S. Simula

INFN - Roma 3

workshop on ***ApeNext: Computational Challenges and First Physics Results***

GGI, Florence (Italy), February 8th-10th, 2007

Lattice QCD and flavor physics (I): determination of V_{us}

1) motivations;
2) (quenched) lattice QCD results for $K \to \pi$ and $\Sigma \to n$ form factors;
3) preliminary results with ApeNext.

thanks to: D. Becirevic, D. Guadagnoli, G. Isidori, V. Lubicz, G. Martinelli, F. Mescia, M. Papinutto, C. Tarantino, G. Villadoro

MOTIVATIONS

* **V_{us}** is a fundamental parameter of the **Standard Model** playing a central role in the **CKM matrix**

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parameterization

$$\lambda = \sin\theta_c = V_{us}$$

* the most accurate **test of CKM unitarity** comes from low-energy s → u and d → u semileptonic transitions

PDG ('06) values

nuclear SFT	⇨	$\lvert V_{ud}\rvert = 0.97377 \pm 0.00027 \quad (\sim 0.03\%)$
$K_{\ell 3}$ decays	⇨	$\lvert V_{us}\rvert = 0.2257 \pm 0.0021 \quad (\sim 1\%)$
inclusive/exclusive B decays	⇨	$\lvert V_{ub}\rvert = 0.00367 \pm 0.00015$

CKM unitarity: $\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 - 1 = -\left(8 \pm 5_{V_{ud}} \pm 9_{V_{us}}\right)\cdot 10^{-3}$

* the most accurate determination of $|V_{us}|$ comes still from $K_{\ell 3}$ decays, even if alternative approaches, like $K_{\mu 2}$ (W. Marciano ('04)) or τ-decays (see A. Pich ('06)), are becoming more competitive

* experiments measure the product $|V_{us}\, f_+(0)| = 0.21686\ (49)$ quite accurately (~ 0.2%) (Flavianet ('06)) and thus the relevant hadronic ingredient is the vector f.f. at zero-momentum transfer $f_+(0)$

* the present uncertainty on $|V_{us}|$ (~1%) is almost totally due to the theoretical uncertainty on $f_+(0)$ (~0.8%)

* the crucial role is played by the conservation of the vector current in the SU(3) limit and by the Ademollo-Gatto (AG) theorem, which guarantees that SU(3)-breaking effects in $f_+(0)$ should appear only at second order in the breaking parameter $(m_s - m_\ell)$

* χPT expansion of the vector form factor at zero momentum transfer, $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation (→ 1)

$f_2 = -0.023$ (LEC and renorm. scale independent)

- f_4 is **AG protected**, $O[(m_s - m_\ell)^2]$, but has **the largest theoretical uncertainties (~ 50%)**

- Leutwyler-Roos ('84): from **quark model** (overlap of K and π wave functions)

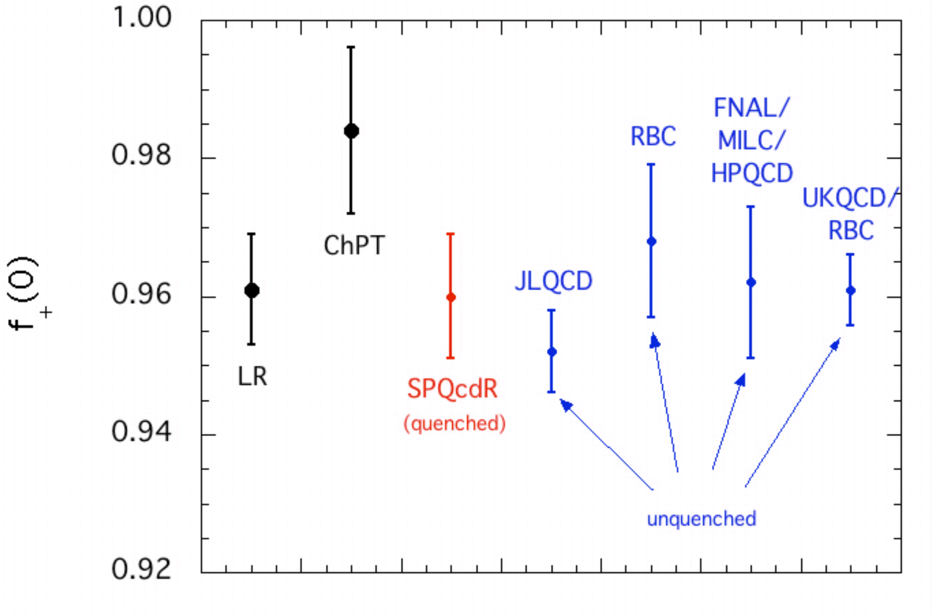
$$f_+(0) = 0.961 \pm 0.008$$

- Post and Schilcher ('01), Bijnens and Talavera ('03), Cirigliano et al. ('06): from **NNLO [O(p^6)] ChPT + 1/N_c expansion**

$$f_+(0) = 0.984 \pm 0.012$$

- SPQcdR coll. ('05): from (quenched) **lattice QCD**

$$f_+(0) = 0.960 \pm 0.005_{stat.} \pm 0.007_{syst.} + \text{quenching error}$$



preliminary unquenched results for $f_+(0)$

JLQCD ('05): 0.952 (6) [N_f=2, M_π~ 500 MeV]

RBC ('06): 0.968 (9) (6) [N_f=2, M_π~ 400 MeV]

FNAL/MILC/HPQCD ('05): 0.962 (6) (9) [N_f=2+1]

UKQCD/RBC ('06): 0.961 (5) [N_f=2+1, M_π~ 400 MeV]

our goal is to reach an accuracy better than 1% with unquenched simulations at M_π < 400 MeV

review of our strategy for $K_{\ell 3}$ decays

1. **high-precision evaluation** of the scalar form factor $f_0[q^2_{max}]$ obtained using a suitable ratio of correlation functions
2. **extrapolation to $q^2 = 0$** to get $f_+(0) = f_0(0)$ (determination of the slope λ_0)
3. subtraction of the leading chiral logs [study of $\Delta f = f_+(0) - 1 - f_2$] and **extrapolation to physical masses**

Step 1: consider the double ratio:

$$R_0 \equiv \frac{C^0_{K\pi}(t_x, t_y) \cdot C^0_{\pi K}(t_x, t_y)}{C^0_{KK}(t_x, t_y) \cdot C^0_{\pi\pi}(t_x, t_y)} \xrightarrow[t_x - t_y \to \infty]{t_y \to \infty,} \frac{\langle \pi | V^0 | K \rangle \cdot \langle K | V^0 | \pi \rangle}{\langle K | V^0 | K \rangle \cdot \langle \pi | V^0 | \pi \rangle} \propto \left[f_0\left(q^2_{\max}\right)\right]^2$$

