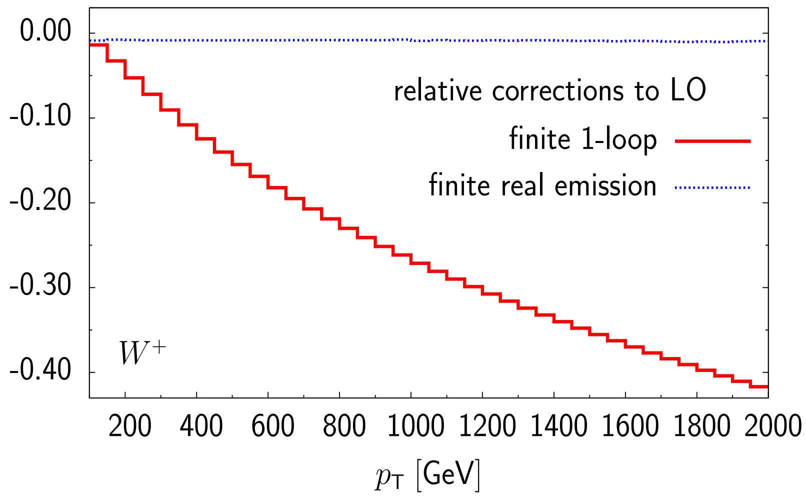
- better than 1% ! (process-dependent)

⇒ use asymptotic expansions for two-loop EW corrections at high energies

Separation of photonic singularities for pp $\rightarrow$ Wj [Kühn, Kulesza, S.P., Schulze (2007)]

Cancellation of virtual-photon divergencies requires real bremstrahlung. Needed techniques (dipole subtraction) not available beyond one loop.





Strategy: gauge-invariant splitting

- $\sigma^{\mathrm{fin}}_{\mathrm{virt}} = \sigma_{\mathrm{virt}}(M_\gamma = M_{\mathrm{W}})$
- $\sigma^{\mathrm{fin}}_{\gamma}$ = virtual-photon singularities +photon bremsstrahlung

One-loop calculation for pp $\rightarrow$ Wj

- $\sigma^{\mathrm{fin}}_{\mathrm{virt}}$ = large negative corrections
- $\sigma^{\mathrm{fin}}_{\gamma} \leq 1\%$ for fully inclusive γ

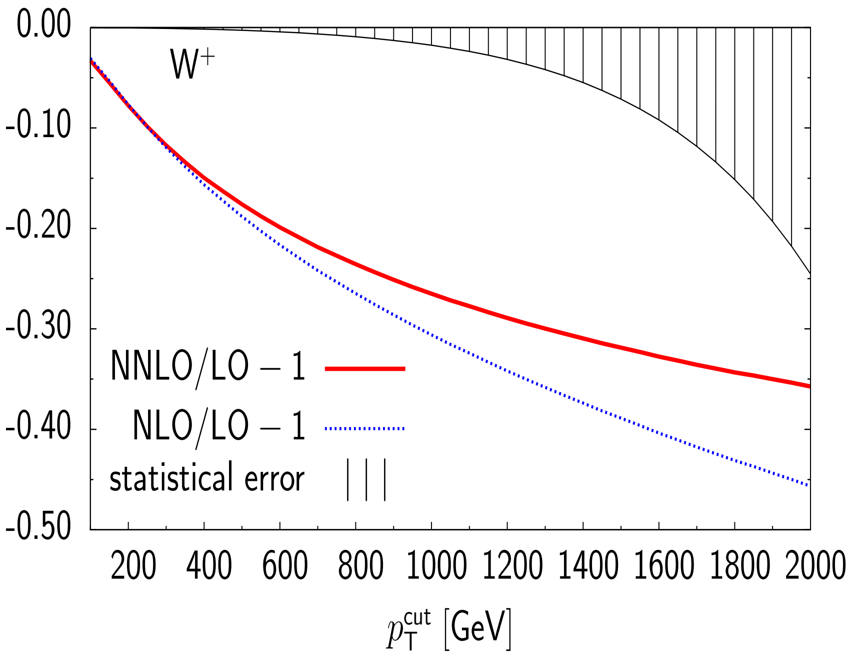
Two-loop LL and NLL corrections for pp→Wj [Kühn, Kulesza,S.P.,Schulze (2007)]

Compact formula for partonic scattering amplitudes ($\bar{q}q' \to Wg$)

$$\overline{\sum}|\mathcal{M}_2|^2 = 4\frac{\alpha^3\alpha_\mathrm{S}}{s_\mathrm{W}^2}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\left\{\left(C_{q_\mathrm{L}}^{\mathrm{ew}}+\frac{C_A}{2s_\mathrm{W}^2}\right)\left[\frac{C_A}{2s_\mathrm{W}^2}\left(\ln^4\left(\frac{\hat{t}}{M_W^2}\right)+\ln^4\left(\frac{\hat{u}}{M_W^2}\right)-\ln^4\left(\frac{\hat{s}}{M_W^2}\right)\right)\right.\right.$$

$$\left.\left.+C_{q_\mathrm{L}}^{\mathrm{ew}}\left(\ln^4\left(\frac{\hat{s}}{M_W^2}\right)-6\ln^3\left(\frac{\hat{s}}{M_W^2}\right)\right)\right]+\frac{1}{3}\left[\frac{b_1}{c_\mathrm{W}^2}\left(\frac{Y_{q_\mathrm{L}}}{2}\right)^2+\frac{b_2}{s_\mathrm{W}^2}\left(C_F+\frac{C_A}{2}\right)\right]\ln^3\left(\frac{\hat{s}}{M_W^2}\right)\right\}$$

Derived from Melles (2001); Denner, Melles, S.P. (2003) (see later)



Size of one-loop and two-loop corrections

- $-27\% + 3\% = -24\%$ at $p_\mathrm{T} \sim 1$ TeV
- $-42\% + 9\% = -33\%$ at $p_\mathrm{T} \sim 2$ TeV

Two-loop $\simeq$ statistical error!

Percent-level precision requires also two-loop EW effects!

- Complete two-loop (analytical or numerical) calculation out of reach

- **Asymptotic high-energy expansion for** $M_W^2/s \ll 1$

$$\alpha^2\left[C_4 \underbrace{\ln^4\left(\frac{s}{M_W^2}\right)}_{\text{LL}} + C_3 \underbrace{\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{NLL}} + \ldots\right]$$

organize calculation according to (formal) hierarchy

$$\ln^4\left(\frac{s}{M_W^2}\right) \gg \ln^3\left(\frac{s}{M_W^2}\right) \gg \ldots$$

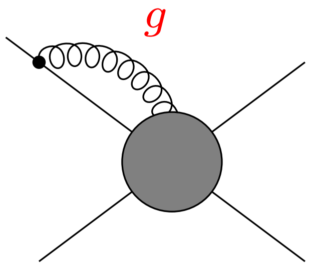
PART 3

Beyond one loop: QCD-inspired resummations

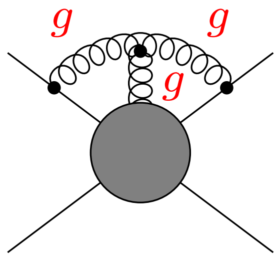
InfraRed Evolution Equation (IREE) for QCD matrix elements

Logarithmic dependence on soft-collinear cut-off $\frac{\partial \mathcal{M}}{\partial \ln(\mu_{\mathrm{T}})} = K(\mu_{\mathrm{T}})\mathcal{M}$

$\mathcal{M}(\mu_{\mathrm{T}}) =$



$+$



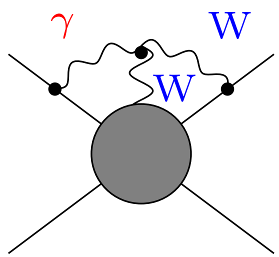
$+ \cdots \quad = \exp\left[-\int_{\mu_{\mathrm{T}}}^{Q}\frac{\mathrm{d}\mu}{\mu}K(\mu)\right]\mathcal{M}(Q)$

$\mathbf{k}_{\mathrm{T}}(\mathbf{gluon}) > \mu_{\mathrm{T}}$

Gribov(1967); Kirschner, Lipatov (1982)

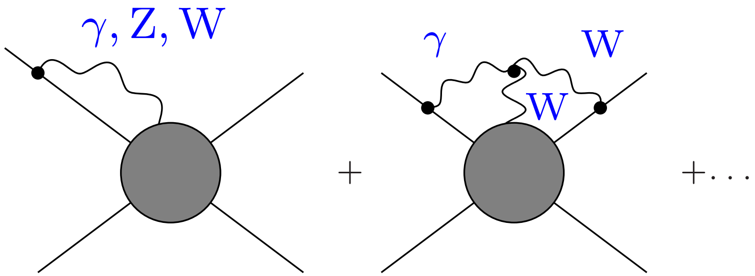
How to deal with mass gap in the electroweak gauge sector?

$M_\gamma = 0 \ll M_{\mathrm{Z}} \sim M_{\mathrm{W}}$:



$\Rightarrow \quad \alpha^2 \frac{1}{\varepsilon} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right)$

$\boxed{\text{SU(2)×U(1) regime: } \mu_\mathrm{T} > M_\mathrm{W,Z}}$

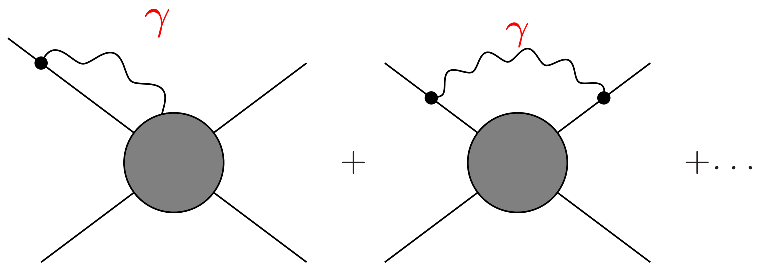


mass gap irrelevant ($M_\gamma = M_\mathrm{Z} = M_\mathrm{W}$)

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_\mathrm{T})} = K_\mathrm{EW}(\mu_\mathrm{T})\mathcal{M}$$

as in symmetric $\mathrm{SU}(2) \times \mathrm{U}(1)$ theory

$\boxed{\text{U(1)}_\mathrm{em} \text{ regime: } \mu_\mathrm{T} < M_\mathrm{W,Z}}$



weak boson frozen ($M_\mathrm{Z}, M_\mathrm{W} = \infty$)

$$\frac{\partial \mathcal{M}}{\partial \ln(\mu_\mathrm{T})} = K_\mathrm{QED}(\mu_\mathrm{T})\mathcal{M}$$

as in QED

$\boxed{\text{IREE prediction: double factorization and exponentiation}}$

$$\mathcal{M}(\mu_\mathrm{T}) = \exp\left\{-\int_{\mu_\mathrm{T}}^{M_\mathrm{W}} \frac{\mathrm{d}\mu}{\mu} K_\mathrm{QED}(\mu)\right\} \exp\left\{-\int_{M_\mathrm{W}}^{\sqrt{s}} \frac{\mathrm{d}\mu}{\mu} K_\mathrm{EW}(\mu)\right\} \mathcal{M}_\mathrm{Born}$$

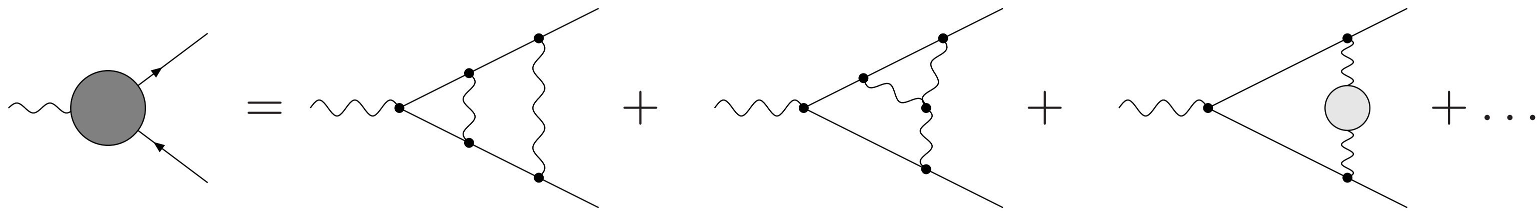
Fadin, Lipatov, Martin, Melles (2000)

Two-loop EW predictions based on IREE approach

Massless $f\bar{f} \to f'\bar{f}'$ processes

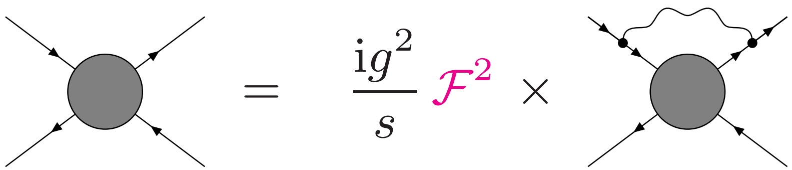
$$\alpha^2 \Bigg[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right) + C_3 \ln^3\left(\frac{s}{M_W^2}\right) + C_2 \ln^2\left(\frac{s}{M_W^2}\right)}_{\text{Kühn, Moch, Penin, Smirnov (2000,2001,2003)}} + \underbrace{C_1 \ln^1\left(\frac{s}{M_W^2}\right)}_{\text{Jantzen, Kühn, Penin, Smirnov (2004,2005)}} \Bigg]$$

Main ingredient: 2-loop log corrections for fermionic vertex [Jantzen, Smirnov (2006)]

$\mathcal{F} =$ 

(A) SU(2)×U(1) Higgs model, $M_\gamma = M_Z = M_W = M_H$, $\sin(\theta_W) = 0 \Rightarrow \ln(s/M_W^2)$

(B) $M_\gamma = 0 \Rightarrow$ check IREE (separation of photonic singularities)

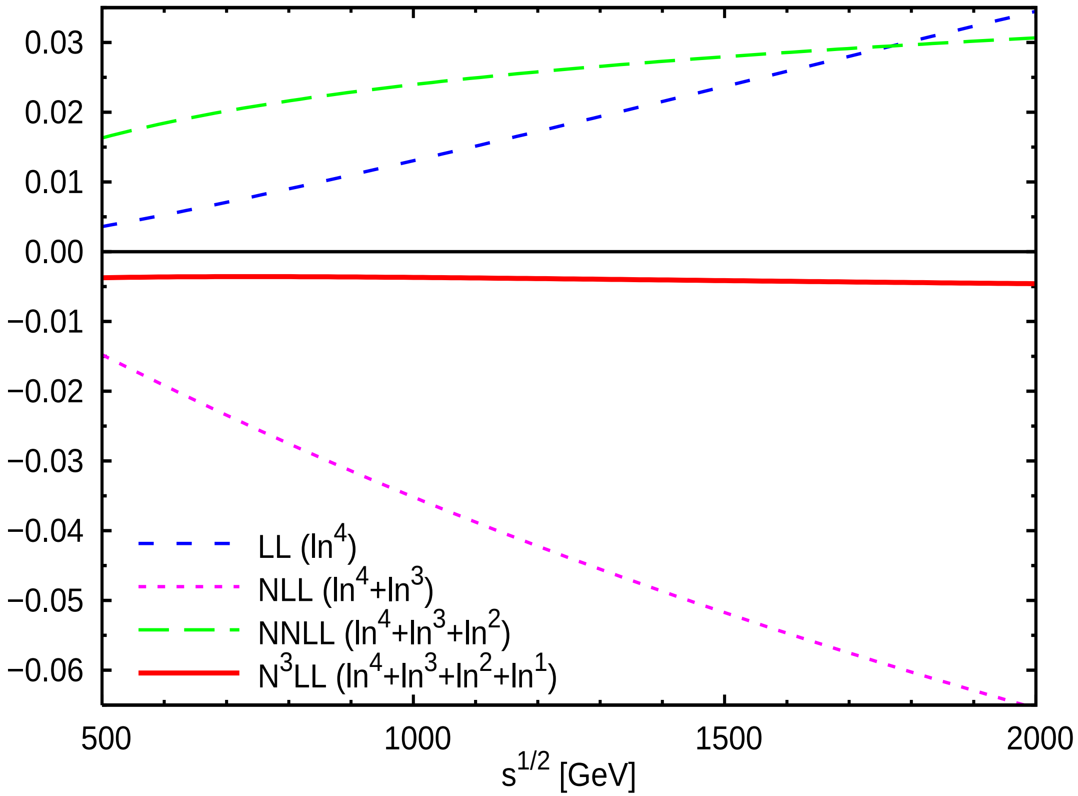
(C) 

(reduced amp. from 2-loop QCD results)

Mueller(1979), Collins(1980), Sen (1981), Botts, Sterman (1987)

Relative 2-loop logarithmic corrections to $\sigma(\mathrm{e^+e^-} \to d\bar{d})$

$$\left(\frac{\alpha}{4\pi \sin^2\theta_w}\right)^2 \left[2.79 \ln^4\left(\frac{s}{M_\mathrm{W}^2}\right) - 51.98 \ln^3\left(\frac{s}{M_\mathrm{W}^2}\right) + 321.34 \ln^2\left(\frac{s}{M_\mathrm{W}^2}\right) - 757.35 \ln^1\left(\frac{s}{M_\mathrm{W}^2}\right)\right]$$



Subleading logarithms

- increasingly large coefficients
- alternating signs

Importance of logarithmic effects

– total 2-loop correction small

+ residual theoretical error $\mathcal{O}(10^{-3})$

Behaviour of log expansion

– oscillating, bad convergence

+ better convergence expected for gauge-boson production

Other two-loop EW predictions based on IREE approach

Arbitrary processes

$$\alpha^2 \left[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right)}_{\text{Fadin, Lipatov, Martin, Melles (2000)}} + \underbrace{C_3 \ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Melles (2001,2002,2003,2004)}} \right]$$

Assumptions

- splitting of EW theory in symmetric SU(2)×U(1) and QED regime
- symmetry breaking completely negligible

Not proven...