(initial and final mesons at rest)

essential features of the double ratio

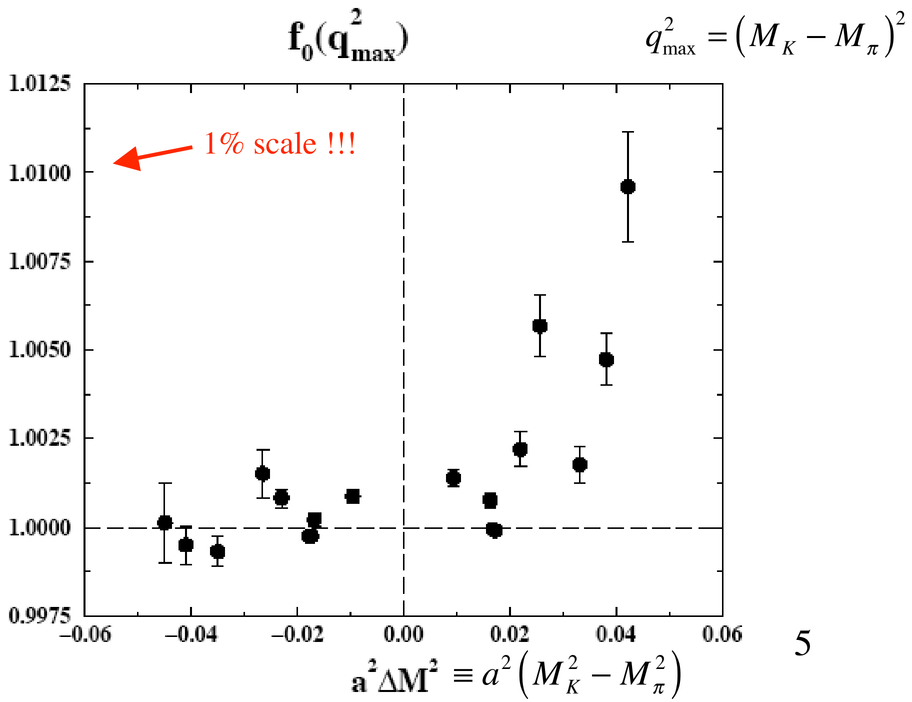
1. independent of renormalization constants;
2. exactly normalized at 1 in the SU(3) limit;
3. statistical fluctuations strongly cancel out;
4. improved at $O[a^2(m_s - m_\ell)^2]$ thanks to the $K \Leftrightarrow \pi$ symmetry.