PART 4

Diagrammatic two-loop calculations based on EW Lagrangian

- check IREE-based predictions
- control additional effects: mixing, M_{W}-M_{Z} difference

Two-loop calculations based on EW Lagrangian

The (few) existing results agree with the IREE

$$\alpha^2 \Bigg[\underbrace{C_4 \ln^4\left(\frac{s}{M_W^2}\right)}_{\text{Melles; Hori,Kawamura, Kodaira (2000)}} + \underbrace{C_3^{\mathrm{ang}} \ln\left(\frac{t}{s}\right)\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Denner, Melles,P. (2003)}} + \underbrace{C_3 \ln^3\left(\frac{s}{M_W^2}\right)}_{\text{S.P. (2004)}} \Bigg]$$

Beenakker, Werthenbach (2000,2002) — arbitrary processes involving $Z, W, H, b, t, \ldots$

Denner, Jantzen, S.P. (2006) — massless $f_1 f_2 \to f_3 \ldots f_n$

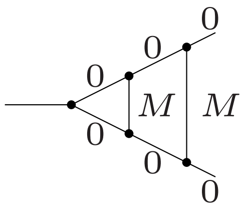
Technology for two-loop NLL for arbitrary processes now available

- automatic algorithm for 2-loop diagrams in NLL approximation
- electroweak collinear Ward identities for process-independent treatment

Algorithm based on sector decomposition [Denner, S.P. (2004)]

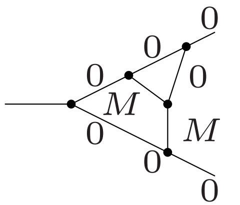
- arbitrary two-loop diagrams in the limit $L = \ln(Q^2/M^2) \gg 1$
- photons and light fermions massless in $D = 4 - 2\epsilon$

$(q_2 p_1)(q_2 p_2) \times$



$$= -\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left[\frac{1}{16\epsilon} L^2 + \frac{1}{24} L^3\right]$$

$(q_2 p_1) \times$



$$= -\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \frac{1}{Q^2} \left[\frac{5}{48} L^4 + \frac{1}{12\epsilon} L^3 + \frac{1-2\gamma_{\mathrm{E}}}{12} L^3\right]$$

- completely automatized to **NLL accuracy**; computing time $= \mathcal{O}(10\mathrm{s})$

Multi-loop integrals with sector decomposition (one-slide summary)

Hepp(1966); Denner, Roth (1996); Binoth, Heinrich(2000); Denner, S.P. (2004)

(A) **L-loop integral with I propagators: Feynman parametrization**

$$G = \int_0^1 \prod_{i=1}^{I} \mathrm{d}\alpha_j\, \delta(1-\sum_{s=1}^{I}\alpha_s) \frac{\Gamma(e)\mathcal{U}(\vec{\alpha})^{-e}}{\left[\mathcal{P}(\vec{\alpha}) + (M^2/Q^2)\mathcal{R}(\vec{\alpha})\right]^f} \quad \text{with} \quad M^2/Q^2 \ll 1$$

(B) **Isolate mass singularities in FP space: sector decomposition**

$$G' = \int_0^1 \prod_{i=1}^{m} \mathrm{d}\beta_i \int_0^1 \prod_{j=1}^{n} \mathrm{d}\alpha_j^f \frac{\mathcal{G}(\vec{\alpha};\vec{\beta})}{\left[\alpha_1\alpha_2\dots\alpha_n + (M^2/Q^2)\mathcal{H}(\vec{\alpha};\vec{\beta})\right]^f} \Rightarrow \ln^n \text{ singularity!}$$

(C) **Extract logarithms: singular α_j-integrations**

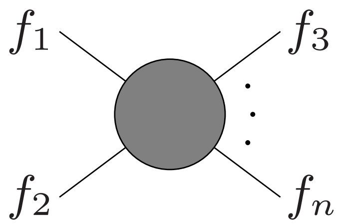
$$G' = \frac{1}{n!}\int_0^1 \prod_{i=1}^{m} \mathrm{d}\beta_i \left\{ \mathcal{G}(\vec{0},\vec{\beta})\ln^n\left(\frac{Q^2}{M^2}\right) + (n-1)\left\{\mathcal{G}(\vec{0},\vec{\beta})\left[\ln[\mathcal{H}(\vec{0},\vec{\beta})] + \sum_{k=1}^{f-1}\frac{1}{k}\right]\right.\right.$$
$$\left.\left. + \sum_{j=1}^{n}\int_0^1 \frac{\mathrm{d}\alpha_j}{\alpha_j}\left[\mathcal{G}(0,\dots,\alpha_j,\dots,0,\vec{\beta}) - \mathcal{G}(\vec{0},\vec{\beta})\right]\right\}\ln^{n-1}\left(\frac{Q^2}{M^2}\right) + \mathcal{O}(\ln^{n-2})\right\}$$

(D) **Compute LL and NLL coefficients: non-singular β_i-integrations**

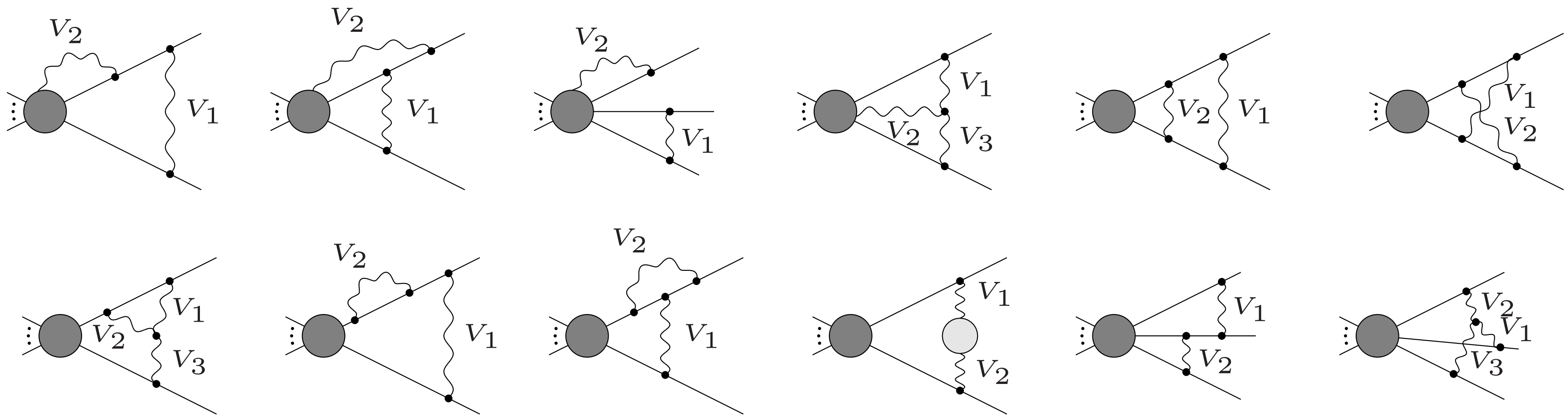
(2-loop diagrams $\Rightarrow$ integrations 2-dimensional and simple)

Computing 2-loop NLL corrections for generic processes

Scattering of n massless fermions [Denner, Jantzen, S.P. (2006)]

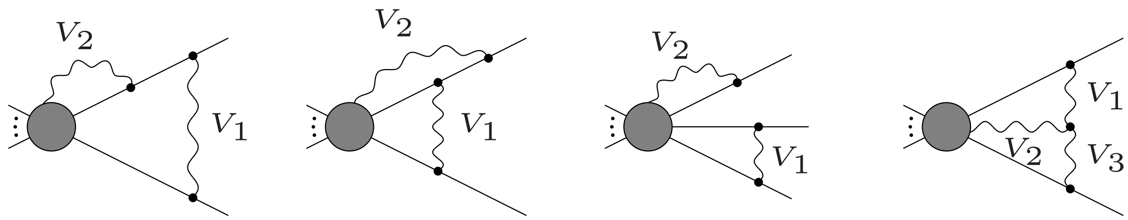


Classification of 2-loop diagrams that yield NLL contributions

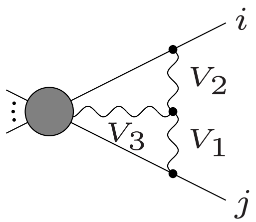


(A) Non-factorizable two-loop diagrams

Collinear gauge bosons coupling to external and internal lines



Collinear Ward identities for spontaneously broken non-abelian theories



$$= \mu_0^{4\epsilon}\int \frac{\mathrm{d}^D q_1}{(2\pi)^D}\int \frac{\mathrm{d}^D q_2}{(2\pi)^D}\frac{4\mathrm{i}e^2 g_2 \varepsilon^{V_1 V_2 V_3}}{(q_1^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)(q_3^2-M_{V_3}^2)(p_i-q_2)^2(p_j-q_1)^2}$$

$$\times \lim_{q_1^\mu\to 0}\lim_{q_2^\mu \to x p_i^\mu}(p_i-q_2)^{\mu_2}(p_j-q_1)^{\mu_1}\Big[g_{\mu_1\mu_2}(q_1-q_2)^{\mu_3}+g^{\mu_3}_{\mu_2}(q_2+q_3)_{\mu_1}-g^{\mu_3}_{\mu_1}(q_3+q_1)_{\mu_2}\Big]$$

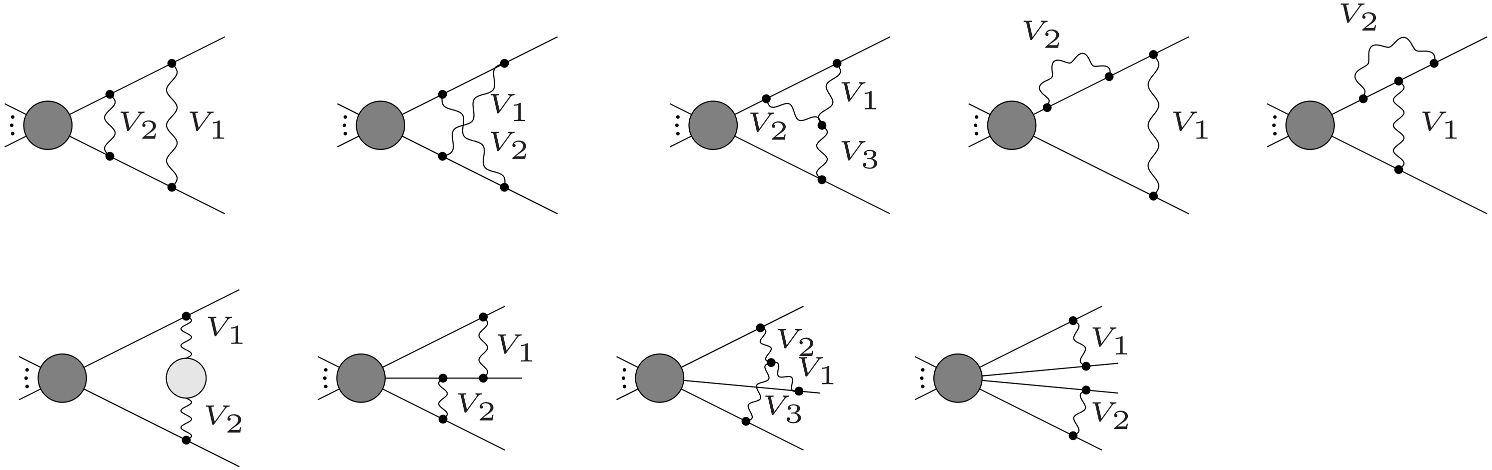
$$\times \sum_{\varphi'_i,\varphi'_j}\Bigg\{ G^{[\underline{\bar{V}_3}\,\underline{\varphi'_i}]}_{\mu_3}(q_3,p_i-q_2)u(p_i,\kappa_i)+\frac{2(p_j+q_2)_{\mu_3}}{(p_j+q_2)^2}\sum_{\varphi''_j} eI^{\bar{V}_3}_{\varphi''_j\varphi'_j}\mathcal{M}_0^{\varphi_1\ldots\varphi'_i\ldots\varphi''_j\ldots\varphi_n}$$

$$+\sum_{\substack{k=1\\k\neq i,j}}^{n}\frac{2(p_k+q_3)_{\mu_3}}{(p_k+q_3)^2}\sum_{\varphi'_k}\mathcal{M}_0^{\varphi_1\ldots\varphi'_i\ldots\varphi'_j\ldots\varphi'_k\ldots\varphi_n} eI^{\bar{V}_3}_{\varphi'_k\varphi_k}\Bigg\} I^{\bar{V}_1}_{\varphi'_j\varphi_j} I^{\bar{V}_2}_{\varphi'_i\varphi_i} = 0$$

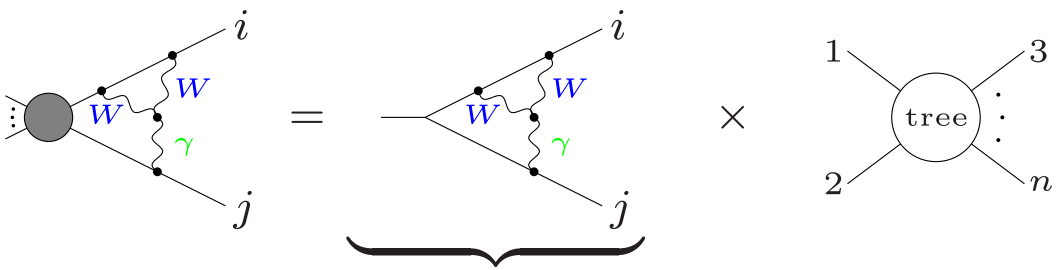
cancellation mechanism permits process-independent treatment

(B) Factorizable two-loop diagrams

Collinear gauge bosons coupling only to external lines

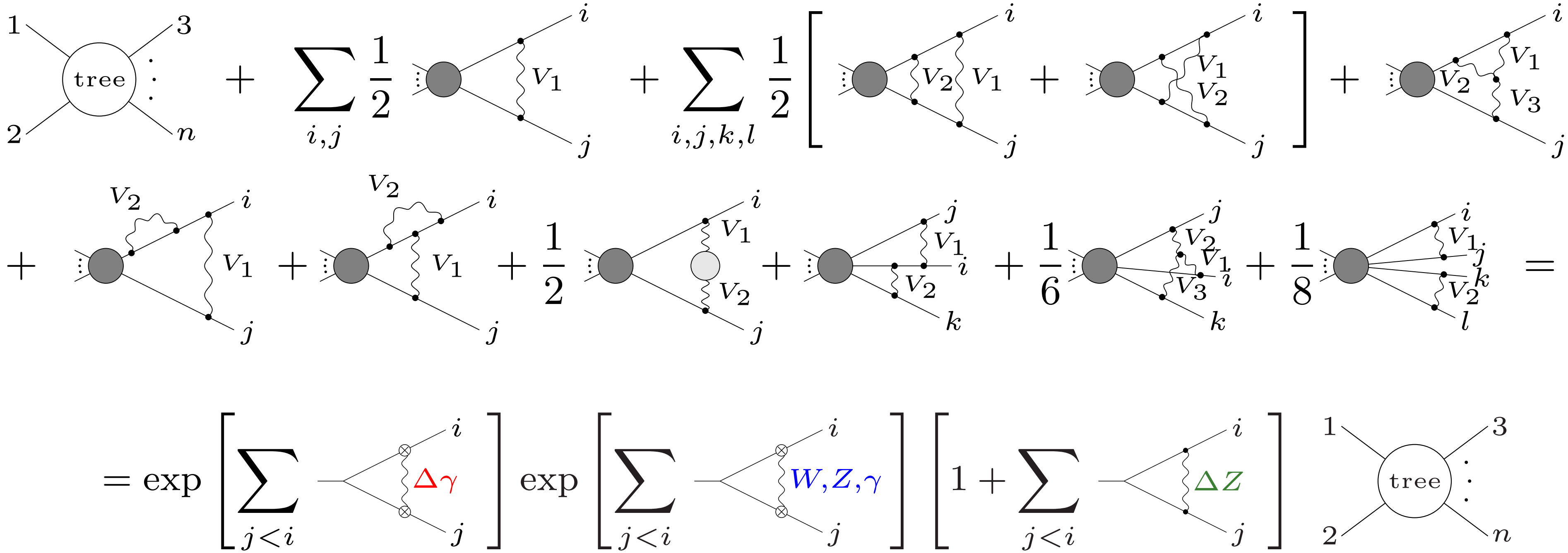


Factorization and explicit calculation using sector decomposition [Denner, S.P. (2004)] and expansion by regions [Jantzen, Smirnov (2006)]



$$\frac{\mathrm{i}e^4\epsilon^{W\bar{W}\gamma}I_i^W I_i^{\bar{W}} I_j^{\gamma}}{s_\mathrm{W}}\left(\frac{s}{Q^2}\right)^{2\epsilon}\left[-\frac{1}{3}L^3\epsilon^{-1}-5L^4-6\epsilon^{-3}-6L\epsilon^{-2}-2L^2\epsilon^{-1}+\frac{2}{3}L^3\right]$$

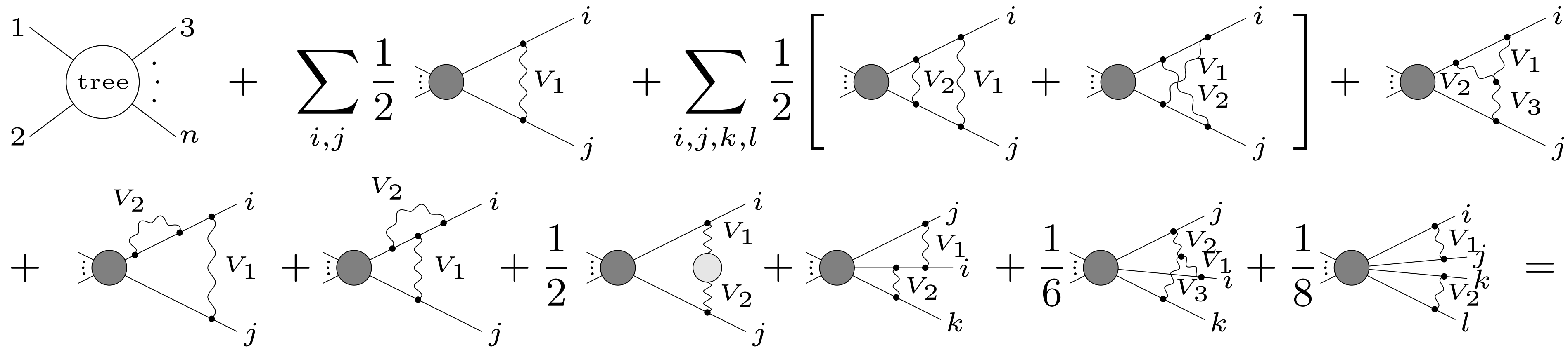
Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$

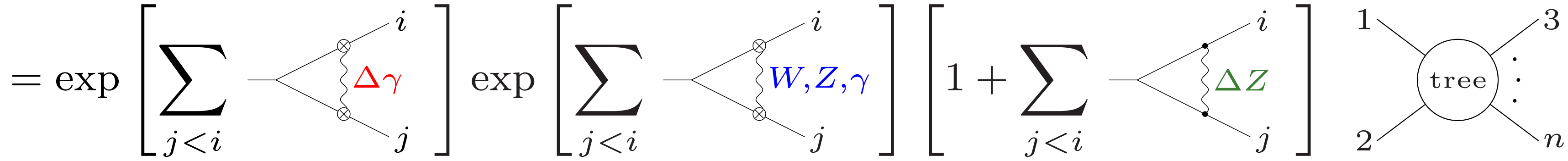


70 inequivalent two-loop diagrams $\Rightarrow$ very simple result!