$\beta = 6.2 \ (a^{-1} \simeq 2.6 \ GeV)$

230 (*quenched*) *gauge confs.*

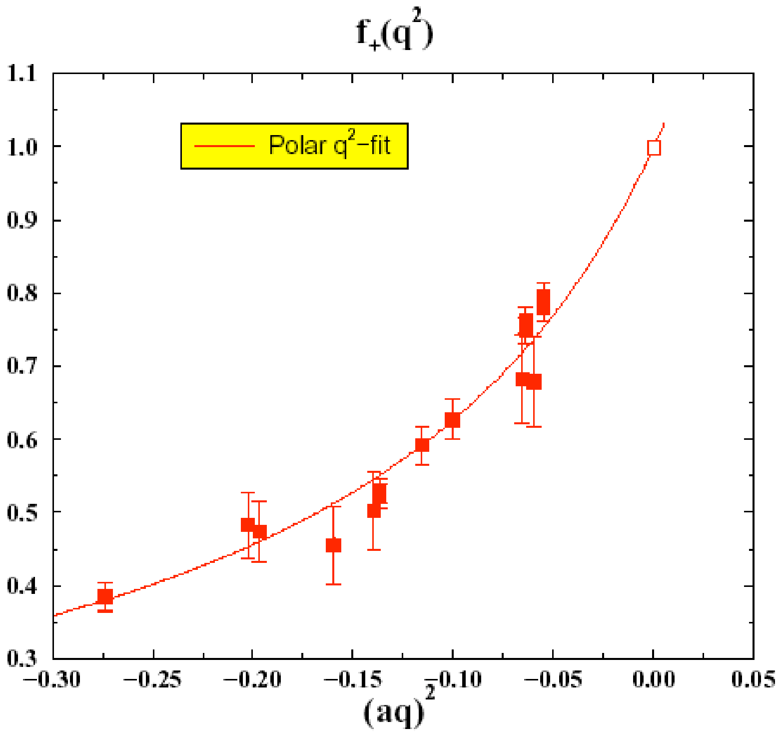
Clover fermions

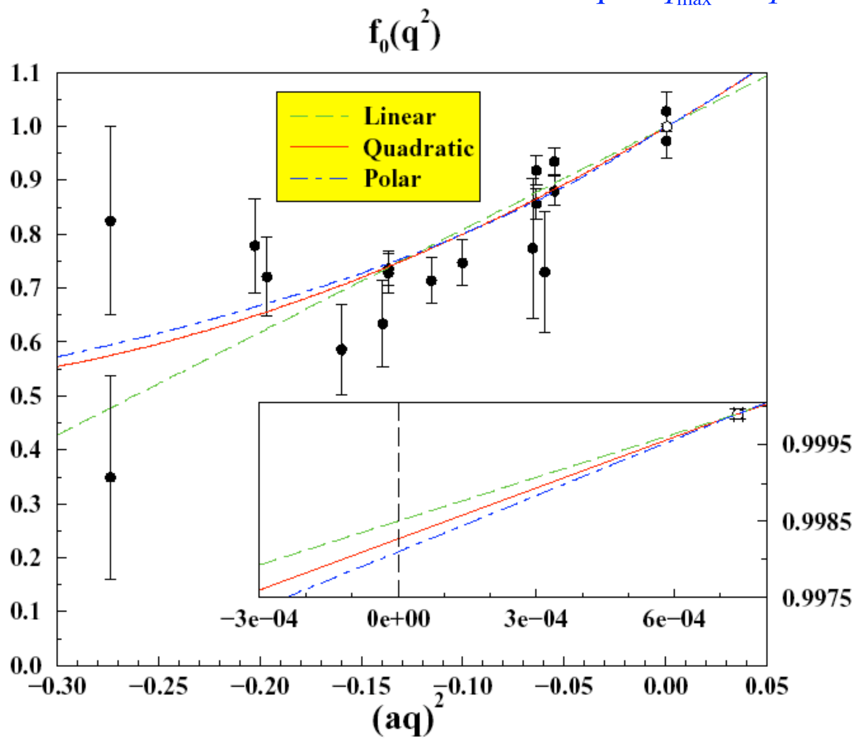
$0.5 < M_{PS}(GeV) < 1$



Step 2: study the momentum dependence of the f.f.'s

extrapolation of $f_0(q^2)$ from $q^2 = q^2_{max}$ to $q^2 = 0$





monopole fit: $f_+(q^2) = f(0) / (1 - \lambda_+ q^2)$

$f(0) = f_+(0) = f_0(0)$

linear fit: $f_0(q^2) = f(0)\,(1 + \lambda_0 q^2)$

quadratic fit: $f_0(q^2) = f(0)\,(1 + \lambda_0 q^2 + c_0 q^4)$

monopole fit: $f_0(q2) = f(0) / (1 - \lambda_0 q^2)$

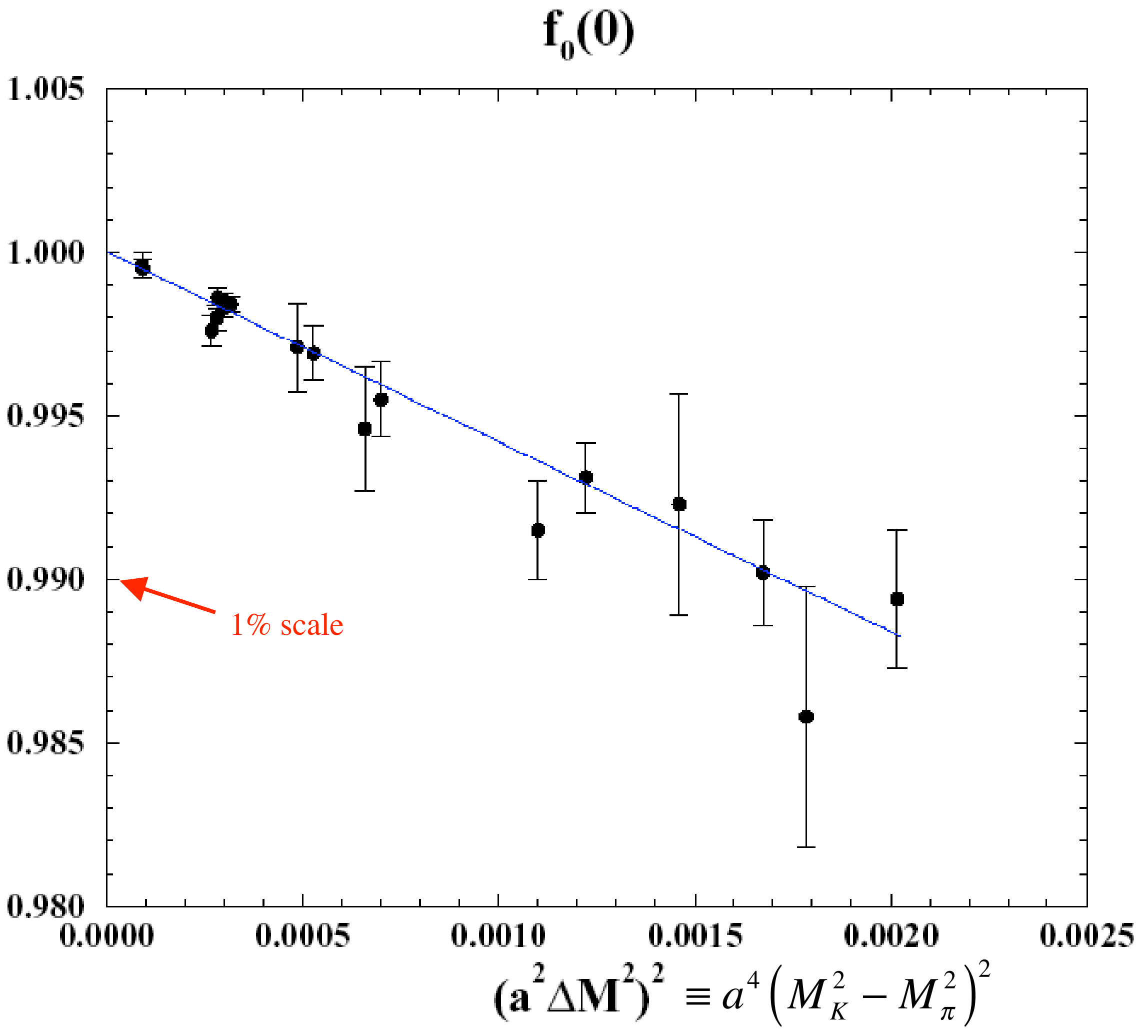
******* comparison of slopes in units of $(M_\pi)^2$ *******

lattice (extrapolated): $\lambda_+ = 0.025 \pm 0.002$ $\lambda_0 = 0.012 \pm 0.002$ → $\approx 15 \div 20\%$

FlaviaNet ('06): $\lambda_+ = 0.02454 \pm 0.00126$ $\lambda_0 = 0.01314 \pm 0.00140$

… and get the vector form factor at zero-momentum transfer

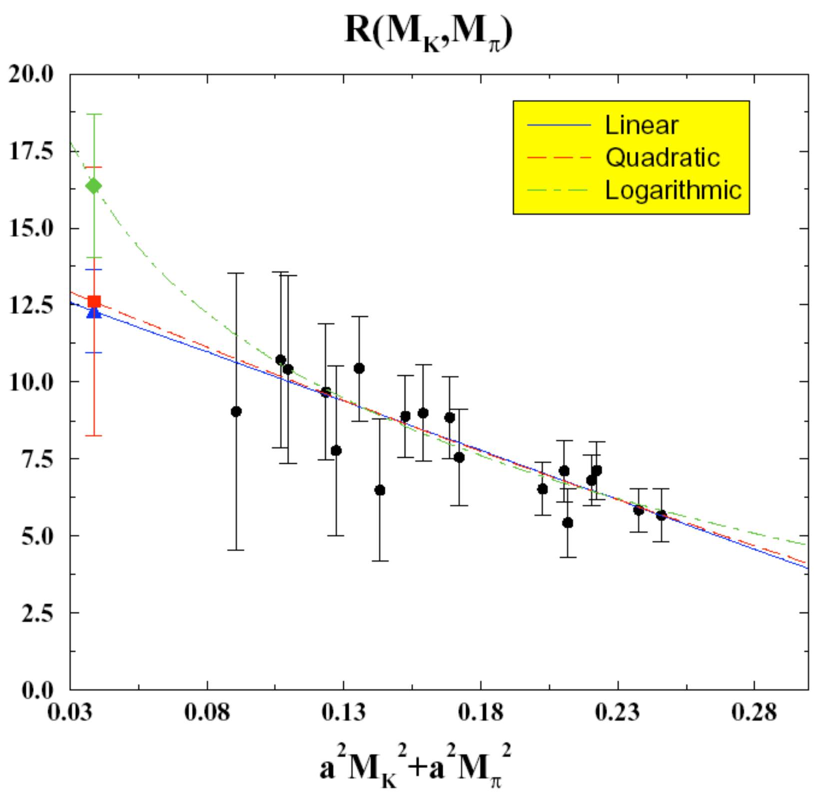
$$f(0) = f_+(0) = f_0(0) = 1 + O\left[\left(M_K^2 - M_\pi^2\right)^2\right]$$ **(AG theorem)**



black dots: results obtained from a quadratic fit in q^2

Step 3: subtraction of leading chiral logs in (quenched) ChPT $\Delta f^q \equiv f(0) - 1 - f_2^q = O\left[\left(M_K^2 - M_\pi^2\right)^2\right]$

and extrapolation of the "AG slope" $R \equiv \Delta f^q \Big/ \left(a^2 M_K^2 - a^2 M_\pi^2\right)^2$ to physical meson masses



linear fit: R = A + B x

quadratic fit: $R = A + B\,x + C\,x^2$

log fit: R = A + B x + C log(x)

$x = a^2 (M_K^2 + M_\pi^2)$

the dominant contributions to the systematic error come from the uncertainties on the momentum and mass dependencies of the (scalar) form factor

$\Delta f^q = -0.017 \pm 0.005_{(stat.)} \pm 0.007_{(syst.)}$ ⇒ $f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{(stat.)} \pm 0.007_{(syst.)} = 0.960 \pm 0.009$

+ quenching error (~ 1%)

V_{us} from hyperon semileptonic decays

- both vector and axial f.f.'s are involved:

$$q = p - p'$$

$$\langle B'(p')|V^\mu|B(p)\rangle = \bar{u}(p')\left\{ f_1(q^2)\gamma^\mu - f_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{M'+M} + f_3(q^2)\frac{q^\mu}{M'+M} \right\} u(p)$$

$$\langle B'(p')|A^\mu|B(p)\rangle = \bar{u}(p')\left\{ g_1(q^2)\gamma^\mu - g_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{M'+M} + g_3(q^2)\frac{q^\mu}{M'+M} \right\} \gamma_5\, u(p)$$

- a total of six (real) f.f.'s for each octect transition, but

$$\Gamma_{rate} \propto |V_{us}|^2 f_1^2 \left\{ 1 + 3\frac{g_1^2}{f_1^2} + 4\frac{M'-M}{M'+M}\frac{g_1}{f_1}\frac{g_2}{f_1} \right\}$$

SU(3) limit

$f_1(0) = CG_{B'B}$

$g_2(0) = 0$

- **$f_1(0)$ is AG protected**, but $g_1(0)$ and $g_2(0)$ are not AG protected

* recent analysis from Cabibbo, Swallow and Winston ('04):

1) the ratio $g_1(0) / f_1(0)$ is extracted from data,

2) SU(3) symmetry is assumed for $f_1(0)$ [as well as for $g_2(0)$].