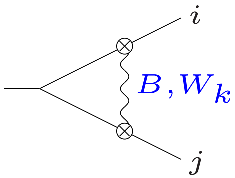
- Two-loop $\equiv$ exp(1-loop)$\times$ Born
- Agreement with IREE [Kühn, Moch, Penin, Smirnov (2000); Melles (2003)]

$$\boxed{\textbf{Two-loop NLL result for } f_1 f_2 \to f_3 \dots f_n}$$





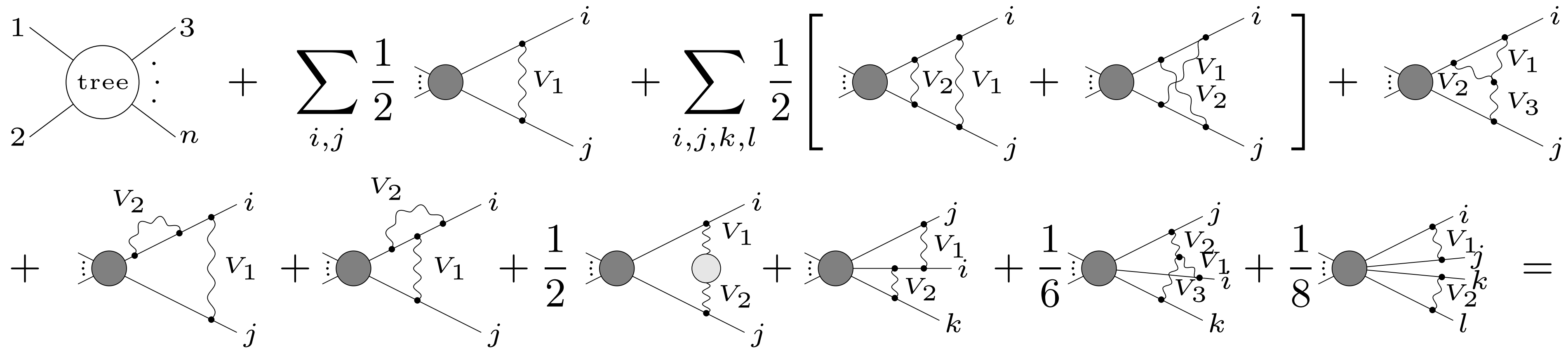
contains only $L = \ln(s/M_W^2)$ and behaves as in a symmetric SU(2)xU(1) theory with $M_W = M_Z = M_\gamma$

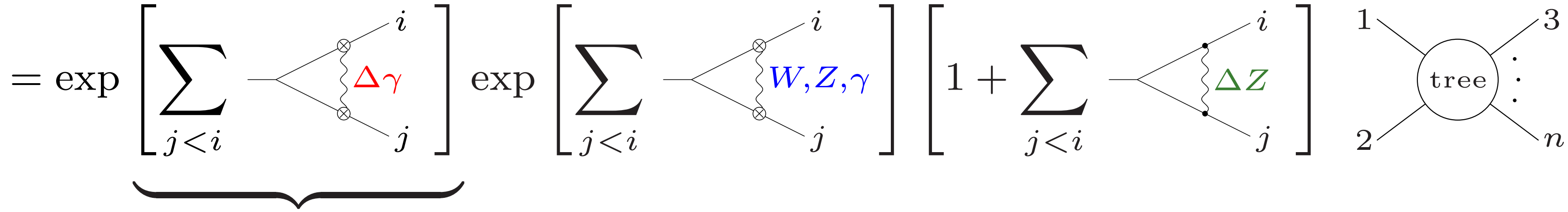


$$= \left(\frac{\alpha}{4\pi}\right) \sum_{V=B,W^a} I_i^{\bar V} I_j^V \Big[\underbrace{L^2 + \frac{2}{3} L^3\epsilon + \frac{1}{4} L^4\epsilon^2 - \left[\frac{3}{2} - \ln\left(\frac{r_{ij}}{s}\right)\right]\left(2L + L^2\epsilon + \frac{1}{3} L^3\epsilon^2\right)}_{K(\epsilon,\, M_W;\, r_{ij})} \Big]$$

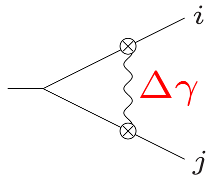
$$+ \left(\frac{\alpha}{4\pi}\right)^2 \frac{1}{2\epsilon}\left[\left(\frac{-s}{\mu^2}\right)^{\epsilon} K(\epsilon, M_W, s) - K(2\epsilon, M_W, s)\right]\left(g_1^2 \frac{Y_i Y_j}{4} b_1^{(1)} + g_2^2 \frac{T_i^a T_j^a}{4} b_2^{(1)}\right)$$

Two-loop NLL result for $f_1 f_2 \to f_3 \ldots f_n$





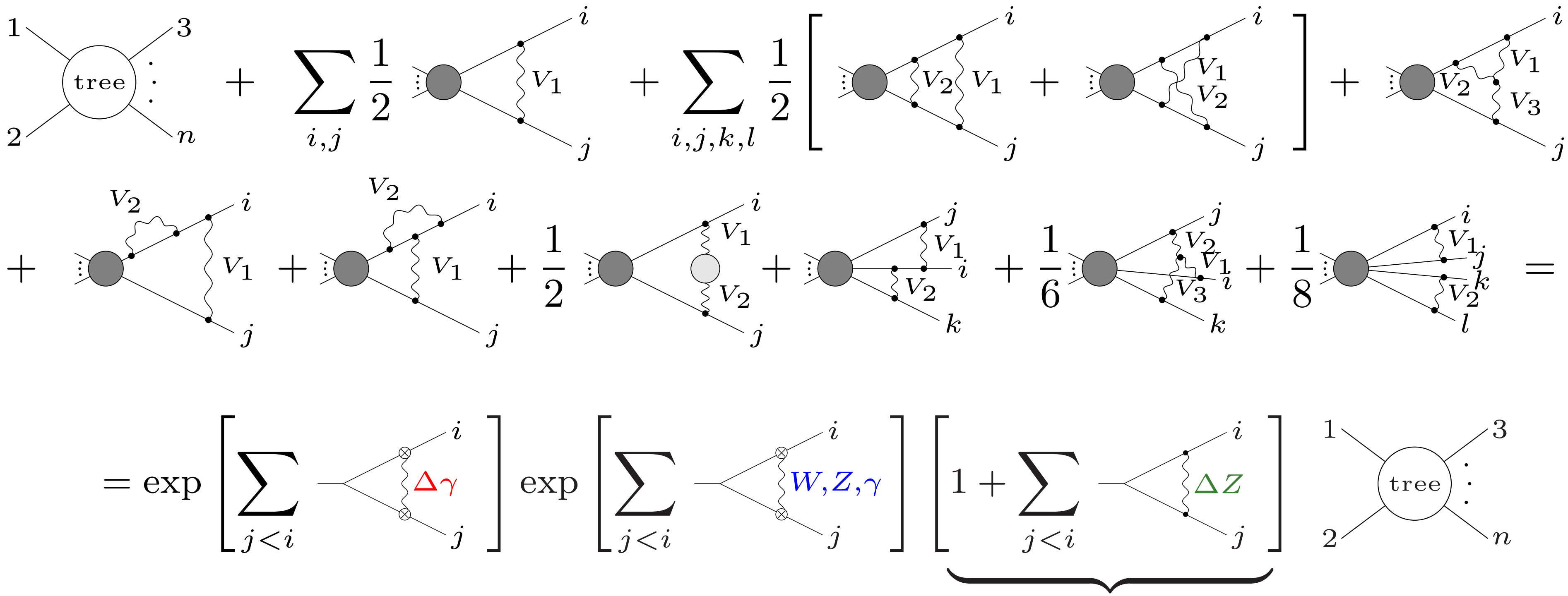
photonic singularities factorize
and behave as in QED



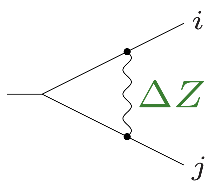
$$= \left(\frac{\alpha}{4\pi}\right) Q_i Q_j \Bigg[\underbrace{2\epsilon^{-2} + 3\epsilon^{-1} - 2\epsilon^{-1} \ln\left(\frac{r_{ij}}{s}\right) - K(\epsilon, M_\mathrm{W}; r_{ij})}_{\Delta K(\epsilon, 0, r_{ij})} \Bigg]$$

$$+ \left(\frac{\alpha}{4\pi}\right)^2 \frac{1}{2\epsilon} \left[\left(\frac{-s}{\mu^2}\right)^{\epsilon} \Delta K(\epsilon, 0, s) - \Delta K(2\epsilon, 0, s) \right] e^2 Q_i Q_j b^{(1)}_\mathrm{QED}$$

Two-loop NLL result for $f_1 f_2 \to f_3 \ldots f_n$



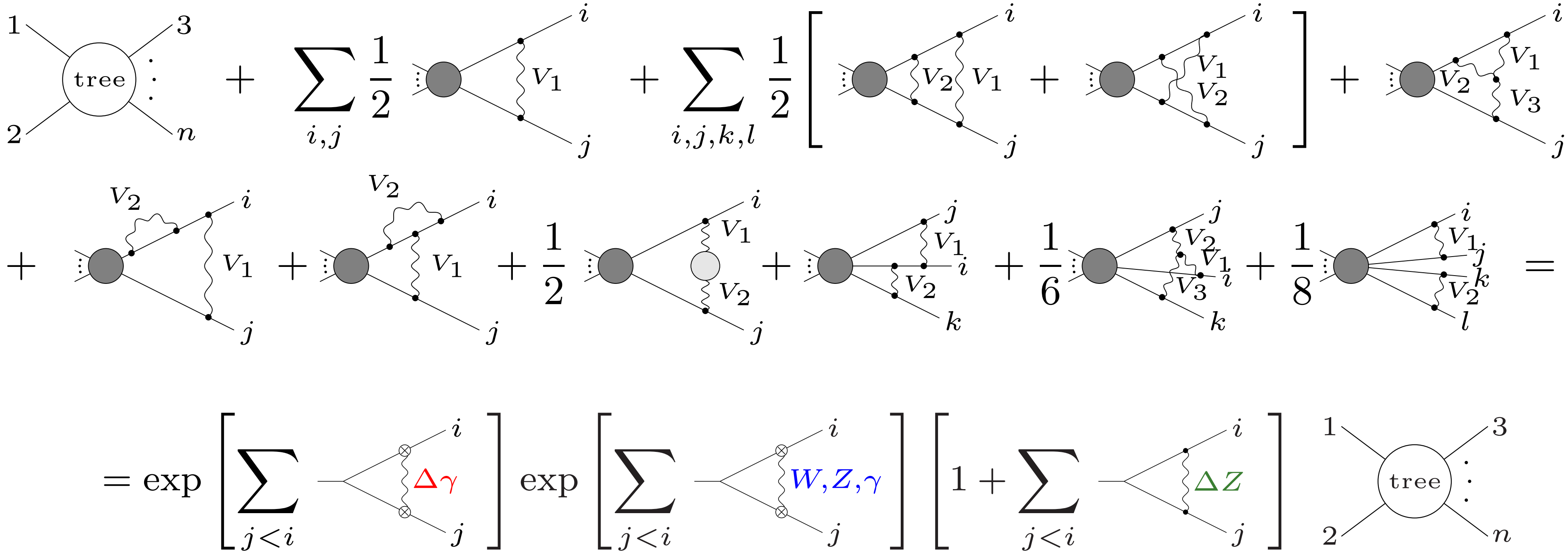
Mixing correction depending on Z-W mass difference



$$= -\left(\frac{\alpha}{4\pi}\right) I_i^Z I_j^Z \ln\left(\frac{M_Z^2}{M_W^2}\right)\left[2L + 2L^2\varepsilon + L^3\varepsilon^2\right]$$

$\Rightarrow \mathcal{O}(10^{-3})$ **effect at two loops**

Two-loop NLL result for $f_1 f_2 \to f_3 \dots f_n$



- massless-fermion scattering well understood
- method developped for general processes with massive fermions and bosons
- work in progress ...

Conclusions

Hard reactions at $Q^2 \sim 1\,\mathrm{TeV}^2$ receive large $\ln(s/M_\mathrm{W}^2)$ EW corrections

- one loop $\gg 10\%$
- two loops $\gg 1\%$ (important for LHC/ILC precision tests and NP searches)

Systematic asymptotic expansions

- strong simplification, process-independent treatment
- good precision (requires subleading logarithms!)

Status and future challenges

- NLLs for any process within reach
- Next important challenges: NNLLs, hard photon bremsstrahlung ...

pp $\longrightarrow$ WW, WZ, ZZ $\longrightarrow$ 4 leptons

	pp $\to WZ \to l\nu_l l'\bar{l}'$		pp $\to ZZ \to l\bar{l}l'\bar{l}'$		pp $\to WW \to l\bar{\nu}_l \nu_{l'}\bar{l}'$	
$M_{\mathrm{inv}}^{\mathrm{cut}}$(leptons)[GeV]	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]
500	-7.4	5.4	-15.0	8.5	-13.8	2.6
600	-9.5	7.5	-18.3	11.9	-15.9	3.7
700	-10.9	9.9	-21.0	15.7	-18.1	4.9
800	-13.3	12.8	-23.8	20.1	-20.2	6.5
900	-15.1	16.2	-26.1	25.3	-22.0	8.3
1000	-16.7	20.2	-28.1	31.2	-23.4	10.4

estimate based on $L = 100\mathrm{fb}^{-1}$ and final states with $l, l' = $ e or μ

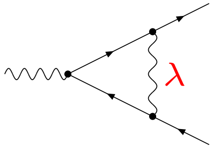
Accomando, Denner, Kaiser (2005)

Electroweak corrections (Δ_{EW}) vs statistical error ($\Delta_{\mathrm{stat}} = 1/\sqrt{2L\sigma_0}$)

- Δ_{EW} and Δ_{stat} grow with M_{inv}(leptons)
- $|\Delta_{\mathrm{EW}}| \gtrsim \Delta_{\mathrm{stat}}$ up to $M_{\mathrm{inv}} \sim 1$ TeV

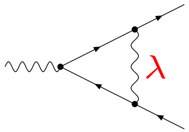
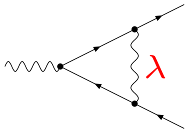
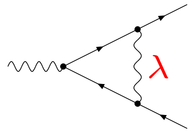
Double logarithms beyond one loop

QED vertex corrections for $s \gg \lambda^2$



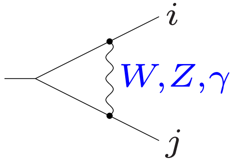
$$= -\frac{\alpha}{4\pi}\ln^2\left(\frac{s}{\lambda^2}\right)$$

Double logarithms beyond one loop: exponentiation

$$\ldots + \frac{1}{2}\Bigg($$  $$\Bigg)^2 + \frac{1}{3!}\Bigg($$  $$\Bigg)^3 + \ldots = \exp\Bigg($$  $$\Bigg)$$

Sudakov (1956)

Back-of-the-envelope estimate of two-loop electroweak logs at $\sqrt{s} \simeq 1\,\mathrm{TeV}$

$$\left(\frac{\delta\sigma}{\sigma_{\mathrm{Born}}}\right)_{2\to 2} \simeq \frac{1}{2}\Bigg(\sum_{j\neq i}$$ 

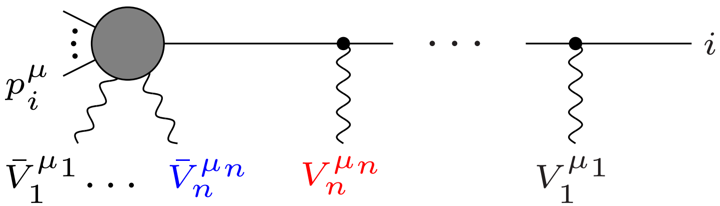
$$\Bigg)^2 \simeq \frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4}\ln^4\frac{s}{M_{\mathrm{W}}^2} \simeq 3.5\%$$

well beyond 1%

Cancellation mechanism for non-factorizable contributions

Soft-collinear fermion-boson vertices (Dirac structure disappears)

$$\lim_{q_k^\mu \to x_k p_i^\mu}$$

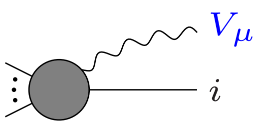


$$= G^{\bar V_1 \cdots \bar V_n \, \underline{i}}_{\mu_1 \ldots \mu_n}(-q_1, \ldots, -q_n, p_i + \tilde q_n)\, u(p_i, \kappa_i)$$

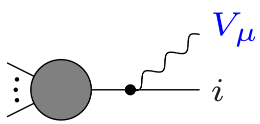
$$\times \frac{-2eI_i^{V_n}(p_i + \tilde q_n)^{\mu_n}}{(p_i + \tilde q_n)^2} \cdots \frac{-2eI_i^{V_1}(p_i + q_1)^{\mu_1}}{(p_i + q_1)^2}$$