Decay	g_1/f_1	V_{us}
$\Lambda \to p e^- \overline{\nu}$	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \to n e^- \overline{\nu}$	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^- \overline{\nu}$	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}$	$1.32^{+.22}_{-.18}$	0.209 ± 0.027
Combined	—	0.2250 ± 0.0027

$f_1^{SU(3)}(0) = CG_{B'B}$

$$\frac{f_1(0)}{f_1^{SU(3)}(0)} = 1 \quad \text{(CSW)}$$

<1 (quark model)

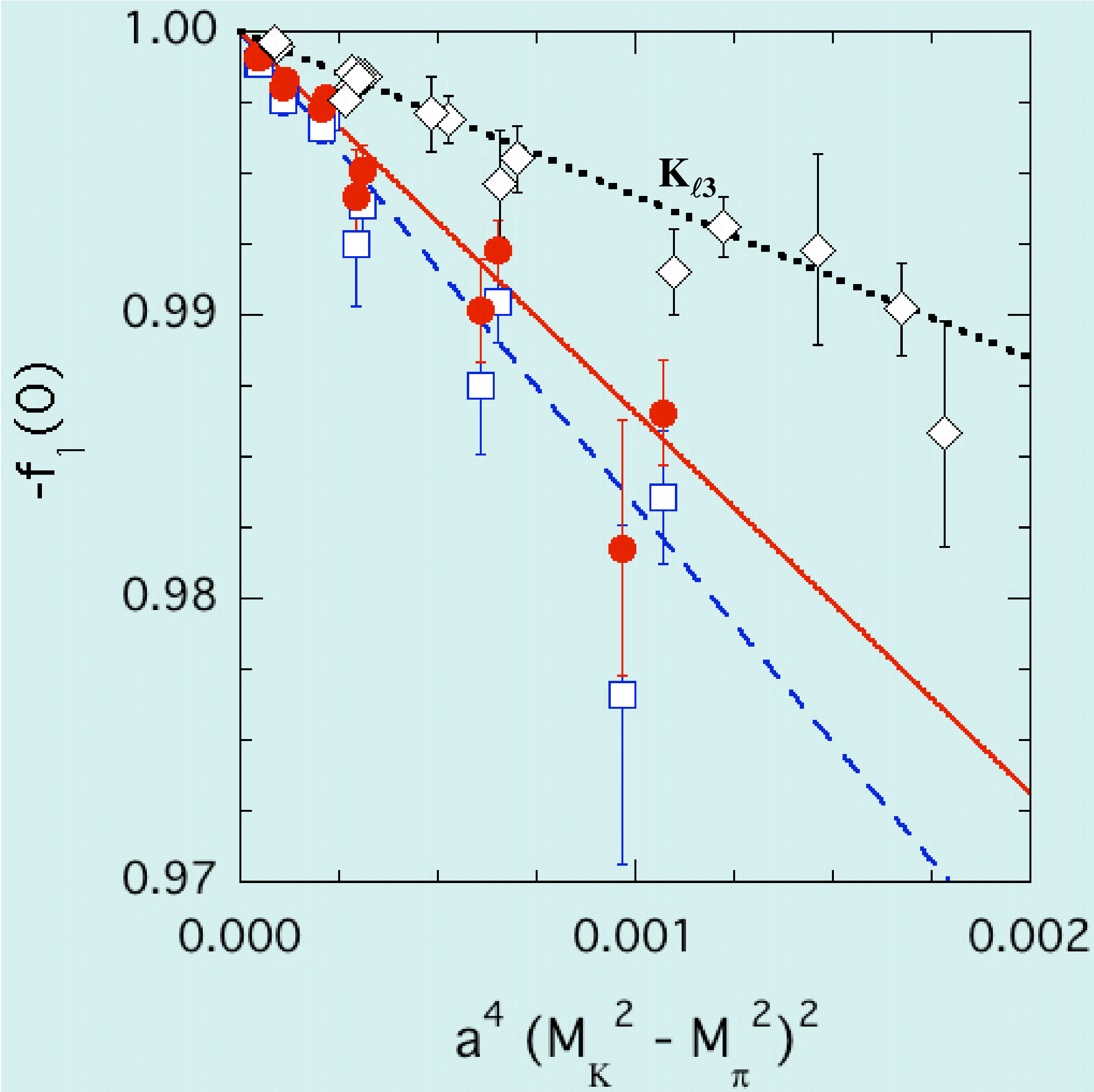
>1 ($1/N_c$ expansion)

* experiments measure the product $|V_{us} f_1(0)|$

- crucial hadronic ingredient: the vector f.f. at zero-momentum transfer $f_1(0)$
- existing analyses make assumptions on the other f.f.'s

(quenched) lattice study of <u>all the f.f.'s</u> for the weak decay $\Sigma^- \to n\, e^-\, \overline{\nu}_e$ [NPB ('06)]

Step 1 + Step 2 ⇒ **get the vector form factor at zero-momentum transfer, $f_1(0)$**