Collinear Ward identities for spontaneously broken non-abelian theories

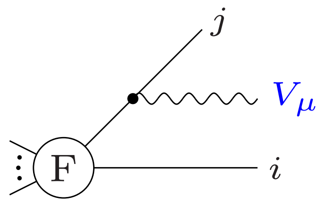
$$\lim_{q^\mu \to x p_i^\mu} q^\mu \times \Bigg[$$



$$-$$



$$- \sum_{\substack{j=1 \\ j \neq i}}^{n}$$



$$\Bigg] = 0$$

Denner, S.P. (2001)

derived from BRS symmetry and valid for arbitrary processes

Origin of $1/\varepsilon$ and $\ln(Q^2/M^2)$ singularities

$$G \propto \int_0^1 \mathrm{d}^I \vec{\alpha}\, \delta(1 - \sum_{r=1}^{I} \alpha_r) \frac{\Gamma(e)}{[\mathcal{U}(\vec{\alpha})]^e \, [\mathcal{F}(\vec{\alpha})]^f}$$

Polynomials ($\mathcal{T}$ = trees, $\mathcal{C}$ =cuts)

$$\mathcal{U}(\vec{\alpha}) = \sum_{\mathcal{T}} \alpha_{\mathcal{T}_1} \ldots \alpha_{\mathcal{T}_L}$$

$$-\mathcal{F}(\vec{\alpha}) = \sum_{\mathcal{C}} s_{\mathcal{C}}\, \alpha_{\mathcal{C}_1} \ldots \alpha_{\mathcal{C}_{L+1}} - \mathcal{U}(\vec{\alpha}) \sum_{r=1}^{I} \alpha_r M_r^2 + \mathrm{i}\varepsilon$$

UV and mass singularities ($s_{\mathcal{C}} = s, t, u < 0$)

$\mathcal{U}(\vec{\alpha}) = 0 \;\Rightarrow\;$ UV sing. $\qquad\qquad$ $\mathcal{F}(\vec{\alpha}) = 0 \;\Rightarrow\;$ mass sing.

Singular regions ($\mathcal{U} = 0, \mathcal{F} = 0$)

$$\{\vec{\alpha} | \alpha_{i_1} = \ldots = \alpha_{i_n} = 0\}$$

Crucial for factorization of singularities in FP space!

Step 2: Sector decomposition

Goal: **factorization of mass singularities** from $\mathcal{F}(\vec{\alpha})$

$$\int_0^1 \frac{\mathrm{d}^{I-1}\vec{\alpha}}{\underbrace{\left[Q^2\mathcal{P}(\vec{\alpha}) + M^2\mathcal{R}(\vec{\alpha})\right]}_{\mathcal{F}(\vec{\alpha})}{}^f \ldots} \Rightarrow \int_0^1 \frac{\mathrm{d}^{I-1}\vec{\alpha}}{\left[Q^2\,\underbrace{\hat{\mathcal{P}}(\vec{\alpha})}_{\neq 0}\,\alpha_1\ldots\alpha_k + M^2\hat{\mathcal{R}}(\vec{\alpha})\right]^f \ldots}$$

Sector decomposition for overlapping singularities $\mathcal{P}(\vec{\alpha})|_{\alpha_1=\ldots=\alpha_k=0} = 0$

(A) **partition of** $[0,1]^{I-1}$ **into sectors** $\Omega_1,\ldots,\Omega_k$

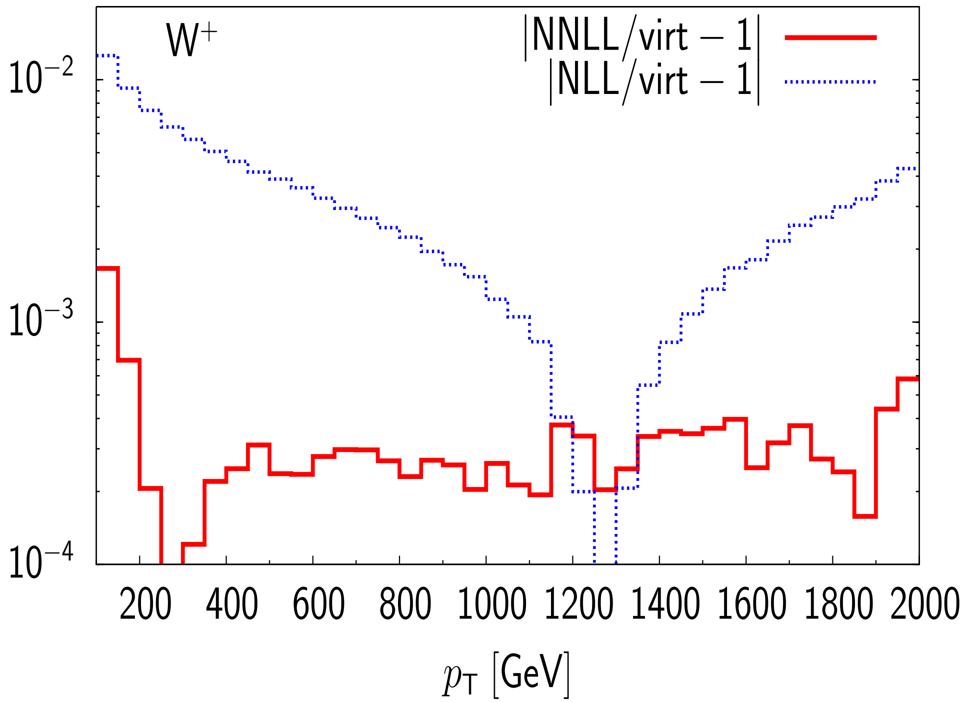
$$\Omega_j = \{\vec{\alpha}|\alpha_1,\ldots,\alpha_k \le \alpha_j\}$$

(B) **remapping** $\Omega_j \to [0,1]^{I-1}$ yields **factorization** in Ω_j-sector

$$\alpha_k \to \alpha_k\alpha_j \ \text{ for } \ k \neq j \qquad \Rightarrow \qquad \mathcal{P}(\vec{\alpha}) \to \alpha_j\,\hat{\mathcal{P}}_j(\vec{\alpha})$$

(C) **iterate** until $\hat{\mathcal{P}}_j(\vec{\alpha}) \neq 0$

Precision of NLL and NNLL approximations for pp $\rightarrow$ W+jet



NLL-error $\lesssim 10^{-2}$, NNLL-error $\lesssim 10^{-3}$

Dominant two-loop contributions for $\bar{q}q \to Zg$

Include LL and NLL terms

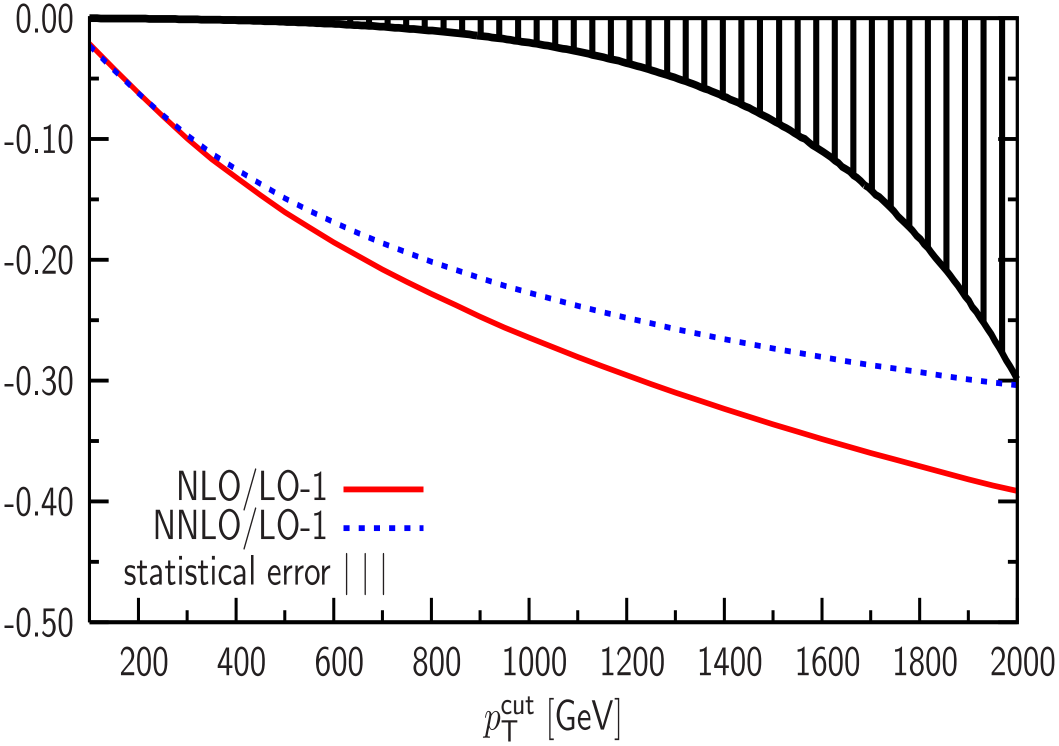
$$
\begin{aligned}
X_1 &= \ln^4\left(\frac{\hat{s}}{M_W^2}\right) - 6\ln^3\left(\frac{\hat{s}}{M_W^2}\right) \\
X_2 &= \ln^4\left(\frac{\hat{t}}{M_W^2}\right) + \ln^4\left(\frac{\hat{u}}{M_W^2}\right) - \ln^4\left(\frac{\hat{s}}{M_W^2}\right) \\
X_3 &= \ln^3\left(\frac{\hat{s}}{M_W^2}\right)
\end{aligned}
$$

derived from general two-loop results for EW logarithms (see later)

$$
\overline{\sum}|\mathcal{M}_2|^2 = \overline{\sum}|\mathcal{M}_1|^2 + 2\alpha^3\alpha_{\mathrm{S}}(N_c^2-1)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}\sum_{\lambda=\mathrm{L,R}}\left\{\frac{1}{2}\left(I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda} + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}\right)\right.
$$
$$
\left.\times\left[I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}X_1 + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}X_2\right] - \frac{T^3_{q_\lambda}Y_{q_\lambda}}{8s^4_{\mathrm{W}}}X_2 + \frac{1}{6}I^V_{q_\lambda}\left[I^Z_{q_\lambda}\left(\frac{b_1}{c^2_{\mathrm{W}}}\left(\frac{Y_{q_\lambda}}{2}\right)^2 + \frac{b_2}{s^2_{\mathrm{W}}}C_{q_\lambda}\right) + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}}T^3_{q_\lambda}b_2\right]X_3\right\}
$$

Melles (2001); Denner, Melles, S.P. (2003)
Kühn, Kulesza,S.P.,Schulze (2005)

1- and 2-loop corrections to $\sigma(p_T > p_T^{\rm cut})$ for pp $\to$ Z+jet



Estimated statistical error

- $\mathcal{L} = 300\,\mathrm{fb}^{-1}$, Z $\rightarrow$ leptons
- $(\Delta\sigma/\sigma)_{\rm stat} \sim 2\%$ at 1 TeV

Size of corrections at $p_{\rm T} \sim 1\,\mathrm{TeV}$

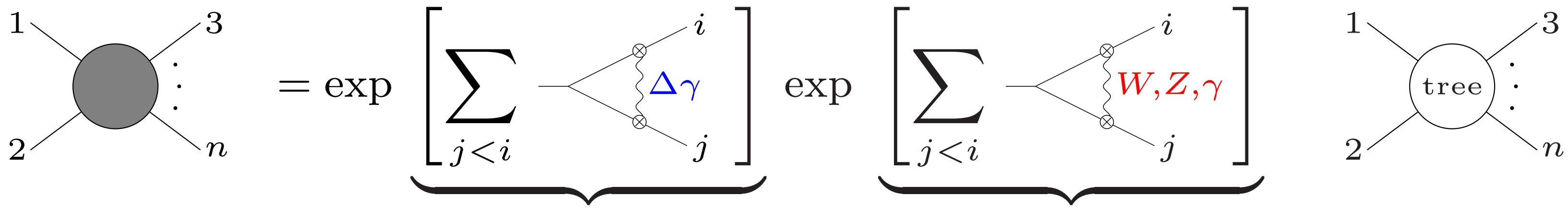
- 1-loop: -26% $\simeq -13\sigma_{\rm stat}$
- 2-loop:+ 4% $\simeq +2\sigma_{\rm stat}$

$\overline{\rm MS}$ **input:** $\alpha_S(M_Z^2) = 0.13,\ \alpha = 1/128.1,\ s_w^2 = 0.2314$

PDFs: LO MRST2001, $\mu_{\rm F} = \mu_{\rm R} = p_{\rm T}$

Resummation formula based on IREE approach

Double exponentiation resulting from $M_\gamma \ll M_W \sim M_Z$



$\ln(M_W/M_\gamma) \equiv$ QED IR sing.

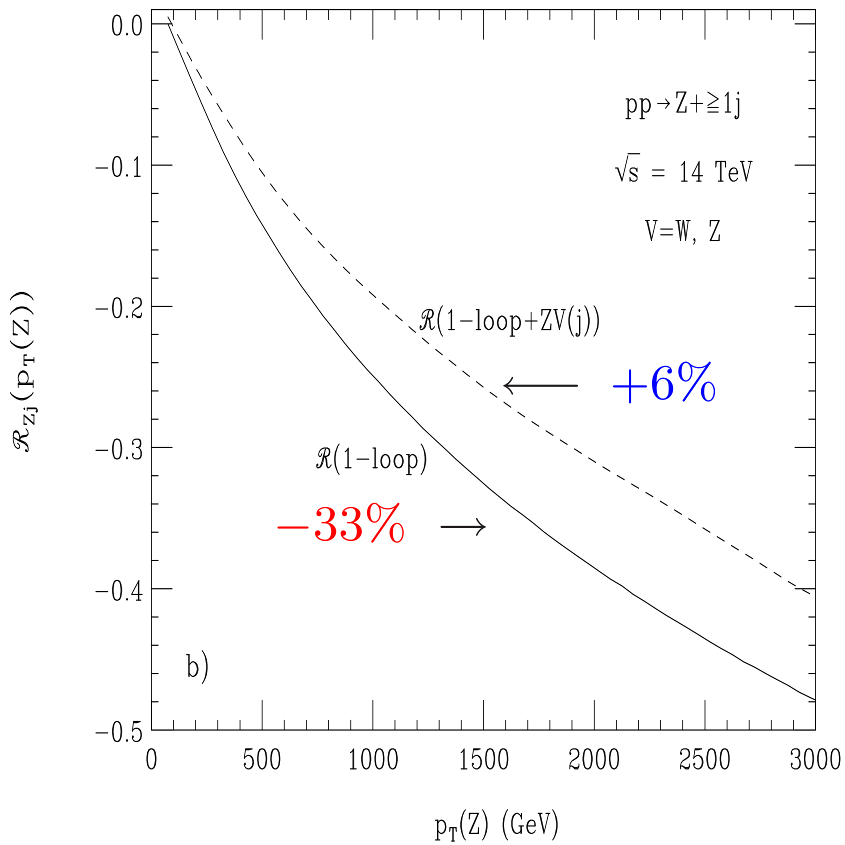
$\ln(s/M_W^2) \equiv \mathrm{SU(2)}\times\mathrm{U(1)}$ theory with $M_\gamma = M_W$

Existing results

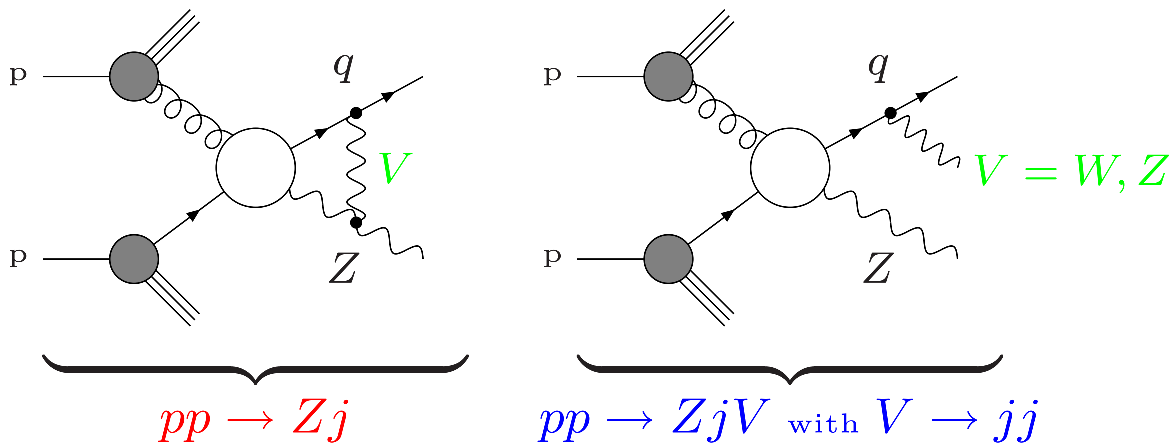
$$\alpha^2\Bigg[\underbrace{\underbrace{C_4\ln^4\left(\frac{s}{M_W^2}\right)}_{\text{Fadin, Lipatov, Martin, Melles (2000)}} + \underbrace{C_3\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Melles (2001–2004)}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_2\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{Kühn, Moch, Penin, Smirnov (2000–2003)}} + \underbrace{C_1\ln^1\left(\frac{s}{M_W^2}\right)}_{\text{Jantzen, Kühn, Penin, Smirnov (2004,2005)}}}_{\text{massless } f\bar{f}\to f'\bar{f}' \text{ processes}}\Bigg]$$

Real W and Z emission for $pp \rightarrow Zj$ [Baur (2006)]

" Since the number of jets is not fixed in a measurement of the Z boson p_T distribution, $\mathcal{O}(\alpha_s \alpha^2)$ ZVj production with $V \rightarrow jj$ has to be included when calculating weak radiative corrections "



Virtual and real $\mathcal{O}(\alpha)$ corr. to $pp \rightarrow Zj$



- W, Z emission can be non-negligible and partially cancel EW virtual corrections
- depends on observable definition and can be reduced by jet veto