$\propto (m_s - m_\ell)^2$

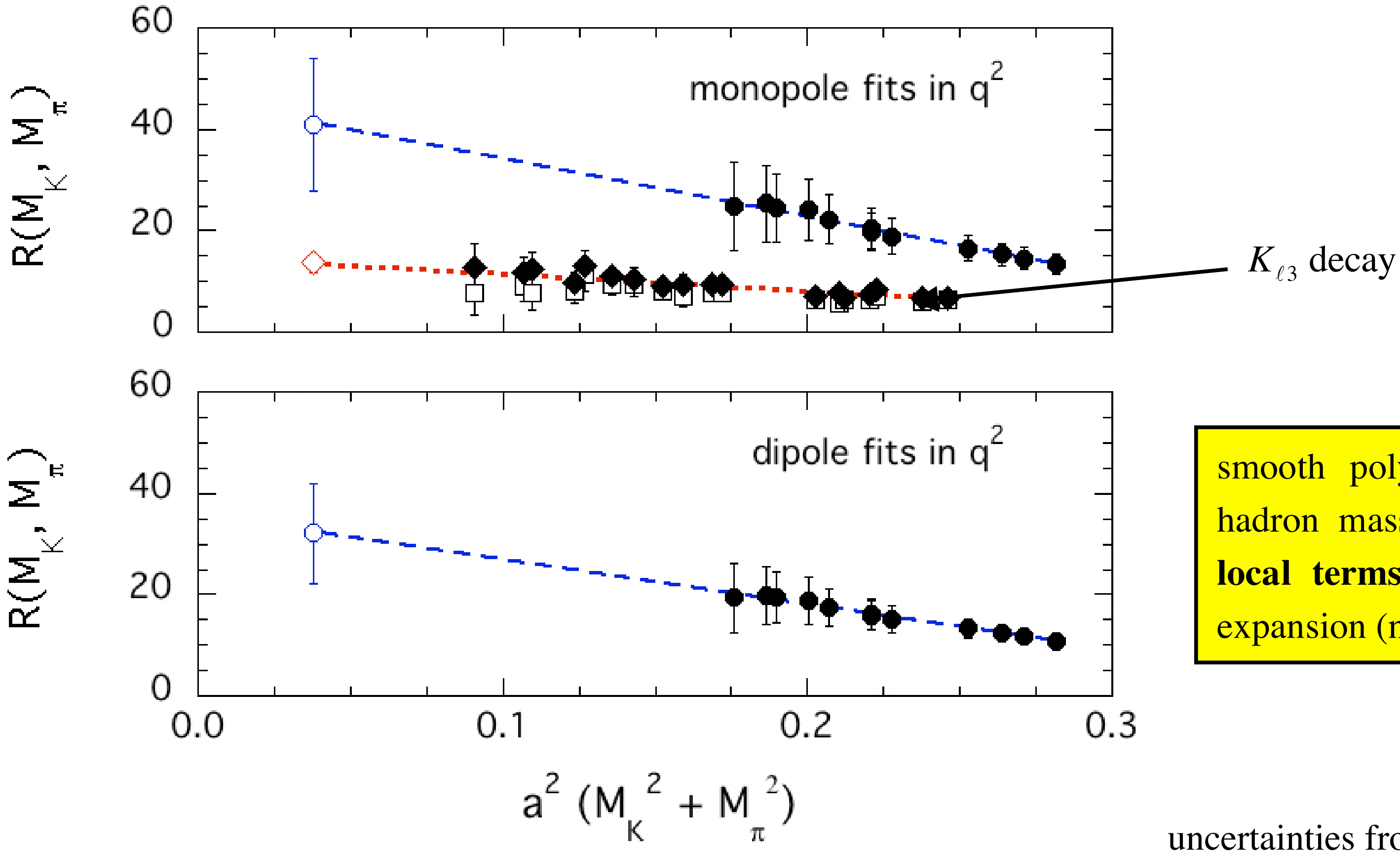
red dots: dipole fits in q^2

blue squares: monopole fits in q^2

black diamonds: $f_+(0)$ results for $K_{\ell 3}$

lines: linear fits consistent with the AG theorem

SU(3)-breaking effects in hyperon decays can be determined on the lattice with a high precision comparable to the one achieved for $K_{\ell 3}$ decays define the AG slope: $R\left(M_K, M_\pi\right) \equiv \frac{f_1(0)+1}{a^4\left(M_K^2 - M_\pi^2\right)^2}$



smooth polynomial fits in the hadron masses ---> estimate of **local terms** only in the chiral expansion (no chiral loops)

uncertainties from q^2 and mass dependencies (~ 2 ÷ 3%)

$$f_1^{\Sigma^- n}(0) = -0.948 \pm 0.015_{stat.} \pm 0.025_{syst.}$$

+ quenching error
+ chiral loops

Chiral corrections in Heavy Baryon ChPT

- recent NLO analysis by G. Villadoro ('06)

$$f_1(0) = f_1^{SU(3)}(0) \cdot \left\{ 1 + \mathcal{O}\left(\frac{M_K^2}{(4\pi f_\pi)^2}\right) + \mathcal{O}\left(\frac{M_K^2}{(4\pi f_\pi)^2}\frac{\pi \delta M_B}{M_K}\right) + \mathcal{O}\left(\frac{M_K^2}{(4\pi f_\pi)^2}\frac{\pi M_K}{M_B}\right) + O(p^4) \right\}$$

← LECs appear at $O(p^4)$ thanks to AG theorem !!!

$f_1(0)/f_1^{SU(3)}(0)$	$f_1^{SU(3)}(0)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(1/M_B)$	All
$\Sigma^- \to n$	-1	$+0.7\%$	$+6.5\%$	-3.2%	$+4.1\%$
$\Lambda \to p$	$-\sqrt{3/2}$	-9.5%	$+4.3\%$	$+8.0\%$	$+2.7\%$
$\Xi^- \to \Lambda$	$\sqrt{3/2}$	-6.2%	$+6.2\%$	$+4.3\%$	$+4.3\%$
$\Xi^- \to \Sigma^0$	$1/\sqrt{2}$	-9.2%	$+2.4\%$	$+7.7\%$	$+0.9\%$

Table 4: *Chiral corrections at the physical point estimated in Ref. [7] for various hyperon decays, adopting $D = 0.804$, $F = 0.463$ and $M_B = 1.151$ GeV.*

***** convergence is quite poor *****

- basic problem: octet-decuplet mixing ($1/2^+$ - $3/2^+$)

if $\Delta >> \Lambda_{QCD} \sim 0.25$ GeV ⇒ decuplet corrections can be reabsorbed into LECs

but, in the real world $\Delta \simeq 0.25$ GeV, and the decuplet-octet-meson coupling constant C≈1.6

one contribution at $O(p^2)$: -3.1%

two contributions at $O(p^3)$: -1.8% from decuplet mass shifts

-38% from octet mass shifts

breakdown of HBChPT ?

* model-independent estimate of leading chiral loops is not yet possible

* assuming that decuplet contributions can be reabsorbed into LECs, one has

chiral loops: $-(4 \pm 4)\%$

local terms: $+\left(5.2 \pm 1.5_{stat.} \pm 2.5_{syst.}\right)\%$

⇒ partial cancellation

our estimate: $f_1^{\Sigma^- n}(0) = -0.988 \pm 0.029_{lattice} \pm 0.040_{HBChPT}$ + quenching error

~3% ~4%

final results for all the f.f.'s at $q^2 = 0$

$f_1(0)$	$-0.988 \pm 0.029_{\text{lattice}} \pm 0.040_{\text{HBChPT}}$	-1.0 (SU(3) limit)
$g_1(0)/f_1(0)$	-0.287 ± 0.052	-0.269 ± 0.047 (SU(3) limit)
$f_2(0)/f_1(0)$	-1.52 ± 0.81	-1.71 ± 0.26 (experiment)
$f_3(0)/f_1(0)$	-0.42 ± 0.22	0.0 (SU(3) limit)
$g_2(0)/f_1(0)$	$+0.63 \pm 0.26$	0.0 (SU(3) limit)
$g_3(0)/f_1(0)$	$+6.1 \pm 3.3$	5.5 ± 0.9 (gen. GT + AWI)

Table 8: *Results of our lattice calculations of the vector and axial form factors for the $\Sigma^- \to n$ transition.*

* small SU(3)-breaking effects on $f_1(0)$ and $g_1(0)/f_1(0)$ (within large errors)
* large SU(3)-breaking effects on $g_2(0)$ (as well as on $f_3(0)$)
* large uncertainties from the extrapolation to physical quark masses and from HBChPT (if applicable at all !)

both $K_{\ell 3}$ and hyperon decays:
* lower quark masses as much as possible;
* improve the precision in the determination of the momentum dependence of f.f.'s

* continuous increase of CPU performances (Moore's law)

* tremendous improvement of algorithms in the last years

($N_f = 2$ Wilson fermions)

2001 (Ukawa): *the Berlin wall*

$$TFlops-years \approx 3.1 \left(\frac{N_{conf}}{100}\right)\left(\frac{L_s}{3\ fm}\right)^5\left(\frac{L_t}{2L_s}\right)\left(\frac{0.2}{\hat{m}/m_s}\right)^3\left(\frac{0.1\ fm}{a}\right)^7$$

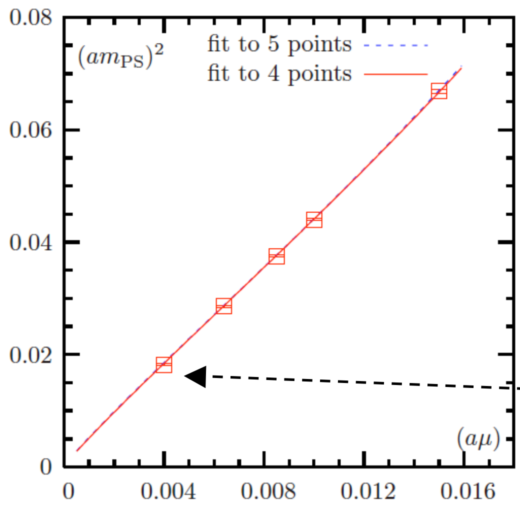
2006 (Del Debbio et al.):

$$TFlops-years \approx 0.03 \left(\frac{N_{conf}}{100}\right)\left(\frac{L_s}{3\ fm}\right)^5\left(\frac{L_t}{2L_s}\right)\left(\frac{0.2}{\hat{m}/m_s}\right)\left(\frac{0.1\ fm}{a}\right)^6$$

* several choices of fermionic actions: standard Wilson, O(a)-improved Wilson, **twisted-mass**, staggered, domain-wall, overlap, …

European Twisted Mass Collaboration

hep-lat/0701012



$N_f = 2$ twisted-mass Wilson fermions at maximal twist

(see Karl's talk)

$M_\pi \sim 300$ MeV

* evaluation of $<\pi|\, V^{\mu} |\, K>$ on the lattice

- three-point correlation function: $C^{\mu}_{K\pi}(t_x, t_y) = \int d\vec{x}\, d\vec{y} \langle 0 | T\left[P_\pi(x) V^\mu(y) P^\dagger_K(0)\right] | 0 \rangle\, e^{-i\vec{p}\cdot\vec{y} + i\vec{p}'\cdot(\vec{y}-\vec{x})}$

- two-point correlation functions: $G_{K(\pi)}(t_x) = \int d\vec{x} \langle 0 | T\left[P_{K(\pi)}(x) P^\dagger_{K(\pi)}(0)\right] | 0 \rangle\, e^{-i\vec{p}_{K(\pi)}\cdot\vec{x}}$

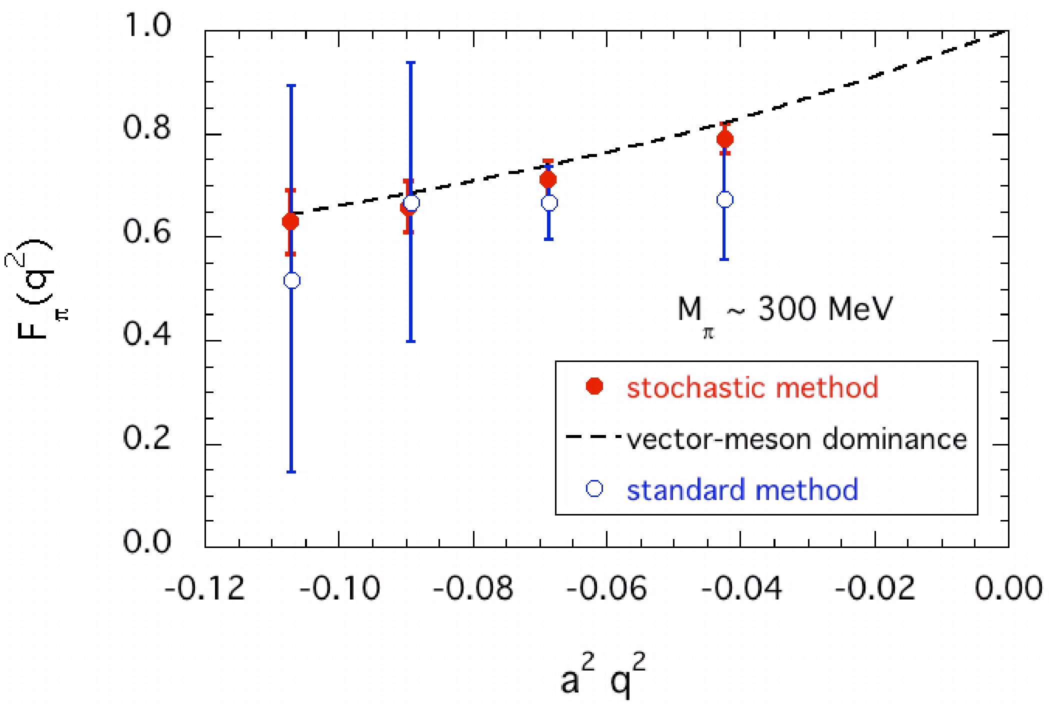
$P_{K(\pi)}(x)$: interpolating PS fields

- plateaux in the ratio: $C^{\mu}_{K\pi}(t_x, t_y) / \left[G_K(t_y) \cdot G_\pi(t_x - t_y)\right] \xrightarrow[t_x - t_y \to \infty]{t_x \to \infty} \propto \langle \pi | V^\mu | K \rangle$

- basic ingredient: the quark propagator $\sum_{\beta, b, z} D^{ab}_{\alpha\beta}(x,z)\, S^{bc}_{\beta\gamma}(z,y) = \delta_{\alpha\gamma}\, \delta_{ac}\, \delta_{xy}$

(thanks to C. Michael)

standard method, point-to-all $S^{ab}_{\alpha\beta}(x,0)$, versus stochastic estimate of all-to-all $S^{ab}_{\alpha\beta}(x,y)$



pion form factor

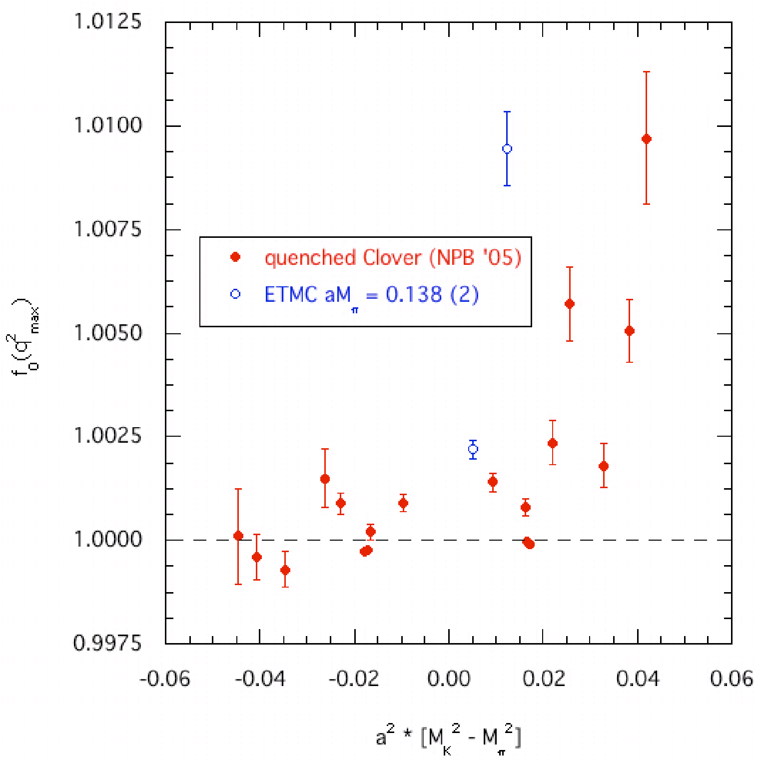
preliminary run on ApeNext

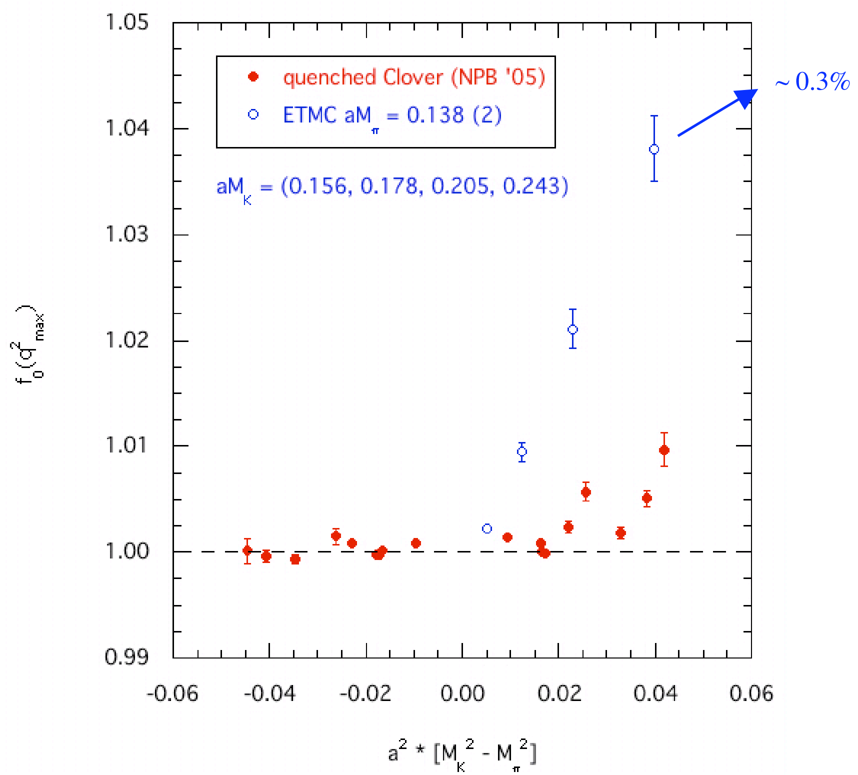
80 N_f=2 gauge configurations

at β= 3.9 and V T = 24^3 48

with a μ_{sea} = a μ_{val} = 0.0040

*** preliminary ETMC results (80 confs) for the scalar form factor $f_0(q^2_{max})$**





CONCLUSIONS

* in the last couple of years the SPQcdR collaboration has shown that lattice QCD can play an important role in the determination of the Cabibbo angle from kaon and hyperon semileptonic decays;

* this achievement has been made possible by the development of a suitable strategy that allows the determination of the vector form factor at zero-momentum transfer, $f_+(0)$, with an overall 1% precision;

* the main limitations are the use of the quenched approximation and of relatively high values of the simulated quark masses ($M_\pi > 500$ MeV);

* the increase of computational power offered by ApeNext and the remarkable improvement of several algorithms on the lattice represent a clear opportunity to remove such limitations;

* using the ETMC unquenched gauge configurations and working at pion masses of ~ 300 MeV an accuracy on $f_+(0)$ well below 1% can be reached, allowing a quite stringent check of the CKM unitarity from low-energy processes.

additional transparencies

*** evaluation of $<\pi|\, V^{\mu} |\, K>$ on the lattice: standard procedure**

- three-point correlation function:

$$C^{\mu}_{K\pi}\left(t_x, t_y\right) = \int d\vec{x}\, d\vec{y} \left\langle 0 \left| T\left[P_{\pi}(x) V^{\mu}(y) P^{\dagger}_{K}(0)\right] \right| 0 \right\rangle e^{-i\vec{p}\cdot\vec{y}+i\vec{p}'\cdot(\vec{y}-\vec{x})}$$

interpolating (local) PS fields

- two-point correlation functions:

$$G_{K(\pi)}(t_x) = \int d\vec{x} \left\langle 0 \left| T\left[P_{K(\pi)}(x) P^{\dagger}_{K(\pi)}(0)\right] \right| 0 \right\rangle e^{-i\vec{p}_{K(\pi)}\cdot\vec{x}}$$

- plateaux in the ratio: $$\frac{C^{\mu}_{K\pi}\left(t_x, t_y\right)}{G_K\left(t_y\right)\cdot G_{\pi}\left(t_x - t_y\right)} \xrightarrow[t_x - t_y \to \infty]{t_y \to \infty,} \propto \left\langle \pi \left| V^{\mu} \right| K \right\rangle$$

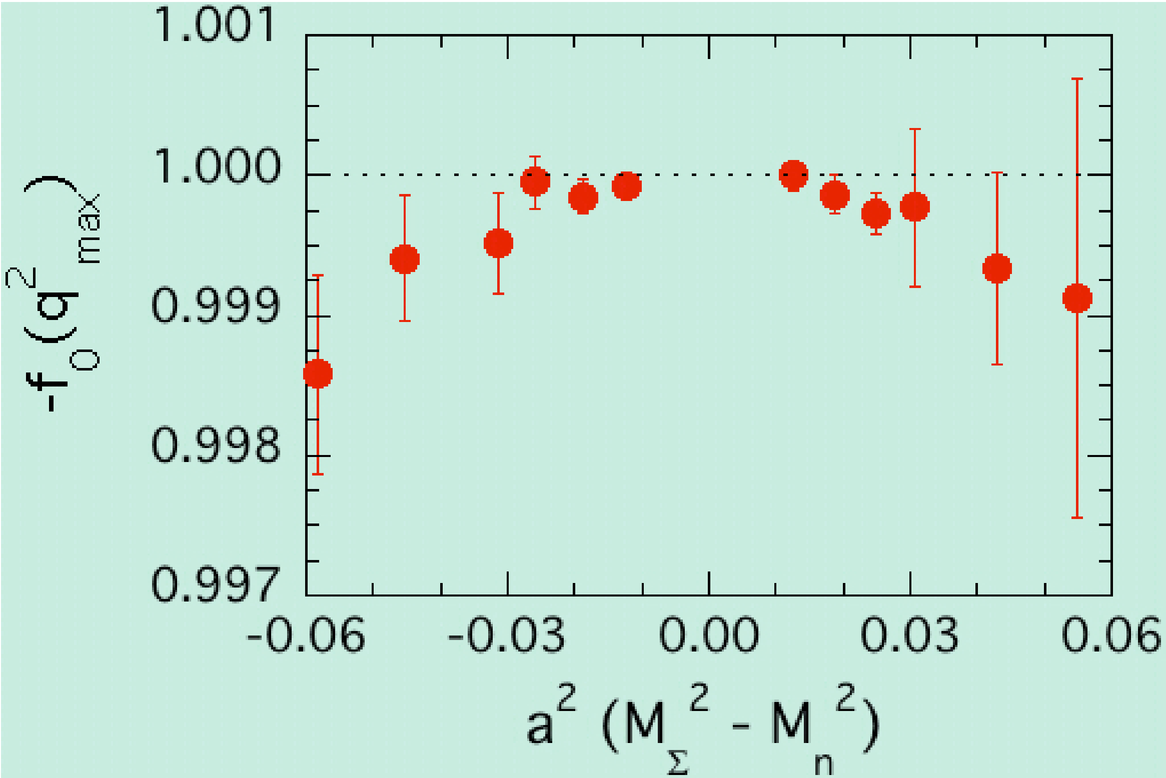
- local V^{μ} renormalizes on the lattice: $\hat{V}^{\mu} \rightarrow Z_V\left(1 + b_V\, a \cdot m_q\right) V^{\mu}$

⇨ need to compute $<\pi|\, V^{\mu} |\, K>$, Z_V and b_V with **huge statistics** to reach **1% level** of accuracy

Step 1: consider the double ratio

$$R_0 \equiv \frac{C^0_{\Sigma n}(t_x,t_y)\cdot C^0_{n\Sigma}(t_x,t_y)}{C^0_{\Sigma\Sigma}(t_x,t_y)\cdot C^0_{nn}(t_x,t_y)} \xrightarrow[t_x-t_y\to\infty]{t_y\to\infty} \frac{\langle n|V^0|\Sigma\rangle\cdot\langle\Sigma|V^0|n\rangle}{\langle\Sigma|V^0|\Sigma\rangle\cdot\langle n|V^0|n\rangle}$$

scalar form factor: $f_0(q^2) \equiv f_1(q^2) + \frac{q^2}{M_\Sigma^2 - M_n^2} f_3(q^2)$ at $q^2_{\max} = (M_\Sigma - M_n)^2$



1.5 < M_B(GeV) < 1.8

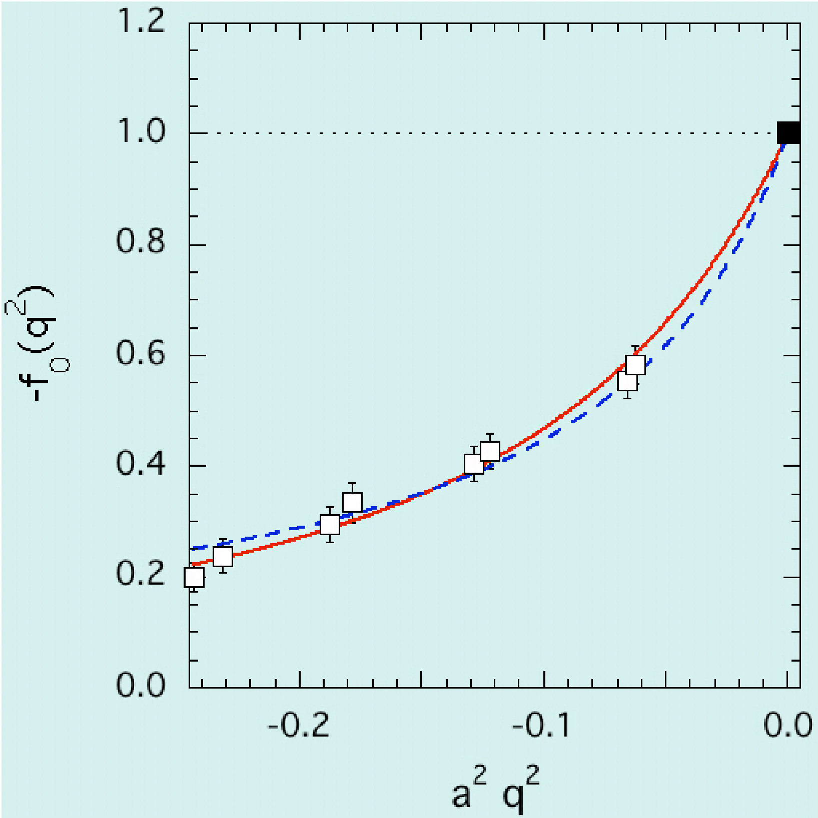
very high-precision !!!

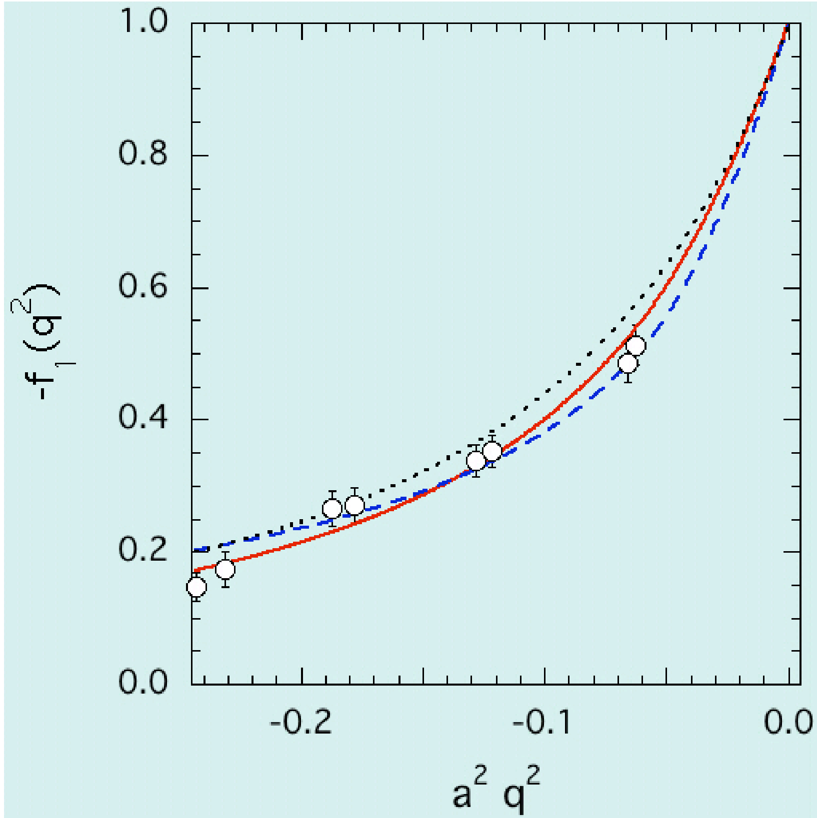
$M_\Sigma \approx A + B(m_s + 2m_\ell)$

$M_\Sigma^2 - M_n^2 \propto (m_s - m_\ell)$

Step 2: study the momentum dependence of the f.f.'s

$$R_{j,0} \equiv \frac{C^{j}_{\Sigma n}(t_x,t_y)\cdot C^{0}_{\Sigma\Sigma}(t_x,t_y)}{C^{0}_{\Sigma n}(t_x,t_y)\cdot C^{j}_{\Sigma\Sigma}(t_x,t_y)} \xrightarrow[t_x-t_y\to\infty]{t_y\to\infty} \frac{\langle n|V^{j}|\Sigma\rangle\cdot\langle n|V^{0}|n\rangle}{\langle n|V^{0}|\Sigma\rangle\cdot\langle n|V^{j}|n\rangle} \Rightarrow \frac{f_0(q^2)}{f_1(q^2)}$$





- - - - monopole fits

$f_{0(1)}(q^2) = f(0) / (1 - \lambda_{0(1)}\, q^2)$

—— dipole fits

$f_{0(1)}(q^2) = f(0) / (1 - \lambda_{0(1)}\, q^2)^2$

…… dipole with M_{K^*}